Ej. 8-20: Calcular el área encerrada por las curvas:

a) $\begin{cases} y = x^2 + 1 \\ y = -x^2 + 9 \end{cases}$

b) $\begin{cases} y = x^2 - 4 \\ y = x - 2 \end{cases}$

c) $\begin{cases} y = x^3 \\ y = 4.x \end{cases}$

d) $\begin{cases} y^2 = x - 1 \\ y = x - 3 \end{cases}$

e) $\begin{cases} y = \frac{1}{2}x^3 \\ y = x + 2 \\ x = 1 \end{cases}$

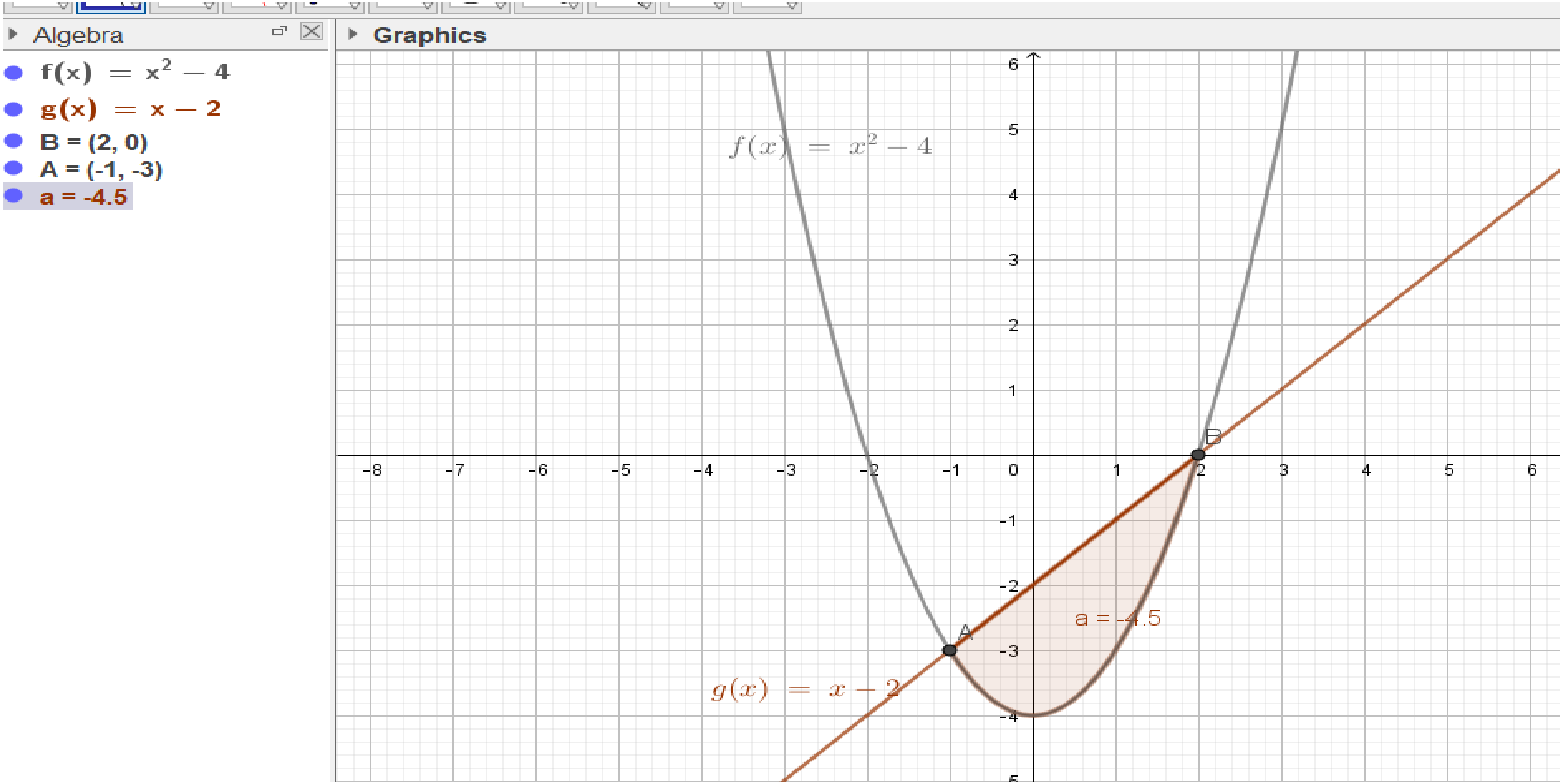
f) $\begin{cases} y = 2^x \\ y = 2^{-x} \\ y = 4 \end{cases}$

g) $\begin{cases} 2y^2 - 3y = x - 1 \\ y^2 - 2y = x - 3 \end{cases}$

h) $\begin{cases} x = 4y^2 \\ x = 4(y-2)^2 \\ eje\ y \end{cases}$

i) $\begin{cases} y^2 = x + 4 \\ eje\ y \end{cases}$

$$b)\begin{cases} y = x^2 - 4 \\ y = x - 2 \end{cases}$$



En primer*a instancia debo sacar las intersecciones*:

$$x^2 - 4 = x - 2$$

$$x^2 - x - 2 = 0$$

$$x_1 I x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$

$$\boxed{x = -1}$$

$$\boxed{x = 2}$$

Ahora calculo el área:

$$A = \int_a^b (g_{(x)} - f_{(x)})\, dx$$

$$A = \int_{-1}^{2} [x - 2 - (x^2 - 4)]\, dx$$

$$A = \int_{-1}^{2} (x - 2 - x^2 + 4)\, dx$$

$$A = \int_{-1}^{2} (-\,x^2 + x + 2).\, dx$$

$$A = -\frac{x^3}{3} + \frac{x^2}{2} + 2x \Bigg| [-1; 2]$$

$$A = -\frac{2^3}{3} + \frac{2^2}{2} + 2.2 - \left[-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2(-1)\right]$$

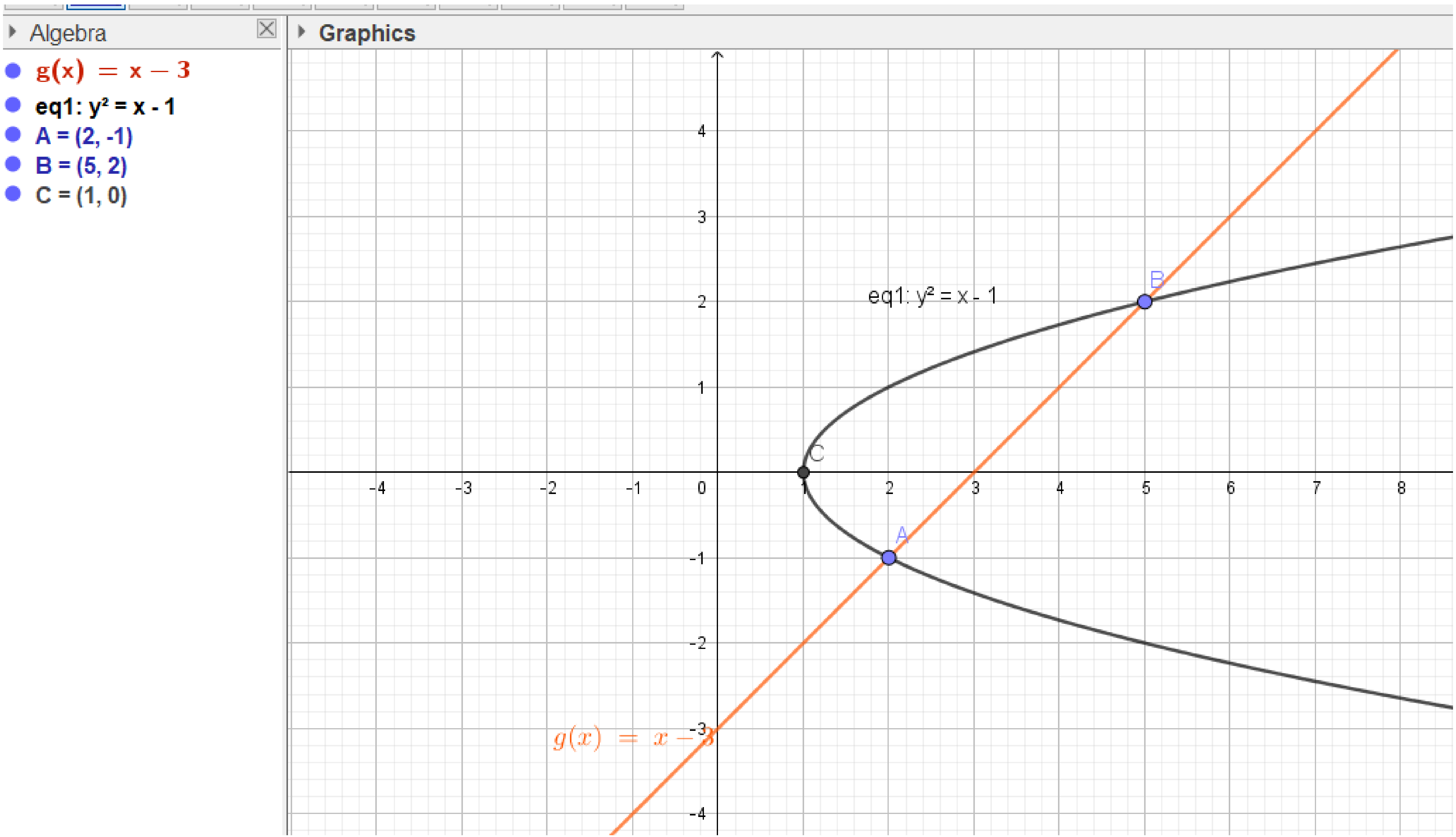
$$A = -\frac{8}{3} + \frac{4}{2} + 4 - \left[\frac{1}{3} + \frac{1}{2} - 2\right] =$$

$$A = -\frac{8}{3} + 2 + 4 - \frac{1}{3} - \frac{1}{2} + 2 =$$

$$A = -\frac{9}{3} + 8 - \frac{1}{2} = 5 - \frac{1}{2} = \frac{10 - 1}{2} = \boxed{\frac{9}{2}}$$

$$d) \begin{cases} y^2 = x - 1 \\ y = x - 3 \end{cases}$$

$Representemos\ sus\ gr\acute{a}ficas.$


Algebra
g(x) = x − 3
eq1: y² = x - 1
A = (2, -1)
B = (5, 2)
C = (1, 0)
Graphics
eq1: y² = x - 1
g(x) = x − 3
A
B
C
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8

En primera *instancia debo sacar las intersecciones*:

$$\pm\sqrt{x-1} = x-3$$

Elevo todo al cuadrado en ambos miembros.

$$\left(\pm\sqrt{x-1}\right)^2 = (x-3)^2$$

$$x-1 = x^2-6x+9$$

$$0 = x^2-6x-x+9+1$$

$$0 = x^2-7x+10$$

	1	-7	10
2		2	-10
	1	-5	0

$$0 = (x - 2).(x - 5)$$

$$x = 2 \quad y \quad x = 5$$

$$A = \int_{a}^{b} (g_{(y)} - f_{(y)}).dy$$

Entonces para este caso despejamos x:

$$\begin{cases} x = y^2 + 1 \\ x = y + 3 \end{cases}$$

$$A = \int_{-1}^{2} [(y + 3) - (y^2 + 1)].dy$$

$$A = \int_{-1}^{2} (-y^2 + y + 2).dy$$

$$A = -\frac{y^3}{3} + \frac{y^2}{2} + 2y \Bigg| [-1;2]$$

$$A = -\frac{2^3}{3} + \frac{2^2}{2} + 2.2 - \left[-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2.(-1)\right]$$

$$A = -\frac{8}{3} + 2 + 4 - \frac{1}{3} - \frac{1}{2} + 2$$

$$A = -\frac{9}{3} + 8 - \frac{1}{2} =$$

$$A = -3 + 8 - \frac{1}{2} =$$

$$A = 5 - \frac{1}{2} = \frac{10 - 1}{2} = \boxed{\frac{9}{2}}$$

Ej. 8-21: Calcular el área de las siguientes regiones

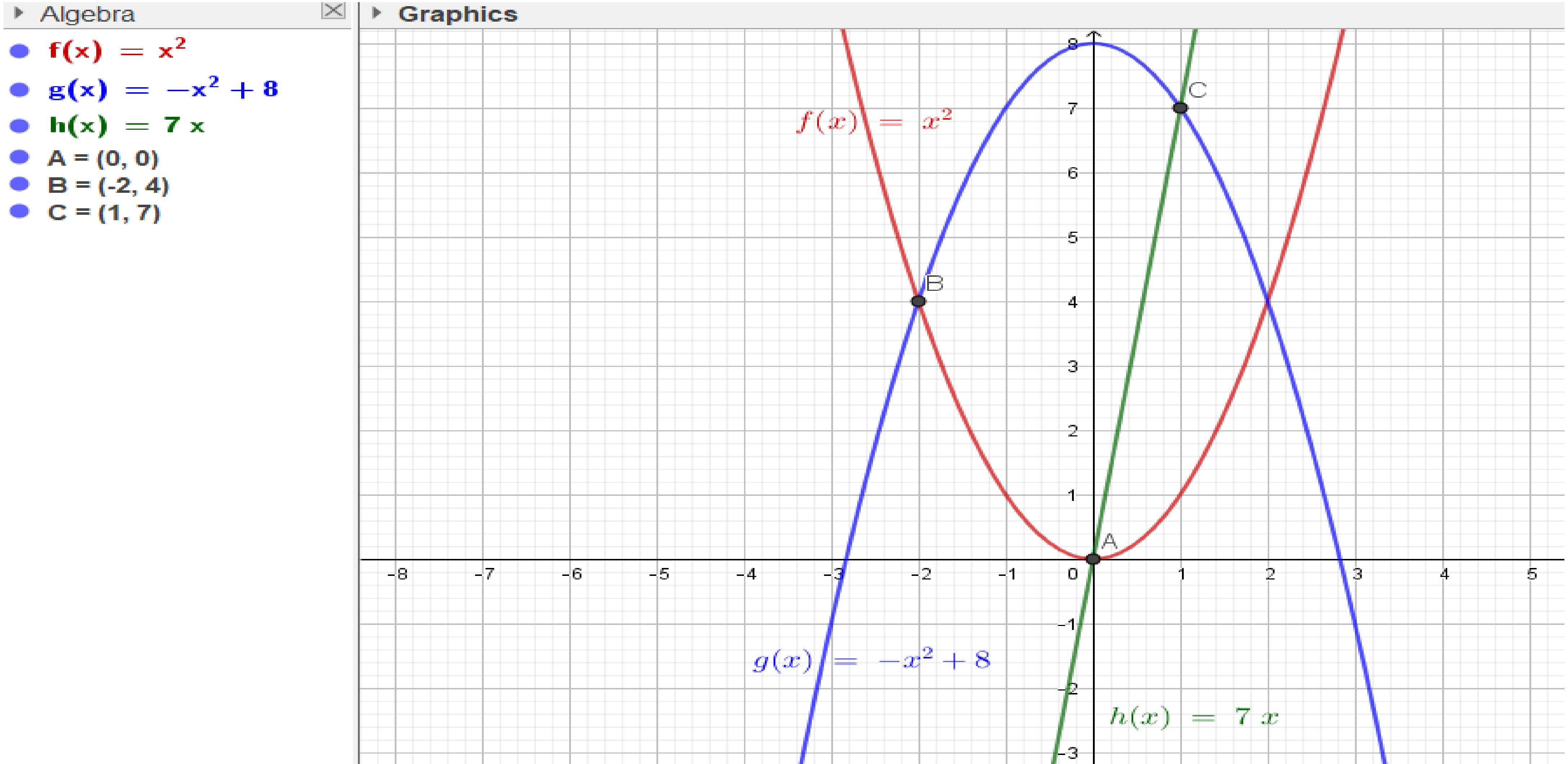
a) $\begin{cases} y \geq x^2 \\ y \leq 3.x + 4 \\ y \leq 4 \end{cases}$

b) $\begin{cases} y \geq x^2 \\ y \leq x + 2 \\ y \leq -x + 2 \end{cases}$

c) $\begin{cases} y \geq x^2 \\ y \leq -x^2 + 8 \\ y \geq 7x \end{cases}$

d) $\begin{cases} y \geq -\frac{1}{3}x + 1 \\ y \leq x + 1 \\ y \geq 5x - 15 \end{cases}$

$$\text{c)} \begin{cases} y \geq x^2 \\ y \leq -x^2 + 8 \\ y \geq 7x \end{cases}$$



$$A_1 = \int_{-2}^{0} (g_{(x)} - f_{(x)}).\, dx$$

$$A_2 = \int_{0}^{1} (g_{(x)} - h_{(x)}).\, dx$$

$$A_T = A_1 + A_2$$

$$A_1 = \int_{-2}^{0} [(-x^2+8) - x^2].\, dx$$

$$A_1 = \int_{-2}^{0} (-2x^2+8).\, dx$$

$$A_1 = -2\frac{x^3}{3} + 8x \Big| [\text{-2;0}]$$

$$A_1 = -2\frac{0^3}{3} + 8.0 - [-2\frac{(-2)^3}{3} + 8(-2)]$$

$$A_1 = -\left(\frac{16}{3} - 16\right)$$

$$A_1 = -\frac{16}{3} + 16 = -\frac{16+48}{3} = \boxed{\frac{32}{3}}$$

$$A_2 = \int_0^1 (-x^2 + 8 - 7x)\,dx$$

$$A_2 = -\frac{x^3}{3} + 8x - 7\frac{x^2}{2}\Big| [0;1]$$

$$A_2 = -\frac{1^3}{3} + 8.1 - 7.\frac{1^2}{2} - [0]$$

$$A_2 = -\frac{1}{3} + 8 - \frac{7}{2} = \frac{-2 + 48 - 21}{6} = \boxed{\frac{25}{6}}$$

$$A_T = \frac{32}{3} + \frac{25}{6} = \frac{64 + 25}{6} = \boxed{\frac{89}{6}}$$