$f(x) = x^3 - 3x^2 - x \qquad df = \mathbb{R}$

$f'(x) = 3x^2 - 6x - 1 \qquad df' = \mathbb{R} \qquad f''(x) = 6x - 6$

Pto crítico

1) $f'(x) = 0 \qquad 3x^2 - 6x - 1 = 0$

$$\frac{6 \pm \sqrt{36 + 12}}{6} \quad \begin{cases} x_1 = \dfrac{6 + \sqrt{48}}{6} \ \checkmark \\[2ex] x_2 = \dfrac{6 - \sqrt{48}}{6} \ \checkmark \end{cases}$$

$f'(x) > 0 \nearrow \qquad f'(x) < 0 \searrow \qquad f \ \nearrow f'(x) > 0$

$x = -1 \qquad x = 0 \qquad x = 3$

$+ \quad | \quad - \qquad - \quad | \quad +$

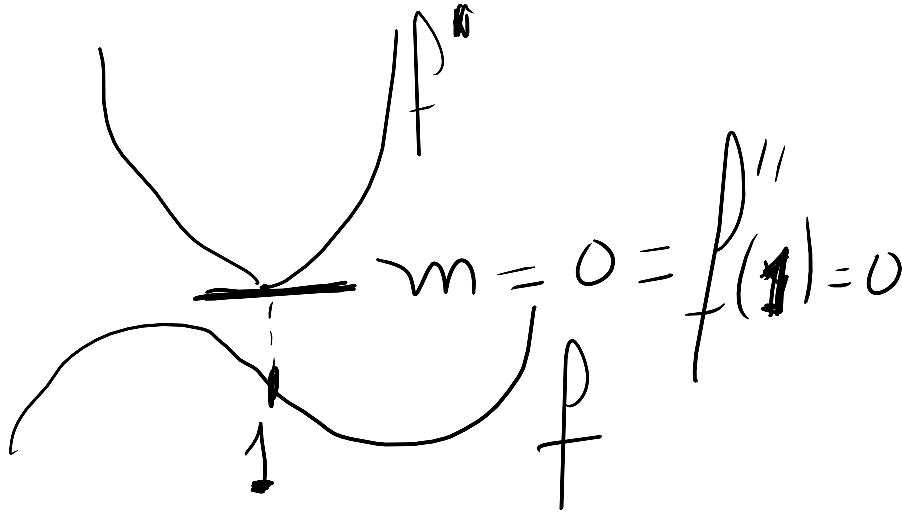
$\frac{6 - \sqrt{48}}{6}$ MÁXIMO $\qquad \frac{6 + \sqrt{48}}{6}$ MÍNIMO

eje x f'

Crece $= \left(-\infty, \frac{6 - \sqrt{48}}{6}\right) \cup \left(\frac{6 + \sqrt{48}}{6}, +\infty\right)$

Decrece $= \left(\frac{6 - \sqrt{48}}{6}, \frac{6 + \sqrt{48}}{6}\right)$

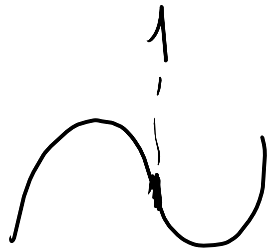
$f''(x) = 6x - 6 \quad d f'' = \mathbb{R}$



Pto crítico Condición necesaria

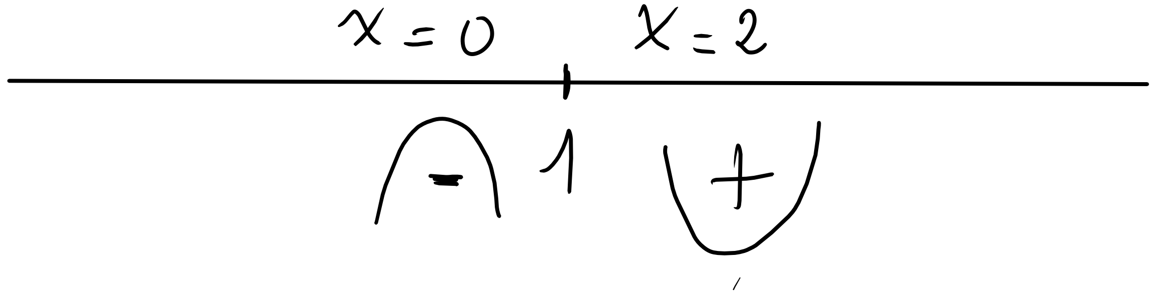
$f''(x) = 0 \qquad 6x - 6 = 0$

$x = 1$



Condición suficiente

$f''(x) < 0 \quad f''(x) > 0$



Concavidad $- = (-\infty; 1)$

Concavidad $+ = (1; +\infty)$

eje x f'' Pto de inflexión

$(1; f(1))$