Optimized For Printing

JET TRANSPORT PERFORMANCE METHODS

Walt Blake
and
the Performance Training Group
Flight Operations Engineering
Boeing Commercial Airplanes

D6-1420
revised March 2009

Copyright Information

NOTICE TO READERS

Welcome to the new JTPM – the 2009 Boeing Jet Transport Performance Methods document. We say "the new" JTPM but actually, the JTPM has been in existence since the early 1960s. However, it was last revised in 1989 – and much has changed since then. In 2007 it was decided to update the book, taking advantage of newer authoring and publishing tools.

This new edition of the JTPM is completely re-written to be as up-to-date as possible in light of changes to technology and the regulatory environment.

We have added an index, making it easy to locate and navigate to the text discussions on almost any performance-related subject. We have completely re-organized the book, breaking it into chapters covering each area of commercial jet transport airplane performance. You'll see that the first group of chapters cover the fundamentals of airplane performance, the second group covers takeoff topics, and the last group of chapters covers the segments of flight following takeoff.

We are making the new JTPM available in two versions. One is optimized for on-screen viewing using the Adobe Reader, the other is optimized for printing paper copies of the document.

This is the version optimized for printing. You may use it to print any portion or all of the document for your own personal use. This is a copyrighted document, however, and reproduction of part or all of the book for any purpose other than your own personal use is not permitted.

We plan to revise the JTPM as needed to keep it up to date. We encourage you to check periodically to ensure that your copy is current. There is a Revision Record at the end of the document.

We sincerely hope that you'll find the new JTPM to be a useful tool for you in your work. We will welcome any comments or suggestions you might care to offer. To do so, please send an email to: flightops.engineering@boeing.com. It will be helpful if you will show "new JTPM" as the subject of the message. Although we won't be able to reply to all such emails, be assured that every one of them will receive our attention.

Table of Contents

Chapter 1: Units and Conversions

Chapter 2: Mass, Weight, and Center of Gravity

Chapter 3: Physics of Air

Chapter 4: The Atmosphere

Chapter 5: Measurement of Altitude

Chapter 6: Measurement of Airspeed

Chapter 7: Near the Speed of Sound

Chapter 8: Flow Near a Surface

Chapter 9: Lift and Drag

Chapter 10: Wings

Chapter 11: Drag Analysis

Chapter 12: Aerodynamic Devices

Chapter 13: Evolution of the Jet Engine

Chapter 14: Jet Engine Fundamentals

Chapter 15: Thrust Ratings

Chapter 16: The Flight – a Performance Overview

Chapter 17: Speeds

Chapter 18: Calculating Takeoff Distances

Chapter 19: Field Length Limit Takeoff Weight

Chapter 20: Takeoff on Non-Dry Runways

Chapter 24: Tire Speed Limit Takeoff Weight

Chapter 25: Brake Energy Limit Takeoff Weight

Chapter 26: Improved Climb Technique

Chapter 27: Reduced Thrust for Takeoff

Chapter 33: Cruise - Engine Failure and Driftdown

Chapter 34: Cruise - Loss of Pressurization

Chapter 35: Descent, Approach, and Landing

Chapter 36: Landing On Non-Dry Runways

Chapter 37: Airplane Performance and the FMCS

Chapter 38: Miscellaneous Topics

Summary of Useful Information

index

Revision Record

Chapter 1: Units and Conversions

Introduction

Aircraft performance engineers use a variety of parameters in their work. A complication to this is the fact that three different systems of units are in common use in the aviation industry: the so-called "English" system, the metric system, and what may be termed the "aeronautical" system. This chapter will list all of the parameters necessary for performance work and will provide units for each of them. Where appropriate, conversions are provided.

Most of the conversion factors shown in the following tables are simple conversions: feet to meters, pounds to kilograms, and the like. But there are sometimes needs for more complex conversions, involving more than one simple conversion. In Discussion 1 in the "Additional Discussion" section at the end of this chapter we show how to develop compound conversion factors.

The tables of units conversions shown in this chapter are also provided in Appendix 1, "Summary of Useful Information For Performance Engineers".

In the following tables of units conversions, it is useful to remember that to obtain X, multiply Y by constant A is equivalent to obtain Y, divide X by constant A.

In the conversion tables, all constants are rounded to five significant figures.

The following list of parameters is in alphabetical order.

Acceleration

Acceleration is the rate of change of velocity. If it's a negative quantity, it is usually termed *deceleration*. It can be expressed as $a = \Delta V \div \Delta t$ or $\frac{dv}{dt}$.

For a detailed discussion of gravitational acceleration, please refer to the chapter entitled "Mass, Weight, and Center of Gravity".

units

Metric system: meters per second per second
English system: feet per second per second
Aeronautical system: knots per second

conversions

to obtain	multiply	by
feet per second per second	knots per second	1.6878
feet per second per second	meters per second per second	3.2808
knots per second	meters per second per second	1.9438

Angles

An *angle* is the measurement of rotation between two intersecting lines.

The units of angles are the same for all units systems. Angles are needed for such things as runway slope, climb path angle, and so on.

units

all systems: degrees, minutes, seconds, radians

If you're unfamiliar with radians as a measurement of angle, discussion 2 in the Additional Discussion section at the end of this chapter will be helpful.

conversions

1 degree = 60 minutes of arc
1 minute of arc = 60 seconds of arc

to obtain	multiply	by
degrees	radians	57.296

Angular rate

Angular rate can be defined as the rate of rotation of some object about a point or axis in space. It is encountered in performance work in several areas: the rotation rate of the earth, which affects the net weight of an airplane; also in specifying rates of turn of an airplane using, for example, units of degrees per second or degrees per minute.

units

All systems: degrees per second, degrees per minute, radians per second

conversions

to obtain	multiply	by
degrees per minute	degrees per second	60.000
degrees per minute	radians per second	3437.7
degrees per second	radians per second	57.296

Area

Area is a measurement of two-dimensional space.

Area is used, for example, in describing the size of a wing as viewed from above it. It is also used in other tasks such as air pressure, which can be defined as force per unit of area, e.g. pounds per square inch.

units

Metric system: square meters, square centimeters.
English system: square inches, square feet, square yards.

conversions

to obtain	multiply	by
square feet	square meters	10.764

Density (mass)

Mass density is defined as the amount of mass of a substance per unit of volume.

units

Metric system: kilograms mass per cubic meter, grams per cubic centimeter
English system: slugs per cubic foot

conversions

to obtain	multiply	by
kilograms mass per cubic meter	slugs per cubic foot	515.38

Density (weight)

Weight density is simply the amount of weight of a substance per unit of volume. It could be air density, fuel density, or a number of other densities frequently encountered in performance work.

One standard for density is the density of pure water at four degrees Celsius; at that temperature, water has a density of exactly one kilogram per liter, or 8.3456 pounds per U. S. gallon at the standard gravitational acceleration. This is the accepted standard in the aviation industry, thus, for our purposes, a specific gravity of 1.000 means a weight density of one kilogram per liter.

units

Metric system: kilograms force per cubic meter, kilograms force per liter
English system: pounds per cubic foot, pounds per gallon
Aeronautical system: pounds per gallon, kilograms per liter

conversions

to obtain	multiply	by
kilograms per cubic meter	kilograms per liter	1000.0
kilograms per cubic meter	pounds per cubic foot	16.018
kilograms per cubic meter	pounds per gallon	119.83
pounds per cubic foot	specific gravity (kilograms per liter)	62.428
pounds per cubic foot	pounds per gallon	7.4805
pounds per gallon	specific gravity (kilograms per liter)	8.3454

Distance or Length

Units of *length* and *distance* are used in virtually all performance calculations: everything from runway length to altitude to trip distance.

units

Metric system: millimeters, centimeters, meters, kilometers.
English system: inches, feet, smoots, yards, statute miles.
Aeronautical system: nautical miles

conversions

1 meter = 100 centimeters = 1000 millimeters
1 foot = 12 inches, 1 yard = 3 feet

By international agreement, the *nautical mile* is defined as exactly 1852 meters. The distance on the earth's surface subtended by an angle of one minute of latitude at the equator is equal to 1855.34 meters; at the poles it is equal to 1849.12 meters. Thus, for practical purposes, it is acceptable to say that one nautical mile is equal to one minute of latitude (<u>not</u> longitude) on a navigational chart at constant longitude. For example, the distance between two points located at 40°N 00°E and 50°N 00°E would be (almost exactly) 600 nautical miles.

to obtain	multiply	by
centimeters	feet	30.480
centimeters	inches	2.5400
feet	kilometers	3280.8
feet	meters	3.2808
feet	nautical miles	6076.1
feet	smoots[a]	5.5833
feet	statute miles	5280.0
kilometers	statute miles	1.6093
kilometers	nautical miles	1.8520
statute miles	nautical miles	1.1508
meters	nautical miles	1852.0
meters	statute miles	1609.3

a. http://en.wikipedia.org/wiki/Smoots

Energy

Energy and *work* share the same units: force multiplied by distance. Energy and work can be broadly defined as the effort that has been expended, or has the potential to be expended, in producing a force for some distance.

Energy can take many forms: potential energy, kinetic energy, chemical energy, and heat energy are some of most common of those. Energy can also be stored, as for example in an electric battery.

Most forms of energy can be converted into other forms of energy. Chemical energy, as for example in liquid fuels such as jet engine fuel or automotive engine fuel, can be converted into the thrust which moves an airplane or the torque which moves an automobile. Potential energy can be converted into kinetic energy. There are many more examples similar to these.

Calories and *British Thermal Units* (BTUs) are units of heat energy, defined as the amount of heat required to raise the temperature of a specified mass of water by a specified increment.

units

Metric system: joules (newton-meters), calories (gram-degree C)
English system: foot-pounds, BTUs (pound-degree F)

conversions

to obtain	multiply	by
calories	BTUs	252.00
foot-pounds	BTUs	778.17
foot-pounds	calories	3.0880
joules (newton-meter)	BTUs	1055.1
joules (newton-meter)	calories	4.1868
joules (newton-meter)	foot-pounds	1.3558

Force or Weight

Force is the effort required to cause an element of mass to accelerate, or to cause an element of mass in motion to decelerate ("retarding force").

Mass, force and acceleration are related by the equation:

$$F = \frac{d(mV)}{dt}$$

where F is force
m is the mass
V is velocity
t is time

More familiarly, when mass is constant we use the equation:

$$F = m\frac{dV}{dt} = ma$$

where a is acceleration

It is essential to remember that in the metric system, "one kilogram" can mean either one kilogram of mass or one kilogram of force; therefore, when specifying units of kilograms, be sure to specify whether it is meant as a unit of mass or a unit of force.

One newton is defined as the amount of force required to accelerate one kilogram of mass at one meter per second per second; one kilogram is the amount of force required to accelerate one kilogram of mass at one standard g – approximately 9.8 meters per second per second. Thus, at the standard gravitational acceleration one kilogram of *mass* has a weight of one kilogram of *force*.

One pound of force is defined as the amount of force required to accelerate one slug of mass at one foot per second per second.

One form of force is that which we refer to as "weight" – force due to gravitational attraction. See the chapter entitled "Mass, Weight, and Center of Gravity" for detailed discussions of mass and weigh.

units

Metric system: newtons, kilograms force
English system: pounds

conversions

1 kilogram = 1000 grams = 1,000,000 milligrams
1 pound = 16 ounces

to obtain	multiply	by
newtons	kilograms	9.8067
newtons	pounds	4.4482
pounds	kilograms	2.2046

Length

See "Distance or Length" above.

Mach number

Mach number, named after physicist Ernst Mach, is defined as the ratio of an airplane's true airspeed to the speed of sound for the atmospheric conditions at which the airplane is flying.

The chapter entitled "Near the Speed of Sound" contains discussions of the speed of sound and Mach number.

Mass

In brief, *mass* is a measure of the amount of matter in an object; it is also that property of a finite body which gives it its *inertia* – that is, its tendency to remain at rest until acted on by a force, or to remain in motion in a straight line at constant speed unless its motion is retarded by a force. The property of mass is independent of location.

The chapter entitled "Mass, Weight, and Center of Gravity" discusses mass in more detail.

It is essential to remember that in the metric system, kilogram units are used for both force and mass.

units

Metric system: kilograms mass

English system: slugs

conversions

to obtain	multiply	by
kilograms mass	slugs	14.594

Power

Power is defined as the rate at which work is done. That is, power is computed by multiplying the force that is expended by the distance over which it is expended and dividing by the time over which it is expended.

units

Metric system: watts (newton-meters per second)
English system: foot-pounds per second, horsepower

conversions

to obtain	multiply	by
foot-pounds per second	horsepower	550.00
watts	foot-pounds per second	1.3558
watts	horsepower	745.70

Pressure

Pressure is force per unit of area.

In the "Additional Discussion" section at the end of this chapter, discussion 3 provides additional discussion about pressure.

units

Metric system: pascals (newtons per square meter)
English system: pounds per square inch, pounds per square foot
Aeronautical system: atmospheres, hectopascals (millibars), inches of mercury, millimeters of mercury

conversions

1 bar = 1000 millibars
1 pascal = 1 newton per square meter
1 pascal = 0.01 hectopascals
1 bar = 100,000 newtons per square meter = 1000 hectopascals

1 hectopascal = 100 newtons per square meter
1 millibar = 1 hectopascal

to obtain	multiply	by
hectopascals	atmospheres	1013.2
hectopascals	millimeters of mercury	1.3332
hectopascals	pounds per square inch	68.948
inches of mercury	atmospheres	29.922
millimeters of mercury	atmospheres	760.00
millimeters of mercury	pounds per square inch	51.715
pounds per square inch	atmospheres	14.696

Specific gravity

Specific gravity is also referred to as “relative density”. That is, it is the ratio of the density of a material compared to the density of water. Most usually, the term “water” is taken to mean pure water at four degrees Celsius.

In aviation, specific gravity is sometimes used to describe the density of jet fuel. A value of approximately 0.805 is a typical jet fuel density.

Using pure water at four degrees Celsius as the reference, the specific gravity of a material is equal to its density in grams per cubic centimeter or kilograms per liter.

See “Density (Weight)” above for conversion factors.

Speed or Velocity

See “Velocity or Speed” below.

Temperature

Temperature, which we commonly think of as the “hotness” of an object, is a measure of the average translational kinetic energy of the molecules within the substance whose temperature is being measured. For a gas such as air, it is a direct measurement of the internal energy of the gas.

Temperature is a fundamental parameter in virtually all performance calculations.

Commonly encountered temperature terms include “OAT” (Outside Air Temperature) and TAT (total air temperature).

For a more complete discussion of temperature, see discussion 4 in the “Additional Discussion” section at the end of this chapter.

units

Metric system: degrees Celsius, degrees Kelvin
English system: degrees Fahrenheit, degrees Rankine

conversions

$$\textit{degrees C} = (\textit{degrees F} - 32) \times \frac{5}{9}$$

$$\textit{degrees F} = 32 + \left(\textit{degrees C} \times \frac{9}{5}\right)$$

$$\textit{degrees K} = \textit{degrees C} + 273.15$$

$$\textit{degrees R} = \textit{degrees F} + 459.67$$

to obtain	multiply	by
degrees Rankine	degrees Kelvin	1.8000

Time

units

All systems: hours, minutes, seconds, microfortnights

conversions

to obtain	multiply	by
minutes	hours	60.000
seconds	minutes	60.000
seconds	microfortnights	1.2096

Velocity or Speed

Velocity, also called *speed*, is the rate of travel – that is, the distance traveled per unit of time.

units

Metric system: kilometers per hour, meters per second
English system: miles per hour, feet per second

Aeronautical system: knots (nautical miles per hour), feet per second, Mach number

conversions

1 knot = 1 nautical mile per hour

to obtain	multiply	by
feet per second	knots	1.6878
feet per second	statute miles per hour	1.4667
knots	smoots per microfortnight	2.7348
statute miles per hour	knots	1.1508

Viscosity

Viscosity may be thought of as the “stickiness” of a fluid – that property of a fluid that causes it to tend to adhere to any solid surface over which it flows.

Oil, for example, exhibits more viscosity than water. Even air exhibits a (very small) degree of viscosity.

Viscosity affects the flow of a fluid over a surface. For that reason, it has some degree of effect on aerodynamic forces.

For further discussion of viscosity refer to the chapter entitled “Flow Near a Surface”.

units

Metric system: newton-second per square meter (pascal-second)
English system: pound-seconds per square foot

conversions

1 pascal-second = 1 newton-second per square meter = 1 kilogram mass per meter-second
1 slug mass per foot-second = 1 pound force-second per square foot
1 kilogram mass per meter-second = 1 newton-second per square meter1

to obtain	multiply	by
pascal-second	pound-second per square foot	47.880
pascal-second	kg force-second per sq. meter	9.8067

Volume

Volume is the measurement of three-dimensional space. It is used frequently for measuring the capacity of a fuel tank, for measurement of liquids such as fuel, or for density measurements such as pounds per cubic foot.

units

Metric system: liters, cubic meters, cubic centimeters
English system: U.S. gallons, imperial gallons, cubic feet, cubic yards, cubic inches
Aeronautical system: U. S. gallons, imperial gallons, liters

conversions

1 liter = 1000 cubic centimeters

to obtain	multiply	by
cubic feet	cubic meters	35.31
cubic inches	gallons	231.00
cubic inches	liters	61.024
gallons	cubic feet	7.4805
gallons	imperial gallons	1.2010
liters	cubic feet	28.317
liters	gallons	3.7854
liters	imperial gallons	4.546 1

Weight or Force

Weight is just one form of force: in this case, it's the force imparted by gravitational attraction. For a more complete discussion of weight, see "Force or Weight" above and in the chapter entitled "Mass and Weight". Weight is normally measured in units of pounds or kilograms.

As discussed above under "Force", it is important to remember that kilograms units can be used for *either* weight (force) or mass. It's essential to specify, when using units of kilograms, whether you are referring to force or mass.

Similarly, the units of pound force and pound mass have both been used in the past, however the latter is not in contemporary use so the term "pound" can safely be taken to mean force.

Work

See "energy".

Additional Discussion

discussion 1: creating compound conversion factors

Throughout most of this chapter, we have provided simple conversions: feet to meters, pounds to kilograms, or the like. But sometimes it's necessary or useful to create other conversion factors not provided in this chapter, and there's a simple method for doing this.

As an example, let's say that we want to convert statute miles per hour to kilometers per hour. We know that the statute mile is defined as exactly 5,280 feet, and that the kilometer is 3,280.8 feet. We can follow this process:

$$\left(\frac{\cancel{statute\ miles}}{hour} \times \frac{\cancel{feet}}{\cancel{statute\ mile}} \times \frac{kilometers}{\cancel{foot}}\right) = \frac{kilometers}{hour}$$

You see that the "statute miles" in the numerator of the first term cancels the one in the denominator of the second term, and that the "feet" in the numerator of the second term cancels the one in the denominator of the third term, leaving kilometers divided by hours, or kilometers per hour. Thus:

$$\frac{kilometers}{hour} = \frac{statute\ miles}{hour} \times \frac{5280}{1} \times \frac{1}{3280.8} = \frac{statute\ miles}{hour} \times 1.6094$$

So to find kilometers per hour, multiply statute miles per hour times 1.6094.

---

discussion 2: angle measurement in radians

Consider Figure 1-1 to the right. We have a circle, and the circle has a radius *r*. We have taken the length of one radius and laid it out along the circumference of the circle. The angle which that makes is, by definition, one radian.

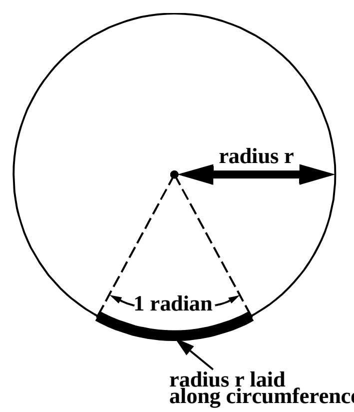


Figure 1-1

Remember that the circumference of a circle is equal to $2\pi r$. You can see then that we could lay out one radius along the circumference only 2π times. Therefore, there are 2π radians in one complete circle so 2π radians is the same as 360 degrees, or one radian is equal to 57.296 degrees.

Radians as a measure of angle are convenient because any angle expressed in radians, when multiplied by the radius of the circle, will give the length of the portion of the circle that the angle marks out.

Referring to the illustration at the right: the length X along a length of circular arc can be found by multiplying the angle θ expressed in radians by the radius r, or the angle θ in radians can be found by dividing X by r.

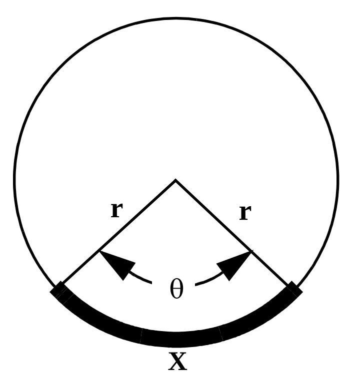


Figure 1-2

Units of radians are particularly useful when dealing with things such as centrifugal force. When the angular rate of rotation ω of an object is expressed in radians per second, the centrifugal force is simply equal to $mr\omega^2$. There is further discussion of this in the chapter entitled "Mass and Weight".

Most performance calculations involving angles require the units of measurement for angles to be in radians. For small angles, the tangent of the angle is approximately equal to the angle expressed in radians.

---

discussion 3: pressure

Think of a gas. If you could examine the gas closely enough, you'd be able to see countless billions of molecules of the gas in random motion.

Now picture that gas as being enclosed within a container. Some of the molecules of the gas, in their random motion, will bounce off the walls of the container, each impact imparting a minute bit of energy to the wall. The effect of these countless tiny impacts is what we feel as *pressure*. If you were to apply heat to the container, the heat energy would convert into increased molecular motion in the gas, so (for a constant volume) the pressure would increase.

Similarly, if you were to add more molecules to the container the number of impacts on the wall would increase, again increasing pressure. Increasing either temperature or density will increase pressure.

Pressure, temperature and volume are all related by the *equation of state* which is discussed in the chapter entitled "Physics of Air".

One way to measure pressure, specifically atmospheric pressure, was devised by the Italian physicist and mathematician Evangelista Torricelli (1608-1647). Torricelli took a glass tube, closed it at one end, and filled it with liquid mercury. Temporarily capping off the open end of the tube to prevent air from entering it, he then inverted the tube so that its capped end was underneath the surface of a volume of liquid mercury in a dish. When the cap was removed from the open end of the tube, now below the surface of the mercury in the dish, the top of the column of mercury in the tube was seen to drop to some height above the level of the mercury in the bowl.

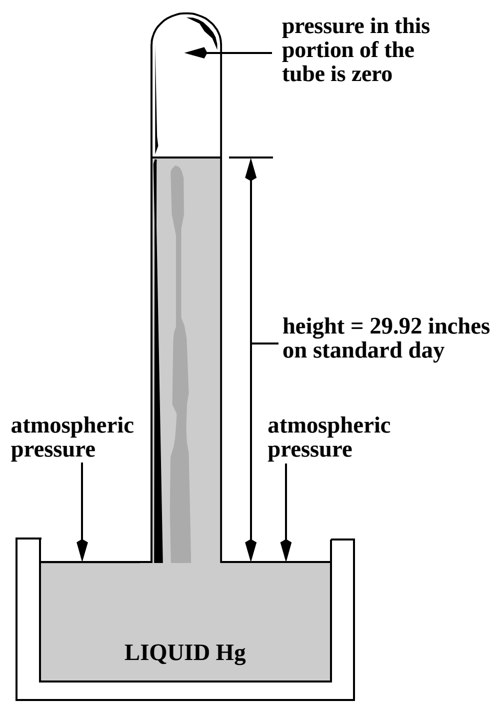


Figure 1-3

Torricelli observed that the height of the column of the mercury varied with the atmospheric conditions and he realized that it was *a direct measurement of atmospheric pressure*.

When you hear atmospheric pressure reported in *inches of mercury* or *millimeters of mercury*, that is exactly what is being measured: the height of the column of mercury. One "standard atmosphere", defined as an air pressure of approximately 14.7 pounds per square inch at sea level, corresponds to 29.92 inches, or 760 millimeters, of mercury.

discussion 4: temperature

Temperature is a complex subject. For our purposes, however, it's sufficient to define it this way: temperature is a measure of what we call "heat" or "hotness". These are indications of the amount of *translational kinetic energy* associated with the random microscopic linear (as distinct from rotational or vibrational) motion of the atoms and molecules of a material. Hotter materials have greater molecular kinetic energies, colder materials have lesser molecular kinetic energies Temperature is not an indication of the total internal energy of a substance, only the kinetic energy.

It's the translational kinetic energy of a substance's molecules which produce what we call "flow of heat" from hotter to colder areas. This "flow" of heat occurs because molecules having greater kinetic energy ("hotter") will pass on some of this energy to molecules having lower energy ("colder") when their molecules collide in their random motion. This flow of heat will cease when all molecules have the same kinetic energy.

There exist two different systems for measuring temperature, and two different temperature scales are in use in each system. The two different systems may be called *absolute temperature* and *relative temperature*, and the two different scales are the English scale and the metric scale.

The system called absolute temperature is based on the concept of *absolute zero*. Absolute zero represents the temperature at which heat energy is at its lowest possible point. The absolute temperature system uses absolute zero as its starting point. That is, zero temperature represents absolute zero.

The relative temperature system is not based on absolute zero. Instead, two unique temperature conditions are selected and arbitrary temperature values are assigned to those conditions. In both the English and metric scales, the unique temperature points which were chosen are the freezing temperature of water and the boiling temperature of water. Since the freezing and boiling points of water are unvarying properties (under specified conditions), they make useful reference points.

On the metric scale of relative temperature, called the *Celsius* scale, the freezing point of water was assigned the value of zero and the boiling point was assigned the value of 100; the range in between the two is thus 100 degrees.

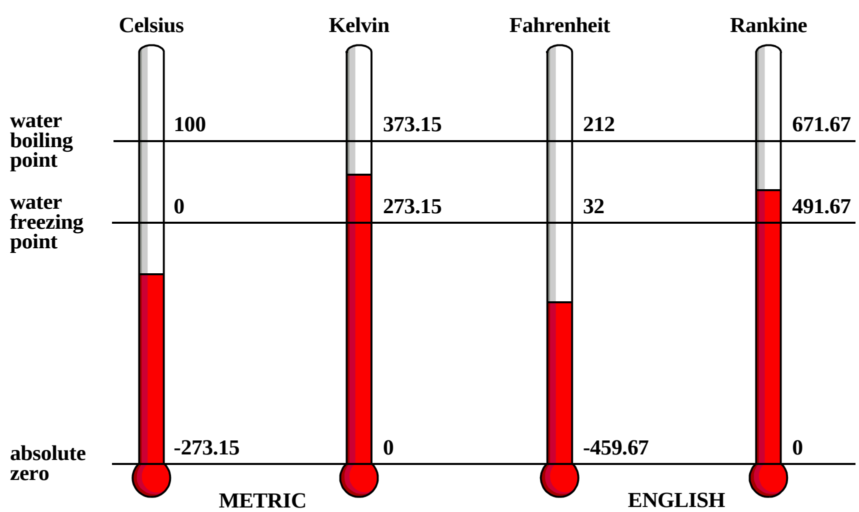


Figure 1-4

On the English scale of relative temperature, called the *Fahrenheit* scale, the freezing point was assigned the value of 32, and the boiling point was assigned the value of 212. Thus the range of temperature between those two points was divided into 180 degrees.

You see that a change of 100 degrees on the Celsius scale represents the same change of heat energy as a change of 180 degrees on the Farhenheit scale. A change of one degree Celsius thus equals a change of 1.8 degrees Fahrenheit.

On the Fahrenheit scale, absolute zero occurs at -459.67 degrees. On the Celsius scale, absolute zero occurs at -273.15 degrees.

The English scale of absolute temperature is called the *Rankine* scale. Absolute zero occurs at zero degrees Rankine, therefore water freezes at +491.67 degrees R and it boils at +671.67 degrees R. That is, Rankine temperature is equal to Fahrenheit temperature plus 459.67 degrees.

The metric scale of absolute temperature is called the *Kelvin* scale. Absolute zero occurs at zero degrees Kelvin, so water freezes at +273.15 degrees K and it boils at +373.15 degrees K. Thus, Kelvin temperature is equal to Celsius temperature plus 273.15 degrees.

It's vital to know that in much of our performance work we need to work in units of absolute temperature. A good example of this is calculating the speed of sound. Only absolute temperature gives a true measure of kinetic molecular energy, and the speed of sound depends on the kinetic energy of the molecules.

Think about it: is 10°C twice as hot as 5°C? No. More hot, yes. Twice as hot – no. In fact, 10°C is twice as hot as minus 131.6°C. Why? Simple: 10°C is equal to 283.15°K. Half of that is 141.6°K. That, when converted back to Celsius, is minus 131.6°C.

In any calculations which involve molecular energy, such as the speed of sound, temperature ratio θ, gas flow characteristics, and many more, you must use absolute units. Only in absolute units can we say, for example, "100 degrees is twice as hot as 50 degrees" or "100 degrees is half as hot as 200 degrees."

On the other hand, all four temperature systems can be used to make additions and subtractions: we can say, for example, "twenty degrees is five degrees hotter than fifteen degrees" or "fifty degrees is ten degrees colder than sixty degrees."

Throughout this book, where necessary we'll tell you which units of temperature should be used.

Chapter 2: Mass, Weight, and Center of Gravity

Introduction

Unless a person becomes involved in certain types of work including airplane performance, that person usually tends to take a casual approach to the concept of *weight*, and probably doesn't pay any attention at all to the concept of *mass*. The difference between these concepts, however, is essential to a performance engineer's work and needs to be clearly understood.

It's easy to think of weight and mass as being almost the same thing. They're related, but they're completely different properties of an object. *Mass* is the essential property in some performance calculations, for example the distance required along the runway to accelerate an airplane to its takeoff speed. This is because an airplane's acceleration during takeoff is a function of the airplane's mass and its thrust, following the relationship $F = ma$. In other performance calculations *weight* may be the essential property, for example when computing the climb gradient of an airplane. This is because the climb gradient depends directly on the lift being created by the airplane's wings, which is equal to the weight.

We're going to talk about mass, then about weight, and finally about gravitational acceleration – which you'll find is much more complex than you might have thought. An airplane's weight, for example, depends on a number of variables including – believe it or not – the direction in which the airplane is flying!

A related topic is the concept of a "center of gravity" which is extremely important when dealing with complex objects such as airplanes. That will be the final topic in the chapter.

Mass

Mass is not a property that can be directly measured. It can only be deduced. For example, you could tie a string to an object, then whirl the object in a circular path at the end of the string. You could measure the tension in the string. Then knowing the string's length r and the angular rate of rotation ω, and knowing that:

$$\textit{tension in the string} = \textit{centripetal force} = mr\omega^2 \qquad \text{(eq. 1)}$$

you could deduce the magnitude of the object's mass.

It's easy to think of mass and size as directly related, but that's just not so. A helium-filled balloon one meter in diameter has much less mass than a ball of lead one centimeter in diameter.

Weight and mass are different in an essential way: the mass of an object doesn't change as its location changes, but the weight of an object can vary as its location changes. An object of known mass will weigh less if it's on the moon's surface than it does on the earth's surface.

Perhaps the easiest way to think of mass is to think of *inertia*. Inertia is the tendency of an object to remain at rest unless acted on by a force – or to remain in motion in a straight line unless a force is applied to it to make it slow down. It's hard to make *massy* objects accelerate (think of trying to push a railroad locomotive). It's easier to make objects having less mass accelerate (think of trying to push a small car).

units of mass

As discussed in the chapter entitled "Units and Conversions", mass is measured in the English units of slugs, or in the metric units of kilograms mass.

Force

We can think of *force* as the effort that is needed to cause a mass to accelerate or to decelerate. If a mass is at rest, we must apply force to make it gain some velocity; if a mass is in motion, we must apply force to make it slow down or stop. The greater the mass of the object we're trying to accelerate or decelerate, the greater will be the required force.

The force, the mass, and the acceleration are related to each other by the equation first expressed by Isaac Newton in his *Philosophiæ Naturalis Principia Mathematica,* published in 1687:

$force = mass \times acceleration$, usually written as F = ma. (eq. 2)

units of force

The chapter entitled "Units and Conversions" discusses force. Force is usually measured in the English units of pounds, or in the metric units of kilograms or Newtons.

Weight

Weight is one form of force, and because it is a force, it follows Newton's equation. Weight is normally measured in the English units of pounds, or in the metric units of kilograms; Newtons, although a unit of force, are not used as a measure of weight.

We tend to think of weight as a simple matter – step on a scale, read your weight. As you'll see, however, if you're a person standing on the surface of the earth your weight depends on the latitude of your location on earth and on your elevation above sea level. If you're a passenger in an airplane, your weight depends on the airplane's latitude and altitude. It also depends on how fast the airplane is flying and in what direction.

While some of these factors are quite small in magnitude and can usually be neglected, precision work such as the accurate determination of lift requirements during flight testing of large commercial jet airplanes does account for all of them. Also, some Boeing software allows you to account for those factors when doing – as one example – airplane cruise fuel mileage checks.

So weight really isn't a simple matter at all. Let's get more specific:

Weight Is Gravitational Force

What we're accustomed to calling "weight" here on earth is just one form of the more general principle of physics called *gravitational force*.

Any two (or more than two, of course, but for simplicity here we're going to be talking about two bodies) are attracted to each other by gravitational force. To this day, nobody understands the cause of gravitational force, but we can predict it very accurately.

Any two objects located anywhere in space and having masses of m_1 and m_2 will be attracted to each other by a force predicted by Newton's universal law of gravitation:

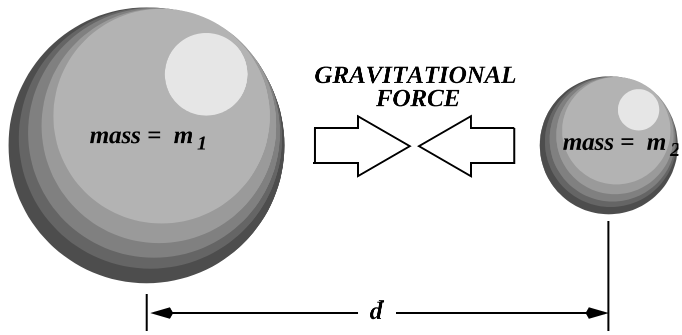


Figure 2-1

$$\textit{gravitational force} = G \times \frac{m_1 m_2}{d^2} \qquad \text{(eq. 3)}$$

where G is the universal gravitational constant
m_1 and m_2 are the masses of the two objects
d is the distance between their centers of mass

In British units:

- the value of G is 3.44×10^{-8} foot^2-pound/slug^2
- m_1 and m_2 are measured in units of slugs
- d is measured in feet

In the metric system:

- the value of G is 6.673×10^{-11} meter^2-Newton/kilogram^2
- m_1 and m_2 are measured in units of kilograms mass
- d is measured in meters

For a discussion of the units of force and mass, you may refer back to the Chapter entitled "Units and Conversions".

An object's "weight" on earth is simply the gravitational force exerted on it resulting from its mass, the mass of the earth, and the distance between their centers.

With that in mind, equation 2 can be re-written as:

$$W = G \times \frac{m \times m_{earth}}{d^2} \qquad \text{(eq. 4)}$$

where W is the weight of the object
G is the gravitational constant
m is the mass of the object
m_{earth} is the mass of the earth
d is the distance from the object's center of mass to the earth's center of mass

Gravitational Acceleration

Since weight is gravitational force, it follows Newton's law, but we're going to re-write that equation slightly differently. Instead of F = ma, let's say instead that:

$$weight = mass \times gravitational\ acceleration \quad \text{or } W = mg \qquad \text{(eq. 5)}$$

where m is the mass of an object
g is the gravitational acceleration
W is the weight of the object

Applying equation 5 to equation 3:

$$W = G \times \frac{m \times m_{earth}}{d^2} = mg$$

or:

$$g = G \times \frac{m_{earth}}{d^2}$$

Let's consider, for now at least, gravitational attraction at the earth's surface. For simplicity, let's refer to it as "weight", since that's the term we usually apply to the force that gravity exerts on an object on earth.

sea level gravitational acceleration on a non-rotating earth

For the moment, we're going to pretend that the earth is not rotating.

Most objects at sea level whose weight we want to know will be very small compared to the size of the earth, so it's sufficiently accurate to say that the distance between their centers of mass and the earth's center of mass is equal to the radius of the earth, denoted by r_{earth} or, more simply, r_e.

Thus, in the absence of all other forces:

$$weight = G \times \frac{m \times m_e}{r_e^2} \qquad \text{(eq. 6)}$$

From that we get:

$$g = G \times \frac{m_{earth}}{r_e^2} \qquad \text{(eq. 7)}$$

effect of the shape of the earth on gravitational acceleration

Complicating the calculation of the earth's gravitational acceleration g is the fact that the earth is not a true sphere. Instead, it's an *oblate spheroid*.

The illustration is exaggerated to make the meaning obvious. The earth's radius is slightly greater at the equator than at the north or south pole. The difference is small, only about 70,000 feet, but it's not negligible.

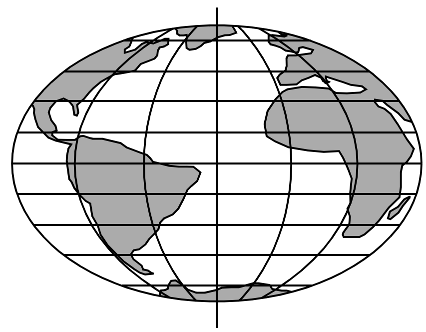


Figure 2-2

The radius of the earth at sea level at any given latitude can be calculated with acceptable accuracy by using the equation for the radius of an ellipsoid:

$$r_e = \sqrt{\frac{a^4 + b^4 \tan^2\varphi}{a^2 + b^2 \tan^2\varphi}} \qquad \text{(eq. 8)}$$

where a is the radius of the earth at the equator, 20,925,780 feet
b is the radius of the earth at the poles, 20,855,696 feet
φ is the latitude, degrees north or south

You've seen that the gravitational acceleration depends on the inverse of the square of the distance between the centers of mass of the object and the earth. It follows that an object at sea level at the equator, where the earth's radius is greatest, will experience less gravitational acceleration than an object at sea level at some latitude above or below the equator, due simply to its greater distance from the earth's center of mass.

Putting it in terms of weight: an object at the equator will weigh less than an object at any latitude above or below the equator. For example, an airplane that weighs 650,000 pounds on the runway at SeaTac Airport near Seattle, north latitude 47.45 degrees, would weigh just over 648,000 pounds on a runway at Changi Airport in Singapore, north latitude 1.35 degrees. *Keep in mind,* *though, that the mass is the same at both locations and hence (for the same thrust) the airplane's acceleration down the runway would be the same at both airports despite the weight difference.*

gravitational acceleration on a rotating earth

The gravitational acceleration experienced by an object located on the surface of the earth can't be found accurately by simply using the equation for shown above. Why not? Because an object located on the earth's surface is moving in a circular path around the earth's axis, due to the rotation of the earth. That rate of rotation, denoted as ω_{earth} or, more simply ω_e, has the value of 7.29212×10^{-5} radians per second.[1]

To illustrate this, imagine that you are able to float in space, looking down on the earth's north pole. Let's further imagine that there's an airplane sitting stationary at the earth's equator.

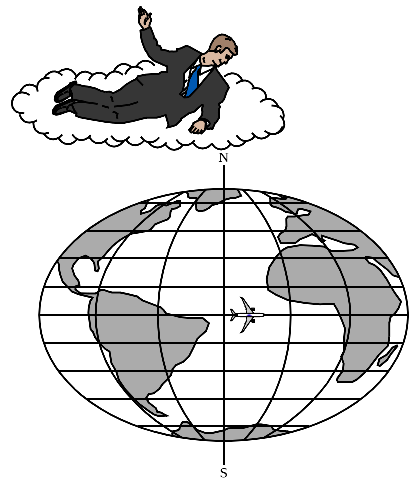


Figure 2-3

1. If you do the arithmetic, you'll find that 7.29212 x 10^{-5} radians per second translates to a rotation of 360° in 23 hours, 56 minutes and 4 seconds for one full revolution. Why isn't it exactly 24 hours?

   23 hours 56 minutes 4.091 seconds is called the "sidereal day", meaning that it's the length of the day referenced to remote stars, *not to our sun.* A solar day is 24 hours, but the earth is rotating at a rate that is very slightly more than that. It actually rotates on its axis approximately 361° in 24 hours. The difference is that the earth's movement in its orbit around the sun has the effect of lengthening the day by four minutes *as measured by sun position*.

   For inertial effects, it's the *sidereal* day that's important, since that's the rate at which the earth is truly rotating in space

As you look down on the earth's north pole, you see it rotating beneath you. You see that the airplane is in motion following a circular path around the earth's axis <u>even though it's stationary on the earth's surface</u>.

The airplane's inertial velocity in its circular path, relative to the axis of rotation of the earth, is $r_e\omega_e$.

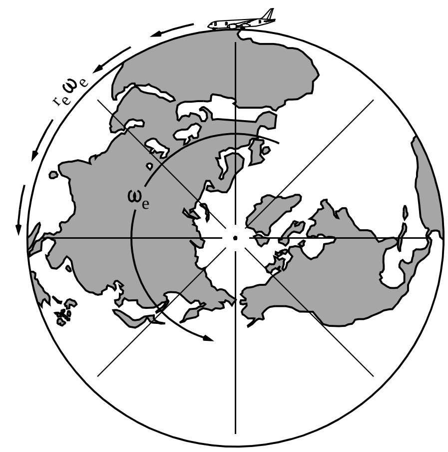


Figure 2-4

This rotation about the earth's axis means that the airplane is experiencing a small centrifugal (outward) acceleration. That acceleration can be computed by using the equation:

$$\textit{centrifugal acceleration} = r_e\,\omega_e^2 \qquad \text{(eq. 9)}$$

In the case of the airplane rotating around the earth's axis, the radius of that circular path depends on the location of the airplane on earth. At sea level at the equator, the radius of the path would be equal to the earth's equator, but north or south of the equator the radius of the path would be less.

Here you see that the radius of the circular path found from the radius of the earth and the angle above the equator – the latitude, denoted by φ:

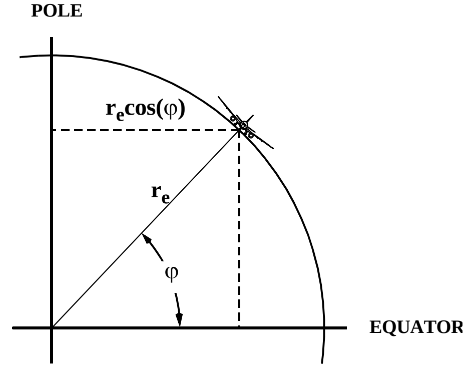


Figure 2-5

$$r = r_e\cos\varphi \qquad \text{(eq. 10)}$$

The centrifugal acceleration created by the earth's rotation is perpendicular to the earth's axis, and its magnitude is

$$r_e\cos(\varphi)\,\omega_e^2$$

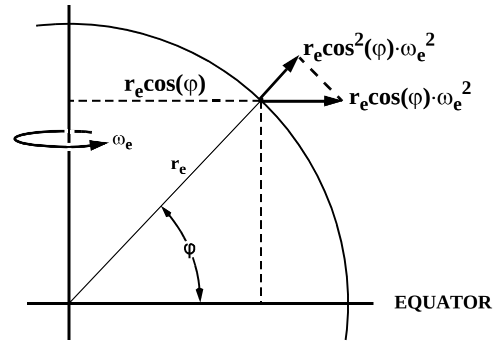


Figure 2-6

The component of that outward acceleration that acts perpendicular to the earth's surface, in the direction opposite to the gravitational acceleration imposed by the mass of the earth and its radius is:

$$centrifugal\ acceleration = r_e cos^2(\varphi)\,\omega_e^2 \quad \text{(eq. 11)}$$

This means that the *apparent acceleration* of the airplane is less than the acceleration predicted by equation 6.

The net acceleration is thus:

$$g = G \times \frac{m_e}{r_e^2} - r_e \omega_e^2 cos^2 \varphi \quad \text{(eq. 12)}$$

We will refer to the first of the two terms in equation 8 as the "Newtonian" or "gravitational" component acceleration. The second term is the "centrifugal" component.

It should be emphasized here that the centrifugal component of weight is extremely small compared to the Newtonian component. From here on, we'll refer to apparent weight as just "weight", since the centrifugal effect is always present and is very small. When doing more precise calculations of weight, however, the centrifugal effect shouldn't be ignored.

gravitational acceleration and reference g

For convenience, the engineering and science communities use a "reference" value of gravitational acceleration, denoted as g_0. The reference values, to seven significant digits in both English and metric units, are

32.17405 feet per second per second
9.806665 meters per second per second

The "reference" value g_0 is a representative sea level mid-latitudes acceleration, and is thus an acceptable value for use when the job it's applied to doesn't demand a high degree of precision. More precise applications will require that g_0 be corrected for a number of other factors, as we'll be discussing in the following sections.

<u>This is important:</u> the g_0 values shown above <u>already account for the centrifugal effect</u> discussed previously. That is, they include both the Newtonian and centrifugal components.

Lambert's equation

"Lambert's equation" provides a simple and acceptably accurate way to calculate the value of *g* for any latitude, at sea level elevation:

$$g_{\varphi, SL} = 32.17244 \times [1 - 2.6373 \times 10^{-3} \cos(2\varphi) + 5.9 \times 10^{-6} \cos^2(2\varphi)] \qquad \text{(eq. 13)}$$

It is important to note that *this equation accounts for both the force of gravitational attraction and the slightly offsetting effect of the earth's rotation, discussed above.*

This value of *g* would be valid only for a stationary object at sea level. What if the object is at some elevation above sea level?

variation of g with latitude and altitude for a stationary airplane

Up to this point, we have been talking about gravitational acceleration at sea level. We have been speaking about the Newtonian component of weight, and the centrifugal component of weight. The height of an object above sea level, which we'll denote as *z*, will affect both of these components.

The equation giving *g* for elevations above sea level, for a stationary airplane, is:

$$g_{\varphi, z} = [g_{\varphi, SL} + \omega_e^2 r_e \cos^2\varphi]\left[\frac{r_e}{r_e + z}\right]^2 - \omega_e^2 (r_e + z) \cos^2\varphi \qquad \text{(eq. 14)}$$

Where: ω_e is the rotation rate of the earth, 7.29212 x 10^{-5} radians per second
z is the height above sea level, feet

For a stationary object at any altitude or latitude, then, finding the value of $g_{\varphi,z}$ requires two steps: first, the use of Lambert's equation to find $g_{\varphi,SL}$ followed by the use of equation 14 above to account for the effect of altitude.

For example, you can compute that an airplane which weighs 800,000 pounds at one standard *g* would weigh instead 797,214 pounds at El Alto Airport at La Paz, Bolivia, elevation 13,313 feet, latitude 16.5 degrees North. Be reminded again, though, that the *mass* would remain the same.

For the derivation of equation 14 above, please see discussion 1 in the "Additional Discussion" section at the end of this chapter.

gravitational acceleration on a moving airplane

The above discussion demonstrates how to find the value of *g* for a stationary airplane at any latitude and altitude. Finding *g* for a moving airplane requires an additional adjustment.

As is the case for a stationary airplane, the value of *g* for a moving airplane has two components. One is the gravitational component which we've already discussed; the other is, again, a centrifugal component but it's more complex in the case of a moving airplane. An airplane can be flying in any direction whereas a stationary airplane is moving in a very simple way: counterclockwise in a circular path as seen from above the north pole.

The velocity of a moving airplane has two components: an east-west component and a north-south component.

The path of an airplane over the ground will be at some angle to true north. This angle is called its true track, and is denoted by the Greek character χ (chi). True track is always a positive number, varying from 0 to 359 degrees, where 0 represents true north, 90 represents true east, and so on.

The direction of the path relative to magnetic north can't be used; we need to know the velocity of the airplane in terms of its component along the lines of latitude and along the lines of longitude. Those are related to true, rather than magnetic, north.

The equation for the centrifugal adjustment is:

$$\Delta g_{centrifugal} = \frac{V_G^2}{(r_e + z)} + 2\omega_e V_G \cos\varphi \sin\chi \qquad \text{(eq. 15)}$$

This adjustment is, of course, subtracted from the gravitational component since it is acting in the direction opposite to that of gravity.

Therefore, for a moving airplane:

$$g = g_{\varphi, z} - \Delta g_{centrifugal} \qquad \text{(eq. 16)}$$

For the derivation of equation 15 above, please see discussion 2 in the "Additional Discussion" section at the end of this chapter.

current practice at Boeing

Current practice at Boeing in publishing airplane flight planning data is to base all published weights on the value of g for an airplane flying at the ground speed for the given Mach number with no wind, on a standard day, at a nominal altitude depending on the weight, at a latitude of forty-five degrees, in a northerly direction. Using these conditions as their basis, published performance data for any weight will represent a reasonably average set of conditions, hopefully rendering more precise calculation of weight unnecessary for most typical operations.

None of the Boeing's published takeoff and landing performance data is adjusted, since it is based on flight test results taken at latitudes near 45 degrees and at slow speeds.

Boeing follows standard industry practice and bases all of its weights on the reference value of gravitational acceleration, g_0. This means that the current weight displayed by a Flight Management Computer (FMC) is always based on the reference value of g.

summary

From the discussions above, it can be seen that the weight of an airplane is a function of a number of variables. Although the effects of the individual corrections to weight may each be relatively small in magnitude, the combined effects can be very noticeable.

To illustrate: an airplane weighing 800,000 pounds on the ground at latitude 45 degrees would weigh 791,932 pounds at the equator flying eastbound at FL330 and 450 knots groundspeed. The same airplane would weigh 799,216 pounds at latitude 60 degrees north flying westbound at the same 450 knots groundspeed.

For typical flight planning work, these corrections can be overlooked, as the corrections are typically less than one percent of airplane weight. When doing more careful calculations, however, such as flight test or cruise performance monitoring, it's advisable to know the airplane's actual weight as exactly as possible because this will more accurately determine the true lift requirement demanded of the wing.

Center of Gravity

The typical Performance Engineer needs at least a basic understanding of the concept of *center of gravity*, usually just referred to as "CG". While the engineer may not be responsible for preparation of that operator's weight and balance load and trim system, he or she may well become interested in the effects of the center of gravity on the airplane's performance. One prime example of this is discussed in the chapter entitled "Alternate Forward CG".

We're going to demonstrate how the *center of gravity* of a group of weights may be found. That group of weights might be simple – a few weights spaced along a beam – or it might be very complex – a fully loaded 747. The principles involved in finding the center of gravity of either of these groups of weights is exactly the same.

To do it, we need to find the total *moment* of the group of weights, and its total weight. Let's first take a minute to review the meaning of "moment".

basics: moment

A very simple definition: a *moment* is a force multiplied by a distance, called a *moment arm*. Suppose for example that we have a wheel that's free to rotate about its center point. Its radius is one foot. We apply a force of 50 pounds at its perimeter, such as a weight hanging down from the wheel. The moment arm of that weight (force) is the distance, from the center of the wheel, at which the force is acting.

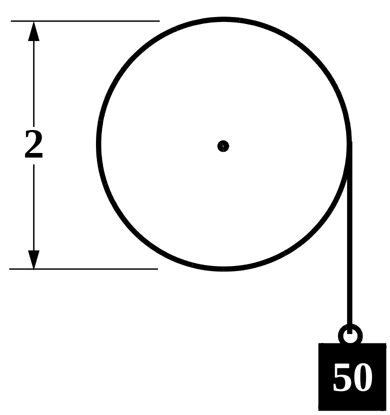


Figure 2-7

The force of 50 pounds, being applied at a moment arm of one foot from the wheel's center, creates a clockwise moment of $50 \times 1 = 50$ *foot-pounds*. That 50 foot-pound moment would cause the wheel to rotate in a clockwise direction. If we didn't want the wheel to rotate, we would have to counteract the moment by applying an equal and opposite moment.

To continue the example, suppose we join a second wheel to the first wheel, so that they rotate together. The radius of the second wheel is one-half foot. If we add now a second weight on the inner wheel as shown, and its weight is 100 pounds, it would then have a counter-clockwise moment of $100 \times 0.5 = 50$ foot-pounds.

Since the new moment is equal and opposite to the first moment, the moments now are in *equilibrium*[1], so the wheels wouldn't rotate.

We need to use moments to calculate a center of gravity, as you'll see next.

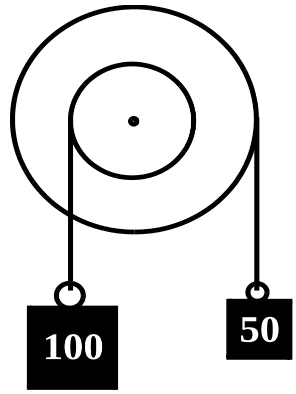


Figure 2-8

finding the center of gravity

For any group of weights that is composed of a number of discrete individual items, it's possible to find one single point that can be used to represent the entire group of individual parts. That single point is referred to as its *center of gravity.*

Let's look at an example. Suppose that we have a uniform beam that is ten feet long and weighs 10 pounds. An example of such a beam would be a board of wood having constant width and constant thickness.

If we want to place the beam on a support such that it will be "balanced" – meaning that the beam will lie motionless on the support in a level attitude – then we should place the support right below a place on the beam that we're going to call its "balance point". Where is that balance point?

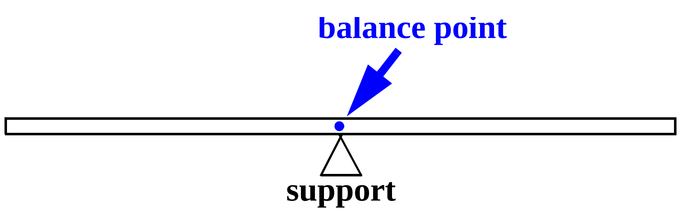


Figure 2-9

That's too easy. The balance point is at the midpoint of the beam. Why? We all know the answer to that one – if a beam is uniform along its length, its balance point will be halfway between its ends – its midpoint.

Now let's place a one-hundred pound weight at each end of the beam. Now where's the balance point?

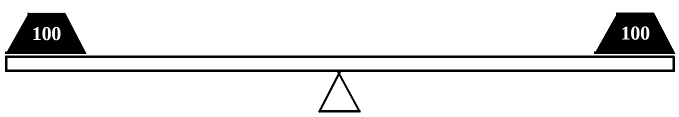


Figure 2-10

1. Equilibrium: "a state of rest or balance due to the equal action of opposing forces."

Same answer: halfway down the length of the beam, at the beam's balance point and equidistant from the two weights.

Now let's make the problem more interesting: We still have the ten-pound uniform beam, but now there are a number of different weights at various locations along its length. We know the amount of each weight, and its location. <u>Now</u> where's the balance point?

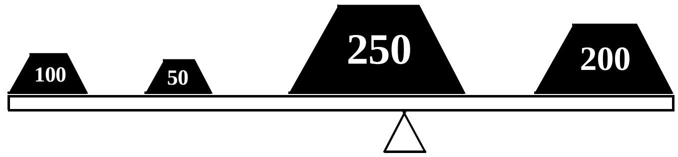


Figure 2-11

Let's change the last illustration to just a simplified representation showing the forces and their locations, as you see to the right.

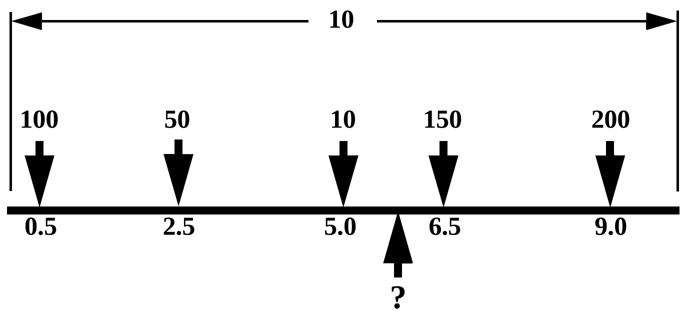


Figure 2-12

The beam is 10 feet long, weighing 10 pounds; four weights weighing 100, 50, 150 and 200 pounds are placed along its length at the distances shown from the left-hand end of the beam. Where should the balance point – the one upward force – be placed, measured from the left-hand end of the beam?

To solve this sort of a problem, we first need a reference point. This reference point, frequently called a *datum*, will serve as a standard location from which to measure the locations of the different weights, and also the center of gravity.

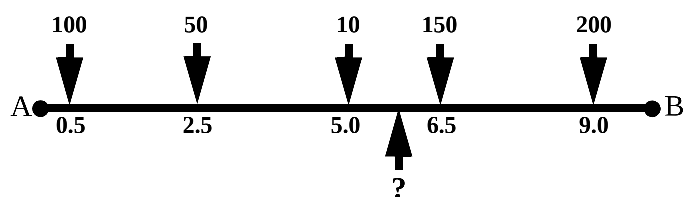


Figure 2-13

For this example, let's put our datum at the left-hand end of the beam, the point marked A.

For this group of weights to be in balance, two conditions must be met:

- the sum of all upward forces must be equal to the sum of all downward forces. This is referred to as vertical equilibrium.
- the sum of all clockwise moments about the datum must be equal to the sum of all counter-clockwise moments about the datum. This condition is referred to as moment equilibrium.

In this example, we have five forces acting downward, and only one acting upward. Thus for vertical equilibrium the upward force must be equal to the sum of the five downward forces, or 510 pounds.

How about the moments? There are five clockwise moments about the datum: the first is 100×0.5 or 50 foot-pounds; the second is 125 foot-pounds; the third weight, the beam itself, has a moment of 50 foot-pounds; the third weight has a moment of 975 foot-pounds, and the fourth weight has a moment of 1800 foot-pounds. The total moment clockwise moment about point A is then 3000 foot-pounds.

The single counterclockwise moment about point A is contributed by the support at the balance point. We know that the upward force at the balance point must be 510 pounds, for vertical equilibrium. To provide a counterclockwise moment of 3000 foot-pounds, to counter the total clockwise moment of all the weights, it must then have a moment arm of 3000 ÷ 510, or 5.88 feet.

What we did to find the balance point was to take the sum of the moments about the datum, and divided it by the sum of the weights:

$$\textit{balance point distance from datum} = \frac{\Sigma \textit{moments about datum}}{\Sigma \textit{weights}}$$

Here's an interesting and sometimes useful fact: <u>you can place the datum anywhere you want to</u>, and you'll still get the correct result. To demonstrate, let's take the moments this time about the <u>right</u>-hand end of the beam, the point marked B: then the total counterclockwise moment of the weights will be:

$$\Sigma\text{moment} = (200\times1) + (150\times3.5) + (10\times5) + (50\times7.5) + (100\times9.5) = 2100 \text{ pound-inches}$$

and the distance to the balance point, <u>measured from point B</u>, is 4.12 feet:

$$\textit{distance} = \frac{\Sigma \textit{moment}}{\Sigma \textit{weight}} = \frac{2100}{510} = 4.12 \textit{ feet}$$

That's the same answer as we obtained using the left-hand end of the beam as a datum, because 5.88 feet from point A is the same as 4.12 feet from point B!

We can take advantage of the fact that the location of the datum is arbitrary: we could place it at the location of one of the weights. Then that weight would have zero moment arm, meaning no moment, saving us one calculation – remembering, of course, that even though it has no moment, it still has <u>weight</u>.

Suppose for example that we place the reference point at the 100-pound weight, one-half foot from point A. The calculation then would be:

$$\textit{distance} = \frac{\Sigma \textit{moment}}{\Sigma \textit{weight}} = \frac{(50\times2) + (10\times4.5) + (150\times6) + (200\times8.5)}{510} = 5.38 \textit{ feet}$$

Does it check with the previous two? Yes, because 5.38 feet from the first weight is the same as 5.88 feet from point A. So you can use this trick when calculating balance points, to save one step of multiplication. It's frequently done in manual load and trim systems.

What's this thing we've been calling the "balance point"? That's the "center of gravity" of the system of weights. It's the single point at which the entire system of weights may be considered as being concentrated, for the purposes of calculating balance.

If you understand the preceding discussion, then you'll understand that the same method can be applied to either our example system of five weights on a beam or to a commercial jet transport airplane. It's only a matter of scale.

Think of a large complex airplane such as a 747. It's made up of more than one million distinct items, each one having its own specific weight and its own specific location. Then add the payload, which might consist of many hundred more items, and add the fuel, the crew, the catering, the blankets, pillows, emergency equipment...the list seems endless.
When computing performance, it's essential to know the weight of the airplane – also, for every single takeoff it's necessary to know the location of the weight relative to the wing's center of lift. When an airplane consists of more than a million discrete items of weight, how do you solve that problem?

The use of centers of gravity for groups of weights can save an enormous amount of effort. Instead of accounting for the weight and distance of every one of the distinct items of weight in a fully loaded 747 – each rivet, each passenger, each piece of cargo, each system component, and on and on, we'll reduce the list to just four items:

- the weight and center of gravity (CG) of the airplane itself at its ready-for-flight condition,
- the weight and CG of the passengers,
- the weight and CG of the cargo,
- the weight and CG of the fuel.

The CG of the airplane at its ready-for-flight condition will include cockpit and cabin crew, catering, passenger service items, everything that makes the airplane ready to go. We're now working with only a few items of known weight and location, instead of millions.

Boeing standard practices

It may prove useful, at some time, for you to know how Boeing specifies the locations of the various items that make up an airplane, and how we designate the location of the airplane's center of gravity.

specifying the location of an item of weight

For weight and balance purposes, it is standard Boeing practice on all models to specify the longitudinal location of all items in terms of *balance arm*. The balance arm of an item is simply its distance, in units of inches, measured from an arbitrary airplane datum.

The datum is customarily placed some distance forward of the nose of the first variant of a model such as the -100. Later variants of the airplane, such as the -200, -300, -400 and so on will retain the same datum location relative to the airplane's mid-section. It's important to understand that:

the distance of the datum from (for example) the main landing gear of the -100 is the same on the -200, -300, and on.

Later variants of a model may have different body lengths, achieved by adding inserts – sometimes called “plugs” – of additional body sections forward and/or aft of the airplane’s midsection. A forward plug, then, has the effect of moving the nose of the airplane forward relative to the mid-section of the airplane. As a result, the datum may be behind the nose of a later variant, even though it’s at the same distance from the landing gear.

To illustrate this, consider the 757-200 and the 757-300. (We never produced a -100 variant.) The balance arms for many of the items of the -300 are different from the balance arms for the same items on the -200.

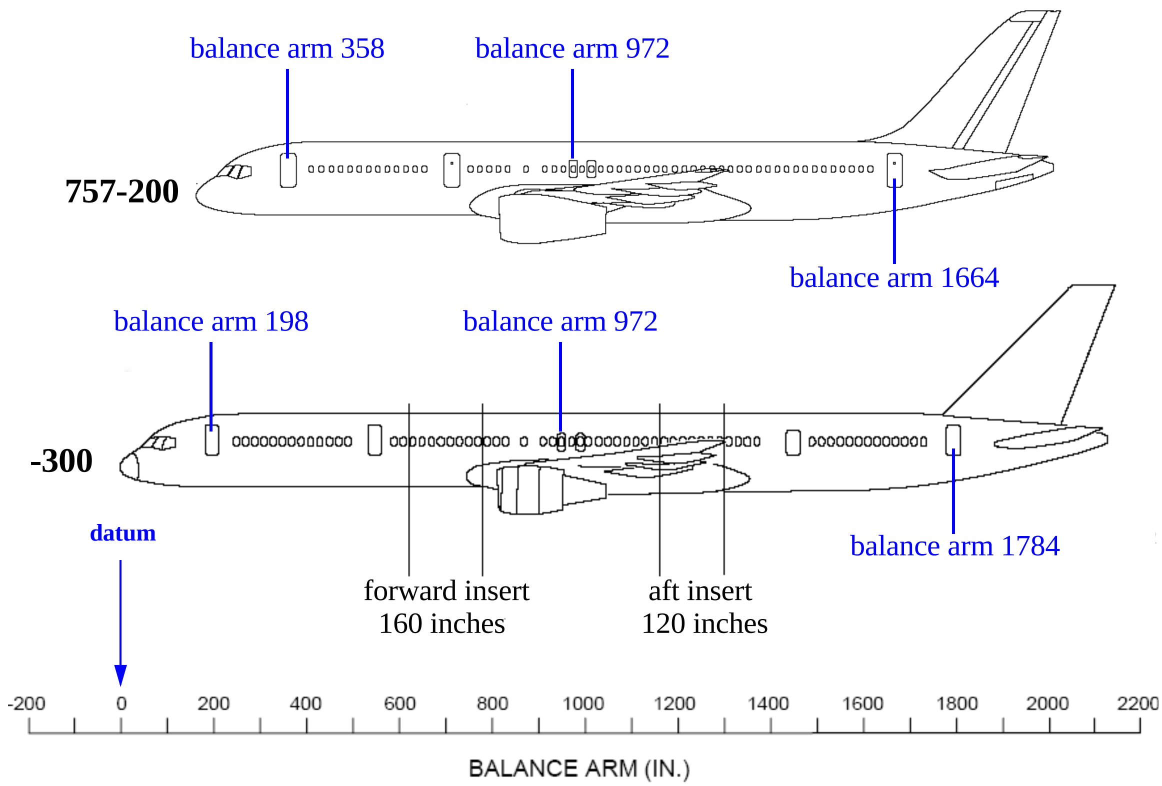


Figure 2-14

For example, the forward entry door is at balance arm 358 in the -200 airplane, but it’s at balance arm 198 in the -300. Why? Because when we designed the -300, we added a 160-inch insert of additional fuselage length forward of the wing, and a 120-inch insert aft of the wing. That moved the forward entry door 160 inches forward, relative to the center section of the body, but the datum didn’t move, so the balance arm of the forward entry door decreased by 160 inches.

For the midsection of the -200 and the -300, the balance arms are the same. For example, the forward overwing emergency exit hatch is at balance arm 972 on both variants.

The rear entry door of the -300 has moved 120 inches farther aft than the its location on the -200 because of the 120-inch insert aft of the wing. So, for example, the rear entry door of the -200 is at balance arm 1664, but for the -300 it's at balance arm 1784.

Data on the balance arms of any particular Boeing airplane model or variant of a model, are contained in the Weight and Balance Control and Loading Manual for that airplane. That document, together with the Supplement to it that is issued containing specific information for each airplane delivered, will assist the engineer in calculating an airplane's center of gravity location.

Now, while it's certainly possible to specify an airplane's center of gravity location in terms of its distance from the datum, the standard Boeing practice for specifying CG location is somewhat different from that.

specifying the location of the airplane's center of gravity

You will see that it is Boeing practice to designate the location of an airplane's center of gravity in terms of *percent MAC*.

In the chapter entitled "Wings", you'll find a detailed discussion of the *Mean Aerodynamic Chord*, or "MAC". We won't repeat the discussion here – please refer to that chapter if you want further discussion of the subject at this time.

For our purposes here, suffice it to say that the MAC is a chord line on the wing at a specific spanwise location. That means that the front and rear ends of the MAC line are at fixed known locations relative to the airplane datum.

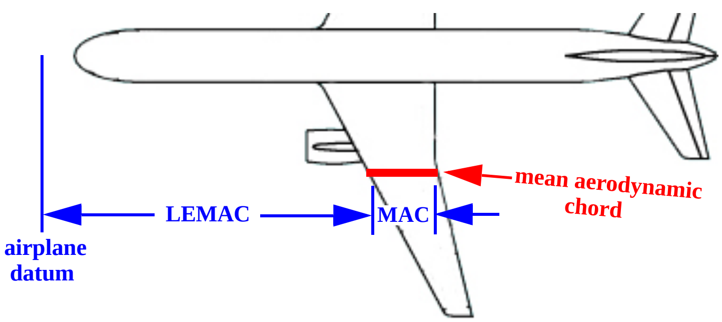


Figure 2-15

The very forward end of the MAC is at a known distance from the airplane datum. That distance is referred to as LEMAC – the Leading Edge of the MAC. The length of the mean aerodynamic chord is simply called MAC.

The forward end point of the MAC line is called 0% MAC, and the aft end point of the MAC is called 100% MAC. Obviously, then, 50% MAC would be halfway between the forward and aft end points of the chord line.

The values of LEMAC and MAC for all Boeing airplanes are provided in Appendix 1 to this book under the heading of "Table of Primary Geometry Parameters For Boeing Airplanes".

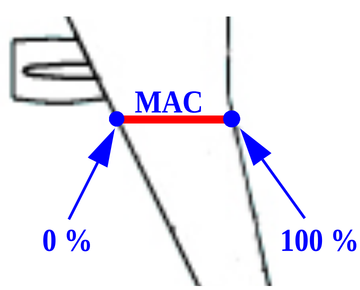


Figure 2-16

In general, the location of the MAC of any specific Boeing airplane model is the same regardless of the variant – that is, MAC and LEMAC are the same for the 757-300 is the same as for the 757-200. There are exceptions to this generality, however, occurring in the case of later variants for which the geometry of the wing has changed from the original, such as the 747-400 compared to the 747-100, -200 and -300, and the 737NG (the -600 through the -900) compared to the 737-100 through the -500. Check the geometry parameters table when necessary, to be sure of using the correct values.

Remember that a balance arm is a distance measured from the airplane datum. LEMAC is also measured from the same datum. We can thus make a simple mathematical equation defining the distance from the datum to any point along the MAC.

The balance arm of an airplane's CG, when expressed in %MAC, can be obtained from the following equation:

$$Balance\ Arm\ =\ LEMAC\ +\left(\frac{\%MAC}{100}\times MAC\right)$$

In this equation, the units of LEMAC and MAC must be the same, yielding a value of balance arm in the same units as LEMAC and MAC.

For example: for both variants of the 757, LEMAC is 991.9 inches and MAC is 199.7 inches. Thus 0% MAC would be 991.9 inches aft of the datum, 100% MAC would be 1191.6 inches aft of the datum, and 25% MAC would be 1041.8 inches aft of the datum.

Additional Discussion

discussion 1: derivation of equation 14

When considering the weight of an object on earth, the location of the object and its height above sea level are important because the distance of the object from the earth's center of mass, which is equal to the earth's radius plus the object's elevation above sea level, affects both the gravitational component of weight and the centrifugal component as well.

The centrifugal component for a stationary object on the earth's sea level surface, we know, is:

$$g_{centrifugal}\ =\ -(\omega_e^2 r_e \cos^2\varphi)$$

We will call the height above sea level z. Thus, for any height above sea level, to find the centrifugal component it would be necessary only to substitute $(r_e + z)$ for r_e.

If we subtract the sea level centrifugal component from $g_{\phi,SL}$ what remains is the sea level gravitational component:

$$g_{gravitational} = g_{\varphi,\,SL} + (\omega_e^2 r_e cos^2\varphi)$$

The sign of the second term in this equation is positive, since we are subtracting a negative value.

Since the gravitational component of *g* is a function of the inverse of the square of the distance from the earth's center, it will be affected thus:

$$g_{grav} \textit{ at altitude z above sea level } = g_{grav,\,SL} \times \left[\frac{r_e}{r_e + z}\right]^2$$

This equation holds true regardless of latitude.

Thus the gravitational component of g at any latitude and altitude will be:

$$[g_{\varphi,\,SL} + \omega_e^2 r_e cos^2\varphi]\left[\frac{r_e}{r_e + z}\right]^2$$

and when we add in the (negative) centrifugal component at altitude z we obtain the value of $g_{\varphi,z}$ which includes both pure gravitational effects and the centrifugal effect due to the earth's rotation:

$$g_{\varphi,\,z} = [g_{\varphi,\,SL} + \omega_e^2 r_e cos^2\varphi]\left[\frac{r_e}{r_e + z}\right]^2 - \omega^2(r_e + z)cos^2\varphi$$

---

discussion 2: derivation of equation 15

The component of the airplane's velocity along the lines of latitude will be $V_{EW} = V_G \sin\chi$, and the velocity along the lines of longitude will be $V_{NS} = V_G \cos\chi$. However, the earth's rotation adds another bit of velocity to the airplane's east-west component. That element of velocity is the same as we've seen before for the stationary airplane: it's equal to $\omega_e\,(r_e + z)\cos\phi$.

Thus, the airplane's velocity components <u>in inertial space</u> are actually:

$$V_{EW} = V_G \sin\chi + \omega_e(r_e + z)\cos\varphi$$

and:

$$V_{NS} = V_G \cos\chi$$

Since these components are at right angles to each other, finding the true inertial resultant velocity is simply a matter of remembering a little trigonometry: in any right triangle, the square of the hypotenuse of the triangle is equal to the sum of the squares of its sides, thus:

$$inertial\ V^2 = (V_G \cos\chi)^2 + [V_G \sin\chi + \omega_e (r_e + z) \cos\varphi]^2$$

$$= [V_G^2 \cos^2\chi] + [V_G^2 \sin^2\chi] + [\omega_e^2 (r_e + z)^2 \cos^2\varphi] + [2\omega_e (r_e + z) V_G \cos\varphi \ \sin\chi]$$

The first two terms of this equation taken together are simply $V^2{}_G$, therefore:

$$inertial\ V^2 = V_G^2 + [\omega_e^2 (r_e + z)^2 \cos^2\varphi] + [2\omega_e (r_e + z) V_G \cos\varphi \ \sin\chi]$$

Remember that the centrifugal acceleration is given by $V^2 \div r$, or in this case by $V^2 \div (r_e + z)$. Therefore the centrifugal component of gravity resulting from the airplane's velocity is:

$$\Delta g_{centrifugal} = \left[\frac{V_G^2}{(r_e + z)} + 2\omega_e V_G \cos\varphi \sin\chi + \omega^2 (r_e + z) \cos^2\varphi \right]$$

You should recognize the last term in that equation as the centrifugal acceleration due to the earth's rotation, *which is already included in* $g_{\phi,z}$ so it can be omitted, leaving the equation for the centrifugal correction as:

$$\Delta g_{centrifugal} = \frac{V_G^2}{(r_e + z)} + 2\omega_e V_G \cos\varphi \sin\chi$$

This correction is, of course, subtracted from the gravitational component since it's acting in the direction opposite to that of gravity.

Chapter 3: Physics of Air

Introduction

In subsequent chapters, we'll be dealing extensively with the atmosphere, aerodynamics, and a number of other topics relating to the behavior of this gas medium within which we all live.

It's necessary to establish some of the fundamental properties of air because we'll be using them as building blocks in the chapters to follow. We're going to introduce to you the concept of air as a fluid, and to some of the characteristics of that fluid.

While you may find this chapter's collection of discussions on the different aspects of the physics of air to be a bit diverse and unrelated, be patient because you'll be seeing them again in later chapters where they're essential for an understanding of airplane performance.

Pressure

We're all familiar, at least in simple terms, with the idea of *pressure*. Pressure can exist in many different media, such as water and air. In airplane performance work, we're constantly dealing with <u>air</u> pressure: air pressure affects lift and drag, and it also affects thrust.

So let's take a few minutes to agree on what <u>creates</u> pressure and on some of pressure's characteristics.

First of all, let's recall that "pressure" is defined as <u>force per unit of area</u>.

Any substance in nature, whether a gas or a liquid or a solid, is composed of countless trillions of molecules of the elements of which the substance is composed. Air, for examples, contains molecules of oxygen, nitrogen and a number of other elements in smaller proportions. From this point onward, let's speak of <u>air</u> pressure, realizing that what we say here about air applies to other substances as well.

All of these air molecules are in constant random motion, moving in all different directions. The amount of molecular motion depends directly on the absolute temperature of the air. When air touches the surface of an object, trillions of the air molecules bounce off each square inch of the surface every second. Each bounce exerts a tiny force on the surface for just an instant. Because there are so many of them, they add together and feel like a steady push that is spread smoothly over all of the surface. This distributed force, measured over a given unit of area, is the air pressure.

Pressure exists everywhere within the volume of the air, not just on the surfaces of objects that the air touches. Think of the air as being divided into small "parcels" – tiny units of volume – with imaginary boundaries. Each parcel pushes out against its neighbors, and the neighbors push back. This interaction between neighboring parcels is always a <u>push</u>, never a pull, because there is no way that bouncing molecules can produce a pull. The pushes exchanged between adjacent parcels are always equal and opposite, in keeping with Newton's third law that states that for every action there is an equal and opposite reaction.

Based on the discussion above, we can say the following:

- Air pressure is omnidirectional. It does not, <u>can</u> not, act in only one direction – it acts in all directions equally.
- There's no such thing as "suction", or <u>negative</u> pressure. All pressures are positive. What we call suction is simply a condition of the <u>difference</u> between a greater positive pressure and a lesser positive pressures. When you suck fluid up from a container through a tube, you're not creating a negative pressure in the tube, you're just <u>reducing</u> the pressure in the tube so that it's less than the pressure in the container; that pressure difference, multiplied by the cross-sectional area of the tube, creates a force that causes the fluid to flow up the tube.
- Pressure depends directly on absolute temperature.

Later in this chapter we're going to be discussing the way that the air pressure in a flow will vary depending on the location within the flow. You'll see, for example, that the air pressure is locally increased where it strikes a body at some velocity. You'll see that where the velocity of a flow of air is locally increased, its pressure is decreased. But there is only one kind of pressure, and that pressure can vary within a flow.

Air At Rest

We're going to discuss first some physical properties of air which is at rest – that is, static, not in motion. Later in this chapter we'll be talking about some characteristics of air in motion.

the hydrostatic equation

In the following chapter entitled "The Atmosphere", we develop the mathematical model predicting the structure of the atmosphere. Fundamental to that model is the "hydrostatic equation", which considers pressure relationships in a fluid that is not moving – that is, a fluid which is static.

Air is composed of gases, and these gases have weight, however small. The air pressure at any point in the atmosphere is a function of the weight of the air above that point. The hydrostatic equation expresses the relationship between weight, pressure and height in this static situation.

Picture a column of air, extending from the earth's surface to the upper limit of the atmosphere, as shown in Figure 3-1 to the right. Let's assume this column of air has a cross-sectional area of *A*.

We take an extremely thin slice through the column; the slice has a height of *dh*. At the top of the slice, let's assume the pressure is equal to some unknown pressure *p*.

The weight of the slice is equal to its mass density ρ multiplied by the acceleration of gravity *g* and by the volume of the slice, which is $A \times dh$.

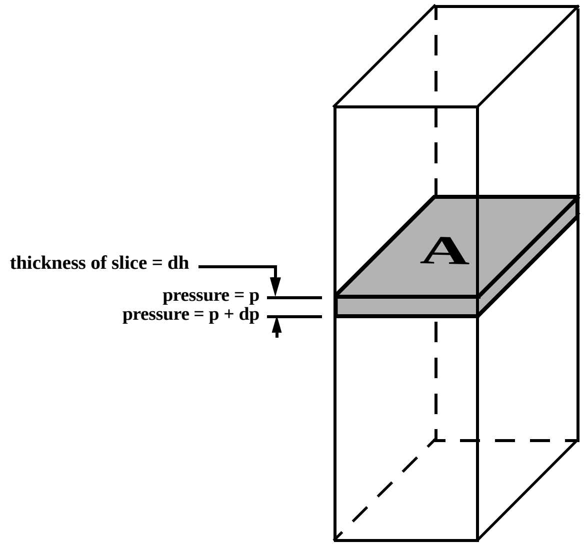


Figure 3-1

The pressure at the <u>bottom</u> of the slice which we will call $p + dp$ (dp being some as yet unknown change in pressure as we move downward) is then equal to the pressure at the top of the slice, PLUS the weight of the slice itself. If this slice of air is not moving (i.e. static) we can equate the vertical forces on the slide as follows:

$$(p + dp) \times A = (p \times A) + (\rho \times A \times g \times dh)$$

which leads to:

$$dp = \rho\, g\, dh \qquad \text{(eq. 1)}$$

Note that in this case, *dh* is positive in the downward direction. If we were taking *dh* as an upward change, then the equation would be:

$$dp = -\rho\, g\, dh \qquad \text{(eq. 2)}$$

We'll be using equation 2 in the next chapter, entitled "The Atmosphere" to develop the equations used to compute atmospheric pressure as a function of height. Keep in mind, though, that equation 2 is valid for *all* static fluids, not just air.

equation of state for air

The mass density of solids and liquids is essentially constant, but the mass density of gases depends on the pressure and temperature.

For air at the temperatures and pressures typical of the atmosphere, the relationship between temperature, pressure, and mass density is accurately described by a simple equation of state called the "ideal gas law":

$$p = \rho RT \qquad \text{(eq. 3)}$$

where p is the pressure in pounds per square foot
ρ is the mass density in slugs per cubic foot
R is a constant called the “specific gas constant”, foot-pounds per slug-°R
T is the temperature of the gas in degrees Rankine

Mass density can be expressed as $\rho = m \div V$ so equation 3 could also be written as:

$$pV = mRT \qquad \text{(eq. 4)}$$

where V is the volume of gas, in cubic feet
m is the mass of the gas in slugs

Equation 3 is very important in the creation of the standard atmosphere model and when re-written into the form of equation 4 it helps in providing us with some insight into some important thermodynamic concepts:

- *Isothermal process:* this is a process in which temperature is a constant. Equation 4 shows that for constant temperature, pressure times volume is fixed. If volume doubles, the pressure will decrease by half. Also, the density will decrease by half.

- *Isochoric process*: this is a process in which volume, and therefore density, is constant. Pressure and temperature are directly dependent on each other in this process. If temperature doubles, the pressure will double.

- *Isobaric process:* this is a process in which pressure is constant. Volume and temperature are directly dependent on each other. If the temperature doubles, the volume must double and the density will be halved.

Two of these three processes are of particular interest to us: the isochoric process and the isobaric process. They will aid us in understanding the specific heats of air and the value of R.

specific heats of air

The *specific heat* of a material is the amount of heat required to raise the temperature of a unit mass of that material by one degree.

For a gas, there are two different ways in which the heating operation may be performed that are of particular interest: at constant volume (isochoric) or at constant pressure (isobaric).

Here's an illustration of the concept of the *isochoric specific heat* of air, which we might also call the specific heat at constant volume.

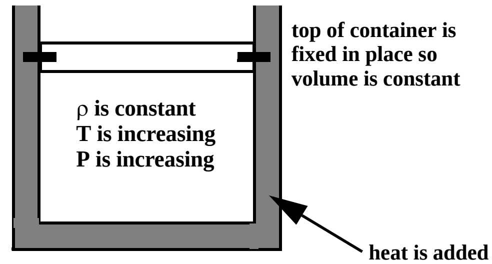


Figure 3-2

A volume of gas is contained in a vessel having rigid walls and a top that is fixed in place, holding the volume of gas constant. Heat is added. The amount of heat required to raise a unit mass of air by one degree at this constant volume is called the isochoric specific heat of air, designated as C_V.

⇒ For air, the value of C_V in English system units is 4290 foot-pounds per slug-°R.

This illustration shows the concept of the *isobaric specific heat* of air, the specific heat at constant pressure. A volume of gas is contained in a vessel, which has rigid walls as before, but also has a frictionless lid that is free to slide upward or downward with no loss of energy. A weight is placed on the lid, so that the pressure on the gas inside the vessel is constant.

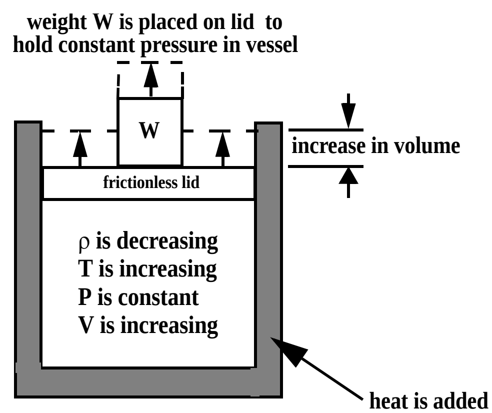


Figure 3-3

Heat is added. When this is done, the lid will move upward as the volume of the gas increases to maintain constant pressure. The amount of heat required to raise a unit mass of air by one degree at this constant pressure is called the isobaric specific heat of air, designated as C_P.

⇒ For air, the value of C_P in English system units is 6006 foot-pounds per slug-°R.

ratio of specific heats, gamma

One of the two reasons for talking so much about the specific heats of air is that it allows us to introduce you to a new aerodynamic parameter called γ (gamma), the ratio of the specific heats.

γ is defined by:

$$\gamma = \frac{Cp}{C_V} \qquad \text{(eq. 5)}$$

γ is important because it appears frequently in high-speed aerodynamics. We'll be pointing that out as it occurs.

⇒ The value of γ for air is 1.4. Since it is a ratio of two specific heats, it is dimensionless.

gas constant, R

The second reason for talking about the specific heats of air is that it allows us to mention once again the specific gas constant for air, called *R*.

The gas constant R is derived from C_P and C_V as follows:

$$R = C_P - C_V \qquad \text{(eq. 6)}$$

We told you earlier that the value of R for air is 1716.5619 foot-pounds per slug-°R in English units. In metric units, the value is 287.0529 Newton-meters per kilogram mass-degree Kelvin.

From the above, you can get a rough physical understanding of the gas constant *R*: it's the physical constant which links pressure, temperature and density to each other.
Remember that $p = \rho RT$.

In a process at constant temperature, it is *R* that governs the relationship between pressure and density. In a process at constant density, it is *R* that governs the relationship between temperature and pressure. In a process at constant pressure, it is *R* that governs the relationship between density and temperature.

Thus, <u>*R* is the central controlling constant for relationships involving air</u>. This constant appears in many of the equations that you'll be seeing in this book.

Air In Motion

the "no-slip" condition

The *no-slip condition* is a result of the way molecules of a fluid interact with solid surfaces. Air molecules, for example, do not typically adhere to a solid surface, but they bounce off the microscopic irregularities of the surface in such a way that their average velocity parallel to the surface must go to practically zero right at the surface.

No matter how smooth or slick a surface feels to the touch, the atoms and molecules of the surface are always as large or larger than the molecules of the gas, and they always effectively stop the parallel motion of the gas molecules.

If we consider air to be a continuous material – a *fluid* – its velocity must go to zero at the surface. That is, it can't "slip" along the surface. Fluids like water and oil behave the same way.

stresses

Very soon, we're going to talking about some of the properties of air, including the property called *viscosity*. Before we do that, let's take a minute to speak in more general terms about some of the types of *stress* that exist, some of which are applicable to air.

In general terms, "stress" can be defined as a measure of the amount of force applied to an object per unit of area.

A very simple example: let's suppose that we have a length of metal bar, having a cross-sectional area of one square inch. We apply an equal pulling force of 100 pounds at each end of the rod. This pulling force creates a *tension stress*, a stress due to tension, of 100 pounds over an area of one square inch, or 100 pounds per square inch.

Suppose instead that we are applying an equal pushing force of 100 pounds at each end of the rod. This pushing force creates a *compression stress* of 100 pounds per square inch.

And there's another kind of stress, called *shear stress*. Look at Figure 3-4 to the right. We see a block of some material, fixed at the base, to which we are applying a force at its top, parallel to the surface. If the body has some elasticity, it will *deform* as shown by the broken lines.

The stress felt within the block of material is shear stress.

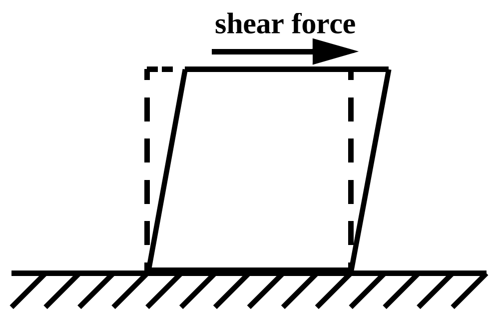


Figure 3-4

viscosity

Shear stresses can exist in fluids as well as in solids. When a fluid is at rest, there is no shearing deformation. However, if a shear force is applied to a fluid, it will flow and deform. In fluids, shearing deformation is resisted by the property of a fluid known as *viscosity*. The greater the viscosity of a fluid, the greater its resistance to shearing deformation.

When a shear force is applied to a fluid it induces shearing stresses, and these stresses are proportional to the rate of deformation. The constant of proportionality between shearing rate and shear stress is called the *coefficient of viscosity.*

viscosity's effect on motion through a fluid

One result of this property of fluids called "viscosity" is to retard – that is, to resist – the flow of a fluid over the surface of a solid, or to retard the motion of a solid through a fluid. We can visualize that easily, based on our experience with everyday happenings.

Visualize, for example, dropping a metal ball into a deep container of water. The ball will fall quite quickly through the water, since the water exhibits relatively little viscosity.

Now, instead of water, imagine that the container is filled with heavy thick oil. If you drop the same ball into that fluid, at first it will accelerate downward. As it accelerates, the viscous forces resisting its motion through the oil will increase rapidly, and the ball will soon reach its "terminal velocity". At that velocity, the viscous forces resisting its fall are just equal to the excess gravitational forces on the ball (the gravitational force in excess of the ball's buoyancy), so there is no net downward force on the ball and thus no further acceleration.

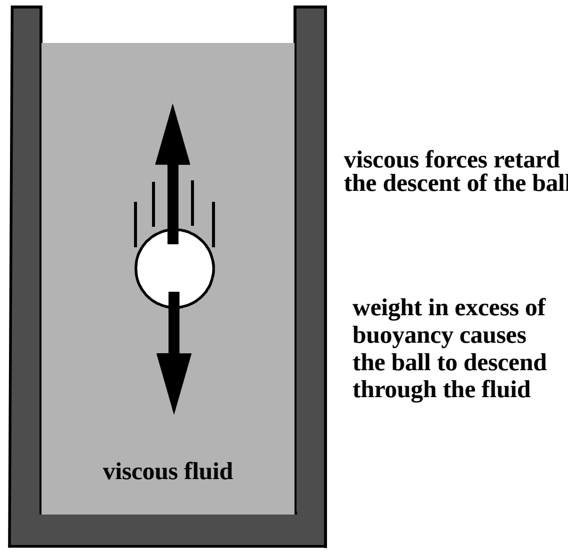


Figure 3-5

We'll return to the subject of viscosity later when we begin the discussion of fluid flow over a flat plate. It's essential to an understanding of the characteristics of fluid flow over a wing.

boundary layers

We're going to return to this subject in the chapter entitled "Flow Near A Surface", but it won't hurt to give you a preview of that discussion here.

Earlier in this chapter, we discussed the "no-slip condition", which we said is a physical reality that causes the velocity of a fluid flow to be zero right at the surface of an object over which it's passing.

In the absence of the no-slip condition, fluid flow velocities near a surface would look like the illustration shown to the right. The velocity of flow would be the same at the surface of the object as it is at any distance away from the surface.

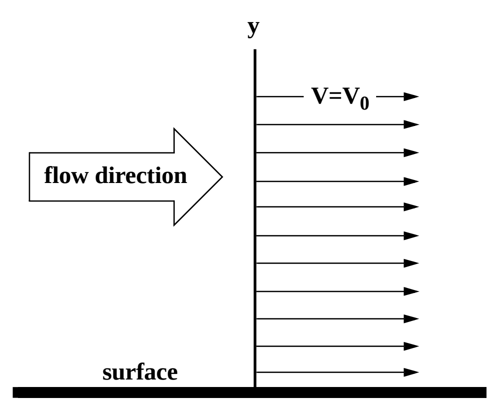


Figure 3-6

In the real world, however, the no-slip condition is a fact of life. Because of that, the velocity of fluid flow near the surface of an object looks like Figure 3-7 to the right.

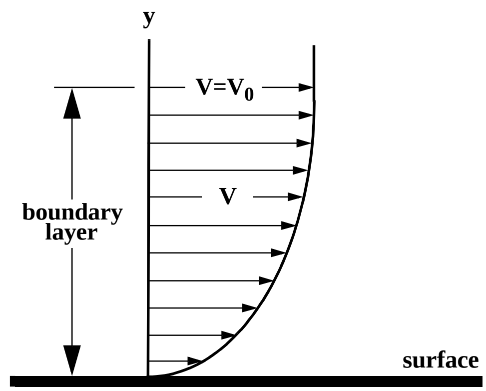


Figure 3-7

At the surface, the flow velocity is zero. Moving away from the surface, we see the effect of viscosity. Any fluid having viscosity resists shear stresses; each imaginary "layer" of the fluid (however thin) is somewhat held back by the layer immediately below it. The result is that as we move away from the surface, the flow velocity gradually increases until it's equal finally to the "free stream" velocity V_0 far away from the surface.

The layer of air between the surface and the height at which the local velocity is finally at the free stream velocity is called the *boundary layer*. The boundary layer, then, is a layer of lower-energy flow close to the surface of an object over which a fluid is flowing. The thickness of the boundary layer depends on the degree of viscosity of the fluid.

air as a fluid

Air can be treated as a fluid because it behaves like a continuous material, in spite of the fact that it is made up of individual molecules and it can't support shearing forces when it is not moving. If a shearing force is applied to it, it too will flow and deform.

streamlines

Begin by imagining a stationary wing, with a steady stream of air flowing over it. All of the conditions are stable and unchanging. That stream of air consists of countless elemental particles of fluid moving in the same direction. The path traced out by any one particle in that fluid stream, in this steady flow, is called a *streamlin*e.

The illustration to the right demonstrates what we mean. The lines traced out are streamlines; here we see the streamlines that would be characteristic of an airflow over an airfoil.

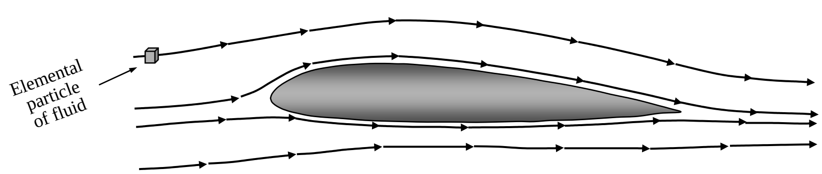


Figure 3-8

In steady flow, there is no flow across a streamline, because by definition the streamline depicts the path taken by the particles.

stream tube

A *stream tube* is just a concept: it's a bundle of streamlines which together form the surface of a closed curve or tube. There can be no flow through the walls of a stream tube because the walls of the stream tube are streamlines. You can think of a stream tube as representing the way the particles of the air or fluid are flowing at any given instant of time.

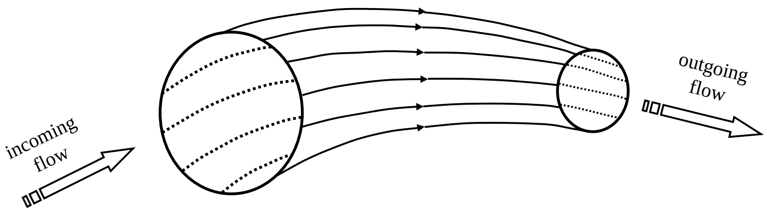


Figure 3-9

Let's discuss some fundamental properties of a fluid as it flows along a streamline or through a stream tube.

incompressible versus compressible flow

Incompressible flow is flow in which the density of the fluid remains constant from point to point in the flow, regardless of any changes in pressure. Liquid flows can be accurately treated as incompressible flows. Flows of air, on the other hand, can be approximated as incompressible only at lower speeds – roughly 200 knots or less.

Compressible flow is flow in which the density of the fluid can change from point to point in the flow. All fluids in real life are compressible to some degree, that is their density will vary as the pressure varies. Air is highly compressible, particularly at high speeds.

conservation of mass

.Suppose we have a mass of fluid flowing through a pipe or stream tube, as shown in the illustration.

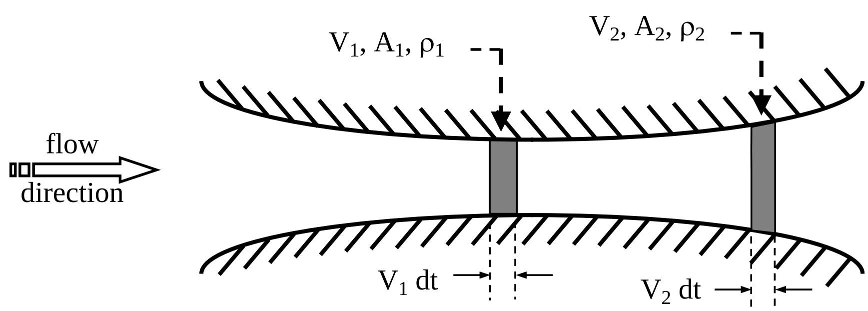


Figure 3-10

How much mass has passed by station 1 in a small amount of time *dt*? The area at station 1 is A_1, the density is ρ_1 and the velocity is V_1. In time *dt* the flow has traveled a distance of $V_1 dt$. The amount of mass that has passed in this time is the amount in the gray section of the illustration. This amount is equal to the density multiplied by the volume. Therefore, the mass that has passed in time *dt* is $\rho_1 A_1 V_1 dt$. From this, we can define the mass flow at station 1 as:

$$\frac{dm}{dt} = \rho_1 A_1 V_1$$

It's logical to say that the amount of mass flowing past any given cross-section of a stream tube per unit of time must be the same as that flowing past any other cross-section, otherwise mass would have to enter or leave the stream tube between the two selected cross sections.

The mass flow <u>leaving</u> the tube per unit of time must be the same as the mass flow <u>entering</u> the tube per unit of time, and clearly it must the same at every point in the pipe or tube.

The principle of *conservation of mass*, then, states that mass flow in a pipe or stream tube per unit of time, dm/dt (sometimes designated as $\dot{m}$ – "m-dot"), can be found from:

$$\dot{m} = \rho \times A \times V \qquad \text{(eq. 7)}$$

where ρ is the mass density
A is the cross-sectional area of the pipe or tube
V is the velocity of the flow.

the continuity equation

Because of the principle of conservation of mass, then, we understand that $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ and, more generally, we can say that for steady fluid flow through a pipe or stream tube, the mass flow per unit time is constant:

$$\dot{m} = \frac{dm}{dt} = \rho A V = constant \qquad \text{(eq. 8)}$$

This is sometimes referred to as the *continuity equation*. It relates density, velocity and cross-sectional area at one section of a stream tube to any other section of the stream tube, based on the principle of conservation of mass.

The continuity equation can be written in another form, using a little differential calculus:

$$\mathrm{d}\rho \mathrm{VA} + \mathrm{dV}\rho \mathrm{A} + \mathrm{dA}\rho \mathrm{V} = 0 \qquad \text{(eq. 9)}$$

Dividing equation 9 by equation 8 will yield:

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \qquad \text{(eq. 10)}$$

This equation is valid for compressible flow, which is flow having varying density. If we assume for the moment that we're dealing with an incompressible flow, and thus that $d\rho$ is zero, equation 10 simplifies to:

$$\frac{dV}{V} = -\frac{dA}{A} \qquad \text{(eq. 11)}$$

Equation 11 is significant because for incompressible flow (or relatively low-speed air flows) it shows that velocity and cross-sectional area are directly related: a convergence of the stream tube (a lessening of the cross-sectional area) will result in an increase in the velocity, and a divergence of the stream tube (an increase of the cross-sectional area) will result in a decrease of velocity. You'll see later that for compressible flow, that's not necessarily true.

the momentum equation

Let's consider an infinitesimally small fluid element moving along a streamline with a velocity *V.* Now look at the forces acting on this fluid element in the x-direction (the direction of motion), neglecting friction and gravity

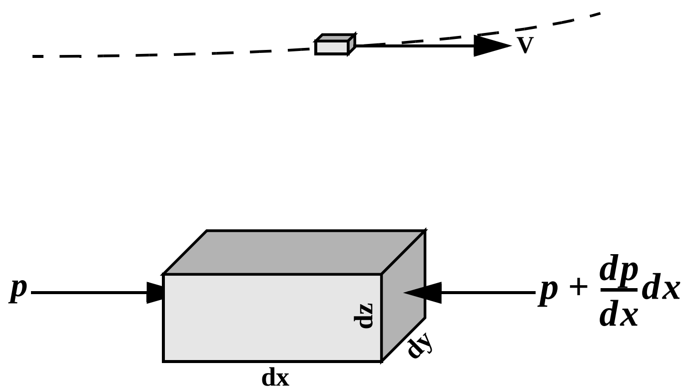


Figure 3-11

The force on the left face of the element will be equal to the pressure multiplied by the cross-sectional area of the face, $dy \times dz$. The force on the right face of the element will be equal to p plus any increase of p along the streamline, $\dfrac{dp}{dx}$, also multiplied by $dy \times dz$, thus the net force on the element will be:

$$net\ force\ =\ (p \times dy \times dz) - \left[\left(p + \frac{dp}{dx} \times dx \right) \times dy \times dz \right]$$

The mass of the element will be its mass density multiplied by its volume, or $\rho \times dx \times dy \times dz$.

The acceleration of the element along the streamline will be: $\dfrac{dV}{dt} = \dfrac{dV}{dx}\dfrac{dx}{dt} = V \times \dfrac{dV}{dx}$.

Remembering that $F = ma$:

$$(p \times dy \times dz) - \left[\left(p + \frac{dp}{dx} \times dx \right) \times dy \times dz \right] = \rho \times (dx \times dy \times dz) \times V \times \frac{dV}{dx}$$

which simplifies to

$$dp = -\rho\ V\ dV \qquad \text{(eq. 12)}$$

This is known as the *momentum equation*, sometimes also called Euler's equation. It's useful to us because it relates a change in pressure to a change in velocity. In compressible flow, ρ is a variable. For incompressible flow, ρ is considered to be a constant.

Bernoulli's equation for incompressible flow

Using the momentum equation as a starting point, we can develop a new equation which is fundamental to aerodynamics: Bernoulli's equation[1]. We'll do that first for incompressible flow, then again for compressible flow.

We must emphasize that the following discussion, leading to the flow equations first expressed by Bernoulli, is for a state of steady flow. The equations are not valid in a situation of unsteady flow in which the velocities and pressures of the flow at any point are changing over time.

First, assuming incompressible flow, we'll accept ρ as a constant.We'll integrate equation 12 between two points along a streamline: point 1 and point 2:

$$\int_{p_1}^{p_2} dp = -\rho\int_{V_1}^{V_2} V dV$$ which, when solved, gives us:

$$p_2 - p_1 = -\rho\left(\frac{V_2^{\ 2}}{2} - \frac{V_1^{\ 2}}{2}\right)$$

Re-arranging that equation yields one form of Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho V_1^{\ 2} = p_2 + \frac{1}{2}\rho V_2^{\ 2} \qquad \text{(eq. 13)}$$

In equation 13, the terms p_1 and p_2 represent the local pressure of the air as it varies in the flow depending on location.

Stating Bernoulli's equation in another form:

$$p_{static} + \frac{1}{2}\rho V^2{}_{local} = constant \qquad \text{(eq. 14)}$$

That is, at every point within a steady flow, the local pressure plus the term $\frac{1}{2}\rho V_{local}^2$ is the same.

At this point, we need to emphasize that Bernoulli's equation does not consider the effects of viscosity. It therefore can't be used to predict flow velocities and pressures at any point lying within a boundary layer.

---

1. Daniel Bernoulli (1700-1782) was a Dutch-born physician and mathematician. He and a colleague, Leonard Euler, were interested in studying the dynamics of fluid flow. Interestingly, his work began with a study of the speed at which blood flows in the human body and its pressure. Out of this study arose the first known method of measuring blood pressure.

static, dynamic and total pressure

In equations 13 and 14, you're exposed for the first time to the idea that there seems to be more than one kind of pressure. Let's discuss this, because it's fundamental to an understanding of aerodynamic flows.

We're going to be talking here about *static pressure, dynamic pressure,* and *total pressure*. Don't let this confuse you. <u>There is only one real physical air pressure, which is "static" pressure</u>. In moving flows, as the velocity of the flow varies from point to point the change in velocity causes a change in the static pressure, following the relationship between velocity and pressure expressed in Bernoulli's equation.

The mathematical term $\frac{1}{2}\rho V_{local}^2$ is seen in Bernoulli's equation. This term is so frequently encountered in aerodynamics that it has its own name and its own symbol: it's called "dynamic pressure" and it has the symbol q, the lower case of the letter Q.

But let's not think of q as being a real physical pressure – it's not. Instead, let's think of it as a mathematical prediction of the way that local pressure varies as a function of the local flow velocity. <u>By seeing how *q* changes in a flow we will know how the pressure will change in the flow</u>.

Let's try an example here: consider a wind tunnel, with a steady flow of air at a constant speed. Across the middle of the wind tunnel, from wall to wall, we have placed a metal bar in the middle of the flow. Far upstream of the bar, we measure the velocity and pressure of the flow: they are V_0 and p_0. These are usually called the *free stream* velocity and pressure.

Where there is a shape placed into a steady flow, the presence of the shape causes the flow to be altered in order to pass around it. The flow will accelerate as it passes, and then it will decelerate so that at a point far downstream of the shape, the velocity of the flow will have returned to V_0.

Where the velocity of the flow <u>accelerates</u> as it passes the metal bar, the term $\frac{1}{2}\rho V_{local}^2$ <u>increases</u>. Equation 14 then tells us that the local pressure will <u>decrease</u>. One unit of <u>increase</u> of the local dynamic pressure above the free stream dynamic pressure means that the local static pressure will be <u>decreased</u> by one unit below the free stream static pressure p_0.

After passing the metal bar, as the velocity of the flow decreases back toward V_0, the local pressure increases until finally, downstream of the bar, it has returned to p_0.

So the dynamic "pressure" is a predictor of the change of static pressure as a function of local flow velocity. What, then is "total pressure"?

Remember that equation 14 says that $p_0 + \frac{1}{2}\rho V_0^2 = p_{local} + \frac{1}{2}\rho V_{local}^2$. Suppose that at some point in the flow, the local flow is totally stopped – that is, its velocity has gone to zero. Then, according to equation 14, the local static pressure will be equal to $p_0 + \frac{1}{2}\rho V_0^2$. That is what is called "total" pressure. Thus, at any point at which the local airflow velocity is zero, the local pressure will be the total pressure, $p_0 + q_0$. So total pressure is simply a prediction of the air pressure at any point at which the local velocity is zero, outside of the viscous boundary layer, in smooth steady-state flow.

Remember that equation 14 is only valid for steady incompressible flows. Air is never really incompressible but for practical purposes it may be considered incompressible below airspeeds of approximately 200 knots. At speeds greater than that, compression of the air in the flow will cause density to change by more than five percent and so compression should no longer be considered negligible.

Bernoulli's equation in compressible flow

In airflow at higher speeds, Bernoulli's equation takes a slightly different form to account properly for density change of the air due to compression. Again, please understand that we are discussing here a condition of steady unchanging flow.

Here too, we must emphasize that the following discussion deals only with flows outside of a boundary layer, as viscosity effects are not considered.

To analyze high-speed flows correctly, it helps to consider the concept of conservation of energy in the flow through a stream tube. That is, understand that energy is neither created nor destroyed, but may be changed in form; still, total energy is constant.

Conservation of energy states that the total energy of a flowing fluid is made up of both thermal energy and kinetic energy. For two locations along a stream tube this can be stated as:

$$C_P T_1 + \frac{1}{2}V_1^2 = C_P T_2 + \frac{1}{2}V_2^2 = constant = C_P T_{total}$$

where C_P is the specific heat of the air at constant pressure
T is the temperature of the air in absolute units
V is the true velocity of the air

The first of these terms, $C_P T_1$ is the heat energy per unit of mass in the flow; the second of the terms, $\frac{1}{2}V^2$, is the kinetic energy per unit mass in the flow.

At this point, we need to recall equations 3, 5 and 6:

$$p = \rho RT \qquad \gamma = \frac{Cp}{C_V} \quad \text{and} \quad R = C_P - C_V$$

Substituting equations 6 into equation 3 for R, dividing top and bottom by C_V, using equation 5, and rearranging, yields:

$$C_P\ T = \frac{p}{\rho} \times \frac{\gamma}{(\gamma - 1)}$$

which, finally, leads to the compressible Bernoulli equation:

$$\frac{\gamma}{(\gamma - 1)} \frac{p}{\rho} + \frac{1}{2} V^2 = \frac{\gamma}{(\gamma - 1)} \frac{p_t}{\rho_t} = \textit{constant along a streamline} \qquad \text{(eq. 15)}$$

You may well be wondering what all this marvelous math will do for you. The short answer is that we rarely use equation 15 in our work as performance engineers – but it forms the basis for something which IS of great importance to us. You'll see that in the chapter entitled "Measurement of Airspeed".

Chapter 4: The Atmosphere

Introduction

The performance of an airplane and its engines depends on the generation of forces by the interaction between the airplane/engine combination and the air mass through which it flies. It's thus necessary to examine the properties of the earth's *atmosphere*: that thin layer of gas surrounding the planet through which we fly.

The earth's atmosphere is made up of a mixture of gases: nitrogen is approximately 78 percent of the atmosphere, oxygen makes up about 21 percent, argon is some 0.9 percent, and the remainder is a mixture of other gases. Water vapor is always present but in varying amounts depending on temperature and relative humidity; it's usually less than one percent at the earth's surface.

The energy of the sun is responsible for heating the earth's atmosphere, but most of this heating is done indirectly – that is, most of the energy goes into heating the earth's surface, which in turn heats the air.

Warm air near the earth's surface rises, expands, decreases in pressure, and cools as altitude increases. Equilibrium is reached at some altitude, above which no more reduction in temperature occurs. This altitude is called the *tropopause*; the region of the atmosphere below the tropopause is called the *trophosphere*, and the region above the tropopause is called the *stratosphere*. Temperature is essentially constant in the stratosphere, up to approximately 65,600 feet, after which it actually begins to increase. This, however, is above the altitude of present-day commercial flights.

The height of the actual tropopause above the earth's surface is variable, rather than a constant. It ranges in height from an average of 4 miles at the poles to 11 miles at the equator. The height of the tropopause varies also with weather changes and exhibits seasonal effects.

The atmosphere is constantly changing. Pressure, temperature and density of the air are affected by a number of factors including pressure patterns associated with frontal systems, surface heating, seasonal effects and so on. Because of this variation from place to place and from day to day, it's virtually impossible to define the performance of an airplane in constant terms. The airplane performance will be different in lower latitudes compared to higher latitudes, on hot days compared to cold days, and so on.

The International Standard Atmosphere

Because of the variability of the atmosphere, it is convenient and helpful to define some standard basis for defining airplane performance. Only by knowing the performance of different airplanes specified <u>in the same atmospheric conditions</u> is it possible to make valid comparisons between them. Thus we want to define something we might call a "standard day", recognizing of course that the performance of any airplane in today's actual current weather conditions will most likely be different from the standard day performance.

The "standard day" definition in common use is that set forth by ICAO, the International Civil Aviation Organization, in the 1960s. It is usually referred to as the International Standard Atmosphere, or ISA. It is a model of the atmosphere determined by averaging data gathered over a long period of time; the data was collected mostly in the mid-latitudes of the northern hemisphere, and therefore the ISA definition is most representative of conditions in these regions. Even though the expected deviations from ISA may be much larger in polar or equatorial regions, the same standard is still used as a reference.

Hereafter in this document, any reference to ISA, ISA conditions, ISA deviation, and other similar terms will refer to the ICAO International Standard Atmosphere. You will also see *ISA* used as a subscript to denote standard day conditions.

This chapter will develop the equations which allow the user to compute atmospheric temperature, pressure and density for any given height.

<u>It must be understood that ISA is just one of any number of possible "standard day" definitions. Since it is the one in widest use in the aviation industry, it's the only definition used by Boeing in performance work and it's used exclusively throughout this document</u>.

tapeline height

When first approaching the subject of the International Standard Atmosphere, it would be reasonable to assume that the atmospheric parameters would be presented as a function of actual height above the earth, sometimes referred to as *tapeline altitude*, *tapeline height* or *tapeline elevation*.

Tapeline height is a physical distance above the sea-level surface of the earth. If it were possible to use a tape measure between the airplane and sea level, it would show this height, hence the name. Tapeline height is also sometimes referred to as *geometric height*.

As you'll see in this chapter, however, it's more practical to use a different form of altitude, referred to as *geopotential altitude*. That concept will be discussed in detail. Suffice it to say for now that for practical purposes, tapeline altitude and geopotential altitude are <u>almost</u> the same. The distinction will be made clear later.

Height of the Tropopause in the Standard Atmosphere

In the discussion above, we defined the tropopause as the height below which lies the troposphere and above which lies the stratosphere.

In the International Standard Atmosphere, the tropopause is defined as occurring at exactly 11,000 meters or 36,089.24 feet.

ISA Temperature, Pressure and Air Density at Sea Level

The International Standard Atmosphere defines the sea level standard day temperature to be 59 degrees F (518.67 °R) or 15 degrees C (288.15 °K). This is given the symbol T_0.

The International Standard Atmosphere defines the sea level standard day air pressure to be 2116.21662 pounds per square foot, or 14.6959 pounds per square inch. This is given the symbol p_0.

The International Standard Atmosphere defines the sea level standard day air density to be 0.00237689 slugs mass per cubic foot. This is given the symbol ρ_0. Since the standard atmosphere is based on the assumption of air as an ideal gas, it follows that $p_0 = \rho_0 R T_0$.

ISA Temperature, Pressure and Density Below the Tropopause

It will be seen that the characteristics of the atmosphere are markedly different above the tropopause compared to those below it. The reason for this is the difference in the variation of temperature with altitude in the stratosphere compared to that in the troposphere. The following paragraphs will explore the characteristics of the troposphere; later we'll discuss the characteristics of the stratosphere, below approximately 65,600 feet.

temperature variation with altitude

Empirical data indicates that in the troposphere the typical variation of temperature with altitude is essentially linear at approximately 3.6 degrees Fahrenheit (two degrees Celsius) decrease per 1000 feet of altitude increase.

The International Standard Atmosphere definition is 0.00356616 degrees F of temperature decrease per foot of altitude increase, or 0.0019812 degrees C per foot.This is called the *lapse rate* of temperature with altitude, and it's denoted by the Greek letter λ.[1]

Please note and remember that the temperature lapse rate in both English and metric units is defined as a function of the altitude <u>in feet</u>. This is simply because in airplane performance work, altitude information is most often specified in feet.

Below the tropopause, then, ISA temperatures can be computed as:

$$ISA\ OAT = 518.67 - 0.00356616 \times h \quad \text{(degrees Rankine)} \qquad \text{(eq. 1)}$$

$$ISA\ OAT = 288.15 - 0.0019812 \times h \quad \text{(degrees Kelvin)} \qquad \text{(eq. 2)}$$

$$ISA\ OAT = 59 - 0.00356616 \times h \quad \text{(degrees Fahrenheit)} \qquad \text{(eq. 3)}$$

1. You'll see in other parts of this book that the term "lapse rate" is sometimes applied to other relationships, such as the lapse rate of thrust with speed.

$$ISA\ OAT = 15 - 0.0019812 \times h \qquad \text{(degrees Celsius)} \qquad \text{(eq. 4)}$$

where h is the true height above sea level in feet.

pressure variation with altitude

In an earlier chapter entitled “Physics of Air” we showed the equation of state and the hydrostatic equation. We can combine them to allow us to develop an equation with which we can model the variation of pressure with altitude.

This equation expresses the standard day pressure p in terms of the sea level standard day pressure, the standard day temperature T and the sea level standard day temperature T_0. Both T and T_0 must both be in absolute units, degrees Rankine or degrees Kelvin:

$$p = p_0 \times \left(\frac{T_{ISA}}{T_0}\right)^{5.25588} = p_0 \times \left(\frac{288.15 - 0.0019812 \times h}{288.15}\right)^{5.25588} \qquad \text{(eq. 5)}$$

You know the sea level standard day air pressure and temperature. You also know you can use equations 1 and 2 to find the standard day temperature at any altitude. You thus have everything you need to find the standard day pressure at any altitude.

To see the detailed derivation of equation 5, refer to discussion 1 in the “Additional Discussion” section at the end of this chapter.

air density variation with altitude

We don’t need to develop an equation for density as a function of altitude. As you’ll see in the next paragraphs, standard day density can be found from the standard day air temperature. Read on.

pressure ratio, density ratio and temperature ratio

At this point, we need to introduce three new variables called δ, σ, and θ, defined as follows:

$\delta = \dfrac{p}{p_0}$ is the “pressure ratio”. p_0 is the sea level ISA air pressure.

$\sigma = \dfrac{\rho}{\rho_0}$ is the “density ratio”. ρ_0 is the sea level ISA air density.

$\theta = \dfrac{T}{T_0}$ is the “temperature ratio”. T_0 is the sea level ISA temperature.

⇒ Something to keep in mind is that the temperature ratio θ only holds true in units of absolute temperature – i.e. degrees Rankine or degrees Kelvin.

relationship between the pressure, density and temperature ratios

There's a direct relationship between the three variables δ, σ, and θ. To see that, recall the equation of state:

$$p = \rho RT$$

For sea level ISA conditions it is also true that:

$$p_0 = \rho_0 RT_0$$

and if you divide the first form of the equation above by the sea level standard day form, you get:

$$\frac{p}{p_0} = \frac{\rho}{\rho_0}\frac{T}{T_0}$$

or:

$$\delta = \sigma\theta \qquad \text{(eq. 6)}$$

Make a point of remembering equation 6 – it's frequently useful, particularly in determining σ once δ and θ are known. <u>This relationship is equally true for standard or non-standard conditions</u>.

the temperature ratio in the ISA troposphere

The variable θ is easy to compute for the standard atmosphere. Equations 1 and 2 give ISA temperature at any altitude in the troposphere. All that's needed is to divide by the corresponding value of T_0, thus:

$$\theta_{ISA} = \frac{518.67 - 0.00356616 \times h}{518.67} \quad \text{in English units, or} \qquad \text{(eq. 7)}$$

$$\theta_{ISA} = \frac{288.15 - 0.0019812 \times h}{288.15} \quad \text{in metric units} \qquad \text{(eq. 8)}$$

where h in both equations is the height above sea level <u>in feet</u>

the pressure ratio in the ISA troposphere

We developed equation 5 above to allow us to compute pressure at any altitude in the troposphere. Rewriting that equation using two of the atmospheric ratios, equation 5 becomes:

$$\delta_{ISA} = \theta^{5.25588} \qquad \text{(eq. 9)}$$

Bear in mind, however, that <u>equation 9 is only valid when the value of θ used is a standard day value</u>.

Knowing a standard day value of θ, computed as shown above, it's easy to find δ: equation 9 provides that relationship.Thus:

$$\delta_{ISA} = \left(\frac{518.67 - 0.00356616 \times h}{518.67}\right)^{5.25588} \quad \text{in English units, or} \qquad \text{(eq. 10)}$$

$$\delta_{ISA} = \left(\frac{288.15 - 0.0019812 \times h}{288.15}\right)^{5.25588} \quad \text{in Metric units.} \qquad \text{(eq. 11)}$$

the density ratio in the ISA troposphere

Finding σ is simple: remember that δ = σθ. Re-written, that becomes σ = δ ÷ θ.

If $\delta_{ISA} = \theta^{5.25588}$ then $\theta^{5.25588} \div \theta$ would just be equal to $\theta^{4.25588}$. Thus:

$$\sigma_{ISA} = \left(\frac{518.67 - 0.00356616 \times h}{518.67}\right)^{4.25588} \quad \text{in English units, or:} \qquad \text{(eq. 12)}$$

$$\sigma_{ISA} = \left(\frac{288.15 - 0.0019812 \times h}{288.15}\right)^{4.25588} \quad \text{in metric units} \qquad \text{(eq. 13)}$$

ISA temperature, pressure and density in the ISA troposphere

Equations 7 through 13 give you everything you need to find the temperature, pressure or density at any altitude in the ISA troposphere. Simply multiply θ by T_0 to get T, multiply δ by p_0 to get p, or multiply σ by ρ_0 to get ρ. You'll find, however, that the values of the temperature, pressure and density ratios are used more frequently in performance work than the actual values of temperature, pressure or density.

ISA Temperature, Pressure and Density Above the Tropopause

Since a large percentage of commercial jet airplane operation takes place above the tropopause, we need now to examine the characteristics of the stratosphere.

temperature variation with altitude above the tropopause

You'll recall that below the tropopause the atmospheric temperature is assumed to decrease linearly with increasing altitude. Above the tropopause, however, empirical data shows relatively little change in temperature with increasing altitude.

The International Standard Atmosphere assumes that temperature remains constant at -69.7 degrees Fahrenheit or -56.5 degrees Celsius until the altitude reaches 20,000 meters (65,617 feet).

The standard day temperature ratio θ above the tropopause is thus constant at $\frac{459.67 - 69.7}{518.67}$ or 0.7519.

pressure variation with altitude above the tropopause

While the fundamental principles of gasses are the same above or below the tropopause, the change in the temperature lapse rate to zero causes the equations seen above to take different forms.

The following equation allows you to calculate the standard day pressure at any altitude above the tropopause:

$$\delta = \frac{p}{p_0} = 0.22336\ e^{-\left(\frac{h - 36089.24}{20805.8}\right)} \qquad \text{(eq. 14)}$$

To see the derivation of equation 14, refer to discussion 2 in the “Additional discussion” section at the end of this chapter.

density variation with altitude above the tropopause

Since the standard day value of θ above the tropopause is constant at 0.7519, and since the relationship δ = σθ remains true above the tropopause, then:

$$\sigma_{ISA} = \frac{\delta_{ISA}}{\theta_{ISA}} = \frac{0.22336\ e^{-\left(\frac{h - 36089.24}{20805.8}\right)}}{0.7519}$$

thus:

$$\sigma_{ISA} = 0.29707\ e^{-\left(\frac{h - 36089.24}{20805.8}\right)} \qquad \text{(eq. 15)}$$

Standard Atmosphere Table

Now all of the information necessary for the creation of a standard atmosphere table has been presented. For convenience, and to help you check your ability to use these equations, here is a table of standard day parameters from sea level up to 45,000 feet.

STANDARD ATMOSPHERE TABLE									
geopotential altitude	temperature		θ	a / a_0	pressure			δ	σ
	OAT °F	OAT °C			in. Hg	lb / ft^2	mb		
0	59.0	15.0	1.0000	1.0000	29.920	2116.3	1013.2	1.0000	1.0000
1000	55.4	13.0	0.9931	0.9966	28.854	2040.9	977.1	0.9644	0.9711
2000	51.9	11.0	0.9862	0.9931	27.820	1967.7	942.1	0.9298	0.9428
3000	48.3	9.1	0.9794	0.9896	26.816	1896.7	908.1	0.8962	0.9151
4000	44.7	7.1	0.9725	0.9862	25.841	1827.7	875.1	0.8637	0.8881
5000	41.2	5.1	0.9656	0.9827	24.895	1760.8	843.0	0.8320	0.8617
6000	37.6	3.1	0.9587	0.9792	23.977	1695.9	812.0	0.8014	0.8359
7000	34.0	1.1	0.9519	0.9756	23.087	1633.0	781.8	0.7716	0.8106
8000	30.5	-0.8	0.9450	0.9721	22.224	1571.9	752.6	0.7428	0.7860
9000	26.9	-2.8	0.9381	0.9686	21.387	1512.7	724.2	0.7148	0.7620
10000	23.3	-4.8	0.9312	0.9650	20.576	1455.4	696.8	0.6877	0.7385
11000	19.8	-6.8	0.9244	0.9614	19.790	1399.8	670.2	0.6614	0.7156
12000	16.2	-8.8	0.9175	0.9579	19.029	1345.9	644.4	0.6360	0.6932
13000	12.6	-10.8	0.9106	0.9543	18.291	1293.7	619.4	0.6113	0.6713
14000	9.1	-12.7	0.9037	0.9507	17.577	1243.2	595.2	0.5875	0.6500
15000	5.5	-14.7	0.8969	0.9470	16.885	1194.3	571.8	0.5643	0.6292
16000	1.9	-16.7	0.8900	0.9434	16.216	1147.0	549.1	0.5420	0.6090
17000	-1.6	-18.7	0.8831	0.9397	15.568	1101.1	527.2	0.5203	0.5892
18000	-5.2	-20.7	0.8762	0.9361	14.941	1056.8	506.0	0.4994	0.5699
19000	-8.8	-22.6	0.8694	0.9324	14.335	1014.0	485.5	0.4791	0.5511
20000	-12.3	-24.6	0.8625	0.9287	13.750	972.5	465.6	0.4595	0.5328
21000	-15.9	-26.6	0.8556	0.9250	13.183	932.5	446.4	0.4406	0.5150
22000	-19.5	-28.6	0.8487	0.9213	12.636	893.7	427.9	0.4223	0.4976
23000	-23.0	-30.6	0.8419	0.9175	12.107	856.3	410.0	0.4046	0.4807
24000	-26.6	-32.5	0.8350	0.9138	11.596	820.2	392.7	0.3876	0.4642
25000	-30.2	-34.5	0.8281	0.9100	11.103	785.3	376.0	0.3711	0.4481
26000	-33.7	-36.5	0.8212	0.9062	10.627	751.7	359.9	0.3552	0.4325
27000	-37.3	-38.5	0.8144	0.9024	10.168	719.2	344.3	0.3398	0.4173
28000	-40.9	-40.5	0.8075	0.8986	9.725	687.8	329.3	0.3250	0.4025
29000	-44.4	-42.5	0.8006	0.8948	9.297	657.6	314.8	0.3107	0.3881
30000	-48.0	-44.4	0.7937	0.8909	8.885	628.4	300.9	0.2970	0.3741
31000	-51.6	-46.4	0.7869	0.8870	8.488	600.4	287.4	0.2837	0.3605
32000	-55.1	-48.4	0.7800	0.8832	8.105	573.3	274.5	0.2709	0.3473
33000	-58.7	-50.4	0.7731	0.8793	7.737	547.2	262.0	0.2586	0.3345
34000	-62.2	-52.4	0.7662	0.8753	7.382	522.1	250.0	0.2467	0.3220
35000	-65.8	-54.3	0.7594	0.8714	7.040	498.0	238.4	0.2353	0.3099
36000	-69.4	-56.3	0.7525	0.8675	6.712	474.7	227.3	0.2243	0.2981
36089	-69.7	-56.5	0.7519	0.8671	6.683	472.7	226.3	0.2234	0.2971
37000	-69.7	-56.5	0.7519	0.8671	6.397	452.4	216.6	0.2138	0.2844
38000	-69.7	-56.5	0.7519	0.8671	6.097	431.2	206.5	0.2038	0.2710
39000	-69.7	-56.5	0.7519	0.8671	5.810	411.0	196.8	0.1942	0.2583
40000	-69.7	-56.5	0.7519	0.8671	5.538	391.7	187.5	0.1851	0.2462
41000	-69.7	-56.5	0.7519	0.8671	5.278	373.3	178.7	0.1764	0.2346
42000	-69.7	-56.5	0.7519	0.8671	5.030	355.8	170.3	0.1681	0.2236
43000	-69.7	-56.5	0.7519	0.8671	4.794	339.1	162.3	0.1602	0.2131
44000	-69.7	-56.5	0.7519	0.8671	4.569	323.2	154.7	0.1527	0.2031
45000	-69.7	-56.5	0.7519	0.8671	4.355	308.0	147.5	0.1455	0.1936

Table 4-1

Pressure Altitude

This seems like the right place to introduce the concept of *pressure altitude*, as it's an important factor in most calculations of airplane performance.

Stated simply: "pressure altitude" is the altitude corresponding to a value of atmospheric pressure following the International Standard Atmosphere relationship of pressure as a function of height.

For example, if we know that today's atmospheric pressure at our location is 29.920 inches of mercury, then by definition that means that the pressure altitude is zero. If we see that the atmospheric pressure is 28.854 inches of mercury, then by definition the pressure altitude is 1000 feet. A pressure of 27.820 inches of mercury means that the pressure altitude is 2000 feet.[1] And so on.

In the troposphere, when the atmospheric pressure is known the corresponding pressure altitude can be calculated using the following equation, which is derived by rearranging equation 5:

$$h_p = 145442.15 \times \left[1 - \left(\frac{p}{p_0}\right)^{0.190263}\right] \qquad \text{(eq. 16)}$$

where h_p is the pressure altitude, in feet
p is the atmospheric pressure
p_0 is the sea level standard day pressure in the same units as p

In the stratosphere, i.e. above 36,089 feet and below 65,617 feet, the corresponding equation would be:

$$h_p = 36089.24 - 20805.8 \times \ln\left[4.47706\left(\frac{p}{p_0}\right)\right] \qquad \text{(eq. 17)}$$

In performance work, pressure altitude is generally more meaningful than true altitude – the exception being, of course, problems involving obstacle clearance. This statement may sound a bit strange, but bear in mind that the performance of an airplane and its engines is a function of air density, which in turn is a function of air pressure and temperature. From a performance standpoint, the airplane and engines neither know nor care what the true height above the ground may be (aside from second-order gravitational effects).

When we speak of ISA conditions, we speak of pressure, temperature, and density as functions of pressure altitude – simply because by definition pressure altitude is related to pressure by the standard day relationship. If you know the air pressure you know the pressure altitude, and vice versa.

1. These values are taken from the International Standard Atmosphere table appearing in this chapter.

Say, for example, that you're located at sea level. Just at this moment, though, due to a low pressure weather pattern the local air pressure is only 29.38 inches of mercury. You are at a pressure altitude of 500 feet, and the airplane and engines will perform just as they would at an elevation of 500 feet on a standard day. You might, on the other hand, have a local air pressure of 30.46 inches of mercury – you are then at a pressure altitude of -500 feet, and the airplane and engines will perform just as they would at an elevation of -500 feet on a standard day.

Geopotential Altitude

Earlier in this chapter when we were developing the equations for pressure, temperature and density as a function of height, we simplified our job by making the assumption that the gravitational acceleration g was constant at the sea level standard value g_0. We talk about this subject in Discussion 1 in the "additional Discussion" section at the end of this chapter.

The standard atmosphere table is therefore modeled on an artificial gravitational field concept – one of constant gravitational acceleration regardless of altitude. This is termed the *geopotential model* and the altitude values shown in the table are referred to as *geopotential altitudes*.[1]

As a result, pressure altitudes are geopotential altitudes in that they're based on the geopotential model of the atmosphere.

Will this simplifying assumption of a constant gravitational field have a real effect on our work? Let's see how much the difference is between geopotential altitude and tapeline altitude.

Geopotential altitude will always be slightly less than tapeline altitude. Why? Say you raise a 1000-pound object to a geopotential altitude of 1000 feet above sea level. Throughout that height increase the gravitational acceleration is assumed to be constant at g_0. At 1000 feet, its geopotential energy would be one million foot-pounds.

On the other hand, suppose that you raise the same object to a tapeline altitude of 1000 feet in the real world, in which gravitational acceleration decreases with increasing altitude. At 1000 feet tapeline altitude the object's potential energy would be slightly *less* than a million foot-pounds, because the gravitational force is very slightly less at a thousand feet than it is at sea level. Thus, to obtain the same potential energy, you would have to raise the mass to a tapeline height very slightly greater than 1000 feet.

1. In this context, the term "geopotential" refers to potential energy. You know that *potential energy* is just one of many forms of energy. Specifically, it's the energy which is "stored", you might say, in an object which is raised above its resting position in a gravitational field. If, for example, you were to lift an object weighting ten pounds and hold it five feet above the ground, it would have a potential energy of 50 foot-pounds, the product of the weight (pounds) and the height (feet). If you then were to release the object and allow it to fall to the ground, the potential energy would convert into kinetic energy – the energy of motion – as it accelerated, and the ten-pound object would strike the ground with 50 foot-pounds of kinetic energy.

The highest altitude at which present-day commercial airplanes operate is 45,000 feet. If you calculate the tapeline altitude corresponding to a geopotential altitude of 45000 feet, at a mid-latitude, you'd find that the tapeline altitude is 45097 feet – only a 97-foot error, and in the conservative direction. At lower altitudes where terrain clearance may be critically important, the error will be less, and still in the conservative direction. For this reason, the conversion from geopotential altitude to tapeline altitude is usually omitted as being unnecessary.

<u>For practical purposes, you can consider the tapeline altitude to be the same as geopotential altitude. The standard day table can be considered to be atmospheric parameters as a function of true altitude, with negligible error. On a standard day, then, pressure altitude is for practical purposes the same as tapeline altitude</u>.

To see further discussion of tapeline altitude, please refer to discussion 3 in the Additional Discussion section at the end of this chapter.

For the remainder of this chapter, we will refer to geopotential altitude only. You understand now that terms "geopotential altitude" and "tapeline altitude" are for practical purposes synonymous.

calculating geopotential altitude from pressure altitude

For much of a performance engineer's work, it's sufficient to know the airplane's pressure altitude. This sort of work includes tasks such as typical flight planning, cruise analysis, fuel mileage monitoring, and the like. For these, the geopotential altitude isn't important.

In certain other performance tasks, however, the knowledge of geopotential altitude truly <u>is</u> important. One good example of this is the calculation of airplane height during driftdown following an engine failure in flight. In those areas of the world having high terrain, the ability to compute geopotential altitude is essential for ensuring compliance with regulatory requirements and safety in ensuring obstacle clearance by the necessary amounts.

You should be forever mindful of the fact, though, that we can never really calculate geopotential altitude accurately. At best, we can calculate what the geopotential height would be following the standard day value of temperature lapse rate. Any deviation of the actual atmospheric temperature conditions from the assumed values will introduce a margin of error into our calculations.

Our earth's atmosphere is a complex and dynamic environment. It would be unreasonable to assume that it neatly follows our assumed temperature lapse rate of two degrees Celsius per thousand feet. Still, this assumption is based on the average of thousands of observations of actual meteorological conditions. Hopefully it is acceptably accurate – there's no other practical way of accounting for the relationship of temperature to altitude.

However, although it's not practical to account for differing lapse rates from day to day, it <u>is</u> practical to account for atmospheres which differ from the ISA temperatures, being either hotter or colder than the standard day.

The relationship between the pressure altitude and the geopotential altitude on a given day depends on the atmospheric temperature on that day. For convenience in calculation, it is usual to express the atmospheric temperature in terms of the ISA deviation, ΔISA. In general:

$$\Delta ISA\ °C = OAT\ °C - (288.15 - 0.0019812 \times h_{geo}) \qquad \text{(eq. 18)}$$

where $OAT\ °C$ is the outside air temperature at the given pressure altitude
h_{geo} is the geopotential altitude in feet

Knowing the pressure altitude and the ISA deviation, the geopotential altitude can be found from:

$$h_{geo} = h_p - 96.0343 \times \Delta ISA°C \times ln(\delta) \qquad \text{(eq. 19)}$$

where h_p is the pressure altitude in feet
$\Delta ISA\ °C$ is the ISA deviation in degrees C
δ is the value of the pressure ratio δ at pressure altitude h_p

Demonstrating the derivation of equation 19 exceeds the scope of this book and is not included.

Over a limited change of pressure altitude, within which we can consider the relationship of pressure to altitude to be linear (say, ten thousand feet or so), we can calculate the corresponding change of geopotential altitude:

$$\Delta h_{geo} = \Delta h_p \left(\frac{T_{ISA} + \Delta ISA}{T_{ISA}} \right)$$

Where T_{ISA} is the average temperature between the two values of pressure altitude.

There is additional discussion of this topic in the chapter entitled “Measurement of Altitude”.

Calculating Atmospheric Parameters for Non-Standard Conditions

Up to this point, we’ve been discussing the International Standard Atmosphere and how you can find the atmospheric parameters in standard day conditions. Sadly, however, not all days are standard days. What to do then?

Most frequently, when we speak of non-standard conditions we’re referring to temperatures which are different from the standard day temperatures for the given altitude. Temperature variation from standard is customarily expressed in terms of <u>the difference between actual and standard</u> – often called *ISA deviation*, ISA Δ or sometimes ΔISA. You will frequently hear terms such as ISA+10°C, ISA-5, or something similar to these. Again, however, remember that a deviation from ISA does <u>not</u> imply any change in the lapse rate. If the sea level temperature at sea level happens to be 25 °C, that signifies an ISA+10°C condition and that ISA deviation will be expected to continue as a constant with increasing altitude – so it will still be ISA+10°C at all other altitudes.

For calculations calling for the temperature ratio θ, simply divide the actual temperature, in absolute temperature units, by 288.15°K or 518.67°R as appropriate.

How about the pressure ratio δ for non-standard days? No, you can not use the ISA equation for δ, that is, $\delta = \theta_{ISA}^{5.25588}$, when the value of θ is based on a non-standard temperature; the equation for δ assumes standard day temperatures and pressures.

In typical low-altitude performance work, such as takeoff and landing performance problems, you'll know the elevation of the airport being studied. In this case, you might choose to assume that the pressure altitude is equal to the elevation, if some degree of inaccuracy is acceptable. On a below-standard pressure day, however, the inaccuracy would be in the unconservative direction.

To be more precise in your work, you'll need more information. You'll need to know either the current atmospheric pressure or the altimeter setting QNH or QFE. If you know the atmospheric pressure, you could divide by p_0 to get δ; you could go to the standard atmosphere table to find pressure altitude.

You can refer to the chapter entitled "Measurement of Altitude" for a complete discussion of altimeter settings and how they can be used to find pressure altitude.

Knowing the temperature and pressure information, it's possible to find the density by use of equation 6: $\delta = \sigma\,\theta$. That equation holds true for either standard or nonstandard conditions.

For performance work other than takeoff and landing, it's probable that you will be given an ISA deviation and a pressure altitude. Referring again to the "Measurement of altitude" chapter, you should know that all flight above the transition altitude will be conducted by reference to altimeters which are set "QNE" – the standard day setting of 29.92 in. Hg or 1013.2 millibars. Thus the altimeter is reading pressure altitude directly, and all air traffic control altitude assignments above the transition altitude will be pressure altitudes.

Additional Discussion

discussion 1: derivation of equation 5

Here is the equation of state again:

$$p = \rho R T$$

where p is the static pressure
ρ is the mass density of the air
T is the absolute temperature
R is the specific gas constant, 1716.5619 ft-lb/slug °R

Now the hydrostatic equation:

$$dp = -\rho\ g\ dh$$

Dividing the second equation by the first one yields:

$$\frac{dp}{p} = \frac{-g dh}{RT} \qquad \text{(eq. 20)}$$

From equations 1 through 4 above, you see that ISA temperatures can be expressed as:

$$T = T_0 - \lambda h$$

where T is the temperature
T_0 is the sea level standard temperature
λ is the temperature lapse rate
h is the height above sea level

Differentiating equation 6 gives:

$$dT = -\lambda\ dh \quad \text{or} \quad dh = -\frac{dT}{\lambda}$$

Substituting equation 7 into equation 6:

$$\frac{dp}{p} = \frac{-g}{RT}\left(\frac{-dT}{\lambda}\right) \quad \text{or:} \quad \frac{dp}{p} = \frac{g}{\lambda R}\frac{dT}{T}$$

To find the relationship between pressure and temperature in the standard atmosphere, integrate that equation, remembering that λ and R are constants and hence come outside the integral:

$$\int \frac{dp}{p} = \frac{1}{\lambda R}\int \frac{g\ dT}{T}$$

The difficulty with this equation is that if we treat g as a variable, it will complicate the solution of the equation. One way we can simplify the job is to assume that g is constant and is arbitrarily set it equal to the sea level standard value g_0. Then it can be brought outside the integration.

Assuming a constant gravitational field introduces something called “geopotential height” discussed in this chapter. Treating g as constant at g_0 yields:

$$\int \frac{dp}{p} = \frac{g_0}{\lambda R} \int \frac{dT}{T}$$

Integrating from p_0 to p and from T_0 to T gives:

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{g_0}{\lambda R}}$$

We know that R, the specific gas constant, is equal to 1716.5619, that g_0 is 32.17405, and that λ is 0.00356616 (all of these in English units, of course). Substituting those values into the equation above gives:

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{5.25588}$$

Since $\frac{T}{T_0}$ is a ratio of temperatures, this equation can be used with either English or metric units. The temperatures, of course, must be in units of absolute temperature (degrees K or degrees R).

---

discussion 2: derivation of equation 14

Recall equation 5 from above:

$$\frac{dp}{p} = \frac{-g\,dh}{RT}$$

since T is now constant at the stratosphere value, and still keeping g constant at g_0:

$$\int \frac{dp}{p} = -\frac{g_0}{RT} \int dh$$

Integrating and substituting the limits:

$$ln\left(\frac{p}{p_{trop}}\right) = -\frac{g_0}{RT_{trop}}(h - 36089.24)$$

The term $\dfrac{g_0}{RT_{trop}} = \dfrac{32.17405}{1716.5617 \times (459.67 - 69.7)} = \dfrac{1}{20805.8}$

thus:

$$ln\left(\frac{p}{p_{trop}}\right) = -\left(\frac{h - 36089.24}{20805.8}\right)$$

or:

$$\frac{p}{p_{trop}} = e^{-\left(\frac{h - 36089.24}{20805.8}\right)}$$

thus:

$$p = p_{trop} \times e^{-\left(\frac{h - 36089.24}{20805.8}\right)}$$

From equation 9:

$$\frac{p_{trop}}{p_0} = \left(\frac{T_{trop}}{T_0}\right)^{5.25588} = \left(\frac{389.97}{518.67}\right)^{5.25588} = 0.22336$$

or:

$$p_{trop} = 0.22336\ p_0$$

then:

$$p = 0.22336 \times p_0 \times e^{-\left(\frac{h - 36089.24}{20805.8}\right)}$$

Thus, finally:

$$\delta = \frac{p}{p_0} = 0.22336\ e^{-\left(\frac{h - 36089.24}{20805.8}\right)}$$

---

discussion 3: calculating tapeline altitude from geopotential altitude

If we assume a constant gravitational field with $g = g_0$, the potential energy of any object raised to a geopotential height h_{geo} would be simply:

$$potential\ energy = m \times g_0 \times h_{geo} \qquad \text{(eq. 21)}$$

If, however, you want to find the energy more precisely, you'd have to integrate its rate of increase of potential energy as its height is increased, in a gravitational field having a decreasing value of g depending on the true height. That requires a little calculus:

$$potential\ energy = m \int_0^{h_{tape}} g\ dh_{tape} \qquad \text{(eq. 22)}$$

To find the relationship between geopotential height and tapeline height, you could set equation 18 equal to equation 19 and work the mathematics.

To simplify the job of solving the mathematics here, an assumption is made. It assumes that the value of gravitational acceleration *g* at any height above sea level is related to the sea level gravitational acceleration by the equation:

$$g\ at\ altitude\ =\ g\ at\ sea\ level \times \left(\frac{earth\ radius}{earth\ radius + height\ above\ sea\ level}\right)^2 \qquad \text{(eq. 23)}$$

If you have read the chapter entitled "Mass and Weight" carefully, you will recognize that equation 20 is not really correct. The equation as written is only valid for pure gravitational acceleration – what you might call the Newtonian component of the force of gravity. It is *not* correct in the real world because it ignores the slight effect that the earth's rotation has on the gravitational acceleration experienced by an object. The effect of earth's rotation is to create a negative incremental change to the gravitational acceleration, called $\Delta g_{centrifugal}$ thus resulting in a net gravitational force that is slightly less than the Newtonian component.

If you're being precise when finding the value of *g* at altitude, you'd first have to correct the sea level value of *g* to a pure Newtonian value by removing the effect of the earth's rotation on an object at sea level. Then you'd ratio the sea level Newtonian value of g up to the altitude you want. Then, finally, you'd add in the (negative) centrifugal effect at altitude to arrive at the net gravitational acceleration at altitude. This process is exact but it's obviously much more work than using equation 20.

By taking equation 20's shortcut method for finding *g* at altitude, you would arrive at the following equation:

$$h_{tapeline} = \frac{h_{geopotential} \times r' \times \left(\frac{g_0}{g_{\varphi,\,SL}}\right)}{\left[r' - h_{geopotential} \times \left(\frac{g_0}{g_{\varphi,\,SL}}\right)\right]} \qquad \text{(eq. 24)}$$

where *r'* is an artificial earth radius, see below for the explanation
g_0 is the standard value of acceleration of gravity, 32.1741 ft/sec^2 or 9.8067 m/sec^2
$g_{\varphi,,SL}$ is the sea level acceleration of gravity at latitude φ

The value of $g_{\varphi,,SL}$ is found by using Lambert's Equation, shown in the chapter entitled "Mass and Weight" and reproduced here for convenience:

$$g_{\varphi,\,SL} = 32.17244 \times [1 - 2.6373 \times 10^{-3} cos(2\varphi) + 5.9 \times 10^{-6} cos^2(2\varphi)]$$

where φ is the latitude in degrees

This "artificial radius" r' used in equation 20 needs a bit of explaining...

As we said above, the shortcut method of finding the value of g at altitude shown in equation 20 is not the accurate way to find it. This simplification introduces an inaccuracy into the result of the equation for $h_{tapeline}$ shown in equation 21. The use of an artificial radius instead of the true earth radius at the latitude of interest is simply a means of correcting the error introduced by the shortcut taken by equation 20. However, this means that the value of the artificial radius r' varies as a function of latitude:

$$r' = \frac{2g_{\varphi,SL}}{3.085462 \times 10^{-6} + 2.27 \times 10^{-9} \times \cos 2\varphi - 2 \times 10^{-12} \times \cos 4\varphi} \qquad \text{(eq. 25)}$$

Obviously, this method for finding tapeline height is complex. To illustrate the sort of magnitude of difference between geopotential height and tapeline height, let's take the reference conditions, in which $g_0 = g_{\varphi,,SL}$

For that case, equation 21 simplifies to:

$$h_{tapeline} = \frac{r' \times h_{geopotential}}{r' - h_{geopotential}} \qquad \text{(eq. 26)}$$

where r' is the radius of the earth used for this calculation, 20855531 feet.

Using equation 23 assuming a geopotential height of 45000 feet, the highest certified operating height for Boeing jet transport airplanes (as of this writing) yields a corresponding tapeline height of 45097 feet. In other words, there's an error of only 97 feet introduced by our use of the geopotential altitude model for the standard atmosphere. The error is much less at the lower altitudes, especially those where obstacle clearance may be a consideration.

For a much more detailed discussion of the international standard atmosphere including the geopotential model, we recommend the document called Item number 72022 published by the International Sciences Data Unit, located in London, England.

Chapter 5: Measurement of Altitude

Introduction

The fundamental principle of altitude measurement is extremely simple: we know that air pressure varies with altitude in a known manner, so we can just measure the ambient static air pressure wherever we are and from this deduce our altitude.

Well, yes, but it's a bit more complex than that, especially if you want to be safe under all reasonably possible atmospheric conditions. In this chapter, we'll look in detail at the measurement of altitude, and we'll discuss possible inaccuracies in our methods of doing so.

Altimeters

An altimeter is simply a device that is designed to sense local atmospheric pressure and from that to deduce and display altitude. Altimeters can be very simple, or quite complex, depending on the desired accuracy.

Let's look first at a very simple altimeter.

the simplest altimeter

In the discussion of pressure in the chapter entitled "Units", we show that a simple device, a tube filled with fluid as shown in Figure 5-1 to the right, can be used to indicate pressure. Knowledge of the density of the fluid and a little arithmetic would allow this device to be calibrated to read pressure quite accurately.

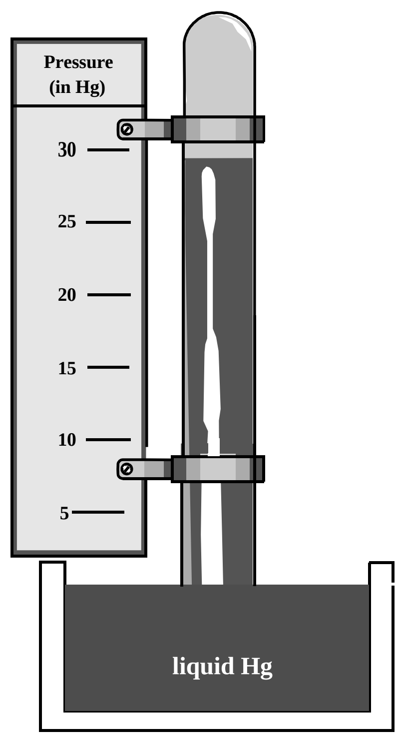


Figure 5-1

Mercury is most frequently used in devices of this type, because its density allows the use of a shorter tube, and because it doesn't evaporate.

For discussion of the calibration of a simple mercury tube pressure gauge, refer to discussion 1 in the "Additional Discussion" section at the end of this chapter.

Having a working pressure gauge, however, isn't the same as having a working altimeter. Yes, we know that air pressure decreases as altitude increases, but how should we relate the indicated pressure to the altitude?

If we want to calibrate our pressure gauge in units of height, <u>we have to assume some relationship between air pressure and height</u>

Once we have established the relationship between pressure and altitude, we can make our pressure gauge display altitude, as shown in Figure 5-2 to the right. This relationship is referred to as the altimeter's *calibration*.

While it's possible to construct an altimeter as shown in this sketch, it's pretty obviously an impractical design for aviation use, because of its size and its fragile nature. We need something more compact and more rugged - and more precise.

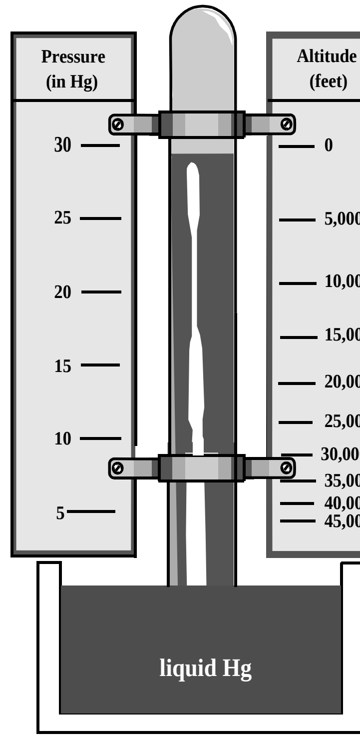


Figure 5-2

the aneroid altimeter

Aneroid means simply "without liquid" – that is, an aneroid altimeter is a mechanical device which isn't based on the height of a column of liquid such as mercury.

The heart of an aneroid altimeter is a sealed bellows chamber, called an aneroid. It is surrounded in the instrument's case by a supply of air at the present atmospheric static pressure taken from outside the airplane. The bellows will expand or compress according to the pressure around it: increased pressure will cause it to compress, reduced pressure allows it to expand. This expansion or contraction causes, through a mechanical gear system, the movement of the instrument's pointer on a scale of airplane altitude.

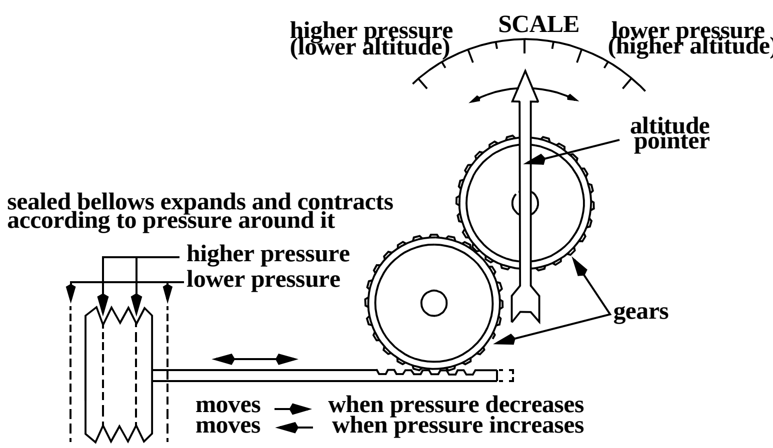


Figure 5-3

Figure 5-3 is obviously a greatly simplified drawing, but it allows us to visualize in general terms how such an altimeter works. A pressure increase, for example, corresponding to a decrease of altitude, causes the aneroid to compress which produces a counterclockwise motion of the altimeter pointer. A pressure decrease corresponding to an increase of altitude allows the aneroid to expand, producing a clockwise motion of the pointer. Altimeters of this type, when well designed for a minimum of mechanical imperfections such as friction, are capable of producing highly accurate readings of altitude based on measurements of static pressure.

Altimeter Calibration

Altimeters currently in use are calibrated following the International Standard Atmosphere relationship between pressure and altitude, as defined in the chapter entitled “The Atmosphere”. Repeating an equation from that chapter:

$$\delta = \frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{5.25588} = \left(\frac{288.15 - 0019812 \times h}{288.15}\right)^{5.25588} \qquad \text{(eq. 1)}$$

where p is the static pressure
p_0 is the sea level standard day pressure, in the same units as p
T is the static temperature at height h on a standard day, in absolute units
T_0 is the sea level standard day temperature, in absolute units
h is the height above sea level in feet

Note that this equation uses metric temperatures, but since it’s a ratio you could equally well use temperatures in the English system: substitute 518.67 for 288.15, and substitute 0.00356616 for 0.0019812

Entering a range of values of h between sea level and the tropopause will allow the calibration of an altimeter in units of height. For example, an airplane height of 27000 feet would correspond to a pressure of 10.17 inches of mercury.

Rearranging equation 1 gives:

$$h = \frac{288.15 \times \left[1 - \left(\frac{p}{p_0}\right)^{0.190263}\right]}{0.0019812} = 145442.15 \times \left[1 - \left(\frac{p}{p_0}\right)^{0.190263}\right] \qquad \text{(eq. 2)}$$

You’ll recognize that equation 2 is valid only for the troposphere. Above the tropopause, the corresponding equation would be:

$$h = 36089.24 - 20805.8 \times \left[\ln\left(4.47708 \times \frac{p}{p_0}\right)\right] \qquad \text{(eq. 3)}$$

Since the ISA pressure at the tropopause is 6.683 inches of mercury, an altimeter reading a static ambient pressure of p greater than that value would indicate the airplane’s height h following equation 2; equation 3 would be used for pressures less than 6.683 inches of mercury.

This is the standard altimeter calibration equation. If entering the value of p in inches of mercury, for example, then p_0 would have to be 29.92; if entering p in hectoPascals, then p_0 would be 1013.2, and so on.

Figure 5-4 shows the relationship between static pressure and indicated altitude – that is, an altimeter's calibration curve – plotted from equations 2 and 3.

A well-designed altimeter following this relationship can produce highly accurate results. There's a problem with this simple relationship, however: <u>it only yields valid altitude readings on a standard day</u>.

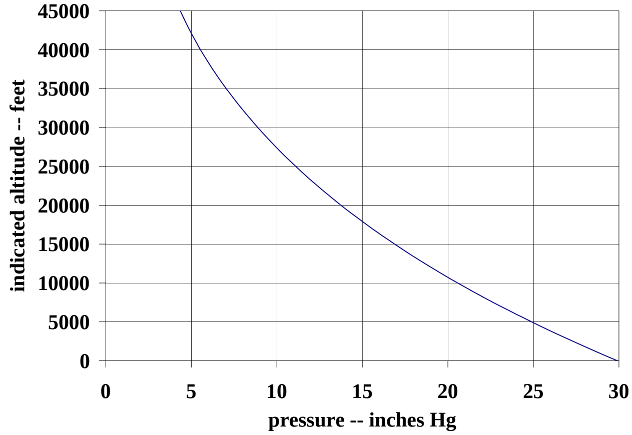


Figure 5-4

Effect of Atmospheric Pressure Variation on Altitude Measurement

The atmosphere in which we live is a dynamic, constantly changing environment. Weather patterns of pressure and temperature are constantly in motion. This fact reveals one weakness of the simple aneroid altimeter described above: it makes no allowance for the normal day-to-day variations in atmospheric pressure.

altimeters having only the standard day calibration

Look at Figure 5-5 to the right, which illustrates three different atmospheric pressure conditions. As examples, we have arbitrarily chosen two values of non-standard sea level pressure: for a high-pressure day we use 30.42 inches of mercury, and for the low-pressure day we are using 29.42 in. Hg. We also assume that the rate of pressure change with height does not differ from the standard day model.

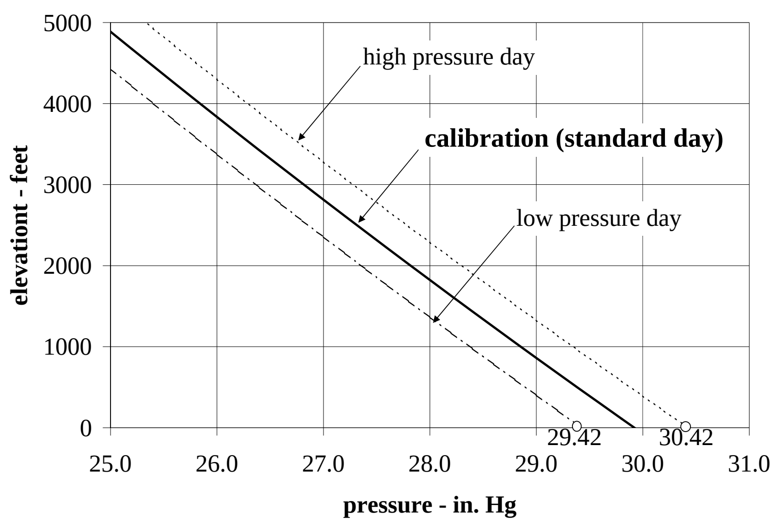


Figure 5-5

While sea level pressure may vary by more than the range shown of plus or minus one-half inch of mercury from standard day, the great majority of operations do fall within this range.

We see that, for a given height elevation, the actual static pressure could be either higher or lower than the standard pressure for that elevation. This indicates how much altitude error can be introduced by an altimeter which uses only the standard day relationship, like the simple aneroid altimeter discussed above.

For example, suppose that an airplane climbs after takeoff until the static air pressure has decreased to 26 inches of mercury. On the high pressure day shown, that would occur at an elevation of approximately 4400 feet. The altimeter which is calibrated to ISA, however, would show a height of approximately 3800 feet – an error of some 600 feet. This error would be conservative, in that it would tell the crew that they're lower than they truly are – and that therefore they must climb higher in order to clear whatever obstacles might exist.

On a <u>low</u> pressure day, however, pilots can be tricked into believing that they're higher than they truly are. Using the same example of 26 inches of mercury, this could occur at an elevation of approximately 3400 feet – <u>but the flight crew is being told by their altimeter that they're 3800 feet above sea level</u>. If there is an obstacle in their path having a height of 3600 feet, the crew might feel they're safely clear of it when in fact they're not.

Another inadequacy of an altimeter having only the standard day calibration is its inability to adjust for varying conditions when the airplane is on the ground.

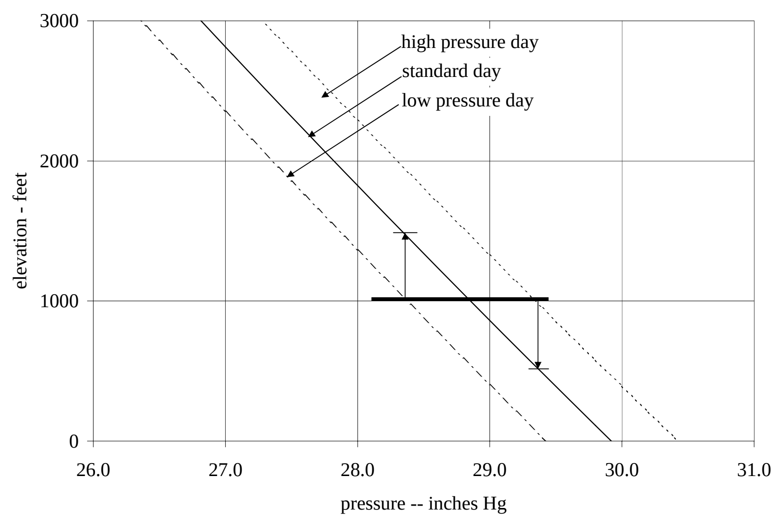


Figure 5-6

Look again at a diagram of pressure versus height for three different conditions, as shown in FIgure 5-6 to the right. For example, for an airport at an elevation of 1000 feet, a high-pressure day like the one shown above would cause the standard day-calibrated altimeter to display an altitude of approximately 550 feet – an error of 450 feet. Similarly, on a low-pressure day, the altimeter will read approximately 1450 feet, again an error of 450 feet.

Since takeoff and landing safety requires the flight crew to know accurately their height above the airport and the local terrain, or above sea level, this inconsistency is obviously unacceptable.

"altimeter settings" for non-standard day pressures

The simple altimeter, calibrated only to ISA conditions, is inadequate for safe operation under all possible conditions. For that reason, all present day altimeters have the ability to adjust for non-ISA pressure conditions. They accomplish this by <u>shifting the altimeter's calibration</u>. In effect, "setting" the altimeter is moving its calibration curve, as shown in Figure 5-3, upward or downward.

Before takeoff and during an approach for landing, pilots are provided with the current altimeter setting either by the control tower or some other source of weather data.

Shown on the right is the face of a typical aneroid altimeter. At the lower left is the adjustment knob, and at the right of the center of the face is the “altimeter setting window”. When given the current correct setting for the altimeter, the pilot will rotate the adjustment knob until the *altimeter setting* is properly shown in the window. Notice that the altimeter setting is shown as a pressure in inches of mercury or hectoPascals (millibars).

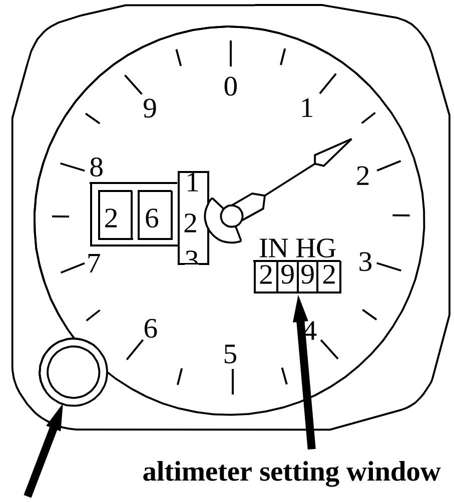


Figure 5-7

Electronic altimeters, which typically present altitude information on the face of an LCD display on the instrument panel, can also be adjusted through the system software.

There are three different altimeter settings in common use; we’ll discuss them one at a time.

altimeter setting “QNH”

The altimeter setting used in most areas of the world when flying at lower altitudes is called *QNH*[1]. <u>When an airplane is sitting on the ground with its altimeter set to QNH, the altimeter pointer will indicate airport elevation</u>.

With the altimeter set to QNH, then, the altimeter will display the airplane’s height above sea level for the local conditions. During departures and approaches, this facilitates a crew’s awareness of their height relative to obstacles in the area, if any, since obstacle heights are shown in published navigation data as heights above sea level.

You can think of QNH as an adjustment which shifts the pressure-height calibration curve up or down as needed to adjust for current local pressure conditions. Look at the following illustration:

1. The letters Q-N-H have no literal meaning. QNH is simply one of the so-called “Q-codes” developed in the early 1900s for greater ease of transmitting information by Morse code. They remain in widespread use today not only in aviation but in a few other fields as well.

In the example shown on the right, we have an airport elevation of 1000 feet. At that elevation on a the standard day the pressure would be 28.85 inches Hg. Let's assume that the actual current pressure at the airport, called the "station pressure", is 28.35 inches Hg.

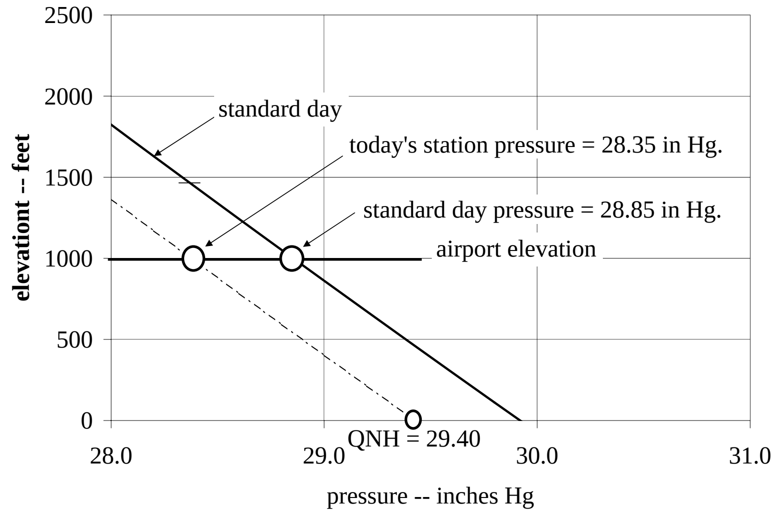


Figure 5-8

In order to ensure that the altimeter displays 1000 feet when the airplane is on the ground at the airport, we need to slide the standard day line downward, in effect, until it passes through the station pressure of 28.35 at an altitude of 1000 feet. We have now shifted the calibration line so that the altimeter will correctly display a height of 1000 feet at the airport for the current conditions. So what is QNH?

From the above example, you see that QNH is specifically the sea level pressure which, following the shape of the standard day calibration curve, will cause the altimeter to correctly display airport elevation when on the ground at the airport, for the current atmospheric conditions.

You can calculate the QNH for any given airport elevation and airport station pressure:

$$QNH = 29.92 \times \left[\left(\frac{station\ pressure}{29.92} \right)^{0.190263} + \left(\frac{airport\ elevation}{145442.16} \right) \right]^{5.25588} \qquad \text{(eq. 4)}$$

Here's a thought: what if you want to know the QNH setting without access to the regular sources of that information? It's simple – take an altimeter, adjust it so that it shows your present elevation, and read the QNH in the altimeter setting window.

converting QNH to pressure altitude

When computing airplane takeoff performance, it's necessary to know the pressure altitude at the airport. For conditions which are close to standard day, it might be acceptable to assume that the pressure altitude is the same as the elevation. But you know by now that on low-pressure days, the air pressure is below standard day – and hence the pressure altitude would be above the airport elevation. Since takeoff performance decreases with increasing pressure altitude, simply assuming that pressure altitude equals elevation would be, on a low-pressure day, somewhat unconservative.

If you know the station pressure, calculation of the pressure altitude is simple – just recall standard day equation 2, repeated here for convenience:

$$h = 145442.15 \times \left[1 - \left(\frac{p}{p_0}\right)^{0.190263}\right] \qquad \text{(eq. 2)}$$

What if instead you know the QNH and the airport elevation?

Remember that when we adjust an altimeter for off-standard pressures, we are shifting the calibration curve up or down as necessary so that it will pass through a point which is the intersection of the airport elevation and the station pressure. By doing so we have set the altimeter calibration so that it will correctly show airport elevation when the airplane is on the ground. We'll repeat the previous illustration here with a few changes.

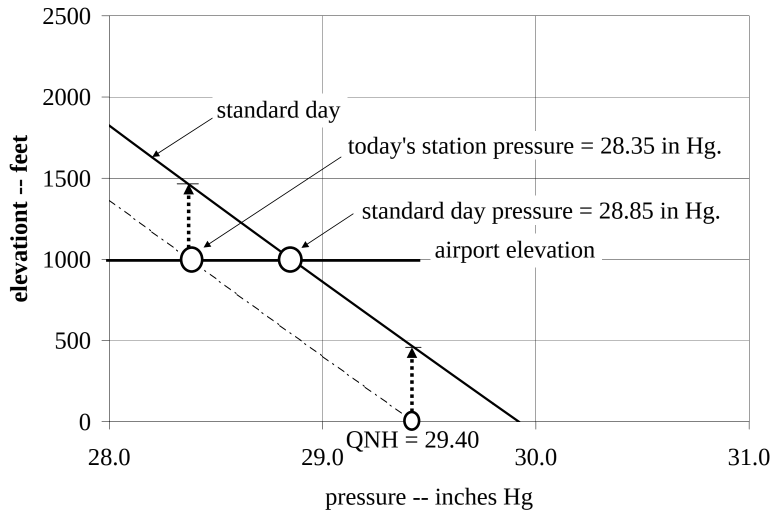


Figure 5-9

If you know the station pressure, then you can directly compute the pressure altitude, as shown above. By that method, what you are doing (mathematically) is drawing a line – shown as a dashed arrow – upward from the station pressure until it strikes the standard day pressure line. At that point you read the altitude on the scale, and since it's the standard day line, that's by definition your pressure altitude. Continuing our example from above, that would give us a pressure altitude of 1484 feet. The pressure altitude is 484 feet above the elevation.

It's possible that you don't know the station pressure, but <u>do</u> know the airport elevation and the QNH. Look at the diagram again, and notice that a second dashed arrow is drawn upward from the QNH to the standard day pressure level. Since we have moved the calibration line straight downward to make it pass through the station pressure point, you'll understand that <u>the vertical space between the two pressure lines is the same at all pressures</u>.

Since you know QNH, you can calculate the pressure altitude at that pressure value. That's simple, using equation 2. You'll find that it's 484 feet. Thus the difference between the elevation and the pressure altitude is 484 feet, and you can see that pressure altitude can be found from:

$$\textit{pressure altitude } h_p = \textit{airport elevation} + 145442.15 \times \left[1 - \left(\frac{QNH}{29.92}\right)^{0.190263}\right] \qquad \text{(eq. 5)}$$

Continuing the example from above for an airport with an elevation of 1000 feet and a QNH of 29.40: using equation 5 would yield a pressure altitude of 1484 feet.

Bear in mind, of course, that although we have shown 29.92 inches of mercury as p_0 in equation 5 above, if you're expressing the sea level standard day pressure in units other than inches of mercury, the value of p_0 must be in the same units.

A simpler but less precise method for finding pressure altitude from QNH and elevation is shown in Figure 5-10 to the right. This table appears in a number of Boeing performance documents including the Boeing Flight Planning and Performance Manuals.

The "QNH to Pressure Altitude" table is made by assuming a linear decrease of pressure with altitude having a magnitude of 1.06 inches of mercury, or 36.1 hectoPascals per thousand feet of altitude change. This is correct between sea level and 1000 feet, and it's sufficiently accurate for other airport elevations.

QNH to Pressure Altitude

QNH (IN. HG.)	CORRECTION TO ELEVATION FOR PRESS ALT (FT)	QNH (MILLIBARS)
28.81 to 28.91	1000	976 to 979
28.91 to 29.02	900	979 to 983
29.02 to 29.12	800	983 to 986
29.12 to 29.23	700	986 to 990
29.23 to 29.34	600	990 to 994
29.34 to 29.44	500	994 to 997
29.44 to 29.55	400	997 to 1001
29.55 to 29.66	300	1001 to 1004
29.66 to 29.76	200	1004 to 1008
29.76 to 29.87	100	1008 to 1012
29.87 to 29.97	0	1012 to 1015
29.97 to 30.08	-100	1015 to 1019
30.08 to 30.19	-200	1019 to 1022
30.19 to 30.30	-300	1022 to 1026
30.30 to 30.41	-400	1026 to 1030
30.41 to 30.52	-500	1030 to 1034
30.52 to 30.63	-600	1034 to 1037
30.63 to 30.74	-700	1037 to 1041
30.74 to 30.85	-800	1041 to 1045
30.85 to 30.96	-900	1045 to 1048
30.96 to 31.07	-1000	1048 to 1052

Example: Elevation = 2500 FT
QNH = 29.48 IN. HG.
Correction = 400 FT
Press Alt = 2900 FT

Figure 5-10

altimeter setting QFE

Flight crews in some regions of the world use a different altimeter setting called *QFE* – another of the Q codes. QFE is fundamentally different from QNH in that when the altimeter is set to QFE on the ground, the altimeter will indicate zero altitude. During takeoff or approach, the altimeter will show the airplane's height above the airport.

Look at the diagram to the right. Remembering that our goal is now to shift the calibration to make the altimeter read zero on the ground. Clearly, QFE is the station pressure.

Both QNH and QFE have their adherents and their critics. Either one is acceptable provided the pilot clearly understands the significance of what is displayed on the altimeter.

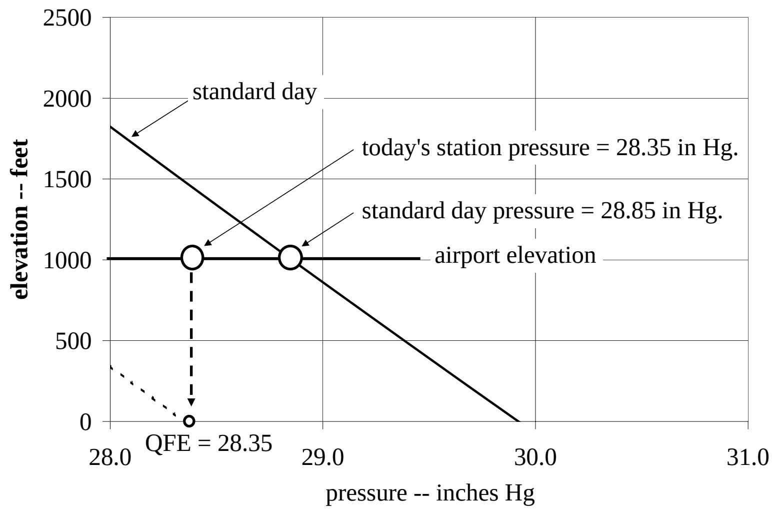


Figure 5-11

converting QFE to pressure altitude

Since QFE is just the station pressure, it is necessary only to use equation 2 to find the pressure altitude.

transition altitude and altimeter setting QNE

Both QNH and QFE have a fundamental shortcoming: they are only valid within a relatively short distance from the airport providing them. This is simply due to the fact that weather patterns are usually localized rather than covering a wide area. A flight departing Chicago for Seattle will use the Chicago QNH for departure and a different Seattle QNH for approach[1]. What should be used in between?

After departure, when passing through the altitude called the "transition altitude" in climb, the crew will set the altimeter to the setting called QNE. There's nothing special about QNE – it's just the standard day setting of 29.92 inches of mercury or 1013.2 hectoPascals. When descending back down through the transition altitude toward the destination, the pilot will set the altimeter to the local QNH provided by the destination airport.

What this means is that all airplanes operating above the transition altitude will have their altimeters set the same way, therefore allowing pilots flying in the same area to know exactly their heights relative to each other. If their altimeters were set to different QNH values, they wouldn't

1. unless, of course, by coincidence the QNH is the same for Seattle as it is for Chicago. While it's possible, it's relatively unlikely that they'll be exactly the same.

have a consistent basis for comparing altitudes. Also, remember that since QNE is the standard day value, when set to QNE all altimeters are reading pressure altitude.

In the United States, the transition altitude is set at 18,000 feet. Other countries observe other transition altitudes. Airplanes flying below the transition altitude will, for the most part, be local flights operating within a relatively small area and thus subject to little pressure change. For long low-altitude flights, the prudent flight crew will keep their altimeter set to the local QNH for the region through which they are then flying.

flight levels

When flying above the transition altitude, altitudes are referred to as flight levels (FL). Flight levels are expressed in hundreds of feet, thus FL330 is a pressure altitude of 33000 feet.

Since the vast majority of commercial flights operate above the transition altitude, cruise altitudes are ordinarily specified in terms of flight levels.

Cold Weather Effects on Altitude Measurement

We have said that all altimeters use the same calibration curve of height versus pressure. Yes, the altimeters do allow for variations in local air pressure by means of the altimeter setting, shifting the calibration curve upward or downward, but the slope of the calibration curve in the altimeter is always the same – it is fixed at the value corresponding to a standard day. At low altitudes, that's a slope of (approximately) one thousand feet of altitude change for a pressure change of one inch of mercury.

The slope of the curve of pressure versus height is a function of the air density, which in turn depends on temperature. For a given pressure, cold air is more dense, hot air is less dense. This density variation has an effect on altimetry. For an analogy to help understand this, look at the following illustration:

The column in the middle consists of a stack of cubes, each having a density of 1 pound per cube. Each cube is one foot on a side, a volume of one cubic foot. The pressure changes by one pound per square foot (psf) for each one foot of height change up or down. Thus for a pressure change of ten pounds per square foot, it would be necessary to change height by ten feet. For this analogy, we'll consider this to be the "standard" condition, analogous to ISA.

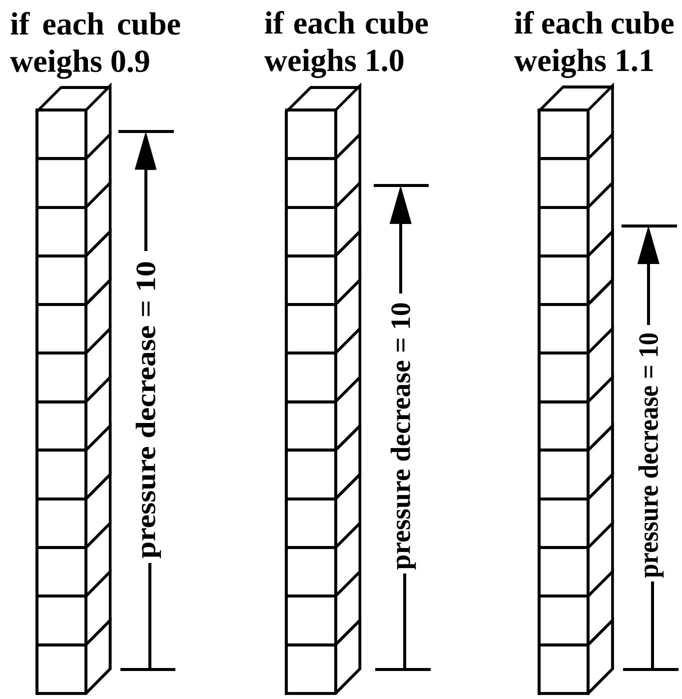


Figure 5-12

The column on the left consists of cubes at a lower density of 0.9 pounds per cube. The pressure changes by 0.9 psf for each one foot of height change up or down. For a pressure change of ten pounds per square foot, it would be necessary to change height by 11.1 feet – more than for the standard condition.

The column on the right consists of cubes at a higher density of 1.1 pounds per cube. The pressure changes by 1.1 psf for each foot of height change up or down. For a pressure change of ten pounds per square foot, it would be necessary to change height by 9.1 feet – *less* than for the standard condition.

In each of these cases, a pressure gauge would show the same pressure change of ten pounds per square foot – <u>but the change of actual height to cause that pressure change depends on the density</u>. Understanding this analogy, here's how it applies to an airplane:

On a standard day, for which an altimeter calibration slope is correct, when the altimeter indicates that the airplane's height is one thousand feet above the airport, it is truly a thousand feet above the airport;

On a hot day, when an altimeter indicates that the airplane is a thousand feet above the airport, it has actually climbed more than that. This is a conservative condition: the airplane is higher than the pilot believes it to be. Because the hot-day altimeter error is in the conservative direction, it is usually neglected.

On a cold day, however, when an altimeter indicates that the airplane is a thousand feet above the airport, it is actually less than that. <u>This is an unconservative condition: the airplane is lower than the pilot believes it to be</u>.

This chart demonstrates the effect of cold days on pressure versus height. Shown is the standard day line and a line of ISA - 35 °C. Assume a sea level airport, where ISA - 35 °C would be equal to -20 °C. You can see on the chart that when the ISA - 35 °C line has achieved a height of 2000 feet, the ISA line (which represents the altimeter) is at approximately 2300 feet. In other words, the airplane at this point is 300 feet lower than the height indicated on the altimeter!

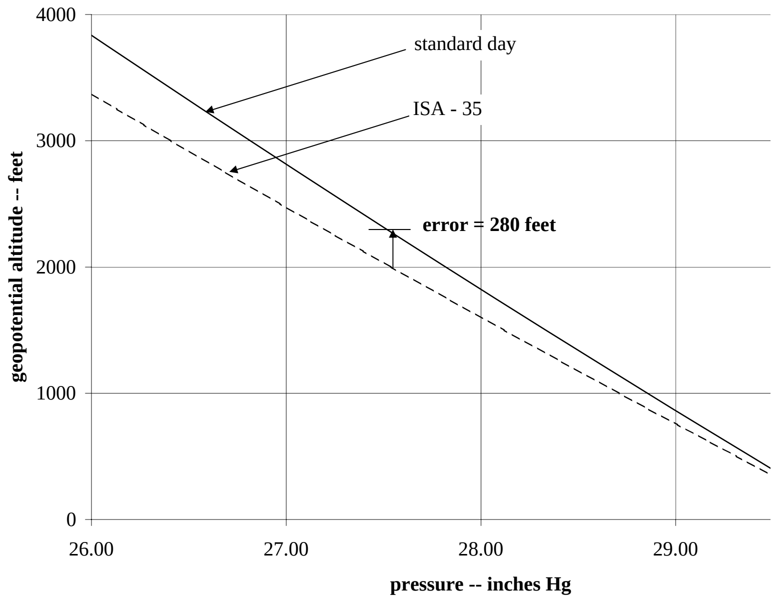


Figure 5-13

Pilots familiar with this shortcoming of altimeters sometimes express it as "cold and low, look out below!" Simply stated: <u>in atmospheric conditions colder than standard, an airplane is lower than the height indicated on the altimeter</u>. If an airplane is flying in an area having obstacles, this could be unsafe because the crew might believe themselves to be safely above the obstacles when in fact they are not.

correcting for altimeter errors on cold days

Since an altimeter has no means of adjustment for different air densities, it is necessary to advise pilots of the conditions under which unsafe operation may occur due to altimeter inaccuracy, and to provide the pilots with the information needed to correct for this inaccuracy.

ICAO has published the following table of data for altimeter error. This table also appears in the Supplementary Procedures section of Boeing Flight Crew Operations Manuals (FCOMs).

airport OAT	**height above elevation of altimeter setting source - feet**													
degrees C	**200**	**300**	**400**	**500**	**600**	**700**	**800**	**900**	**1000**	**1500**	**2000**	**3000**	**4000**	**5000**
0	20	20	30	30	40	40	50	50	60	90	120	170	230	280
-10	20	30	40	50	60	70	80	90	100	150	200	290	390	490
-20	30	50	60	70	90	100	120	130	140	210	280	420	570	710
-30	40	60	80	100	120	140	150	170	190	280	380	570	760	950
-40	50	80	100	120	150	170	190	220	240	360	480	720	970	1210
-50	60	90	120	150	180	210	240	270	300	450	590	890	1190	1500

Table 5-1

The use of this table is as follows: say, for example, that there is an obstacle 1000 feet above the airport elevation, and that the temperature at the airport today is -20 °C. The pilot should add 140 feet to the obstacle height in order to obtain the actual altimeter reading when just climbing past the height of the obstacle. <u>Should the flight crew forget this and believe themselves to be adequately clear of the obstacle when their altimeter reads 1100 feet, they might well fly the airplane into the obstacle</u>.

This table is computed for a sea level takeoff airport, and is slightly conservative for airports above sea level. All values are rounded up to the next ten feet.

Airlines that regularly operate in cold-weather conditions are familiar with this source of altimeter error and publish guidance information such as the above for their flight crews.

To see the derivation of the cold-weather altimeter error table, refer to discussion 2 in the Additional Discussion section at the end of this chapter.

Additional Discussion

discussion 1: calibrating a simple mercury tube pressure gauge

For equilibrium, the downward force of the mercury in the tube must be just exactly equal to the upward force, otherwise the level of the mercury in the tube would change. Putting that a bit differently: <u>at the level of the mercury in the bowl</u>, the downward pressure must equal the upward pressure.

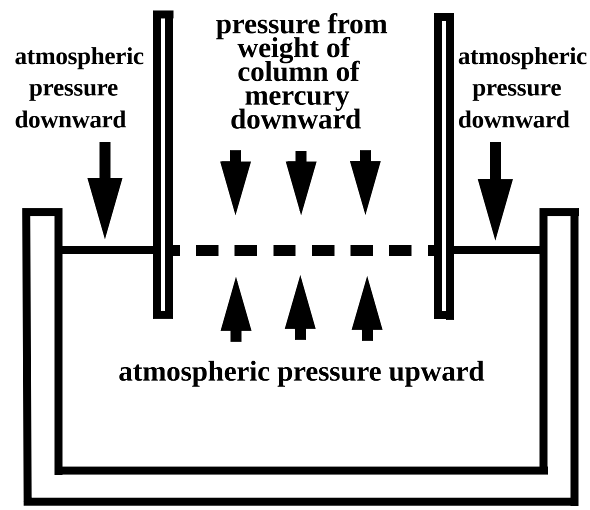


Figure 5-14

The downward pressure is simply the weight of the mercury above the level in the bowl. The upward pressure at that level is the same as the atmospheric pressure, since the pressure exerted on the surface of the mercury in the bowl is propagated through the mercury and upward into the tube, and at the level of mercury in the bowl, the upward pressure would be the same as the outside pressure at the same level in the bowl.

Suppose that the atmospheric pressure pushing down on the mercury in the bowl is 2116.2 pounds per square foot. That's the same as 14.7 pounds per square inch.

The mass density of mercury is 26.38 slugs per cubic foot. Assuming a standard value of g, that works out to 848.75 pounds per cubic foot or 0.4913 pounds per cubic inch.

Imagine (although it's not very realistic) that we have a stack of cubes of mercury, each cube being one inch on each side – a volume of one cubic inch. Each cube therefore weighs 0.491 pounds. What if you have ten of these cubes stacked on top of each other in a column? The pressure being exerted at the bottom of the stack would be 4.91 pounds, the weight of ten cubes, on an area of one square inch – therefore, a pressure of 4.91 pounds per square inch. The pressure at the bottom of a column of mercury, therefore, is simply the height of the column in inches multiplied by 0.491; or, conversely, the height of the column in inches would be the pressure at the bottom of the column divided by 0.491. This is true regardless of the cross-sectional area of the tube.

A pressure of 5 pounds per square inch would occur if the column of mercury in the tube is 10.18 inches high; a pressure of 14.7 psi would occur if the column of mercury is 29.92 inches high. Standard day atmospheric pressure at sea level would be indicated by a column height of 29.92 inches, or 760 millimeters. Sound familiar? That’s how atmospheric pressure, essential in doing airplane performance calculations, is reported by the weather department.

---

discussion 2: derivation of the cold weather altimeter corrections

In the chapter entitled “The Atmosphere”, the equation describing the rate of change of atmospheric pressure with increasing altitude was shown to be, for the ISA standard day definition:

$$\frac{dp}{dh_p} = -\frac{g_0\, p}{RT_{ISA}}$$

where p is the atmospheric pressure
h_p is the pressure altitude
g_0 is the standard gravitational acceleration
R is the gas constant for air
T_{ISA} is the standard day temperature

Over a limited change of pressure altitude, within which we could consider the relationship of pressure to height to be linear, that equation could be simplified to:

$$\Delta h_p = -\Delta p \times \frac{R\, T_{ISA}}{p}$$

where Δp is the change of pressure
Δh_p is the change of pressure altitude corresponding to the change of pressure

What would be the pressure-altitude relationship for a non-standard day? The only difference would be the temperature at the starting altitude. In this case, though, the altitude would NOT be pressure altitude, since pressure altitude and height are the same only on a standard day. In the case of the non-standard day, the equation would yield pressure change in terms of geopotential height, thus:

$$\Delta h_{geo} = -\Delta p \times \frac{R\,(T_{ISA} + \Delta ISA)}{p}$$

where ΔISA is the ISA deviation for the non-standard day.

Dividing the first of these two equations by the second gives:

$$\frac{\Delta h_p}{\Delta h_{geo}} = \frac{T_{ISA}}{T_{ISA} + \Delta ISA} \qquad \text{(eq. 6)}$$

The second of these equations confirms what we already know: when ΔISA is negative, meaning a day colder than standard, pressure altitude increases faster than geopotential altitude, telling us again that <u>on cold days the indicated altitude will be higher than the actual altitud</u>*e*.

If you play with the equations some more, you can find that the difference between pressure and geopotential heights is:

$$\Delta h_{geo} - \Delta h_p = \Delta\ h_{geo} - \Delta\ h_{geo}\left(\frac{T_{ISA}}{T_{ISA} + \Delta ISA}\right)$$

which gives:

$$\Delta\ h_{geo} - \Delta\ h_p = \Delta h_{geo} \times \left(\frac{\Delta ISA}{T_{ISA} + \Delta ISA}\right) \qquad \text{(eq. 7)}$$

Remember, please, that the value of T_{ISA} must be in absolute units.

When we showed you the ICAO altimeter correction table above under the heading of “correcting for altimeter errors on cold days” we took an example of a sea level airport with a reported OAT of -20 °C, and we said we had an obstacle 1000 feet high. What would be the altimeter reading when passing 1000 feet?

First of all, -20 °C at sea level would be an ISA deviation of -35 °C

We’re assuming, remember, that the slope of pressure versus height is linear – which is acceptably accurate over relatively small height increments. But for best accuracy using this linear assumption, when calculating the value of T_{ISA} let’s use the temperature at the half-way point up the climb, which would give the average slope of pressure versus height over that height gain.

Thus the ISA temperature at 500 feet, half-way up the 1000-foot height gain from sea level would be $288.15 - 0.0019812 \times 500$ or 287.16 °K, giving

$$\Delta h_p = 1000 \times \left(\frac{287.16}{287.16 - 35}\right) = 1139\ feet$$

Thus, it would be necessary to climb to an altimeter height of 1139 feet in order to be clear of a 1000-foot obstacle. There is a cold-weather altimeter error of 139 feet, which agrees with the value of 140 feet that we found in the ICAO table (which is always rounded up to the next ten feet).

If you wanted to derive a cold-weather altimeter error table like the ICAO one shown above, you could use equation 4. For the same example conditions, use of equation 7 would yield:

$$\Delta\, h_{geo} - \Delta\, h_p \;=\; 1000 \times \left(\frac{-35}{287.16 - 35} \right) \;=\; -139 \textit{ feet}$$

which shows an altimeter error of 139 feet due to the cold weather for these example conditions.

[It is worth mentioning here that ICAO used a value of 273 instead of 273.15 when converting to absolute temperature, and used a temperature lapse rate of 0.00198 instead of 0.0019812. This difference causes very small but noticeable discrepancies when computing altimeter error values rounded up to the next ten feet, as the ICAO table shows.]

Chapter 6: Measurement of Airspeed

Introduction

In the chapter entitled "Measurement of Altitude" we showed you how static pressure, sensed at the airplane's location, can be used to produce an indication of altitude for the pilot. In a similar fashion, we can produce an indication of airspeed for the pilot. In this case, however, we need to know more than just static pressure.

In this chapter, we will be describing how airspeed may be measured on an airplane. You'll see, however, that the subject is made more complicated by the fact that we can't measure an airplane's actual speed through the air, because of the limitations of our measurement method; we'll therefore be discussing the different forms of airspeed that you will encounter in performance work.

The Simplest Airspeed Indicator

Just as we introduced you to "the simplest altimeter" in the chapter entitled "Measurement of Altitude", we could also construct very easily an airspeed indicator using nothing more than a glass tube and some fluid.

the manometer

A common way of displaying and measuring the pressure of a gas such as air is by use of a "*manometer.* In its simplest form, a manometer is simply a U-shaped glass tube with open ends, partially filled with a liquid which is usually colored to make it more visible.

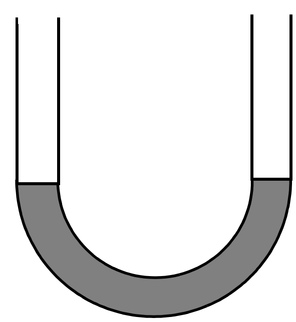


Figure 6-1

It's intuitively obvious that if the pressure is the same at both ends of the tube, then the fluid level will be equal across the U. If, however, one end of the U-tube is supplied with a pressure which is different from the pressure supplied to the other end, the fluid level would be different between the two sides of the U. And, obviously, the difference in the height of the fluid level between the sides would be an indicator of the difference between the two pressures.

We can calculate the amount of the difference between the two pressures by measuring the difference in height of the two levels and knowing the density of the fluid.

Call the difference in height Δh, and call the density of the fluid ρ_{fluid}.

Recalling the hydrostatic equation introduced earlier, the difference between the two pressures, P_2 and P_1, can be calculated as:

$$P_2 = P_1 + \rho_{fluid}\, g\, \Delta h$$

where g is the local acceleration of gravity.

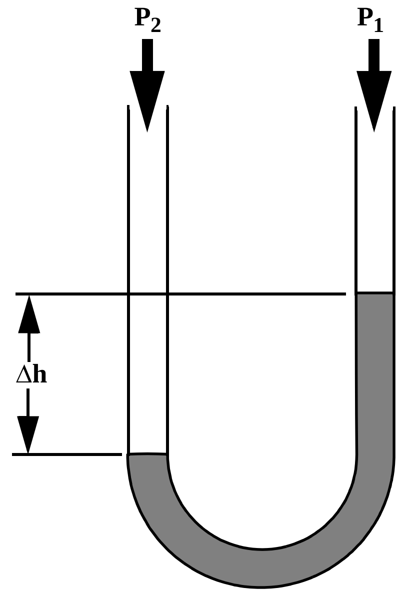


Figure 6-2

Rearranging:

$$P_2 - P_1 = \rho_{fluid}\, g\, \Delta h \qquad \text{(eq. 1)}$$

Bear in mind that we can't deduce the individual values of P_2 and P_1, only the difference between them. But for the purpose we're about to discuss, that's enough.

Recall equation 14 from the chapter entitled "Physics of Air", repeated here for convenience. (Remember that equation 14 is only valid under the assumption of an incompressible flow.):

$$p_{static} + \frac{1}{2}\rho_{air} V_{true}^2 = p_{total} = \textit{constant along a streamline}$$

We can re-state that as:

$$p_{total} - p_{static} = \frac{1}{2}\rho V_{true}^2 \qquad \text{(eq. 2)}$$

From equation 2 you can see that if we could sense the total pressure and the static pressure acting on an airplane in flight, we could apply those two pressures to the ends of a U-tube manometer. At relatively low airspeeds, the Δh of the fluid would then be a direct function of the airplane's speed V_{true} and the speed could be computed from:

$$\frac{1}{2}\rho_{air} V_{true}^2 = \rho_{fluid}\, g\, \Delta h \quad \text{and thus:}$$

$$V_{true} = \sqrt{\frac{2\,(\rho_{fluid}\; g\; \Delta h)}{\rho_{air}}} \qquad \text{(eq. 3)}$$

the Pitot-static probe

So how shall we sense the total and static pressures?

The device used on contemporary airplanes is called a *Pitot* or *Pitot-static probe,* depending on the design[1]. Here's a simplified diagram of a Pitot-static probe.

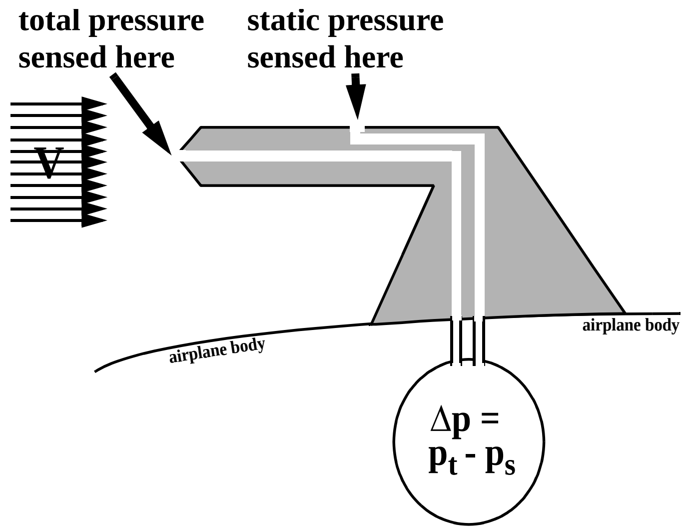


Figure 6-3

The probe is typically mounted on the side of the airplane's fuselage. The orifice at which the total pressure is sensed is held far enough from the fuselage of the airplane to keep it outside the local boundary layer.

In another type of installation, the probe contains only the total pressure sensing orifice, and the static sensing ports are located on the fuselage of the airplane. In this case, the probe is referred to as a "Pitot probe" rather than a "Pitot-static" probe.

1. The Pitot tube is named after its inventor, Henri Pitot (1695-1771), who was a French hydraulic engineer. He became interested in the flow velocities of water in rivers and canals and invented the Pitot tube for measuring those velocities.

typical Pitot and Pitot-static probe installations

This photograph illustrates typical locations of Pitot-static probes. For Boeing airplanes, this applies to the 737 airplanes through the -500 model, to all 747 models, and to all 767 models.

Figure 6-4

This photograph shows a typical installation of a Pitot probe, with the flush static ports located farther aft on the fuselage.

This type of installation is used on the 707-300, all 727 models, all 737-600, -700, -800 and -900 models, and all 757 and 777 models.

Figure 6-5

impact pressure and dynamic pressure

The difference between the total pressure and static pressures sensed on the airplane, (p_t - p_s), is called *impact pressure*. In the chapter entitled "Physics of Air", however, we defined (p_t - p_s) as dynamic pressure, $\frac{1}{2}\rho V^2$. What's the difference?

Impact pressure and dynamic pressure are the same at low speeds, below 200 or 250 knots, in which speed range the density of the air is considered to be unaffected by the airplane's speed. At higher speeds, the air density around the airplane *is* affected by compressibility and the term $(p_t - p_s)$ is no longer equal to $\frac{1}{2}\rho V^2$. Just remember that *impact pressure* is defined as $(p_t - p_s)$ whereas dynamic pressure q is defined as the kinetic energy of the airflow, $\frac{1}{2}\rho V^2$.

When computing aerodynamic forces, you'll use dynamic pressure frequently. For airspeed measurement, however, which must cover both the lower speeds where there's no compressibility effect and the faster airspeeds where there IS some compression, we'll use the term "impact pressure" to denote $(p_t - p_s)$.

Present-day Airspeed Measurement

Clearly, although a manometer tube could be connected to a Pitot-static probe and used as an airspeed indicator, it wouldn't be a very practical device. Airspeed indicators in use today are either mechanical devices, very similar to an altimeter, or are electronic - however, they both do operate by sensing total and static pressures and computing airspeed from the difference between them.
In mechanical airspeed indicators, the instrument case is pressurized with the static pressure, and the aneroid inside the case is provided with the total pressure. The aneroid expands or contracts therefore as a function of the *difference* between the two pressures. Expansion or contraction of the aneroid is mechanically converted into movement of the pointer of the instrument on the scale of airspeed, much as the expansion or contraction of the aneroid in a mechanical altimeter is converted into a display of altitude.

In electronic airspeed indicators, an air data computer is provided with both the static and total pressures and the airspeed is electronically computed following an equation for airspeed as a function of the two pressures.

the airspeed equation

In the chapter entitled "Physics of Air", we showed the equation called "Bernoulli's equation for incompressible flow". We stated that flow could be treated as incompressible only up to approximately 200 or 250 knots.

We also showed a different form of Bernoulli's equation which is valid for compressible flow – that is, all real flows. It is this latter equation which is at the heart of airspeed systems in modern-day airplanes, as their operating speeds are substantially above 250 knots for most of their operating time.

Here's "Bernoulli's equation for compressible flow" repeated for convenience:

$$\frac{\gamma}{(\gamma - 1)}\frac{p}{\rho} + \frac{1}{2}V^2 = \frac{\gamma}{(\gamma - 1)}\frac{p_t}{\rho_t} = constant$$

This equation needs to be re-written to solve for velocity. The derivation is a bit complex and will not be shown here. If you want to see the derivation, refer to discussion 1 in the “Additional Discussion” section at the end of this chapter.

Here’s the equation:

$$V_{true} = \sqrt{\frac{2\,\gamma\,p_s}{(\gamma - 1)\,\rho_s}\left[\left(\frac{p_t - p_s}{p_s} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \qquad \text{(eq. 4)}$$

where γ is the ratio of specific heats C_P/C_V. You’ll recall that its value for air is 1.4
p_s is the static pressure at the airplane
$(p_t - p_s)$ is the impact pressure sensed by the Pitot-static system
ρ_s is the density of the air at the airplane.

This equation is the basis of the calibration of all modern-day airspeed systems. There are two problems with using equation 4 in measuring airspeed, however:
First, equation 4 requires us to know the impact pressure, the static pressure, and the air density. The first two are measurable at the airplane, but the air density is not.

Second, it’s impossible to sense the precise values of static and total pressures at the airplane, simply because the Pitot-static tube, or Pitot tube and static port, are mounted on the airplane body and hence are in a flow of air which is affected by the presence of the airplane. The sensed values of p_t and p_s therefore have some small inaccuracy. Most of this inaccuracy is in the measurement of the static pressure.

The following paragraphs discuss how these problems are overcome.

calibrating an airspeed indicator

We have said that an airspeed indicator is simply a device which moves a pointer across a scale of speed in response to the magnitude of the impact pressure $(p_t - p_s)$ sensed by the Pitot-static system of the airplane.

An airspeed indicator may be a mechanical device, as in most light airplanes and older commercial jet transport airplanes. In more modern airplanes, however, airspeed indicators are electric or electronic, displaying airspeed in accordance with an electrical signal provided by an air data computer device. The function of the air data computer is to correct for the inaccuracy inherent in sensing static pressure and to compute the airspeed electronically; it then sends an electric signal to the airspeed indicator for display digitally or by the pointer of the electric airspeed indicator.

The airspeed indicators of modern-day commercial jet airplanes, therefore, don’t suffer from the inability of the Pitot-static system to sense static pressure precisely.

An airspeed indicator is calibrated following this equation:

$$airspeed = \sqrt{\frac{2\,\gamma\,p_0}{(\gamma-1)\,\rho_0}\left[\left(\frac{p_t-p_s}{p_0}+1\right)^{\frac{\gamma-1}{\gamma}}-1\right]} \qquad \text{(eq. 5)}$$

You'll recognize equation 5 as being simply equation 4 with the sea level standard day values of static pressure p_0 and density ρ_0 substituted for p_s and ρ_s.

This can be re-written as:

$$airspeed = \sqrt{\frac{2\,\gamma\,p_0}{(\gamma-1)\,\rho_0}}\sqrt{\left[\left(\frac{p_t-p_s}{p_0}+1\right)^{\frac{\gamma-1}{\gamma}}-1\right]} \qquad \text{(eq. 6)}$$

Since in equation 6 the variables p_0, ρ_0, and γ are all constant, they can be evaluated and grouped together as a single constant. (Recall that p_0 is 2116.26616 pounds per square foot, ρ_0 is 0.002377 slugs per cubic foot, and γ has the value 1.4).

Equation 6 yields the airspeed in feet per second. Since, however, we work most frequently with airspeeds expressed in knots, we can insert another constant, 1.6878, which is the conversion from feet per second to knots.

With these values, equation 6 becomes:

$$airspeed = 1479.1\sqrt{\left[\left(\frac{p_t-p_s}{p_0}+1\right)^{\frac{\gamma-1}{\gamma}}-1\right]} \qquad \text{(eq. 7)}$$

Where: $1479.1 = \dfrac{\sqrt{\dfrac{2\gamma p_0}{(\gamma-1)\rho_0}}}{1.6878}$

This constant, 1479.1, appears in other equations as well.

calibrated and indicated airspeed

Since the equation used to calibrate an airspeed indicator uses the sea level standard day values of static pressure and density, <u>an airplane's airspeed indicator does not display the airplane's true airspeed</u>.

The airspeed displayed by modern electric or electronic airspeed indicators is called the *calibrated airspeed*, sometimes referred to as CAS or V_C. A calibrated airspeed in units of knots is sometimes called KCAS – knots calibrated airspeed.

Thus, calibrated airspeed is the speed computed following equation 7, with exact knowledge of the impact pressure.

On airplanes that don't have air data computers, however, the airspeed is displayed on mechanical airspeed indicators which are provided with impact pressure from the Pitot-static system. In this case the displayed airspeed is referred to as "indicated airspeed", IAS or V_I.

Also, modern airplanes do have backup "standby" airspeed indicators which are mechanical devices provided with impact pressure from the airplane's Pitot-static system. All of these airspeed indicators thus have an error due to the imprecision of measuring pressures at the airplane.

Indicated airspeed, in other words, is different from calibrated airspeed in that it contains a small amount of error, called *position error*, or, on more recent Boeing airplanes, *static source error.*

position error

Position error/static source error is that amount of inaccuracy in the speed displayed by a mechanical airspeed indicator resulting from the inability of the Pitot-static system to sense precisely the free-stream static air pressure. This error is given the abbreviation ΔV_P.

The position error correction is added to the indicated airspeed to compensate for pressure sensing inaccuracy. After correcting for position error, the revised value of airspeed is the calibrated airspeed.

Thus:

$$V_C = V_I + \Delta V_P \qquad \text{(eq. 8)}$$

equivalent airspeed

We have seen that calibrated airspeed is the speed computed by equation 7 depending on the sensed impact pressure, using the sea level standard day values of static pressure and density. Therefore, if an airplane is operating at exactly sea level standard day pressure and density conditions, the calibrated airspeed will be the airplane's true speed.

Whenever operating at conditions other than those of sea level standard day pressure and density, however, calibrated airspeed could be said to have an altitude error, and a density error.

If calibrated airspeed is corrected for the altitude error by using the actual value of static pressure rather than the sea level standard day one, then it will have a different value and a different name. Calibrated airspeed corrected for the altitude effect is called "equivalent airspeed" and is designated as V_e or EAS. Equivalent airspeed measured in knots is frequently denoted as KEAS.

The equation for equivalent airspeed in terms of pressures is:

$$V_e = \sqrt{\frac{2\,\gamma\, p_s}{(\gamma-1)\,\rho_0}\left[\left(\frac{p_t - p_s}{p_s} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \qquad \text{(eq. 9)}$$

compressibility correction

In the preceding paragraphs, we have said that the difference between calibrated and equivalent airspeed is due to the static pressure term in the airspeed equation. This difference is sometimes provided in the form of a correction called the *compressibility correction*, designated as ΔV_C and defined as:

$$V_e = V_C - \Delta V_C \qquad \text{(eq. 10)}$$

This correction, although commonly called the compressibility correction, shouldn't be confused with going from incompressible to compressible flow equations. Recall that the airspeed equation is based on Bernoulli's equation for compressible flow. The correction is more truly a correction for the effect of altitude on displayed airspeed.

Look at the chart to the right. You see that this correction is quite significant at higher speeds and altitudes. However, for typical takeoff and landing speeds and altitudes, the correction is typically less than one half knot.

For typical cruise speeds and altitudes, ΔV_C will usually exceed 16 knots or so.

This correction is determined purely by comparing the values of airspeed from equations 7 and 9 and is valid for all airplanes.

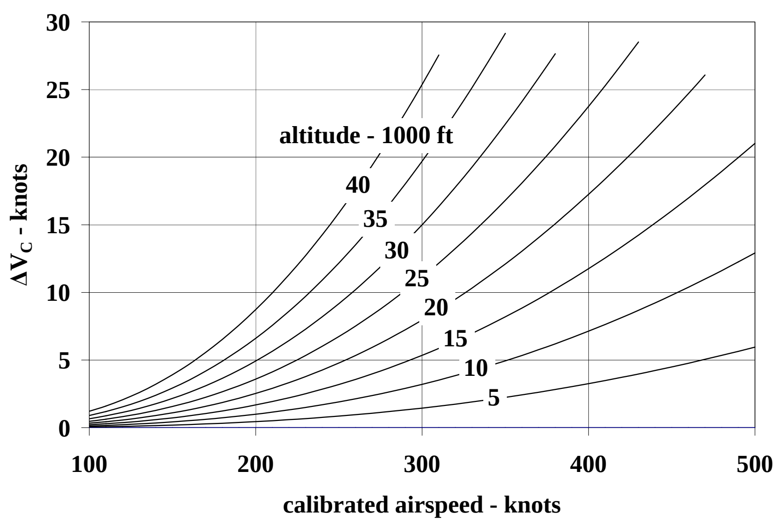


Figure 6-6

A more complete version of this chart is included for convenience in the appendix to this book.

true airspeed

We showed above that calibrated airspeed differs from true – actual – airspeed in that any value of air pressure other than the sea level standard day value introduces an error and, further, that any value of air density other than the sea level standard day value introduces an additional error.

Equivalent airspeed differs from true airspeed in that, while properly accounting for the effect of air static pressure, it still has an error because it uses the sea level standard day value of air density.

To find true airspeed from calibrated airspeed, therefore, it's first necessary to account for the correct value of p_s – that gives us equivalent airspeed – and then to account for the correct value of air density ρ. Only then will we know the true airspeed.

We already have shown how to calculate the equivalent airspeed: by subtracting ΔV_C from the calibrated airspeed. How do we go from equivalent airspeed to true airspeed?

Recall that true airspeed is given by:

$$V_{true} = \sqrt{\frac{2\gamma p_s}{(\gamma-1)\rho_s}\left[\left(\frac{p_t-p_s}{p_s}+1\right)^{\frac{\gamma-1}{\gamma}}-1\right]} \qquad \text{(eq. 4)}$$

and equivalent airspeed is given by:

$$V_e = \sqrt{\frac{2\gamma p_s}{(\gamma-1)\rho_0}\left[\left(\frac{p_t-p_s}{p_s}+1\right)^{\frac{\gamma-1}{\gamma}}-1\right]} \qquad \text{(eq. 9)}$$

From these two equations, we see that finding the true airspeed from the calibrated airspeed is just a matter of replacing the value of ρ_0 in the equation for EAS with the value ρ_s. That's easy: just multiply V_e by $\sqrt{\frac{\rho_0}{\rho_s}}$.

The term $\frac{\rho_0}{\rho_s}$ should be familiar to you: that's the reciprocal of the density ratio σ, which is $\frac{\rho}{\rho_0}$.

Therefore:

$$V_{true} = \frac{V_e}{\sqrt{\sigma}} \qquad \text{(eq. 11)}$$

equivalent airspeed and dynamic pressure "q"

Equivalent airspeed seems to be just an intermediate step between calibrated airspeed and true airspeed. Why not just create a correction to go directly from calibrated airspeed to true airspeed, and skip equivalent airspeed entirely?

First of all, we need temperature in order to calculate true airspeed; second, equivalent airspeed has a unique relationship with dynamic pressure. From the chapter entitled "Physics of Air", recall the following equation:

$$q = \frac{1}{2}\rho V_{true}^2$$

Remember also that $\sigma = \frac{\rho}{\rho_0}$ and therefore $\rho = \sigma\rho_0$ thus $q = \frac{1}{2}\rho_0\sigma V_{true}^2$

Recall from above that $V_{true} = \frac{V_e}{\sqrt{\sigma}}$ so $V_e = V_{true} \times \sqrt{\sigma}$ and $\sigma V_{true}^2 = V_e^2$

Substituting:

$$q = \frac{1}{2}\rho_0 V_e^2 \qquad \text{(eq. 12)}$$

From equation 12, you see that dynamic pressure is a direct function of equivalent airspeed – and that a constant equivalent airspeed produces a constant dynamic pressure q. These facts are very useful when computing aerodynamic forces, as you'll see in the chapter entitled "Lift and Drag".

Summarizing:

Let's summarize the different airspeeds we've examined in this chapter, and show their relationships.

1. Indicated airspeed, IAS or V_I, is the speed displayed on mechanical airspeed indicators.

$\Downarrow$

Apply position error correction ΔV_P: $V_I + \Delta V_P = V_C$

$\Downarrow$

2. Calibrated airspeed, CAS or V_C, is indicated airspeed corrected for position error. On airplanes having electric or electronic displays of airspeed, CAS is the speed displayed.

$\Downarrow$

Apply "compressibility" error ΔV_C: $V_C - \Delta V_C = V_e$

$\Downarrow$

3. Equivalent airspeed, EAS or V_e, is calibrated airspeed corrected for static pressure.

$\Downarrow$

Correct for the density effect: $\frac{V_e}{\sqrt{\rho}} = V_{true}$

True airspeed is equivalent airspeed corrected for density.

Useful Airspeed Conversion Equations

In performance work, it's frequently necessary to convert airspeed from one form to another. The following are some useful equations

Please note that equations 10 through 13 yield airspeeds in knots.

between CAS and EAS: (This requires knowing δ)

$$V_e = 1479.1\sqrt{\delta\left[\left(\frac{1}{\delta}\left\{\left[1+0.2\left(\frac{V_C}{661.4786}\right)^2\right]^{3.5}-1\right\}+1\right)^{\frac{1}{3.5}}-1\right]} \qquad \text{(eq. 13)}$$

$$V_C = 1479.1\sqrt{\left[\left(\delta\left\{\left[1+\frac{1}{\delta}\left(\frac{V_e}{1479.1}\right)^2\right]^{3.5}-1\right\}+1\right)^{\frac{1}{3.5}}-1\right]} \qquad \text{(eq. 14)}$$

between CAS and TAS: (this requires knowing δ and θ)

$$V_{true} = 1479.1\sqrt{\theta\left[\left(\frac{1}{\delta}\left\{\left[1+0.2\left(\frac{V_C}{661.4786}\right)^2\right]^{3.5}-1\right\}+1\right)^{\frac{1}{3.5}}-1\right]} \qquad \text{(eq. 15)}$$

$$V_C = 1479.1\sqrt{\left[\left(\delta\left\{\left[1+\frac{1}{\theta}\left(\frac{V_{true}}{1479.1}\right)\right]^{3.5}-1\right\}+1\right)^{\frac{1}{3.5}}-1\right]} \qquad \text{(eq. 16)}$$

between EAS and TAS: (this requires knowing σ, or δ and θ)

$$V_{true} = \frac{V_e}{\sqrt{\sigma}} \qquad \text{(eq. 17)}$$

or $$V_{true} = V_e\sqrt{\frac{\theta}{\delta}} \qquad \text{(eq. 18)}$$

$$V_e = V_{true}\sqrt{\sigma} \qquad \text{(eq. 19)}$$

or $$V_e = V_{true}\sqrt{\frac{\delta}{\theta}} \qquad \text{(eq. 20)}$$

Additional Discussion

discussion 1: derivation of equation 4

For convenience, the compressible Bernoulli equation is repeated here:

$$\frac{\gamma}{(\gamma-1)}\frac{p}{\rho} + \frac{1}{2}V^2 = \frac{\gamma}{(\gamma-1)}\frac{p_t}{\rho_t} = constant$$

Remember that the Pitot-static system measures impact pressure $p_t - p_s$. Thus we need to rearrange the Bernoulli equation to a form which uses impact pressure.

Rearranging the incompressible Bernoulli equation yields:

$$\frac{1}{2}V_{true}^2 = \frac{\gamma}{\gamma-1}\left(\frac{p_t}{\rho_t} - \frac{p_s}{\rho_s}\right)$$

At this point, we need to introduce something called the “isentropic flow equation”. This is based in thermodynamics and is beyond the scope of this document so will not be proven here, but briefly:

- “isentropic flow” means flow which is both adiabatic and reversible; in turn,
- “adiabatic flow” is one in which no heat is added or lost to or from sources external to the flow. For the purposes of airspeed measurement, the flow is considered to be adiabatic, and
- “reversible flow” is one in which there are no losses to friction or other causes of loss of energy. For airspeed measurement, the flow is considered to be reversible.

One way the isentropic flow equation can be stated is:

$$\frac{p_t}{p_s} = \left(\frac{\rho_t}{\rho_s}\right)^{\gamma} \quad \text{which can be rearranged to} \quad \rho_t = \rho_s\left(\frac{p_t}{p_s}\right)^{\frac{1}{\gamma}}$$

Substituting into the equation above:

$$\frac{1}{2}V_{true}^2 = \frac{\gamma}{\gamma-1}\left(\frac{p_t}{\rho_s\left(\frac{p_t}{p_s}\right)^{\frac{1}{\gamma}}} - \frac{p_s}{\rho_s}\right) = \frac{\gamma}{(\gamma-1)\rho_s}\left(\frac{p_t}{\left(\frac{p_t}{p_s}\right)^{\frac{1}{\gamma}}} - p_s\right)$$

Further re-arranging:

$$\frac{1}{2}V_{true}^2 = \frac{\gamma}{(\gamma-1)\rho_s}\left(\frac{p_t \times p_s}{p_s\left(\frac{p_t}{p_s}\right)^{\frac{1}{\gamma}}} - p_s\right) = \frac{\gamma p_s}{(\gamma-1)\rho_s}\left(\frac{\frac{p_t}{p_s}}{\left(\frac{p_t}{p_s}\right)^{\frac{1}{\gamma}}} - 1\right)$$

This yields:

$$\frac{1}{2}V_{true}^2 = \frac{\gamma p_s}{(\gamma-1)\rho_s}\left[\left(\frac{p_t}{p_s}\right)^{1-\frac{1}{\gamma}} - 1\right] = \frac{\gamma p_s}{(\gamma-1)\rho_s}\left[\left(\frac{p_t}{p_s}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} - 1\right]$$

So:

$$V_{true}^2 = \frac{2\gamma p_s}{(\gamma-1)\rho_s}\left[\left(\frac{p_t}{p_s}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} - 1\right]$$

and therefore:

$$V_{true} = \sqrt{\frac{2\gamma p_s}{(\gamma-1)\rho_s}\left[\left(\frac{p_t}{p_s}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} - 1\right]}$$

The only step remaining is to realize that:

$$\frac{p_t}{p_s} = \frac{p_t - p_s}{p_s} + 1$$

Substituting that into the equation above yields the final form of the airspeed equation in terms of impact pressure, static pressure and density:

$$V_{true} = \sqrt{\frac{2\gamma p_s}{(\gamma-1)\rho_s}\left[\left(\frac{p_t - p_s}{p_s} + 1\right)^{\left(\frac{\gamma-1}{\gamma}\right)} - 1\right]}$$

Chapter 7: Near the Speed of Sound

Introduction

In this chapter, we'll be looking at the concept of *sound*. We'll look at the speed at which sound travels, and some characteristics of aerodynamic flow near the speed of sound. We'll also introduce you to a number of new aerodynamic variables that are needed in performance calculations.

What Is Sound?

What we call *sound* is simply our perception of rapid fluctuations in the pressure of the air, sensed by our ears. This rapid fluctuation of air pressure can be caused by any of a countless number of sources.

Perhaps the easiest source to think of would be something like a bell. When it rings, its vibration causes it to send out pulses – "waves" – of slightly increased air pressure followed by "troughs" of slightly reduced air pressure.

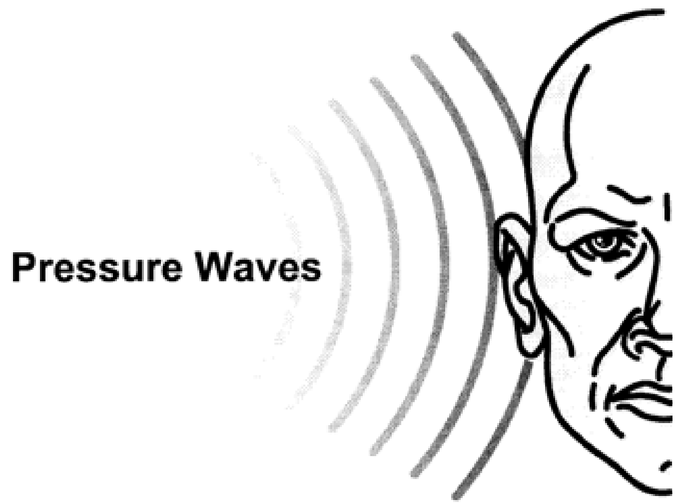


Figure 7-1

The spacing between these waves depends on the size of the bell. If the bell is smaller, the pressure waves will be closer together and we'll perceive the note as a high note; if, on the other hand, the bell is bigger, the waves will be farther apart and we'll perceive the note as a low note.

It's not really the *spacing* between the waves that we're sensing, it's *the number of waves per second* that we perceive as the note of the sound. The number of waves per second is inversely related to the spacing between the waves: closer-spaced waves mean that more waves will reach the listener per unit of time, waves spaced farther apart means that fewer waves will reach the listener per unit of time.

The number of waves per second is referred to as the *frequency* of a sound. Frequency is measured in cycles per second, referred to as hertz, abbreviated as Hz, named after German physicist Heinrich Hertz (1875-1894).

For example, when playing the note of middle C on a standard piano keyboard, the vibrating piano string sends out sound waves with a frequency of 261.6 hertz. If that frequency were doubled, we would perceive the note as one octave above middle C; if that frequency were halved, we would perceive the note as one octave below middle C. Most humans can readily hear sounds having frequencies between approximately 20 hertz and 16,000 hertz. Higher frequencies can be heard by most people, provided the loudness is above the hearing threshold level.

The Speed of Sound

So sound is nothing more than a series of pressure pulses traveling through the air. Just as waves in water travel at a predictable speed, so do waves of pressure in a gas such as air. The rate at which a pressure wave travels in a gas depends on how easily that gas can be compressed. The more easily a gas may be compressed, the slower will be the velocity of a pressure wave traveling in that gas.

It can be shown that the speed of sound in air, denoted as *a*, can be predicted from the equation

$$a = \sqrt{\gamma R T} \qquad \text{(eq. 1)}$$

where γ is the ratio of specific heats
R is the gas constant for air
T is the absolute temperature

If you want to see the derivation of equation 1, please refer to Discussion 1 in the "Additional Discussion" section at the end of this chapter.

Units are important here – you must take care to be consistent. First of all, it's essential to remember that the temperature of the air <u>must</u> be in units of <u>absolute</u> temperature, either degrees Kelvin or degrees Rankine. That's simply because the transmission of sound through a gas such as air depends on the amount of molecular activity of the gas molecules. That degree of activity is a function of absolute temperature. Molecular activity is zero at 0 °K (or 0 °R if using English units), not at 0 °C or 0 °F.

Second, care must be taken to use the value of R appropriate for the units of temperature. In English units, we already know that for air R has the value of 1716.5619.

If we use, for example, a value for *T* of 518.67 degrees Rankine (sea level standard day temperature in °R) then using equation 1 would give us a value for *a* of:

$$a_0 = \sqrt{\gamma R T} = \sqrt{1.4 \times 1716.5619 \times 518.67} = 1116.45 \textit{ feet per second}$$ [1]

Notice that in the equation above, since we used the sea level standard day temperature the speed of sound is shown as a_0. That's because the subscript *0* again implies the sea level standard day

1. How far away is that thunderstorm? Count the number of seconds between the flash of the lightning and the sound that follows it. Divide by five, and that's the approximate distance to the storm, in statute miles. Sure, because the speed of sound at the earth's surface is (approximately) one-fifth of a statute mile per second. Or, divide by three to get the approximate distance in kilometers...

condition, just as we use it in other variables such as p_0, T_0 and ρ_0. So the sea level standard day speed of sound a_0 is 1116.45 feet per second or 661.4786 knots.

If you wanted to derive the speed of sound in metric units, recall that the value of R then will be 287.0529 and T_0 will be 288.15. Remember also that γ is dimensionless, being the ratio of C_P divided by C_V so it's equally valid in either English units or metric units. For air, recall that the value of γ is 1.4. Thus, to find the speed of sound in metric units:

$$a_0 = \sqrt{\gamma R T} = \sqrt{1.4 \times 287.0529 \times 288.15} = 340.294 \text{ meters per second}$$

Let's simplify equation 1 further. Since both γ and R are constants, <u>the speed of sound is truly a function only of temperature</u>. The speeds of sound for two different temperatures will have the ratio of the square roots of the temperatures:

$$\frac{a_1}{a_2} = \frac{\sqrt{\gamma R T_1}}{\sqrt{\gamma R T_2}} = \sqrt{\frac{T_1}{T_2}} \quad \text{and thus:} \quad a_1 = a_2\sqrt{\frac{T_1}{T_2}}$$

For convenience, let's use the sea level standard day temperature, which we call T_0, as the value of T_2 in the equation above, and the sea standard day speed of sound a_0 as the value of a_2. We'll substitute those into the equation above, giving:

$$a_1 = a_0\sqrt{\frac{T_1}{T_0}}$$

From the chapter entitled "The Atmosphere" you will remember that the term $\frac{T}{T_0}$ is what we call θ. This allows us to make equation 1 into a really simple form:

$$a = a_0\sqrt{\theta} \qquad \text{(eq. 2)}$$

where a is the speed of sound at any temperature
a_0 is the sea level standard day speed of sound
θ is the temperature ratio $\frac{T}{T_0}$

Thus:

$$a = 1116.45\sqrt{\theta} \quad \text{for the speed of sound in feet per second} \qquad \text{(eq. 3)}$$

or:

$a = 661.4786\sqrt{\theta}$ for the speed of sound in knots.(eq. 4)

Since θ is a ratio of absolute temperatures, it will be the same regardless of whether you're working in English or metric units.

Mach Number

In commercial jet transport airplane performance work, the speed of sound has a profound effect, because the speed of the airplane compared to the speed of sound will affect the drag of the airplane, as you'll see in a later chapter.

Because the speed of an airplane compared to the speed of sound where it's flying is so frequently used, it's given the special name *Mach number*[1] and is designated as M.

Mach number, then, is defined as:

$$M = \frac{true\ airspeed}{speed\ of\ sound} = \frac{V_{true}}{a_0\sqrt{\theta}} \qquad \text{(eq. 5)}$$

Airspeed Conversions Involving Mach Number

NOTE: all of these conversion equations assume that the input and output will both be in units of either knots or Mach number.

between Mach and TAS: (this requires knowing θ)

$$M = \frac{V_{true}}{661.4786\sqrt{\theta}} \qquad \text{(eq. 6)}$$

$$V_{true} = 661.4786 \times M\sqrt{\theta} \qquad \text{(eq. 7)}$$

between Mach and CAS: (this requires knowing δ)

$$M = \sqrt{5\left[\left(\frac{1}{\delta}\left\{\left[1 + 0.2\left(\frac{V_C}{661.4786}\right)^2\right]^{3.5} - 1\right\} + 1\right)^{\frac{1}{3.5}} - 1\right]} \qquad \text{(eq. 8)}$$

1. Mach number is named after Ernst Mach (1838-1916) who was an Austrian-Czech physicist. His important explorations in the field of supersonic velocity resulted in a paper on this subject published in 1877. Mach deduced and experimentally confirmed the existence of shock waves at high speeds.

$$V_C = 1479.1\sqrt{\left(\{\delta[(0.2M^2+1)^{3.5}-1]+1\}^{\frac{1}{3.5}}-1\right)} \qquad \text{(eq. 9)}$$

between Mach and EAS: (this requires knowing δ)

$$M = \frac{V_e}{661.4786}\sqrt{\frac{1}{\delta}} \qquad \text{(eq. 10)}$$

$$V_e = 661.4786 \times M\sqrt{\delta} \qquad \text{(eq. 11)}$$

Aerodynamic Flow Near the Speed of Sound

Most commercial jet airplanes, as of the time of this writing, are cruising at speeds of Mach 0.86 or less. You might well ask, then, why this emphasis on aerodynamics near Mach 1?

It's true that the airplane may be cruising at Mach 0.85, but as you'll see in another chapter, at those speeds the aerodynamic flow over the wing will be a mix of subsonic (slower than the speed of sound) and supersonic (faster than the speed of sound) flows. Thus it's a good idea to get at least a fundamental idea of the characteristics of aerodynamic flow near Mach 1.

Aerodynamic flows near the speed of sound display some unique characteristics. To help understand them we need to introduce two new equations.

First, it's helpful to know how velocity will change with area in a stream tube. We have already looked at that for incompressible flow, repeated here for convenience from the chapter entitled "Physics of Air":

$$\frac{dV}{V} = -\frac{dA}{A}$$

The equation above is based on the assumption that the density of the air is constant. Now that we're close to the speed of sound, however, that equation becomes more complex because the assumption of incompressibility is no longer valid and compressibility becomes an important factor. Here's the above equation, re-written to include the effects of compressibility:

$$\frac{dV}{V} = \frac{1}{(M^2-1)} \times \frac{dA}{A} \qquad \text{(eq. 12)}$$

To see how equation 12 is derived, refer to discussion 2 in the "Additional Discussion" section at the end of this chapter.

What equation 12 tells us is that for compressible flow at subsonic speeds, $M<1$, dV/V is a negative number for a positive value of dA/A. This simply means that in subsonic flow, as the area of a stream tube increases, flow velocity will decrease; conversely, as stream tube area decreases, flow velocity will increase. That's not a surprise, we already knew that from the continuity equation.

But it also tells us that for supersonic speeds, $M>1$, dV/V will be a positive number for a positive value of dA/A. This fact is something we might not expect: that in supersonic flow, as area increases, velocity increases! Similarly, as area decreases, velocity decreases!

It is also helpful to know how static pressure will change with area in a stream tube. Here's that equation:

$$\frac{dp}{p} = \frac{\gamma M^2}{(1 - M^2)} \frac{dA}{A} \qquad \text{(eq. 13)}$$

If you want to see how equation 13 is derived, refer to discussion 3 in the "Additional Discussion" section at the end of this chapter.

Equation 13 also shows us that the behavior of supersonic flows is completely different from what we would expect in subsonic flows. In subsonic flows, we know that as stream tube area decreases, static pressure will decrease, and as stream tube area increases, static pressure will increase again. In supersonic flows the opposite is true: as area decreases, static pressure will increase, and as area increases, static pressure will decrease.

It's worth pointing out here that even though air behaves somewhat differently at high speeds than we're accustomed to at lower speeds, the overall pressure-velocity relationship is still valid, even at speeds greater than the speed of sound. That is, even when the Mach number is greater than one, when velocity increases static pressure decreases, and when velocity decreases the static pressure increases.

flow through a nozzle

With this knowledge in mind, let's look at the behavior of air in a nozzle. One source defines *nozzle* this way:

> *A nozzle is a mechanical device designed to control the characteristics of a fluid flow as it exits from an enclosed chamber into some medium...A nozzle is often a pipe or tube of varying diameter, and it can be used to direct or modify the flow of a liquid or gas. Nozzles are frequently used to control the rate of flow, speed, direction, and/or the pressure of the stream that emerges from them.*[1]

1. *from Wikipedia, the free encyclopedia, http://en.wikipedia.org/wiki/Nozzle*

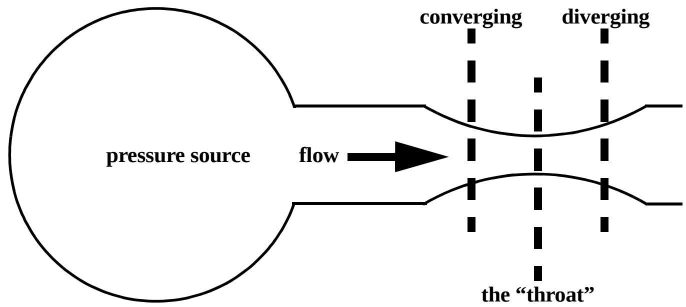


Figure 7-2

The illustration shows a simple system made up of a “pressure source” and a nozzle. One of the simplest examples of a pressure source is a tank containing air at some pressure greater than the ambient pressure outside the tank. The pressure source will send a flow of air along the pipe toward a nozzle which at first becomes narrower – “converging”, and then becomes wider again – “diverging”. The narrowest point of this *converging-diverging nozzle* is called “the throat”.

Suppose we have air at relatively low pressure stored in the source. When released, it will move toward the nozzle. As it enters the converging portion of the nozzle, the flow will accelerate. As it accelerates, the static pressure will decrease.

At the throat, the flow velocity will be at its greatest. Then, as it flows through the diverging portion of the nozzle, the flow will slow down again and the static pressure will increase. Equations 12 and 13 confirm this.

Suppose now that we gradually increase the pressure at the source. Clearly, as the source pressure increases the velocity at the throat will increase correspondingly. At some value of source pressure, referred to as the “critical” source pressure, we can expect the flow velocity at the throat to reach Mach 1, the speed of sound. What happens then?

Simply this: as the flow enters the diverging section past the throat, <u>it will continue to accelerate</u>, and the pressure will continue to decrease. Equations 12 and 13 also confirm this.

This seems to contradict what we have discussed up to this point, as well as seeming to go against common sense. However, no laws of physics have been violated. In fact, the velocity <u>must</u> increase, to maintain constant mass flow, because the gas laws show us that the air density decreases very rapidly after passing the throat as the nozzle diverges and supersonic flow begins.

From equation 13 and the continuity equation we find that:

$$\frac{d\rho}{\rho} = -M\frac{dV}{V}$$

This equation illustrates an important point: density always decreases as speed increases. At speeds greater than Mach 1.0, however, density decreases more rapidly than velocity increases. This balance occurs to maintain conservation of mass.

The problem is that up until this point, we’ve pretty much ignored density changes when thinking about flows. At these speeds, density change becomes a significant factor and flows do things that are markedly different from the behavior of slower flows.

The characteristics of converging-diverging nozzles discussed above can be utilized to advantage if we want to create supersonic velocities. This is how some supersonic wind tunnels operate. In a typical installation, large storage tanks are pumped up to high pressures, then the air is released, it flows into the nozzle, and at the nozzle the flow is accelerated to the desired supersonic test speed.

Normal Shock Waves

The discussion above shows how we can achieve supersonic flows of air. We haven't yet seen how we can <u>decelerate</u> that flow back to subsonic speeds on the way down to zero velocity where we started. No flow can go on indefinitely – sooner or later, somewhere, it has to slow down and, ultimately, come to rest.

Consider the simple supersonic wind tunnel you just learned to make. What happens to the flow after the test section, where you place the model of the airplane, wing, or whatever, in order to measure the aerodynamic forces in supersonic flow? Somewhere that supersonic flow has to decelerate – it can't just go on indefinitely.

Well, let's try this: exhaust the wind tunnel's flow into a very big empty chamber which is ventilated to atmospheric pressure. Clearly, this large empty chamber will contain air at essentially zero velocity, so somewhere after the test section, as the flow approaches the exhaust, it must decelerate back down to subsonic speeds and ultimately come to rest.

This deceleration from supersonic to subsonic flow will occur at an extremely thin stationary wave called a *normal shock wave*.

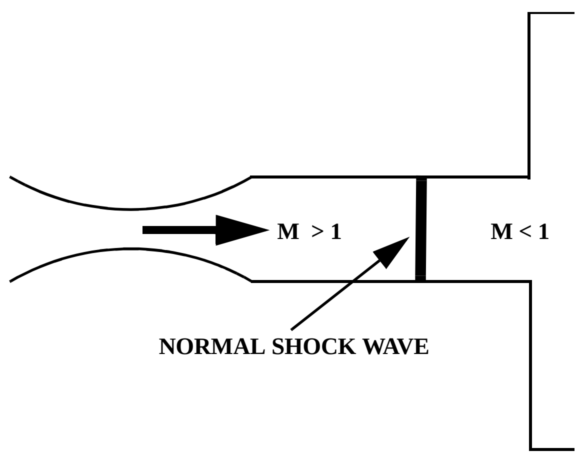


Figure 7-3

A "normal" shock wave is called that simply because it is "normal" to – perpendicular to – the direction of the flow. It's a rapid and drastic transition from supersonic to subsonic flow, creating a virtually instantaneous change in velocity, pressure, density and temperature – hence the term "shock", because of its suddenness. The shock wave is extremely thin. Across a normal shock, the velocity will decrease, and the static pressure, density and temperature will all increase. The shock wave is a very rapid, violent change to these parameters of the flow.

The location of the normal shock wave will depend on the pressure at the exhaust. Lower exhaust pressures will move the shock wave farther downstream, higher exhaust pressures will cause the shock wave to be farther upstream.

A shock wave is <u>not</u> an isentropic process. Much energy is lost as the flow transitions through the shock wave with this energy being lost in the form of heat. Because of the loss of energy in a shock wave, it's important in the design of the commercial jet airplane to attempt to minimize the size of the shock waves which form on it in high-speed flight.

Although in these paragraphs we're speaking of normal shock waves as they occur in nozzles or pipes, normal shock waves will also occur in open areas such as the upper and lower surfaces of the wing of a commercial jet airplane cruising at subsonic speeds. This will occur when the airplane's velocity is sufficiently high to drive airflows over the wing to supersonic speeds[1]. You'll see more about that in a subsequent chapter.

In a later chapter, you'll see very clearly just how drastically the airplane performance can be affected by shock waves.

Total Temperature, Pressure and Density

You'll recall that in the chapter entitled "Physics of Air", when we presented Bernoulli's equation for incompressible flow we introduced you rather briefly to the concepts of "static pressure", "total pressure" and "dynamic pressure". To review:

- We explained that when air is not moving, it exhibits only "static" pressure, which is the pressure we're accustomed to thinking about in normal life – for example, the pressure of the atmosphere which surrounds us.
- We went on to say that when air is in motion, the motion of the flow adds kinetic energy to the air, and that added kinetic energy is felt as an additional component of pressure, called dynamic pressure.
- We said that the sum of static pressure plus dynamic pressure is called total pressure, that is, the pressure that will be felt if a flow is brought to a complete standstill, such as occurs at the orifice of a Pitot probe.
- Finally, we told you that along a streamline total pressure is constant. As velocity increases along a streamline, static pressure decreases and dynamic pressure increases, keeping total pressure constant. As velocity decreases along a streamline, static pressure increases and dynamic pressure decreases, again keeping total pressure constant.

In a low-speed flow, which is usually taken as meaning Mach 0.3 or less, the air is compressed by a negligible amount, so we tend to forget about total density and temperature. As Mach numbers increase, however, compression of the air increases rapidly. This compression of the air results in measurable increases in its density and temperature as well as its pressure.

The concept of total pressure as the pressure at a point at which the flow is brought to rest, is just as true for high-speed flows as it is for low-speed flows. The compression of the fluid, however,

1. In fact, if you're lucky enough to be sitting in flight by a window above a wing, and the lighting conditions are just right, you can actually *see* the shock wave. The compression of the air in the shock wave causes a slight diffraction of the light passing through it. The shock wave can be seen as a faint straight line, ever so slightly darker than the background color, extending some distance above the wing. Watch for it next time you're sitting by the wing.

complicates the calculation of the total pressure: it's no longer simply the static pressure plus the dynamic pressure so a new equation for total pressure is needed. Further, the higher speeds introduce the need to account also for *total temperature* and *total density*, neither of which were necessary in low-speed flows.

The following paragraphs discuss these three parameters.

total temperature

When the velocity of a fluid flow is high enough to cause compression of the fluid, the temperature of the fluid will be increased.

At locations where the velocity of a flow is brought to a rest, the local temperature will be at its total value, simply because that's the point in a flow at which the pressure is at its total value as we've seen before.

One of the most noteworthy points at which flow velocities are brought to zero is at the "stagnation point", near the leading edge of a wing. (Because of this fact, wing leading edges of high-speed airplanes will be heated to temperatures substantially above the atmospheric static temperature value.[1])

The equations for total temperature in a compressible flow is:

$$T_{total} = T_{static} \times \left(1 + \frac{\gamma - 1}{2} M^2 \right) = T_{static} \times (1 + 0.2\ M^2) \qquad \text{(eq. 14)}$$

By this time, you should realize that in equation 14, it's essential to express the static temperature in absolute units, Kelvin or Rankine. The total temperature will then be in the same units. Celsius and Fahrenheit units may not be used in this equation.

You will very often see T_{total} referred to as "TAT", for "Total Air Temperature". Both terms appear frequently in performance calculations.

On commercial jet transport airplanes, knowledge of the air temperature outside the airplane is necessary as that's one of the parameters affecting the thrust settings on the engines. It's difficult, however, to measure static temperature accurately so high-speed airplanes measure the total temperature using a *TAT probe* and from this the static temperature can be computed by the air data computer using equation 14.

If you want to see the derivation of equation 14, please refer to discussion 4 in the "additional discussion" section at the end of this chapter.

1. An excellent example of this is the Lockheed SR-71"Blackbird", formerly operated by the United States Air Force, which was capable of sustained speeds exceeding Mach 3. Flying at an altitude of 80,000 feet at Mach 3, for example, the total temperature on a standard day would be greater than 330 °Celsius!

ram rise

An expression that is sometimes used by performance engineers is *ram rise*. This simply means the amount of temperature *increase* above the static temperature resulting from the airplane's velocity.

Looking back at equation 14, you'll see that it could be re-written as

$$T_{total} = T_{static} + (T_{static} \times 0.2\ M^2)$$

You can see immediately from this that the ram rise can be calculated simply from

$$ram\ rise = T_{static} \times 0.2 M^2 \qquad \text{(eq. 15)}$$

Again, it's essential to remember that this equation requires T_{static} in absolute temperature units.

As examples: an airplane cruising at Mach 0.85 at FL370 on a standard day (OAT = 214.8 °K) would experience a ram rise of 31 degrees K; the example we saw just a moment ago of the SR-71 "Blackbird" in cruise at Mach 3 at FL800 on a standard day (OAT = 216.6 °K) would experience a ram rise of 390 degrees K!

total pressure and total density

In a manner similar to equation 14 for the total temperature, shown above, equations can be developed for total pressure and total density as a function of Mach number:

$$p_{total} = p_{static} \times \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma-1}{\gamma}} = p_{static} \times (1 + 0.2\ M^2)^{3.5} \qquad \text{(eq. 16)}$$

$$\rho_{total} = \rho_{static} \times \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma-1}} = \rho_{static} \times (1 + 0.2\ M^2)^{2.5} \qquad \text{(eq. 17)}$$

Because of the loss of energy in a normal shock wave, the total pressure and the total density will decrease across the shock, whereas the total temperature remains constant.

total temperature, pressure and density ratios

In performance calculations we sometimes use the total values of temperature, pressure and density, but even more often we will use the total temperature, pressure and density *ratios*.

You remember, from the chapter entitled "The Atmosphere" that we discussed the variables δ (defined as p/p_0), σ (defined as ρ/ρ_0) and θ (defined as T/T_0). Since p, ρ and T have total values

p_{total}, ρ_{total} and T_{total} as well as static values, we can also define δ_{total}, σ_{total}, and θ_{total}. We do that by dividing p_{total}, ρ_{total} and T_{total} by their sea level <u>static</u> values, thus:

$$\delta_{total} = \frac{p_{total}}{p_0} = \delta_{static} \times (1 + 0.2\ M^2)^{3.5} \qquad \text{(eq. 18)}$$

$$\sigma_{total} = \frac{\rho_{total}}{\rho_0} = \sigma_{static} \times (1 + 0.2\ M^2)^{2.5} \qquad \text{(eq. 19)}$$

$$\theta_{total} = \frac{T_{total}}{T_0} = \theta_{static} \times (1 + 0.2\ M^2) \qquad \text{(eq. 20)}$$

As it is for low-speed flows in which $\delta = \sigma\ \theta$, it is equally true in high-speed flows that

$$\delta_{total} = \sigma_{total} \times \theta_{total} \qquad \text{(eq. 21)}$$

In following chapters, you'll see how all of these parameters appear in performance calculations.

Additional Discussion

discussion 1: derivation of equation 1

For the sake of this discussion, let's say that you're an observer whose sole job is to watch the conditions of the air immediately in front of you. Something causes a sound wave to pass you – some sharp loud noise causes a pulse of air to pass, and you observe the behavior of the air as it passes.

The wave passes you with a velocity *a*, which you'll remember is the designation for the speed of sound. Ahead of the wave, the pressure, density and temperature of the air are p, ρ, and T respectively.

Behind the wave, the source of the sound has caused the air pressure, density and temperature to change slightly, to values $p+dp$, $\rho+d\rho$, and $T+dT$. Since pressure, temperature, density and velocity are all related, then as you'd expect there's a slight change to the velocity as well; behind the wave, it's $a+da$.

We're going to examine what happens to a unit of area of that advancing wave front. That area won't change as the sound wave passes.

Two equations were developed in the chapter entitled "Physics of Air" which will help us to find out how to compute the velocity of the sound wave, a. One is the "continuity equation", repeated here, substituting the speed of sound a for velocity *V*:

$$\frac{d\rho}{\rho} + \frac{da}{a} + \frac{dA}{A} = 0$$

Since the area A is constant, this becomes

$$\frac{d\rho}{\rho} + \frac{da}{a} = 0 \qquad \text{and therefore } a = -\rho\frac{da}{d\rho}$$

The other useful equation is the "momentum equation" which says (again substituting *a* for *V*):

$$dp = -\rho\ a\ da \qquad \text{and thus:} \qquad da = -\frac{dp}{\rho a}$$

Combining these two yields:

$$a^2 = \frac{dp}{d\rho} \qquad \text{or} \qquad a = \sqrt{\frac{dp}{d\rho}}$$

We need one more fact before we can solve for a in a useful equation.

In the chapter entitled "Measurement of Airspeed" we mentioned briefly something called the "isentropic flow equation". Once again, without dwelling on the theory behind it, since that's a bit beyond the scope of this book, here it is:

$$\frac{p_t}{p_s} = \left(\frac{\rho_t}{\rho_s}\right)^{\gamma} \text{ and out of this comes: } \frac{p}{\rho^{\gamma}} = constant$$

Because we can assume that the flow across the pressure wave is isentropic, we can substitute this into the equation for a:

$$\frac{dp}{d\rho} = \frac{d}{d\rho}(c\rho^{\gamma}) = c\gamma\rho^{\gamma-1}$$

And since:

$$c = \frac{p}{\rho^{\gamma}} \qquad \text{then:} \qquad \frac{dp}{d\rho} = \left(\frac{p}{\rho^{\gamma}}\right)\gamma\rho^{\gamma-1} = \frac{\gamma p}{\rho}$$

Recall the equation of state:

$$p = \rho RT$$

Substituting this into the equation above gives:

$$\frac{dp}{d\rho} = \gamma RT \quad \text{and, finally,} \quad a = \sqrt{\gamma RT}$$

---

discussion 2: derivation of equation 12

Recall the continuity equation, from the chapter entitled “Physics of Air”:

$$\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0$$

We have also seen, in the “Additional Discussion” section, that $a^2 = \dfrac{dp}{d\rho}$

One more: remember the “momentum equation” which states that: $dp = -\rho V dV$

Substituting:

$$a^2 d\rho = -\rho V dV \quad \text{or} \quad \frac{d\rho}{\rho} = -\frac{VdV}{a^2}$$

substituting this into the continuity equation:

$$\frac{dA}{A} + \frac{dV}{V} + \frac{VdV}{a^2} = 0$$

Multiply the third term of that equation by *V/V* and substitute *M* for *V/a*, giving:

$$\frac{dV}{V}(1 - M^2) = -\frac{dA}{A} \quad \text{and thus:} \quad \frac{dV}{V}(M^2 - 1) = \frac{dA}{A}$$

$$\frac{dV}{V} = \frac{1}{(M^2 - 1)} \times \frac{dA}{A}$$

---

discussion 3: derivation of equation 13

The “momentum equation” can be re-written as:

$$VdV = -\frac{dp}{\rho} \quad \text{thus:} \quad \frac{VdV}{V^2} = -\frac{dp}{\rho V^2} \quad \text{therefore:} \quad \frac{dV}{V} = -\frac{dp}{\rho V^2}$$

Remember also that we saw above that:

$$a = \sqrt{\frac{\gamma p}{\rho}} \quad \text{thus} \quad \rho = \frac{\gamma p}{a^2} \qquad \text{(eq. 14)}$$

Substituting equation 9 into equation 8 yields:

$$\frac{dV}{V} = -\frac{dp}{\gamma p} \times \frac{a^2}{V^2}$$

You recognize the last term of that equation as $\frac{1}{M^2}$

Thus:

$$\frac{dV}{V} = -\frac{dp}{\gamma p} \times \frac{1}{M^2} \qquad \text{(eq. 15)}$$

Substituting equation 10 into equation 7 yields:

$$\frac{dp}{\gamma p M^2} = \frac{dA}{A} \times \frac{1}{(1 - M^2)}$$

and therefore:

$$\frac{dp}{p} = \frac{\gamma M^2}{(1 - M^2)} \frac{dA}{A} \qquad \text{(eq. 16)}$$

---

discussion 4: derivation of equation 14

Start with the incompressible Bernoulli equation, repeated here for convenience:

$$\frac{\gamma}{\gamma - 1} \frac{p_1}{\rho_1} + \frac{1}{2}V_1^2 = \frac{\gamma}{\gamma - 1} \frac{p_2}{\rho_2} + \frac{1}{2}V_2^2$$

Total temperature and total density are experienced at locations where velocity is zero, thus:

$$\frac{\gamma}{\gamma - 1} \frac{p_{total}}{\rho_{total}} = \frac{\gamma}{\gamma - 1} \frac{p}{\rho} + \frac{1}{2}V^2$$

Recall from above that: $a = \sqrt{\dfrac{\gamma p}{\rho}}$ and therefore: $a^2 = \dfrac{\gamma p}{\rho}$ and: $a_{total}{}^2 = \dfrac{\gamma p_{total}}{\rho_{total}}$

Substituting:

$$\frac{a_{total}^2}{\gamma - 1} = \frac{V^2}{2} + \frac{a^2}{\gamma - 1}$$

Divide both sides by a^2:

$$\frac{1}{\gamma - 1}\,\frac{a_{total}^2}{a^2} = \frac{1}{2}\,\frac{V^2}{a^2} + \frac{1}{\gamma - 1}$$

We know that: $a = \sqrt{\gamma RT}$ therefore: $a^2 = \gamma RT$ and it follows that: $a_{total}^2 = \gamma RT_{total}^2$

Then: $\dfrac{a_{total}^2}{a^2} = \dfrac{T_{total}}{T}$. We also know, by definition, that: $M^2 = \dfrac{V^2}{a^2}$

Substituting:

$$\frac{1}{\gamma - 1}\,\frac{T_{total}}{T} = \frac{M^2}{2} + \frac{1}{\gamma - 1} \quad \text{and thus} \quad \frac{T_{total}}{T} = 1 + \frac{\gamma - 1}{2}\,M^2$$

Thus:

$$T_{total} = T \times \left(1 + \frac{\gamma - 1}{2}\,M^2\right)$$

Chapter 8: Flow Near a Surface

Introduction

In this chapter, we want to introduce you to the subject of viscous and non-viscous ("inviscid") flows near and over a surface such as a wing. We'll be introducing you to a parameter called the "Reynold's number" which will be seen later as part of the calculation of the drag of an airplane. We're going to keep the discussion relatively simple; when we're done you'll understand how viscosity affects flows near a surface, and we'll have introduced you to laminar and turbulent boundary layers. Then you'll be ready for the following chapters which discuss flow over an airfoil.

Viscosity

In the chapter entitled "Physics of Air" we described viscosity as a fluid's resistance to shearing deformation. We noted that the resistance is in the form of a shear stress that is proportional to the rate of shearing deformation and that there is no shear stress when the fluid is at rest.

We also talked about how fluid molecules interact with a solid surface in such a way that the fluid velocity goes to zero at the surface, something we call the "no-slip condition".

The shear stress that results from viscosity is often called the *internal friction* within a fluid, and it's sometimes described as the friction of layers of fluid sliding past each other. But we should be careful to remember that fluids aren't really make of layers, and that there isn't really any sliding going on. Fluids undergo continuous shearing deformations, and the shear stress that results is kind of like sliding friction, but it isn't the result of any sliding. And, of course, the no-slip condition means that there is no sliding between the fluid and a solid surface.

Now let's look at what happens when fluid flows past a flat surface. The fluid is free to deform, and the velocity can therefore vary with distance from the surface. We'll look first at how the velocity would vary if the no-slip condition weren't there, and then at how it actually varies in real fluids with the no-slip condition.

fluid flow over a surface without a no-slip condition

If we had some imaginary kind of surface that didn't impose a no-slip condition on the fluid, the fluid would be able to slide effortlessly along the surface and would not be required to undergo any shearing deformation. The velocity would be constant with distance from the surface, as illustrated in Figure 8-1. We'll call this the velocity of the local free stream, V_0.

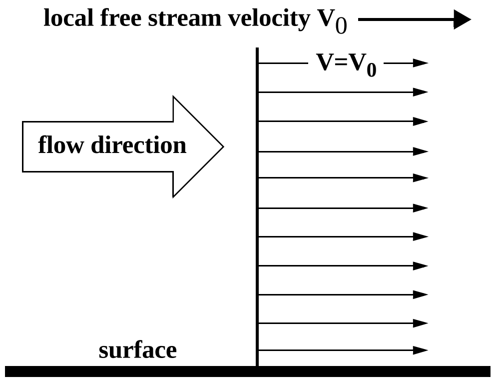


Figure 8-1

fluid flow over a surface with a no-slip condition

Flow over a real surface obeys the no-slip condition that the velocity must go to zero at the surface. The distribution of velocity (the velocity profile) is as shown in Figure 8-2, in which the velocity starts at zero at the surface and increases smoothly from there upward, gradually approaching the free stream value V_0.

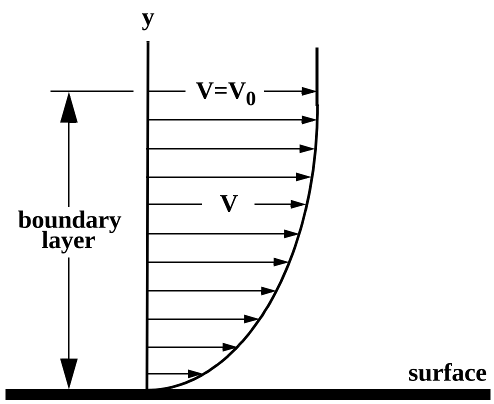


Figure 8-2

The fluid at any given distance from the surface is undergoing shearing deformation at a rate proportional to the velocity gradient. The entire layer of flow that is being deformed in this way is called the *boundary layer.* It is the part of the flow that is affected by the presence of the surface, through the no-slip condition and viscosity.

The shearing deformation in the boundary layer causes shear stresses. Any layer of fluid within the boundary layer is being dragged backward by the shear stress at its bottom, and is being dragged forward by the shear stress at its top. Because of the curvature of the velocity profile shown in FIgure 8.2, the retarding force at the bottom of any layer is stronger than the pulling-forward force at the top, and the layer is thus slowed down. The boundary layer thus grows thicker as it flows along, as shown in Figure 8-3 below.

So when fluid is forced to move past a solid surface, a boundary layer forms in which the fluid undergoes a shearing deformation because it can't slip, and the shearing deformation results in shear stress, because of viscosity. At the interface where the fluid touches the surface, the shear stress is transmitted to the surface and is called the *skin friction* even though there is no slipping of the fluid relative to the surface. The skin friction is a major part of aerodynamic drag.

coefficient of viscosity μ

In a non-uniform flow like the one shown above, there exists shear stress within the fluid. This stress, denoted as τ is a function of the viscosity coefficient, denoted as μ, and the slope of the curve of velocity versus height:

$$\tau = \mu \frac{dV}{dy} \qquad \text{(eq. 1)}$$

where μ is the *viscosity coefficient*.

viscosity coefficient μ *for air*

The viscosity coefficient for air is assumed to vary with absolute temperature in accordance with *Sutherland's equation*:

$$\mu = \frac{0.3125059 \times 10^{-7} \times T^{1.5}}{T + 120} \qquad \text{(eq. 2)}$$

Where T is the absolute temperature in degrees K

The units of μ in equation 2 are slugs per foot-second, English units, but please be reminded that the temperature in Sutherland's equation is entered in degrees Kelvin.

The sea level standard day values of μ are: 3.7373×10^{-7} slug/ft-sec in English units
1.7894×10^{-5} kg/m-sec in metric units

You'll observe from equation 2 that for air, viscosity increases as temperature increases. This is contrary to what you might expect, since you probably have observed that some fluids, such as motor oil, exhibit viscosity which decreases with increasing temperature.

Because of this effect of temperature on the viscosity of air, you'll see in a later chapter that total airplane drag is greater on hot days than on cold days.

laminar and turbulent boundary layers

We have defined a *boundary layer* as that finite layer of fluid next to a surface which is slowed relative to the local free stream velocity of the flow.

The thickness of the boundary layer depends on a number of factors: the viscosity of the fluid, the distance that the fluid flow has traveled along the surface from the point at which it first came into contact with the surface, and the type of flow in the layer.

We said "the type of flow in the layer" for a reason: there are really two different kinds of boundary layers:

- The *laminar boundary layer,* characterized by smooth flow, and

- The *turbulent boundary layer* in which flow is, as the name implies, turbulent and chaotic.

The aerodynamic theory concerning boundary layers is extremely complex and the subject is still not fully understood. It’s beyond the scope of this book to try to teach boundary layer theory, and this knowledge wouldn’t really be useful to the performance engineer anyhow. Let’s summarize what you should know this way:

- Aerodynamic flows over a surface begin as laminar flows; at some point, the flow will transition from laminar to turbulent flow;
- The point at which the transition occurs depends on many factors;
- Turbulent boundary layer flow causes a large increase in the shear forces at the surface. For this reason, from a drag standpoint the design of aerodynamic surfaces should be such as to delay the transition from laminar to turbulent flow as far as possible;
- Most aerodynamic flows over the typical commercial jet transport airplane are turbulent flows;
- The thickness of the boundary layer depends on the viscosity of the fluid, the distance along the surface, and the nature of the flow in the boundary layer;
- The thickness of the boundary layer affects the pressure distribution around an object. The result of this fact will be seen later on when we analyze the components of drag;
- Transition from laminar to turbulent flow depends, among other things, on a parameter called the *Reynolds number,* which we’ll define below;
- The aerodynamic drag of an airplane thus depends, to some small degree, on the Reynolds number for a given set of flight conditions.

One way in which laminar and turbulent flow are often demonstrated is by examining the way smoke rises from the end of a lit cigarette. You have probably observed this yourself: at first, the smoke rises smoothly in neat streamlines but at some point a few inches above the cigarette the flow suddenly changes and becomes disorganized, chaotic. The first part of the flow is the laminar flow, then after the transition point the flow is turbulent.

This illustrates that turbulent boundary layer flow is not the result of high speed in a flow. It can occur at low speeds also, such as the example given of cigarette smoke. Laminar to turbulent transition is a complex phenomenon which depends on a number of factors.

The flow over a surface, which could be a flat plate or anything similar such as an airplane wing, looks something like Figure 8-3. We have greatly exaggerated the thickness of the boundary layer and the width of the transition region for the purposes of this illustration.

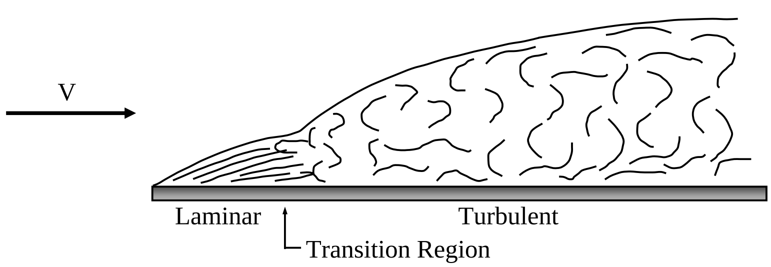


Figure 8-3

At the leading edge of the plate, the flow is laminar. The boundary layer begins at zero height at the leading edge, and gradually increases in thickness as the distance from the leading edge increases. This is the laminar region. At some point, the laminar flow becomes unstable and wave-like disturbances begin to grow. When these disturbances become large enough, they break down into the chaotic motion we call turbulence. This process is called *laminar-to-turbulent transition* and the region in which it takes place is called the *transition region*.

Beyond the transition region, the flow exhibits entirely unsteady chaotic fluid particle motion. This region is called the turbulent boundary layer region.

skin friction drag in the laminar and turbulent regions

The total drag of an airplane is the sum of a number of different components of drag. We'll be discussing the components of drag in a later chapter.

One of those components is called *skin friction drag*. It might just as well be called "viscous shear force drag", for that's what causes it. Remember that it's the nature of viscosity to retard flow across a surface. That "retardation", that viscous shear force, is – in a word – drag.

We showed you earlier in this chapter a sketch of the velocity profile of the air in the boundary layer. You saw that it was characterized by zero velocity at the interface between the surface and the fluid, and then the velocity gradually increased until at some height above the surface it was equal to the local free stream velocity.

The velocity profile in a turbulent boundary layer is markedly different: again the velocity is zero at the surface-fluid interface, but with increasing height above the surface the velocity increases much more rapidly, then it flattens out as it approaches the free stream velocity.

We also showed you that the shear force τ depends directly on the slope of the velocity profile, dV/dy. That slope is less in the laminar region, greater in the turbulent region.

This tells us that shear forces at the surface-fluid interface in the turbulent boundary layer are substantially greater than in the laminar boundary layer. For this reason, the viscous drag of the air over the surface is greater in the turbulent region than it is in the laminar region. For this reason, as first stated above, from a skin friction drag standpoint its desirable to strive for laminar flows to the greatest degree possible.[1]

Reynolds number

The thickness of the boundary layer, in both the laminar and turbulent regions, as well as the transition location, can be predicted and expressed in terms of the parameter mentioned above called the *Reynolds number*, RN. Reynolds number is a dimensionless parameter which is extremely important in viscous flow. It's defined as:

1. To make the subject more confusing, you'll see in a later chapter when we analyze all the components of drag that one component of drag, called *pressure drag*, can actually be reduced by causing the boundary layer to transition to turbulent flow sooner rather than later.

$$\text{Reynolds number } RN = \frac{\rho VL}{\mu} \qquad \text{(eq. 3)}$$

Where ρ is the density of the fluid
V is the free-stream velocity
μ is the viscosity of the fluid
L is a distance

The Reynolds number is often described as indicating the ratio of inertial forces to viscous forces. This isn't quite right. The Reynolds number does not indicate the ratio of the detailed forces, but it does indicate how fast the effects of viscosity spread away from the surface – that is, how thick the boundary grows. For example, if the Reynolds number is increased through an increase in velocity on a surface of given length, the boundary layer generally becomes thinner. The location of laminar-to-turbulent transition depends on the Reynolds number, among other things.

critical Reynolds number

In equation 3, you saw that one variable affecting Reynolds number is *L*, a unit of length or distance. That helps us to estimate the location for the transition from laminar to turbulent flow.

Looking at the sketch on the right, showing flow over a flat surface, we have set L equal to zero at the leading edge of the plate. Thus as the flow progresses downstream from the leading edge, L is increasing and therefore the Reynolds number is increasing as well.

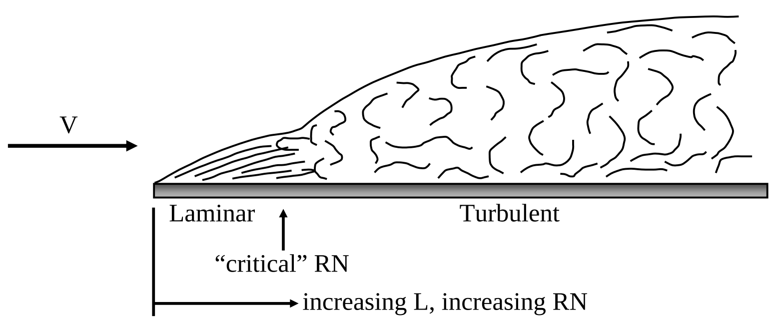


Figure 8-4

At some point, we will have reached a value of RN at which the process of transition to turbulent flow is complete. This is often called the *critical Reynolds number.*

The value of the critical Reynolds number is a function of many variables. It is beyond the scope of this book to attempt to analyze the topic further. Empirical data shows that for ideal conditions, on a typical airfoil, the critical Reynolds Number is on the order of several million. One class of airfoils known as *laminar flow airfoils* are specifically designed to delay the transition from laminar to turbulent and by doing so to reduce drag.

We'll be discussing airplane drag in a later chapter, and you'll see there how we can apply a correction to the calculated drag called $\Delta C_{D_{RE}}$ in order to account for viscosity effects. For now, it's sufficient that you have a rough idea of laminar and turbulent boundary layers, Reynolds number, and the influence of viscosity on airplane drag.

Chapter 9: Lift and Drag

Introduction

In this chapter, we'll be introducing you to the concepts of drag and lift. Probably you already have a fundamental idea of those two parameters, which are certainly two of the four most important parameters in all performance work. Here, we're going to show you the sources of lift and drag, and how they are reduced to a "force coefficient" format for ease of use.

It is not the purpose of this chapter to discuss in detail exactly what causes aerodynamic lift and drag. There are a number of excellent college-level textbooks which explore this subject thoroughly.[1] In this chapter, we're just going to present a simplified discussion and show how the force and moment coefficients are calculated and used.

Ideal and Real Fluids

We're about to examine the aerodynamic forces generated by a fluid flow over a shape. We'll begin by discussing the flow over a cylindrical shape, then we'll look at a simple airfoil shape.

We will start by discussing how the shape behaves in a flow of *ideal* fluid, then we talk about the behavior of the same shape in a *real* fluid. Doing this helps us to understand the nature of the forces which are generated on an airfoil by a fluid flow.

Here's what we mean by ideal and real:

- An *ideal* fluid is one that is incompressible, and which displays no viscosity.
- A *real* fluid is one that is compressible, and does display viscous properties.

Flow over a Cylinder

flow over a cylinder in an ideal fluid

We'll begin by discussing a simple flow: fluid flow over a cylinder. By *cylinder,* we mean a long object having a cross-sectional circular shape of a constant radius along its length. An example of this might be a circular rod of wood or some other substance.

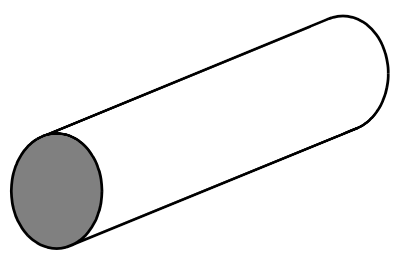


Figure 9-1

If we could look at the streamlines of a non-viscous flow around a cylinder, they would look something like this sketch.

1. We can recommend in particular "Introduction to Flight", Fifth Edition, by John D. Anderson, Jr., published by McGraw-Hill, ISBN 0-07-282569-3, available through Amazon.com and other sources.

Point 1 on the cylinder is the "stagnation point" which we mentioned in the previous chapter. The streamline which comes to the cylinder at the stagnation point comes to a complete stop. The pressure, temperature and density at that point are all at their total values: p_t, T_t and ρ_t.

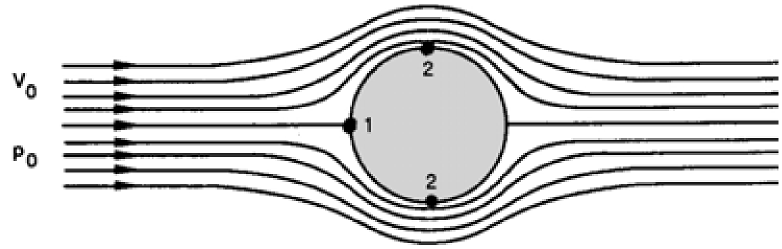


Figure 9-2

All streamlines above the stagnation point streamline will pass up and over the cylinder. All streamlines below the stagnation point streamline will pass down and under the cylinder. At the points marked 2, the width seen by the flow reaches its maximum value.

Bernoulli's equation predicts that where the velocity is greatest the static pressure will be at its lowest value. Let's map the distribution of static pressure around the cylinder, as shown in the following diagram.

The arrows represent the pressure acting on the surface of the cylinder. The length of the arrows show the magnitude of the pressure at that point. Note that all arrows are perpendicular to the surface of the sphere because pressure only acts perpendicular to a surface.

The direction of the arrows indicates whether the pressure is greater than or less than the free stream pressure p_0. An inward-facing arrow denotes locally higher pressure, an outward-facing arrow denotes locally lower pressure.

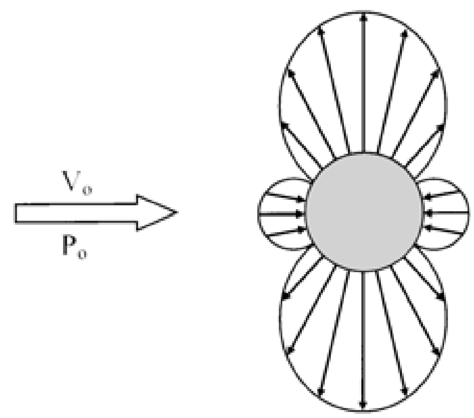


Figure 9-3

As you would expect, the pressure at the sides of the cylinder are markedly reduced from the free stream pressure because of the locally increased velocity of the flow over the cylinder. The pressure at the front of the cylinder is locally increased.

Because there is no viscosity in this illustration, the pressure distribution at the back of the cylinder is the same as it is at the front. This is simply because, in the absence of viscosity, the flows will flow effortlessly around the cylinder, leaving the flow symmetric front-to-back.

In this example, since the pressures in the vertical direction are symmetric, and the flows in the horizontal direction are symmetric, <u>there is no fluid dynamic force developed by the cylinder in this non-viscous flow</u>.

flow over a cylinder in a viscous fluid

Clearly, the assumption of inviscid flow is unrealistic, because it predicts that there will be no net horizontal force generated by the flow over the cylinder. Let's show the flow again, this time assuming a flow in a viscous medium such as air.

This is more realistic. Note that in the viscous flow, the pressure distribution at the front and at the sides of the cylinder is essentially the same as in the non-viscous flow, but at the rear of the cylinder the pressure is now negative. There is now a net rearward force.

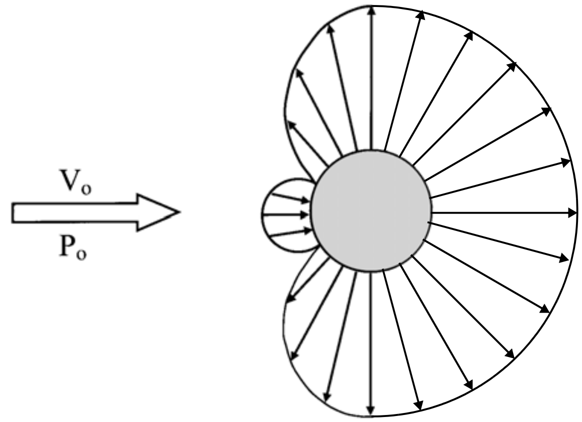


Figure 9-4

lift and drag defined

At this point, we are going to define two new terms: *lift* and *drag*. You've probably heard of lift and drag, of course, but let's give them rigorous definitions:

- *Lift* is defined as a force produced by the flow of a viscous fluid over a body, acting in a direction perpendicular to the free stream direction of the flow.
- *Drag* is defined as a force produced by the flow of a viscous fluid over a body, acting in a direction parallel to the free stream direction of the flow.

lift and drag forces on the cylinder in viscous flow

In the pressure distribution diagram shown above, the pressures are symmetric in the vertical direction, so the cylinder is not generating any lift – any force perpendicular to the free stream flow direction.

However, in the diagram above you can see that there *is* a net horizontal force – drag –because the pressures acting rearward are greater than the pressures acting forward. The cylinder is therefore generating drag, but not lift.

components of drag on the cylinder

The discussion above has stated that the cylinder, in viscous flow, generates drag. In fact, it generates two separate components of drag:

- *pressure drag* is the component of drag due to the asymmetric distribution of pressures in the direction parallel to the direction of flow of the free stream;
- *skin friction drag* is also being generated by the cylinder. You'll recall that we discussed skin friction drag in the chapter entitled "Flow Near a Surface", saying that it is the result of viscous shear forces.

separation of flow over the cylinder

In the illustration at the right, flow is shown again over a cylinder. As the flow approaches the cylinder and as it begins its path around the cylinder, the flow remains smooth, and a laminar boundary layer will prevail.

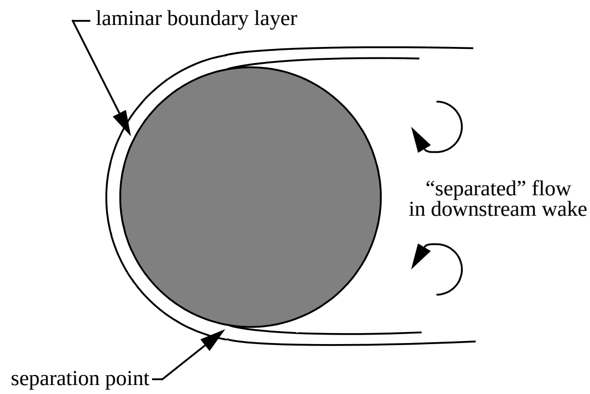


Figure 9-5

The boundary layer, for the conditions of this example, will remain laminar until some point after it has passed the peak of its path. At that point, called the *separation point*, the boundary layer separates from the cylinder and dissipates downstream. What's left after the boundary layer has separated from the cylinder is a large area of "separated" turbulent flow, referred to as the *wake*. A "wake" can be defined as that region of turbulence immediately behind a solid body caused by the flow of fluid around the body. In this area of separated flow, static pressure is low compared to the free stream pressure.

The end result of this sort of flow with early separation is that <u>the skin friction drag component is very small but the pressure drag component is very large</u>, due to the large separated wake downstream of the cylinder. The skin friction drag component is small because the boundary layer is laminar.

Compare the illustration above to this one:

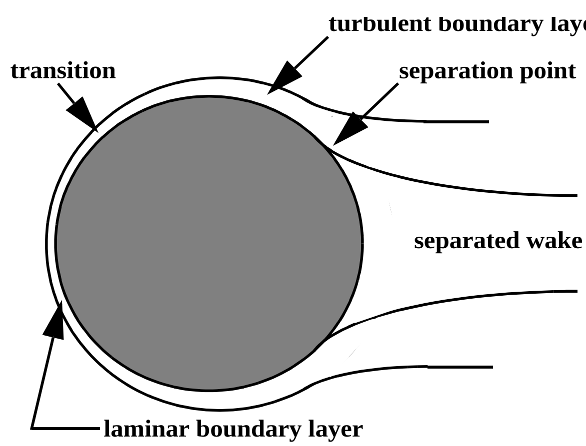


Figure 9-6

In this flow, the laminar boundary layer has transitioned to a turbulent state very early. The point at which the boundary layer will separate from the cylinder is much farther along in the flow. The separated wake is much smaller.

In this case, the skin friction drag is greater than it was in the earlier case, because we know that turbulent boundary layers create greater levels of skin friction drag. On the other hand, though, the pressure drag will be much less since the separated wake is much smaller.

<u>The total drag of this second case will be less than the total drag of the first case above</u>.

Reynolds number effect on separation point and drag

What's the difference between the two cases? <u>The Reynolds number</u>. In the second case, a higher Reynolds number causes the boundary layer to transition to turbulent sooner. However, a turbulent boundary layer, because of its greater viscous forces, will tend to remain attached to the body longer. It will separate from the body farther along the streamline, leaving a smaller separated wake.

As a result, in a blunt shape like this cylinder, it can be advantageous to seek an early transition to turbulent flow.[1]

streamlining for improved aerodynamic drag

The examples we've been examining of flow over a cylinder demonstrate that blunt shapes such as a cylinder will have relatively strong separated wakes with subsequent high values of pressure drag. This is why you'll seldom see blunt shapes on airplanes. Instead, designers will reduce pressure drag by "streamlining" a blunt shape.

For example, in place of the cylinder that we've been studying, we could substitute something better designed for reduced drag.

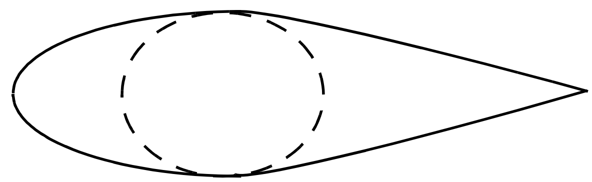


Figure 9-7

A "streamlined" shape like this one, although having the same thickness as a cylinder, will have much less drag – a savings of perhaps as much as 80% of drag. Why? The design produces a much more favorable pressure distribution which will delay the separation of the flow until far downstream along the body. As a result, the separated wake behind the shape will be much less strong, and skin friction drag will predominate.

Depending on the Reynolds number, a streamlined shape will have an optimum ratio of its length to its thickness. If the length-thickness ratio is too low, meaning that the shape is too thick in comparison to its length, then pressure drag would predominate; if the length-thickness ratio is too high then there will be excessive amounts of area and friction drag will predominate. Typical optimum length-thickness ratios are around 2.5.

summarizing:

The main things to learn from the discussion up to this point is this:

- Viscosity provides two sources of drag.
- The first source of drag is skin friction drag from viscous shearing forces. You'll recall that viscous skin friction drag is discussed in the chapter entitled "Flow Near a Surface"
- The second source of drag is "pressure drag" due to the change in the overall pressure distribution around the shape.
- Careful design can reduce total drag by finding the best compromise between skin friction drag and pressure drag.

1. *This is why golf balls have dimples*! The dimples will cause the boundary layer to transition sooner to a turbulent condition, resulting in a smaller separated wake and hence less drag. The dimples make the ball capable of traveling farther when struck than a ball not having dimples. A ping-pong ball has more drag than a golf ball!

Flow Over a Symmetric Streamlined Shape

flow over a symmetric streamlined shape in a non-viscous fluid

Now that you've seen flow over a cylinder, let's examine flow over a symmetric streamlined shape. First, we'll look at it in non-viscous flow.

In the drawing to the right, you see that, as in the case with the cylinder in non-viscous flow, the pressure distribution is symmetric, both vertically and horizontally. As a result, there is no lift or drag generated by the airfoil.

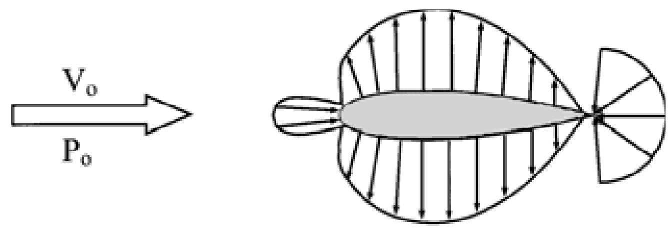


Figure 9-8

flow over the symmetric streamlined shape in a viscous fluid

If we add viscosity, the pressure distribution is different, as you would expect after seeing the example of flow over the cylinder.

Now, in this case of the symmetric streamlined shape at no angle to the flow, the lift is still zero, but there *is* a net horizontal force, the drag of the shape.

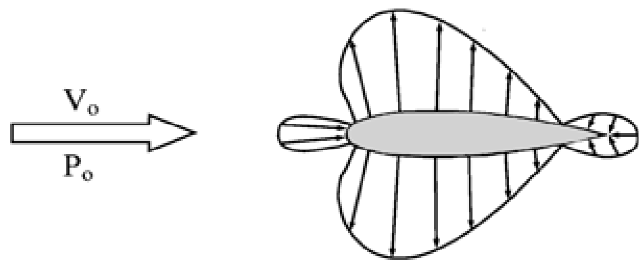


Figure 9-9

Airfoils

Up to this point in the chapter, we haven't seen any discussion of lifting force. That's simply because we've been looking at symmetric shapes, and, in the case of the symmetric shape just above, the shape has been presented at no angle to the direction of fluid flow.

While the streamlined shape shown above is actually capable of developing lift force when placed at an angle to the flow, it's not well-designed for typical commercial jet transport airplane applications.[1] Now it's time to look at some real practical airfoils to see their lift and drag characteristics.

airfoil definitions

When looking at airfoils, some terminology is needed.

1. However, it's worth pointing out that some aerobatic airplanes do have symmetric airfoils, since they need to be capable of operating as well in inverted (upside down) flight as in normal flight attitudes.

The *mean line* is a line equidistant from the upper and lower surfaces. The *chord line* is the straight line joining the intersections of the mean line with the leading and trailing edges of the airfoil. It is usually referred to simply as the *chord* of the airfoil. *Camber* is the term used to describe the curvature of the airfoil. The *maximum camber* is the greatest deviation of the mean line from the chord line.

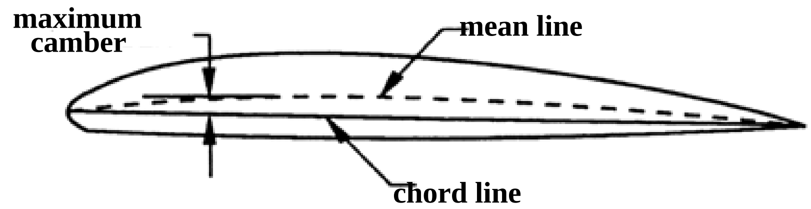


Figure 9-10

The precise shape of an airfoil is described by a series of *stations* and *ordinates*. Stations and ordinates are specified in units of *percentage of the chord* of the airfoil. Here's an illustration of that:

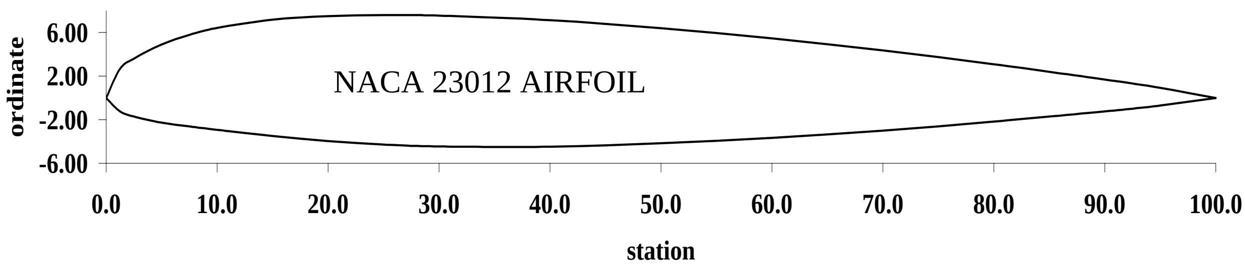


Figure 9-11

UPPER SURFACE		LOWER SURFACE	
station	ordinate	station	ordinate
0.00	0.00	0.00	0.00
1.25	2.67	1.25	-1.23
2.50	3.61	1.25	-1.71
5.00	4.91	5.00	-2.26
7.50	5.80	7.50	-2.61
10.00	6.43	10.00	-2.92
15.00	7.19	15.00	-3.50
20.00	7.50	20.00	-3.97
25.00	7.60	25.00	-4.28
30.00	7.55	30.00	-4.46
40.00	7.14	40.00	-4.48
50.00	6.41	50.00	-4.17
60.00	5.47	60.00	-3.67
70.00	4.36	70.00	-3.00
80.00	3.08	80.00	-2.16
90.00	1.68	90.00	-1.23
95.00	0.92	95.00	-0.70
100.00	0.00	100.00	0.00

Table 9-1

angle of attack α

The *angle of attack* of an airfoil, denoted by the Greek letter α (alpha), is defined as the angle that the airfoil's chord line presents to the free stream flow direction (and hence to the path of an airplane through the air), as shown at right. It is absolutely essential to keep this definition clearly in mind. <u>Angle of attack has no relationship to airplane attitude, only to the angle of the airfoil relative to the airflow</u>. The airplane could be climbing, flying level, descending, or even flying inverted – and the angle of attack would still be the angle of the airfoil to the free stream direction or airplane flight path, <u>not</u> to the horizon or any other reference.

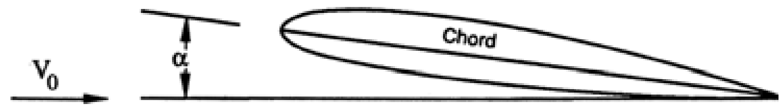


Figure 9-12

How Lift Is Produced

The flow of a fluid obeys known principles, which govern the interrelationships between velocity and pressure. Bernoulli's equations are one expression of these principles.

Placing an airfoil into a stream of fluid – air being a fluid, remember – forces the fluid to deform and change its course so as to flow around the airfoil. As a result of this deformation of the flow, the velocity and pressure of the airflow in the vicinity of the airfoil are influenced following the physical principles of fluid flow. A pattern of pressure distribution – a "pressure field" is established around the airfoil, and a related "velocity field" is also established. These fields are mutually dependent – the pressure is related to the velocity, and the velocity is related to the pressure.

The pressure field exhibits a diffuse region of relative low pressure above the airfoil and a diffuse region of relative high pressure below, as shown by the minus and plus signs in Figure 9-13 to the right.

The minus signs indicate pressure lower than ambient, and plus signs indicate pressure higher than ambient. The tighter the spacing of the symbols, the larger the pressure difference.

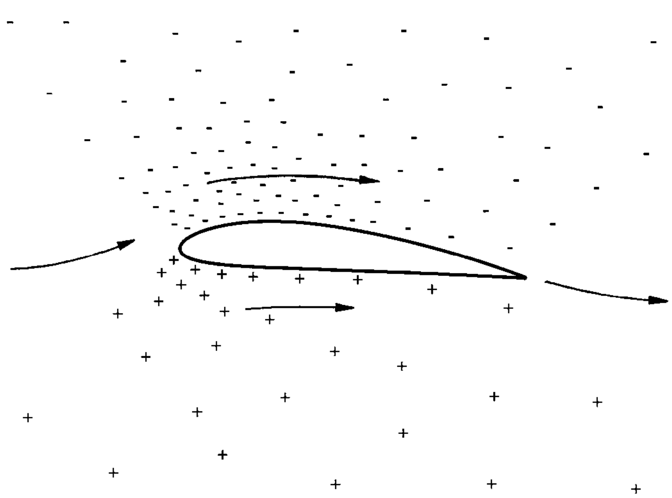


Figure 9-13

The pressure on the bottom of an airfoil is a function of the airfoil's shape, including its thickness, and on the angle of attack. On some airfoils, the wing's lower surface will experience pressure greater than ambient, on some other airfoils the lower surface pressure will be less than the ambient pressure.

The velocity field exhibits increased velocities above the airfoil. Below the airfoil, again depending on airfoil shape and angle of attack, the velocity field will exhibit either increased or reduced velocities.

The direction of the velocity at the rear of the airfoil is slightly downward.

As you have seen previously, a symmetric airfoil at zero angle of attack will create a symmetric pressure distribution above and below the wing and hence there will be no net vertical force. Commercial jet transport airplanes, however, use asymmetric airfoil shapes which will produce a net upward pressure force even at zero angle of attack because the amount of reduction of pressure above the wing exceeds the amount of pressure reduction on the lower surface. On some airfoils that exhibit pressures greater than ambient on the lower surface, the reduced pressure above the wing is supplemented by that increased pressure below. In either case, the pressure distribution creates a net upward pressure force – the force we call lift.

The downward turning of the flow is a direct result of the airfoil shape and angle of attack. To produce the downward turning, the airfoil surfaces, especially the aft part, must have a predominantly downward slope. Thus, to produce lift, the airfoil must have either camber or a positive angle of attack, or a combination of the two.

The pressure differences, the downward turning of the flow, and the changes in flow speed are all necessary for the production of lift. They support each other in a reciprocal cause-and-effect relationship, and none would exist without the others. The pressure differences transmit the lift to the airfoil, while the downward turning of the flow and the changes in flow speed sustain the pressure differences.

For those readers who would like to see further explanation of this force we call "lift", the Additional Discussion section at the end of this chapter provides more reading on the subject.

flow over an airfoil made visible

Figure 9-14 at the right shows the pattern of flow of air over an airfoil. The lines are streamlines, made visible by injecting smoke into the flow upstream of the airfoil.[1]

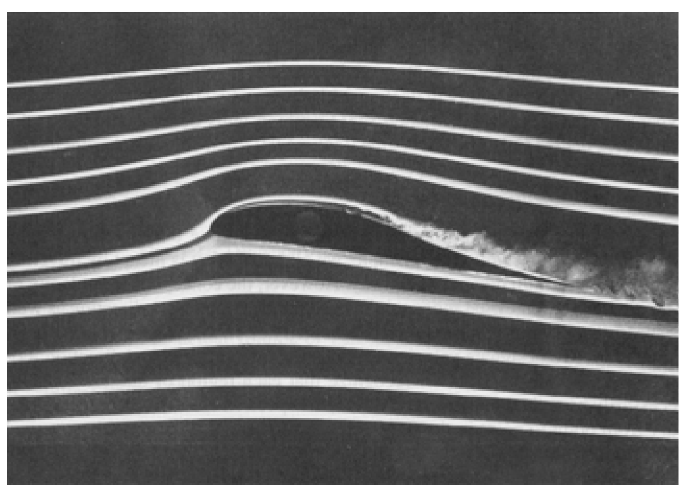


Figure 9-14

You can see very clearly how the flow divides so that some streamlines go up and over the airfoil and the remainder go down and under the airfoil.

If you could see every single streamline, you'd see that there's one that doesn't go up and over or down and under. It comes to the leading edge of the airfoil and simply stops. It divides the up-and-over airflow streamlines from the down-and-under streamlines. The point at which it contacts the leading edge and stops is called the *stagnation point*.

You can see in the photograph above exactly where the stagnation point is, at the airfoil leading edge, in between the first up-and-over streamline and the first down-and-under streamline.

What's also clear in this figure is that ahead of the airfoil the airflow is brought slightly upward from its free stream direction. Also, you can see that the flow leaving the wing's trailing edge is being given a downward component to its direction. What is <u>not</u> visible in the figure is the fact that the airflow is traveling much faster as it passes over the upper surface than it is as it passes under the lower surface, and it reaches the trailing edge sooner. It is not true that when an airflow divides to go over and under an airfoil a particle of air going up-and-over will reach the trailing edge at the same time as a particle going down-and-under.

pressure distribution over an airfoil

[Note: Previous discussions have shown both viscous and non-viscous flows. The concept of non-viscous flows was useful in showing, by comparison, the effects of viscosity. We're now finished with non-viscous flows however and all following discussions will assume normal viscous flows.]

Note the pressure distribution on a typical airfoil at an angle of attack α.

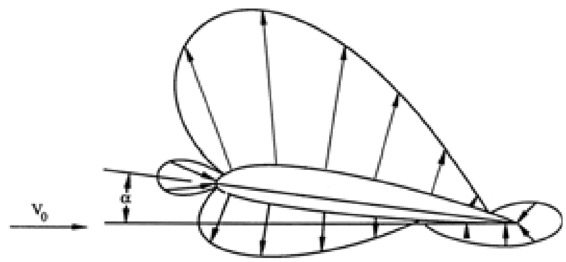


Figure 9-15

Clearly, the pressures on the upper surface of the airfoil are substantially less than on the lower surface. You can see also that there is a small localized high pressure area at the airfoil's leading edge, and another small localized high pressure area at the trailing edge.

Here is that diagram repeated, but here we show a single force – the "resultant" –which is equivalent to the summation of all of the distributed lift force. This resultant acts through a point referred to as the CP, or Center of Pressure.

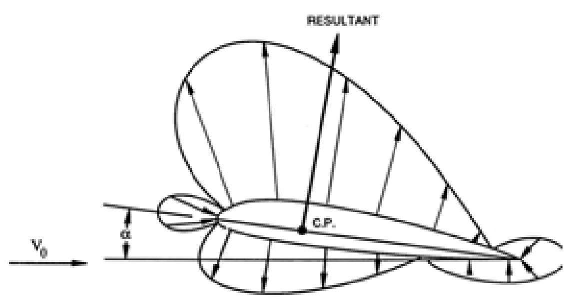


Figure 9-16

Because the force resultant is acting through the center of pressure, there is no moment tending to rotate the airfoil about that point.

1. This photograph of flow over a Lissaman 7769 airfoil at 10 degrees angle of attack was taken in a smoke tunnel at Notre Dame University by Dr. Thomas J. Mueller, Roth-Gibson Professor Emeritus, Aerospace & Mechanical Engineering, and is reproduced here with his kind permission.

The resultant force can be resolved into two components: one perpendicular to the flow – lift – and the other one parallel to the flow – drag.

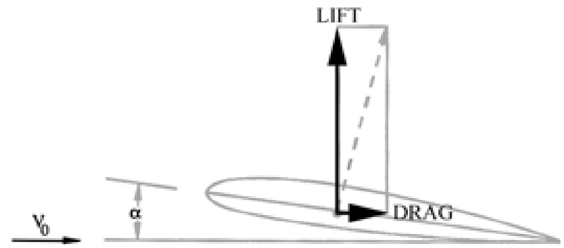


Figure 9-17

The center of pressure is not a constant point, however, rather it moves depending on the angle of attack. This makes life more difficult for aerodynamicists who might be trying to compute the forces and moments on an airplane in flight throughout its operating envelope.

For convenience, it's customary to consider the lift and drag forces as being centered at a point which is on the chord line 25% back from the leading edge. This is usually referred to as "the quarter-chord" point, often designated as "C/4" or ".25C". Arbitrarily moving the lift and drag vectors from the center of pressure to the quarter-chord point, however, creates a complication.

Let's explain this by using an analogy. Suppose that you have a beam of uniform width and thickness. Its center of gravity will be halfway along its length.

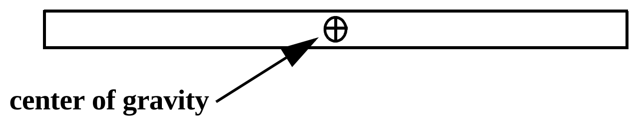


Figure 9-18a

If you place a support point directly underneath the center of gravity, the beam will be in balance.

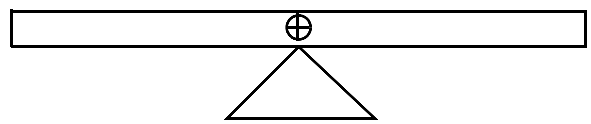


Figure 9-18b

If, however, you want to place the support point at some other location, the beam will no longer be in balance.

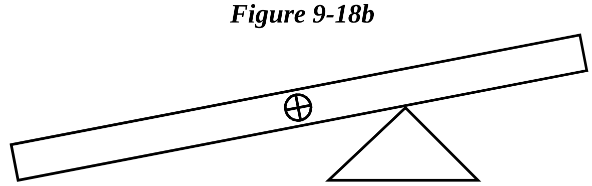


Figure 9-18c

To restore the balance, it will be necessary to add some moment at the support point.

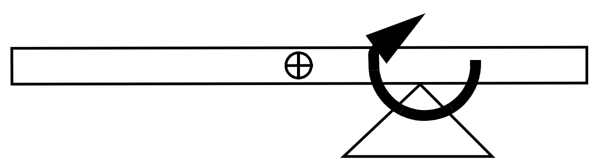


Figure 9-18d

Following this analogy, you'll see that the two figures below are equivalent:

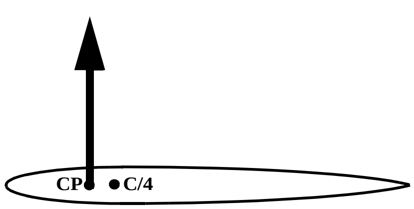


Figure 9-19a

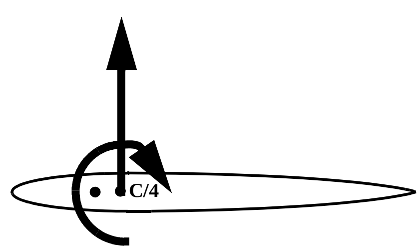


Figure 9-19b

For the reason shown above, the standard practice for data published for the aerodynamic characteristics of airfoils is to include the lift, the drag, and the moment about the quarter-chord point. For calculating the airplane pitching characteristics resulting from the lift force on the wings, it's then possible to consider the center of lift to be fixed at the quarter-chord.

Force and Moment Coefficients

The forces and moments generated by an airfoil are the function of many variables. Those include:

- flow velocity
- airfoil area
- air density
- The forces also depend on the shape of the airfoil, the viscosity, the angle of attack α, and the airflow compressibility.

For a given airfoil design, it would be very convenient if we could represent the lift, drag and moment values for different angles of attack in a manner which is independent of airfoil area, velocity, and air density. That would then allow us to compute the lift, drag and moment values for that airfoil for <u>any other</u> velocity, air density, or airfoil area.

We can do this by showing the airfoil's characteristics in terms of *force and moment coefficients.*

force coefficients

Aerodynamic forces may be simplified to the form of a force coefficient:

$$C_F = \frac{F}{qA}$$

Where C_F is a dimensionless coefficient of force
q is the dynamic pressure
A is a representative area, such as wing area

We'll use this general form to create coefficients of lift and drag, thus:

$$\textit{coefficient of lift } C_L = \frac{\textit{lift force}}{qS}$$

and:

$$\textit{coefficient of drag } C_D = \frac{\textit{drag force}}{qS}$$

Where S is a representative area, usually the wing area

These coefficients may be expressed in several different ways, depending on the input parameters, as follows:

$$C_L = \frac{lift}{\frac{1}{2}\rho V_{true}^2 S} \qquad \text{(eq. 1)}$$

and:

$$C_D = \frac{drag}{\frac{1}{2}\rho V_{true}^2 S} \qquad \text{(eq. 2)}$$

Where ρ is the air density in slugs per cubic foot
V_{true} is the true airspeed, in feet per second
S is the reference wing area, in square feet

Another form, using σ instead of ρ:

$$C_L = \frac{lift}{\frac{1}{2}\sigma\rho_0 V_{true}^2 S} = \frac{841.4 \times lift}{\sigma V_{true}^2 S} \qquad \text{(eq. 3)}$$

and:

$$C_D = \frac{drag}{\frac{1}{2}\sigma\rho_0 V_{true}^2 S} = \frac{841.4 \times drag}{\sigma V_{true}^2 S} \qquad \text{(eq. 4)}$$

Where σ is the density ratio ρ/ρ_0

In equations 3 and 4, the constant 841.4 comes from: $841.4 = \frac{1}{0.5 \times 0.002377}$. You'll recognize the term 0.002377 as ρ_0, the sea level standard day value of air density.

If working with V_{true} in knots, instead of feet per second (which, for commercial jet transport performance engineers, is more often the case):

$$C_L = \frac{295.4 \times lift}{\sigma V_{true}^2 S} \qquad \text{(eq. 5)}$$

and:

$$C_D = \frac{295.4 \times drag}{\sigma V_{true}^2 S} \qquad \text{(eq. 6)}$$

In equations 5 and 6, the constant 295.4 comes from:

$$295.4 = \frac{841.4}{1.6878^2}$$

in which you'll recognize the constant 1.6878 as the constant used to convert feet per second to knots (1 knot = 1.6878 feet per second).

Remembering that: $V_e = V_{true}\sqrt{\sigma}$ and therefore: $\sigma V_{true}^2 = V_e^2$ yields another form of the equations:

$$C_L = \frac{295.4 \times lift}{V_e^2 S} \qquad \text{(eq, 7)}$$

and:

$$C_D = \frac{295.4 \times drag}{V_e^2 S} \qquad \text{(eq. 8)}$$

Equations 7 and 8 are useful when you know the equivalent airspeed V_e instead of the true airspeed V_{true}. In performance work, this is frequently the case. For example, if you're asked to calculate the coefficient of lift for a given weight at some given value of indicated airspeed: knowing IAS, you'd apply the position correction ΔV_P and the compressibility correction ΔV_C and this would yield V_e. Thus if you know the indicated airspeed you don't need to know the air pressure and temperature in order to calculate ρ or σ to use in computing lift or drag coefficients.

If you know the Mach number instead of the airspeed, there's another form of the equations:

$$C_L = \frac{lift}{1481.4 \times M^2 \delta S} \qquad \text{(eq. 9)}$$

and

$$C_D = \frac{drag}{1481.4 \times M^2 \delta S} \qquad \text{(eq. 10)}$$

What's this constant 1481.4, and where does the δ come from? Look back at equation 5:

$$C_L = \frac{295.4 \times lift}{\sigma V_{true}^2 S}$$

Remember that: $V_{true} = Ma_0\sqrt{\theta}$ and so: $V_{true}^2 = M^2 a_0^2 \theta$. Substitute that into equation 5 and you'll get:

$$C_L = \frac{295.4 \times lift}{\sigma\ M^2 a_0^2 \theta\ S}$$

You know that $\delta = \sigma\theta$, so the equation above becomes:

$$C_L = \frac{295.4 \times lift}{M^2 a_0^2\ \delta\ S}$$

Since equation 5 is in terms of knots, we need to know a_0 in knots, which you'll remember is 661.4786 knots. Thus:

$$C_L = \frac{295.4}{661.4789^2} \times \frac{lift}{M^2 \delta\ S} = \frac{lift}{1481.4\ M^2 \delta\ S}$$

pitch and yaw moment coefficients

Although typical performance engineers use pitch moment data rarely, it's quite common to use the yawing moment coefficient when doing calculations of performance with an engine inoperative.

Moments can be expressed as coefficients in the same way as the lift and drag coefficients, with one significant difference.

Remember that we said that force coefficients are *dimensionless*. That simply means that a force coefficient has no units: not feet, not seconds, not degrees – no units at all. Look at one of the equations for lift coefficient, for example: the numerator has the units of force; in the denominator we have dynamic pressure, which has units of force per unit of area, multiplied by wing area. Thus the denominator also has units of force. The lift coefficient then, being force divided by force, has no units. It's a dimensionless number, a ratio.

Moments are different. Recall that a moment is a force multiplied by a distance. When we're working with moments, then, in order to keep the moment coefficient dimensionless it's necessary to put some value of distance (or length) in the denominator. The definition of a moment coefficient is therefore:

$$C_M = \frac{moment}{q \times A \times L}$$

Where L is a representative length used to make the coefficient dimensionless.

For airplane performance work:

$$C_M = \frac{moment}{\frac{1}{2}\rho V_{true}^2 S \times C} \quad \text{(eq. 11)}$$

Where C is the wing chord

In the equation above, the wing chord C is the representative length customarily used to make the coefficient dimensionless. In this equation, the velocity is true airspeed in feet per second

In a manner similar to the force coefficients above, we can re-write equation 11 in several other variants.

For true velocity in feet per second, using σ:

$$C_M = \frac{841.4 \times moment}{\sigma V_{true}^2 S\ C} \quad \text{(eq. 12)}$$

For true velocity in knots:

$$C_M = \frac{295.4 \times moment}{\sigma V_{true}^2\ S\ C} \quad \text{(eq. 13)}$$

For equivalent airspeed in knots:

$$C_M = \frac{295.4 \times moment}{V_e^2\ S\ C} \quad \text{(eq. 14)}$$

For speed in Mach number:

$$C_M = \frac{moment}{1481.4 \times M^2 \delta\ S\ C} \quad \text{(eq. 15)}$$

Lift and Drag as Functions of Angle of Attack

In order to do airplane performance work, we'll need to know the lift, drag and moment characteristics of the airplane over a wide range of speeds. As you'll see, speed and angle of attack are inversely related: more speed means less angle of attack, and vice versa. This means that to cover a range of speeds, we need lift, drag and moment data over a range of angles of attack.

lift coefficient versus angle of attack – the "lift curve"

The diagram at the right shows a typical curve of lift coefficient versus angle of attack. Several things are worth pointing out as you look at this:

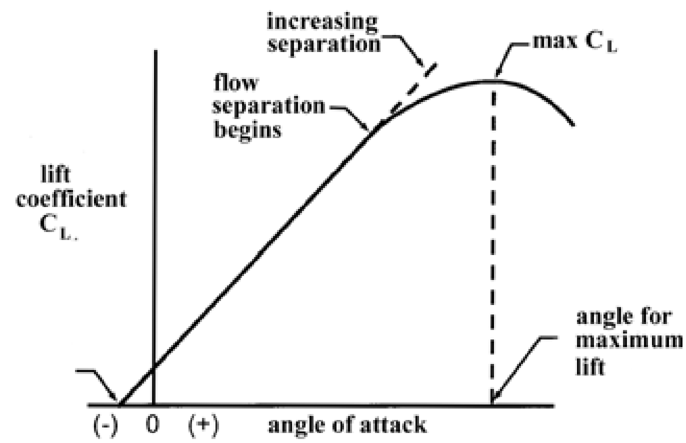


Figure 9-20

- Even at zero angle of attack, for a typical airfoil, there will be some small amount of lift. This would not be the case for a symmetric airfoil, but it's true for most typical airfoils in modern use;
- The slope of the lift curve is essentially linear at the lower angles of attack;
- As the airfoil is taken to progressively higher angles, the airflow will begin to separate from the airfoil's upper surface. If the angle of attack is increased beyond that point, flow separation will become increasingly severe until at some point the lift curve will reach a peak, beyond which the airfoil cannot generate more lift;
- Beyond the point for maximum lift, the airfoil is said to be *stalled*. The airflow in the stalled condition is mostly separated and is highly turbulent. In an airplane, as angles of attack increase toward the stalled condition, the turbulent flow from the wing will strike the airplane fuselage and its horizontal tail causing a condition called *buffet*, or *low-speed buffet*, characterized by a shaking of the airplane. The speed at which the buffet begins is called the *initial buffet speed*. You'll be seeing more about that in a later chapter.

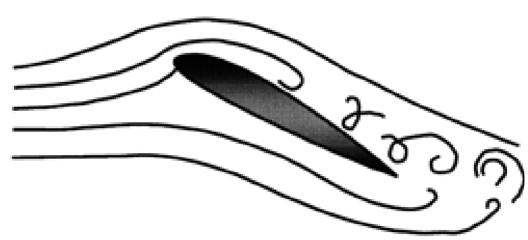


Figure 9-21

drag coefficient versus angle of attack

In the same manner as you saw above for the lift curve, we can also plot the drag coefficient versus angle of attack. Noteworthy here are the following points:

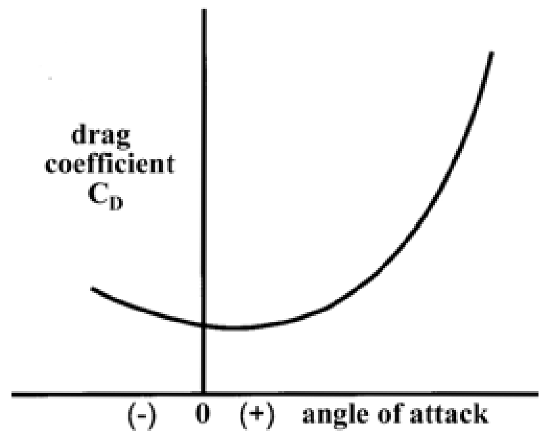


Figure 9-22

- At zero angle of attack, there will be a finite amount of drag, although it will be relatively small. You should expect this: any airflow over an airfoil will involve a certain amount of skin friction drag, even though at very low angles the pressure drag may be negligible.
- It's possible that the angle at which the least amount of drag occurs will not be zero, but some small positive angle. For a symmetric airfoil, we would expect the minimum drag to occur at zero angle of attack, but for a typical commercial jet transport airplane wing, that may not be the case.
- As the angle of attack increases, drag increases more and more rapidly, particularly when the flow separation begins and turbulent wakes begin to cause substantial pressure drag.

the drag polar – a performance engineer's best friend

It's sometimes useful to know the lift coefficient as a function of the angle of attack – for example, if you're asked "what body attitude should the pilot see on his flight director in flight for specified conditions?"

The greatest amount of work we do, however, is made possible by the use of a *drag polar.*

A drag polar is simply a plot of lift coefficient versus drag coefficient, for a range of angles of attack. A typical drag polar will look something like the figure at the right.

A high angle of attack will be characterized by a high lift coefficient as well as a high drag coefficient, and that combination would be toward the high-coefficient right-hand end of the polar curve. Low angles of attack will be at low drag and lift coefficients, and those points will fall at the low end of the polar curve.

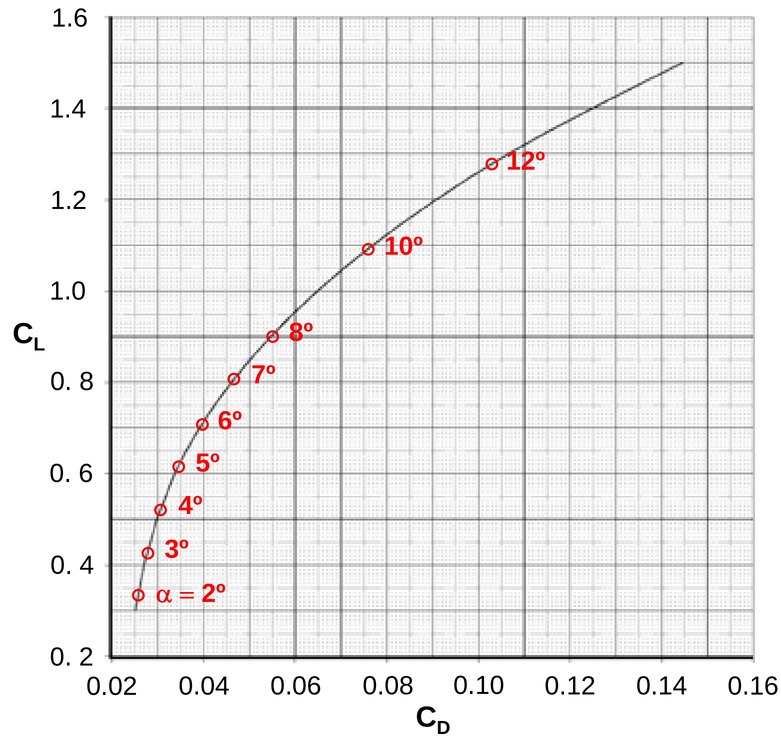


Figure 9-23

The slope of a line drawn from a point at which both C_L and C_D are zero (the origin of the graph) to any point on the polar curve will be representative of the *lift-to-drag ratio* of that airfoil. Higher slopes are better, since that tells you that the drag will be less for a given amount of lift.

The highest lift-to-drag ratio will occur at the point where a line from the origin is just tangent to the curve. In this example, that occurs at about 5.6 degrees angle of attack, at a C_L of about 0.66 and a C_D of about 0.0365. That's a lift-to-drag ratio (L/D) of 18 – not bad until you learn that high-performance sailplanes may have L/Ds greater than 50.

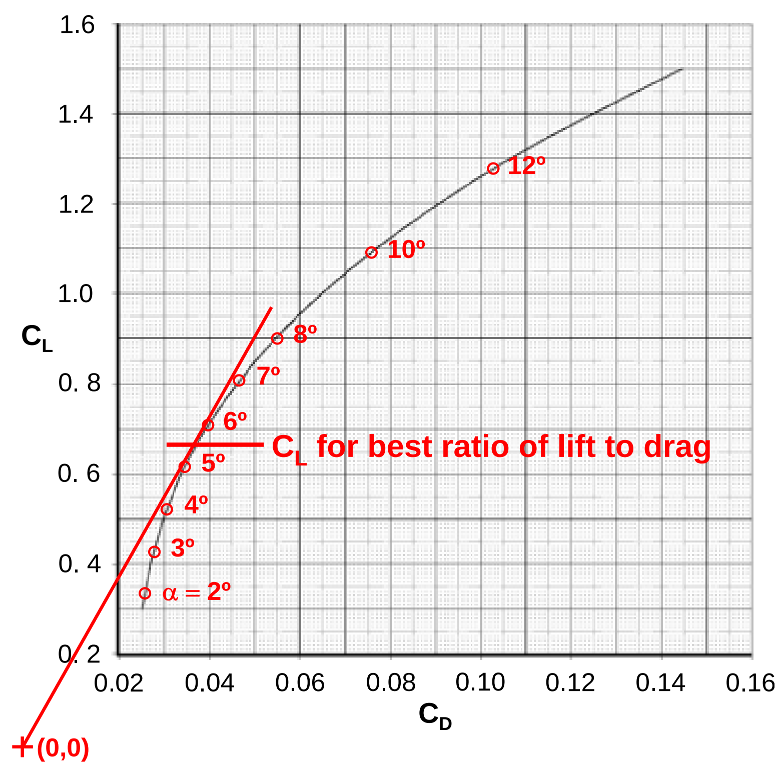


Figure 9-24

You'll see very clearly in the chapter entitled "Climb Angle and Rate of Climb" just how important the L/D ratio is.

In the next chapter, we'll apply what you've learned here to real wings on real airplanes.

Additional Discussion

The following additional discussion of lift is provided by Boeing's Aerodynamics Staff. We acknowledge their contribution to this chapter with thanks.

lift

Earlier in this chapter, we said that lift is the result of the flow and velocity fields that are produced when an object such as an airfoil is inserted into a fluid flow. We saw that a region or "cloud" of reduced pressure is produced above an airfoil, and a cloud of increased pressure is produced below an airfoil. We saw also that the airflow leaving the trailing edge of the airfoil has a downward component of velocity.

The clouds of low and high pressure above and below the airfoil are "confined" to a limited area. "Sustaining" the pressure differences essentially means maintaining this spatial "confinement", both vertically and horizontally. Specifically, interaction with the downward deflection of the flow provides vertical confinement of the pressure differences, and interaction with the changes in flow speed provides horizontal confinement.

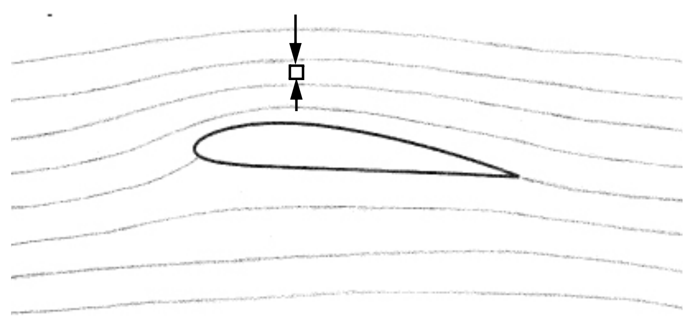


Figure 9-25

Consider how the downward deflection of the flow does its part. Any fluid parcel following the downward curvature of the flow within the region of downward turning, as shown in Figure 9-25, is being accelerated downward, and according to Newton's second law, must have a net downward force applied to it. The force is supplied by higher pressure above the parcel than below, as shown by the two vertical arrows of unequal length pushing on the parcel in the illustration. For this parcel, the pressure difference is just a reflection of the fact that the pressure varies gradually in the vertical direction within the cloud of low pressure above the airfoil, between the ambient pressure far above the airfoil and the low pressure at the airfoil's upper surface.

The interaction between pressure and downward turning is reciprocal: the pressure difference acting on a fluid parcel causes the parcel's path to be deflected downward, and the pressure difference is sustained because the fluid parcel has mass and therefore resists having its path deflected from a straight line.

Now consider how the changes in flow speed provide the horizontal "confinement". Consider the fluid flowing from in front of the airfoil into the cloud of low pressure above the upper surface, as illustrated by the fluid parcel on the left in Figure 9-26 to the right. This parcel sees lower pressure on the downstream side than on the upstream side, as illustrated by the horizontal arrows of unequal length, and is given a net push in the direction of the flow. It is therefore accelerated to a higher speed, as indicated by the longer arrow above the upper surface in Figure 9-13 earlier in this chapter. Likewise, the parcel on the right in Figure 9-26 here is leaving the region of low pressure and is being slowed back down. On the other hand, a parcel passing through the cloud of high pressure below the airfoil experiences the opposite sequence: it is slowed down and then speeded back up.

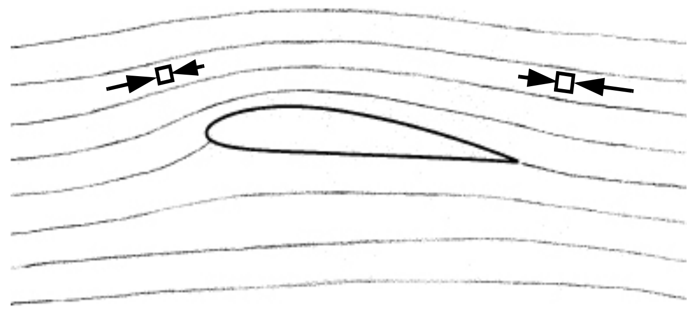


Figure 9-26

These changes in flow speed are consistent with Bernoulli's equation. The relationship is reciprocal: the differences in pressure in the horizontal direction cause the changes in flow speed, and the fluid's resistance to acceleration, because of its mass, sustains the pressure differences.

To summarize, we've seen that downward deflection of the flow and different flow speeds above and below the airfoil are both necessary accompaniments to lift production. They are both parts of the reciprocal cause-and-effect relationship that sustains (or "confines") the pressure differences. The pressure differences cause the flow to change speed and direction, and the changes in flow speed and direction cause the pressure differences to be sustained. To understand this interaction correctly we must see the causation as working both ways. And of course the fact that the fluid has mass is crucial to the interaction. This circular cause-and-effect that supports lift might seem a bit like "something for nothing" or "perpetual motion", but it's not. Lift follows naturally from the laws of physics and from the fact that the flow follows the contours of the airfoil surface.

faulty explanations of lift

Like most phenomena in fluid dynamics, lift is surprisingly difficult to explain in simple physical terms. Many of the explanations that have been widely circulated are incomplete, and there are numerous popular misconceptions. The following is a discussion of some of this faulty lore.

How the two main popular explanations are incomplete

The two most widely circulated explanations of lift are the momentum-based (Newtonian) and the Bernoulli-based. Both contain elements of the truth, but neither provides a complete explanation.

A momentum-based explanation starts with the downward turning of the flow that we saw in figure 9-12(2). The downward deflection means that the fluid is being accelerated downward and given downward momentum. To accelerate the fluid downward, the airfoil must exert a downward force on the fluid, as required by Newton's second law and the fact that the fluid has mass. Then according to Newton's third law, for every action there is an equal-and-opposite reaction, and the fluid must therefore exert an upward force on the airfoil. Thus if we know that there is downward turning of the flow, Newton's laws dictate that there must be lift. This explanation is correct as far as it goes, but is incomplete in that it doesn't explain how the force is actually transmitted to the airfoil by the pressure, and it takes for granted that there is downward turning, without explaining how the airfoil can impart downward turning to such a wide swath of the flow. To understand how the flow turning really comes about, we must do as we did above and look at the flow in more detail, including the reciprocal cause-and effect relationship between the pressure differences and the changes in flow speed and direction.

A Bernoulli-based explanation starts by arguing that the flow over the upper surface is speeded up, either because the path length over the upper surface is longer, or because of an "obstacle", "hump", or "Venturi" effect. Because of the higher speed, the pressure over the upper surface must be lower, via Bernoulli's principle, and thus there is lift. Explanations of this type are incomplete in that they don't adequately or correctly explain what causes the flow to speed up. The longer-path-length explanation is simply wrong (see below, under "Popular misconceptions"). The "obstacle", "hump", or "Venturi" explanations are better, but only a little. They often mention "pinching" or "necking down" of the flow over the upper surface, but they don't provide a convincing physical reason for the pinching. A common fault in all of these explanations is that they imply that a speed difference can arise from causes other than a pressure difference, and that the speed difference then causes a pressure difference, via Bernoulli's principle. This is a misconception that we'll discuss below.

There are at least three schools of thought among proponents of these explanations. One is that only one or the other can be correct. Another is that both are correct, and that they apply to two different kinds of lift. A third is that both are correct, and either one suffices to explain lift in general. These are all misconceptions that we'll discuss below.

Popular misconceptions

We've seen that lift generation involves subtle cause-and-effect relationships, so it shouldn't be surprising that many of the attempts to explain it to a popular audience have made errors of one kind or another. To solidify our understanding and make it less likely we'll be taken in by incorrect ideas, let's identify some of the misconceptions and consider where they went wrong.

One-way causation: This is a misconception we've already discussed, that a velocity difference can be deduced first, based on some argument that does not depend on the pressure, and that a pressure difference follows, via Bernoulli's principle.

This implication that the causation runs in only one direction is not consistent with the physics of fluid flows. If you try to explain a speed difference without referring to the pressure difference, you'll inevitably get the reasons for the speed difference wrong. One example of this, an erroneous reason for high velocity over the upper surface of an airfoil, is the next item on our list.

Longer path length and equal transit time: This is an argument that is widespread in explanations aimed at the layman. It is assumed that the upper surface of the airfoil is more convex than the lower surface, and that the path the fluid must follow around the upper surface is therefore longer than the path around the lower surface. It is further assumed that fluid parcels that are split apart at the leading edge to traverse the upper and lower surfaces must rejoin at the trailing edge. Thus fluid parcels negotiating both paths must do so in equal time, and we conclude that the velocity over the upper surface must be higher than that over the lower surface.

First, this isn't a proper kind of physical explanation. Just saying that something has to arrive somewhere at a particular time doesn't explain why the thing might speed up. To explain why something speeds up, you need to identify and explain the force that makes it speed up. And this explanation is wrong on another level. There is no reason why fluid parcels that start together ahead of the airfoil must rejoin at the trailing edge, and in fact, they generally don't. A parcel that traverses near the upper surface typically arrives at the trailing edge well ahead of one that traverses near the lower surface. So no difference in path length is required, and there are many situations in which lift is produced without a difference. And on airfoils where there is a difference, it is typically much too small to explain the speed difference that actually occurs when lift is produced.

<u>Bernoulli is right, and Newton is wrong, or vice-versa</u>: Some proponents of the Bernoulli-based explanations argue that lift is produced solely by a pressure difference, via Bernoulli's principle, and that there is no downward momentum imparted to the fluid (Newton). Some proponents of momentum-based explanations argue the opposite: that imparting momentum is everything (Newton), and that the Bernoulli principle is not applicable.

We've seen that a pressure difference between the upper and lower surfaces and downward turning of the flow are both essential parts of the picture.

<u>Bernoulli and Newton are both right, and they explain two different kinds of lift</u>: This line of argument maintains that "Bernoulli lift" and "reaction lift" represent two distinct physical mechanisms.

There is only one kind of lift, and explaining it requires both a pressure difference and downward turning.

<u>Bernoulli and Newton are both right, and either one suffices</u>: According to this line of argument, the Bernoulli-based and the momentum-based explanations are just different but equivalent ways of looking at the same thing.

Again, we've seen that a complete explanation must refer to both the pressure difference and the imparting of downward momentum.

<u>Invoking the Coanda effect as the reason the flow is able to follow the curved surfaces of the airfoil</u>: Some explanations argue that viscosity plays a crucial role in enabling the flow to turn and follow the curved upper surface of the airfoil. They refer to this purported coupling between viscosity and flow turning as the Coanda effect.

This reflects a misunderstanding of the role of viscosity in fluid flows and of what the Coanda effect actually entails. As fluid flows over the surface of an airfoil, there is no direct coupling between viscosity and flow turning, and none is needed. Viscosity plays a significant role in lift generation only in the immediate vicinity of the airfoil trailing edge, by preventing the flow from going around the trailing edge from the lower surface to the upper surface. The real Coanda effect refers to the tendency of a turbulent jet flow with higher energy than the surrounding fluid to attach itself to an adjacent surface and to follow the surface even if it is curved. This is not so much a viscous effect as it is an indirect effect of the jet turbulence. It arises because of the tendency of jet flows to entrain surrounding fluid, and it plays no role in ordinary airfoil flows.

Invoking a starting vortex and circulation around the airfoil: This is an explanation that appears in several aerodynamics texts and has been repeated on popular web sites. It starts with the observation that when an airfoil starts its motion through the fluid, a starting vortex is left behind. The formation of the starting vortex is accompanied by the establishment of a "circulatory flow," or circulation, around the airfoil, which is responsible for the lift via the Kutta-Joukowski theorem.

This is not a proper physical explanation for two reasons. First, it requires a non-technical reader to accept several advanced mathematical and aerodynamic theorems on faith. Then, although the explanation is mathematically and logically correct, the logical argument runs in a direction that is mostly opposite to that of physical cause and effect, which is misleading. The starting vortex and the circulation are actually more properly seen as byproducts of the lift than as causes.

The low pressure on the upper surface pulls upward on the airfoil: Many popular explanations of lift describe the effect of low pressure in these terms. The idea of the airfoil being pulled upward has a strong intuitive appeal, but it is incorrect.

Pressure, especially in air, is always a push, never a pull. The pressure on the upper surface of an airfoil pushes downward on the airfoil, but the higher pressure on the lower surface pushes upward harder, and net effect is lift.

Not acknowledging the importance of angle of attack: Some explanations, such as the one based on the longer-path-length-and-equal-transit-time argument, never mention the angle of attack.

The angle of attack is a key factor that determines how much lift an airfoil produces at a given flow speed and is an essential ingredient in achieving controlled flight.

Why there have been so many misconceptions

Explaining lift in physical terms is more difficult than most people realize, and the difficulty is inherent in the basic nature of fluid mechanics. We are dealing with countless little parcels of fluid that move in coordination with their neighbors and exert forces on their neighbors, all while separately and simultaneously obeying Newton's second law. It simply isn't possible to look at an airfoil and deduce, by mental effort alone, what flow pattern satisfies the physical laws everywhere at once. There are too many simultaneous relationships to keep track of.

This kind of complexity isn't easy to deal with mathematically, either. Mathematically expressing all the relationships a fluid flow must satisfy results in a set of partial-differential equations called the *Navier-Stokes equations*. By solving these equations we can predict in detail what the flow around an airfoil does and how much lift is produced. But solving the equations means mathematically determining how the pressure and the flow velocity vary throughout a large volume of space surrounding the airfoil. For any given flow situation it requires lengthy calculations that are practical only on a high-speed computer. Computer programs are available that can make such calculations routinely. However, all they do is provide a simulation of what would happen in a real flow; they don't provide a physical explanation of how it happens.

Simplified theories have also been developed, such as potential-flow theory, which ignores viscosity, and the Kutta-Joukowski theorem, which relates lift to a circulatory component of flow around the airfoil, but these don't provide a direct physical explanation for lift either.

So we see that predicting the existence of lift using nothing but the properties of the fluid and the laws of physics would essentially require solving the Navier-Stokes equations or the potential-flow equation for the flow around the airfoil, which is not something we can do in our heads. Explaining what happens, with words and simple diagrams instead of laborious calculations, thus requires some prior knowledge of what the flow does. The explanation above started with knowledge of some basic features of the pressure and velocity fields around an airfoil, and then showed how the pressure field and the velocity field support each other in a manner consistent with the laws of physics, including the proper reciprocal cause-and-effect relationships.

Faulty explanations often assume too little prior knowledge and then try to do more than is logically possible by mental effort alone. As a result, they tend to leave important things unexplained (such as what really causes the high velocity over the upper surface) and to resort to logical fallacies such as one-way causation.

Chapter 10: Wings

Introduction

In the previous chapter, we looked at airfoils and saw how they generate lift and drag. The discussion was quite general and didn't deal at all with the characteristics of real airplane wings. Instead, we were talking about what aerodynamicists call *two-dimensional wings*.

A two-dimensional wing is one which has no wingtips. It's sometimes referred to as a wing of infinite span. Think, for example, of having an airfoil in a wind tunnel for testing. That airfoil might occupy the full width of the tunnel, going from one wall of the tunnel test section all the way across to the opposite side. Testing of this nature is quite common when it's desired to know only the exact lift, drag and pitch characteristics of an airfoil, without consideration of wingtip effects and the like.

Now it's time to look at real wings on real airplanes. Real wings are called "three-dimensional" because they have wingtips – that is, they have a finite span. While real wings generate lift and drag in the same way as two-dimensional airfoils, they also have some unique characteristics that aren't seen in two-dimensional wings.

Three-Dimensional Wing Terminology

In the previous chapter we introduced you to the terminology relating to airfoils, such as chord, camber, and so on. To look at real wings on real airplanes, we need some additional terminology.

wingspan

Refer to the illustration on the right.

Wingspan is the term for the wingtip-to-wingtip dimension of the airplane.

The symbol used for wingspan is *b*.

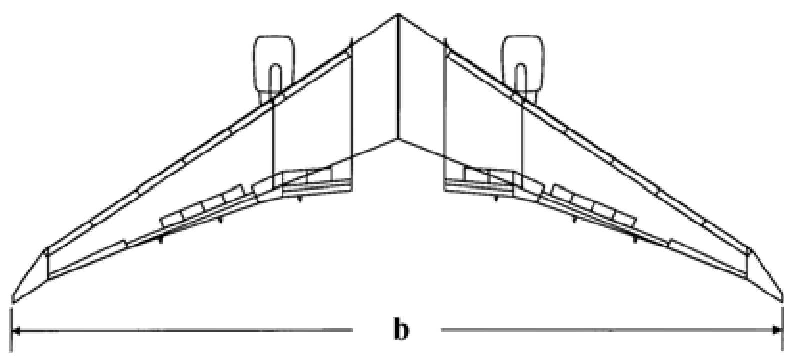


Figure 10-1

taper ratio

Taper ratio is the term given to the ratio of the wing chord at the wingtip (C_T) divided by the chord at the wing root (C_R). The symbol used to designate taper ratio is the lower-case Greek letter λ (lambda).

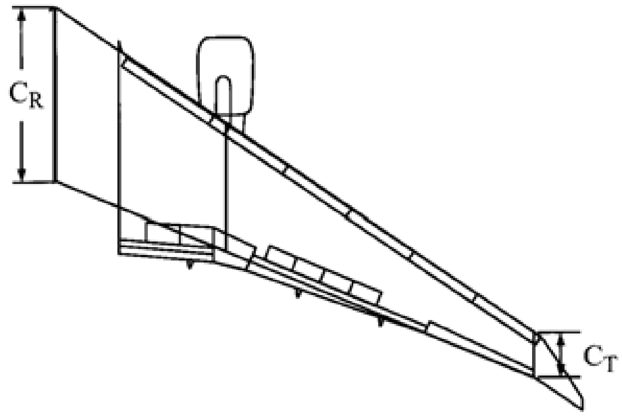


Figure 10-2

In the case of a wing having a complex planform, it is possible to simplify the shape to a simple "trapezoidal wing". The root chord is then the base of the trapezoid at the airplane's centerline, and the tip chord is at the peak of the trapezoid, the wingtip.

In the illustration, λ would be found from:

$$\lambda = \frac{C_T}{C_R}$$

quarter-chord line

The *quarter-chord line* is a line drawn along the span of the wing one-fourth of the chord behind the leading edge. It's usually denoted as *C/4*.

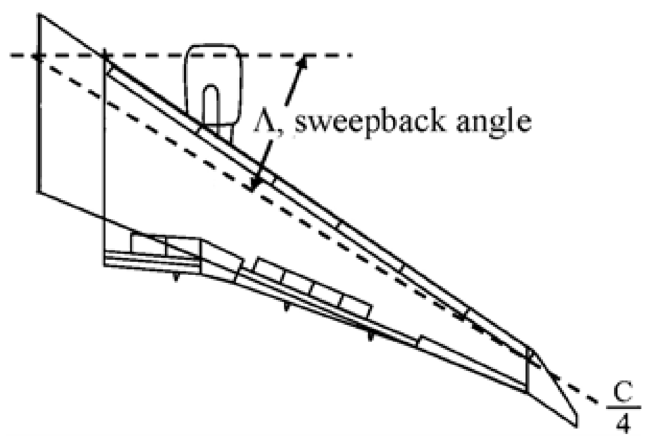


Figure 10-3

sweepback angle

The *sweepback angle*, or simply "wing sweep", is the angle between a line perpendicular to the plane of symmetry of the airplane and the quarter-chord line of each airfoil section. The sweepback angle is denoted by the upper-case Greek letter Λ (Lambda).

reference wing area

You'll recall from the previous chapter that we frequently need to know the "wing reference area" called S or S_{ref} in order to calculate force and moment coefficients.

There are two different conventions for defining wing area:

- *trapezoidal wing area* means that the wing is treated as purely trapezoid in shape, ignoring any wing root fillets, leading edge "gloves", or the like. Refer to the illustration above.

- *Effective area* is the most commonly used definition of wing area at Boeing for aerodynamic and performance work. This accounts for the complete projected outline of the wing.

A little more discussion of wing area is in order here:

Notice that we call the wing area the “reference” wing area. It is not, nor does it need to be, an exact value of wing area. Provided that the same value of area is used consistently, any value could be used.

For convenience, Boeing does not usually use different wing area values for variants of a given airplane model. For example, the published S_{ref} for the 747-100 is 5500 square feet. The same value is used for the 747-400, despite the fact that the wingspan of the -400 is almost 16 feet greater due to the addition of extended wingtips and winglets.

aspect ratio

Aspect ratio, abbreviated as *AR*, is defined the square of the wing’s span divided by its area, $\frac{b^2}{S}$. As such, it’s a measure of the relative narrowness of the wing compared to its span. For a rectangular wing, the aspect ratio would be equal to the ratio of the span to the chord.

For reasons that we’ll discuss later in this chapter, high-efficiency wings such as those on high-performance sailplanes have very high aspect ratios. That means that the span of the wing is very long compared to its chord. Commercial jet transport airplanes, on the other hand, typically have much lower aspect ratios for reasons of structural weight and fuel-carrying capability.

The aspect ratio of the 747 wing is approximately 7, and is approximately 8 for the 757 and 767. A high performance “open-class” sailplane may have an aspect ratio in excess of 40![1]

mean aerodynamic chord

Mean Aerodynamic Chord, or *MAC*, is the most difficult of the wing parameters to explain.

MAC is a value for the theoretical effective average chord of a constant-chord wing used for the purpose of looking at pitching moment characteristics. It is also used for weight and balance issues – more specifically, for denoting the location of the airplane’s center of gravity (CG). CG is usually specified in percent of MAC. With the exception of weight and balance tasks, MAC appears rarely in everyday performance work.

The pitching moment characteristics of a real wing depend on many factors, including sweep, taper, twist and camber. However, it’s acceptably accurate and customary to use a simple geometric definition of MAC as a parameter for correlating pitching moment data.

1. For example, the Schleicher ASW-22 sailplane has a wingspan of 87.2 feet and an area of 179.8 square feet. The aspect ratio is thus 42.3 and the average wing chord is only 2.1 feet!

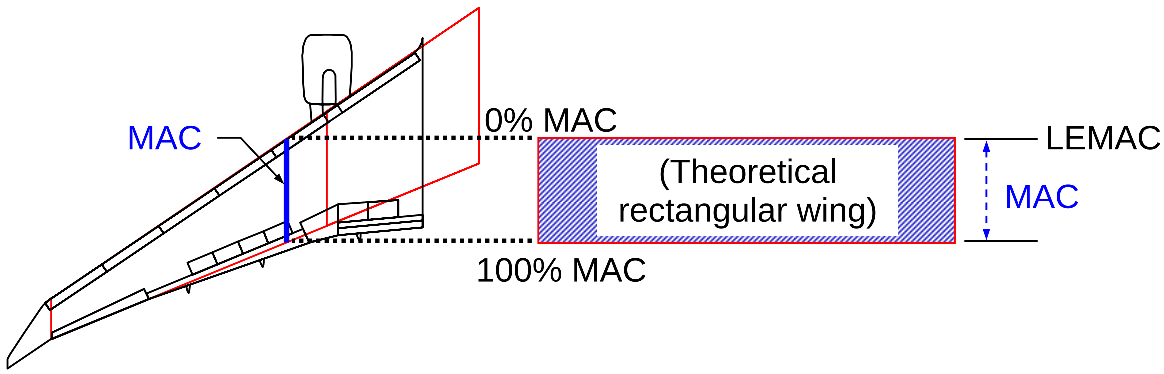


Figure 10-4

Rigorous calculation of the geometric definition of MAC can be computed from the equation:

$$MAC = \frac{\int_0^b c^2 db}{S_{ref}} \qquad \text{(eq. 1)}$$

Where c is the chord of the wing at any location along the span
b is the wing span
S_{ref} is the reference wing area.

For a trapezoidal wing, the value of MAC can be found from

$$MAC = \frac{2}{3} C_R \left(\frac{\lambda^2 + \lambda + 1}{\lambda + 1} \right) \qquad \text{(eq. 2)}$$

Where λ is the taper ratio
C_R is the chord of the wing at the root

For weight and balance work, the airplane CG is usually specified in *% MAC*, percent of MAC. A center of gravity at 0% MAC would be at the very front edge of the MAC, whereas a CG at 100% MAC would be at the trailing edge of the MAC.

In order to know percent MAC, it's necessary to know the length of the MAC, and the location of its front edge, which Boeing refers to as *LEMAC* (leading edge of the MAC). These are published in the Boeing weight and balance documents.

angle of incidence

Most wings have an "angle of incidence" relative to the longitudinal axis of the airplane. This angle of incidence is the angle between the wing chord line at the root and the airplane longitudinal axis.

An angle of incidence is usually incorporated into a design for reasons of reducing body drag in cruise, or providing a more comfortable passenger cabin attitude in cruise,

twist

Some airplanes have wings which are *twisted* such that the angle of incidence at the root of the wing is different from the angle at the tip. This is usually done to enhance the stalling characteristics of a wing, or to tailor the lift distribution. The wing of the 747, for example, is twisted by 3.5 degrees, having an incidence of 2 degrees at the root and -1.5 degrees at the tip.

Lift Distribution

We have spoken about "two-dimensional wings" (wings having no wingtips) and "three-dimensional wings", wings having tips. Obviously, the latter is more relevant to our work with commercial jet transport airplanes. But let's look at both, by way of illustrating some of the significant properties of real airplane wings.

two-dimensional wings

For a wing having no tips, the distribution of lift will be uniform along the span.This would be ideal, as it would mean that the entire wing is working equally. Unfortunately, it's unrealistic in the world of real wings on real airplanes.

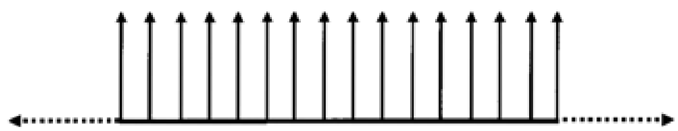


Figure 10-5

three-dimensional wings

Real-world wings <u>do</u> have wingtips, an unfortunate fact from the performance standpoint, since there will always be some loss of performance around the wingtips. The distribution of pressure looks more like this diagram, with the inboard wing working much harder than the outboard section of the wing.

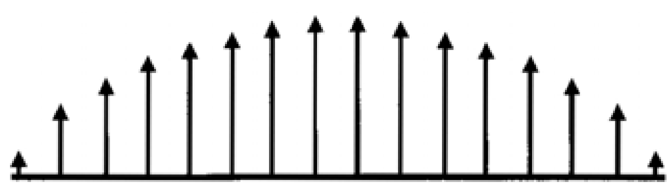


Figure 10-6

effect of wing planform on lift distribution

Wing planform has a marked effect on the distribution of lift along the span, shown here:

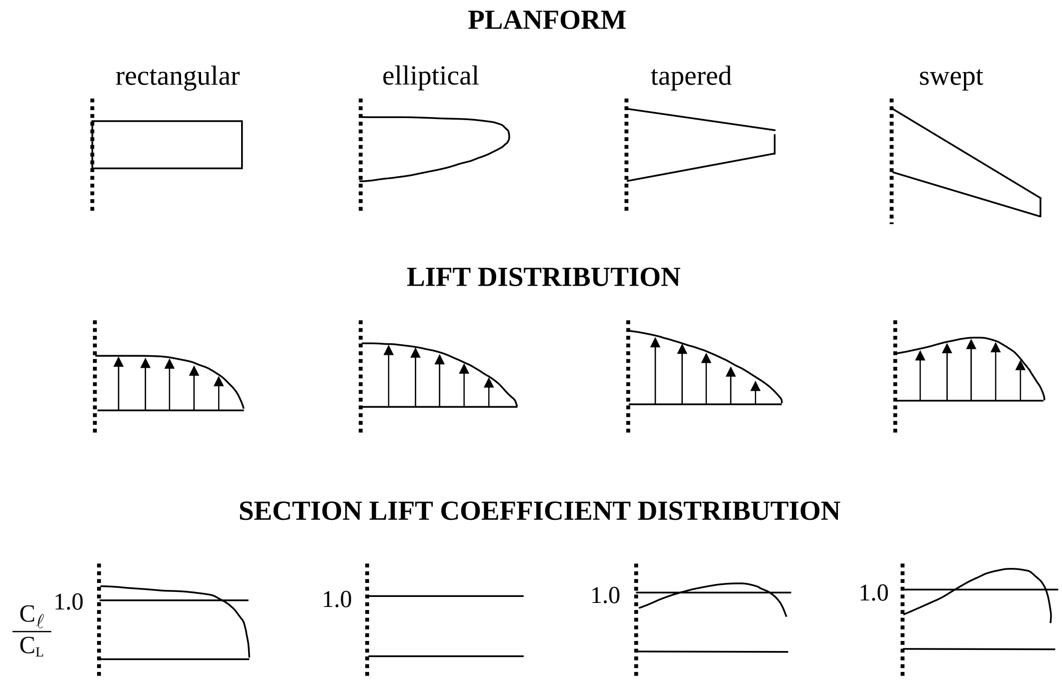


Figure 10-7

In the illustration above, the term “section lift coefficient” requires a bit of explanation. If you could take a very thin spanwise slice of the wing (a “section”), of width *db*, and if you could measure the aerodynamic force generated by that thin slice of wing, then the section lift coefficient could be calculated by dividing the lift force of the slice by the product of the dynamic pressure and the area of the slice. The area of the slice would be the product of its width *db* and the wing chord at the location of that slice. A diagram, as above, of section lift coefficient along the span can show how well the lift is distributed in the spanwise direction.

If the lift distribution along the span varies in the same way as the chord is varying, then the section lift coefficient would be a constant.

A *rectangular wing* has a fairly uniform lift distribution until approaching the wingtip, when it decreases rapidly. It has higher induced drag than the elliptical wing.

An *elliptical wing* planform can be shown by analysis to be the ideal shape from the standpoint of lift distribution. It has a constant section lift coefficient. Theoretically, <u>it will have the minimum possible amount of induced drag</u>. Some very successful airplanes have been designed with elliptical wing planforms.[1]

---

1. Noteworthy among those is the British Supermarine Spitfire of World War II. For that airplane, the elliptical wing had two major advantages: superior aerodynamic performance and a greater wing chord allowing it to carry eight machine guns. On the other hand, it was more difficult to manufacture because of the complex planform.

A *tapered wing* has relatively poor lift distribution, with the middle portion of the wingspan doing the greatest amount of work. It has drag advantages since its lift distribution is more nearly elliptical than either the rectangular wing or the swept wing.

The *swept wing* also has an inferior lift distribution, but that planform has great advantages at higher Mach numbers, as you'll see later on in this chapter.

You've probably noticed that in the paragraphs above, we referred to something called "induced drag". This is another kind of aerodynamic drag; we'll be discussing it very soon in this chapter.

Stalling Characteristics of Different Wing Planforms

We spoke in the preceding chapter about "stall" saying that it is a condition, occurring at very high angles of attack, in which the wing ceases to be able to generate adequate lift due to extensive separation of flow from the wing's upper surface.

Stall characteristics are also a function of wing planform, as shown below. The cross-hatched areas show the progression of the stalled portion of the wing as the angle of attack is increased:

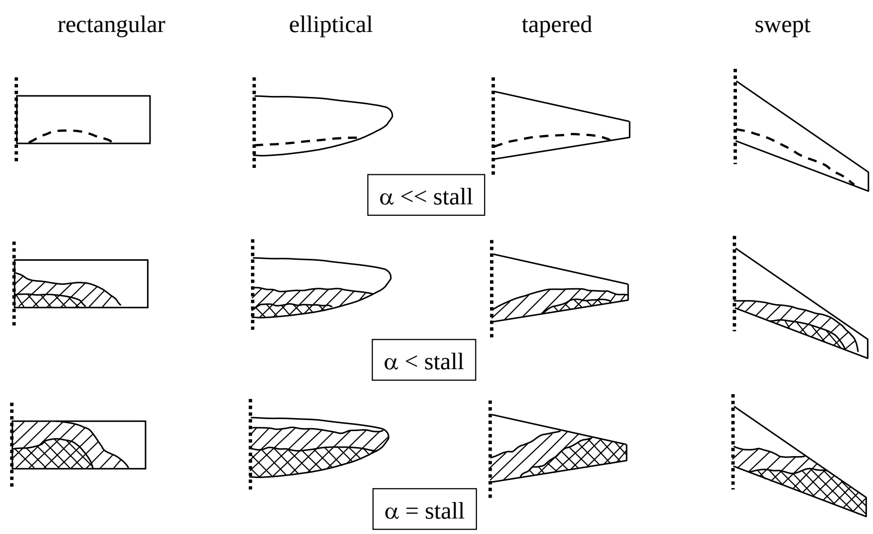


Figure 10-8

The rectangular wing has good stalling characteristics but poor high-speed performance. Non-elliptical lift distribution creates higher induced drag than an elliptical wing.

The elliptical wing tends to stall all at once.

The tapered wing has poor stalling characteristics because the tip stalls first.

The swept wing also has poor stalling characteristics because the tip stalls first. The principal advantage of the swept wing is its superior aerodynamic performance at high speeds.

Drag Due to Lift – Induced Drag

Previously, we have discussed several sources of aerodynamic drag: skin friction drag which results from viscous shear forces, and pressure drag, the result of separated turbulent wakes downstream of aerodynamic shapes. These two components of drag will occur on two-dimensional wings just as much as on three-dimensional "real" wings.

There's one component of aerodynamic drag however which is unique to three-dimensional wings: *induced drag*. It's sometimes referred to as *drag due to lift*, because it's exactly that. Remember that the air pressure on the lower surface of a wing in flight is greater than it is on the upper surface. This results in a flow of air from the lower surface toward the upper surface, around the wing tips, as illustrated here.

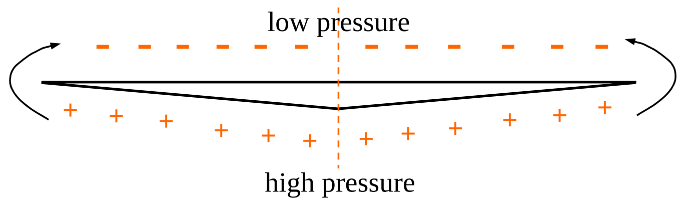


Figure 10-9

That flow from below to above around the tips has an effect on the flow over wings. It induces a spanwise component of the flow direction: toward the wingtips on the lower surface, toward the wing root on the upper surface.

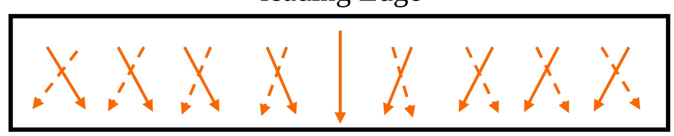


Figure 10-10

If you could visualize the airflow over the wings of an airplane in flight when looking at the airplane from directly ahead of it or behind it, you would see something like this:

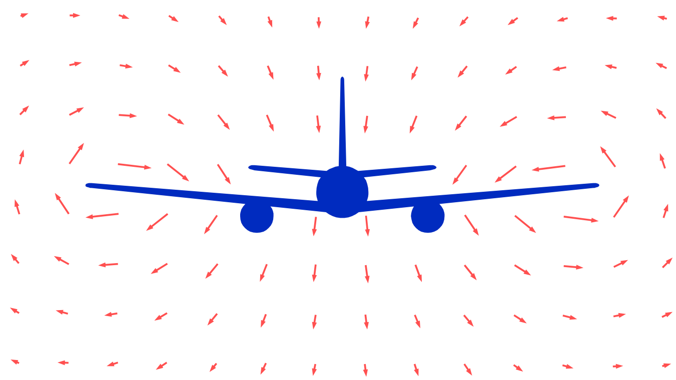


Figure 10-11

Here's a great photograph illustrating just how powerful that flow of air around the wingtips can be.[1]

The photograph shows the flow behind an approaching 777, as demonstrated by the patterns seen in the clouds through which the airplane has flown. Very clearly seen are the counter-rotating wingtip vortices; note that the vortex field persists far behind the airplane and also that the field descends with increasing distance.

Figure 10-12

cause of induced drag

Aerodynamicists have several ways of describing the reason for induced drag. We won't dwell on this, as it's of academic interest only. Suffice it to say that it's clear from the photograph above that there is considerable energy in the wingtip vortices of an airplane. This is an energy loss to the airplane, which must be compensated for by added thrust. Hence it's considered to be one component of the airplane's total drag.

The induced drag can be calculated from the following equation:

$$\textit{induced drag} = \frac{L^2}{\pi e q b^2} \qquad \text{(eq. 3)}$$

Where L is the lift
e is the "span efficiency factor"
q is the dynamic pressure
b is the wingspan

The "span efficiency factor" is a function of planform. Elliptical wings have an e value of 1.00; for other planforms, e varies from 0.85 to 0.95. Note also that the induced drag is a function of the square of the lift.

Notice also that the induced drag is an inverse function of the square of the wingspan, thus explaining why high-performance airplanes such as sailplanes typically have long wingspans.

1. This photograph is by Mr. Steve Morris of AirTeamImages and is reproduced here with his kind permission.

To express the induced drag in a dimensionless coefficient form, you need to recall that:

$$C_{D_{induced}} = \frac{induced\ drag}{qS}$$

and that:

$$L = C_L Sq \quad \text{and therefore} \quad L^2 = C_L^2 q^2 S^2$$

Finally remember that the aspect ratio AR is defined as $AR = \frac{b^2}{S}$.

Playing around with the equations then will yield:

$$C_{D_i} = \frac{C_L^2}{\pi e AR} \qquad \text{(eq. 4)}$$

Equation 4 shows that induced drag is inversely affected by aspect ratio, and that therefore the best thing to do to achieve a wing having low induced drag is to maximize the aspect ratio. That's true, but the increase of aspect ratio should be accomplished <u>without a decrease in the wing area</u>. Simply reducing the wing chord would increase the aspect ratio (good) but failing to increase the span at the same time so as to hold the same wing area will result in an increase in the lift coefficient (bad).

Winglets and other wingtip treatments are designed and installed for the sole purpose of reducing wingtip energy losses. We'll be discussing them in a following chapter.

Effect of Mach Number on Drag

high-speed flow on a wing

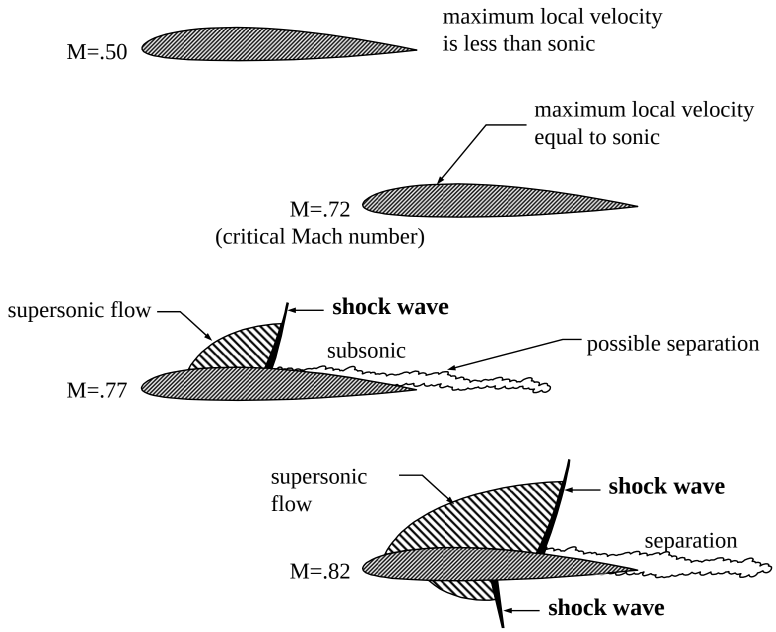


Figure 10-13

Refer to the illustration to the right. These four figures show the effect of local velocities of flow over a wing.

When the Mach number is low, the local velocities on the wing are at all points less than the speed of sound (*subsonic*).

At a Mach number referred to as the *critical Mach number,* the local velocity at some point on the wing's upper surface just becomes equal to the speed of sound. There is no area of supersonic flow. The critical Mach number is affected by a number of factors, the principal one of which is the thickness of the wing relative to its chord. Thicker wings will have lower critical Mach numbers.

As the Mach number increases above the critical Mach number, an area of supersonic flow develops. At each point in that region, the local velocity of flow is greater than the speed of sound. This region is sometimes referred to as the *supersonic bubble*.

Since we are dealing here with subsonic commercial jet transport airplanes, the free stream velocity ahead of the airplane is subsonic, and therefore the supersonic flow on the wing's upper surface must somewhere decelerate back to subsonic velocity. In the chapter entitled "Near the Speed of Sound" we said that a shock wave will form where a flow decelerates back to a subsonic velocity. Thus, above the critical Mach number, there will be a region of supersonic flow on the wing which will increase in size as the airplane's velocity increases, and which ends at a shock wave.

The shock waves in this instance are not <u>normal</u> shock waves such as we discussed in the "Near the Speed of Sound" chapter. Instead, they are *oblique shock waves*.

If the airplane's speed becomes high enough, supersonic flow will also develop on the wing's <u>lower</u> surface, in addition to the region of supersonic flow on the upper surface.

Shock waves have the effect of increasing the airplane's drag in two ways:

First, there is an energy loss through the shock wave. This energy loss is, in effect, an increase of drag;

Second, as illustrated in the sketch to the right, the shock wave has the effect of increasing the thickness of the boundary layer due to the pressure rise across the shock. There is a resulting increase in pressure drag. The shock wave may also trigger separation of the flow from the wing, but this doesn't occur in normal cruise, only at higher Mach numbers or at increased load factors.

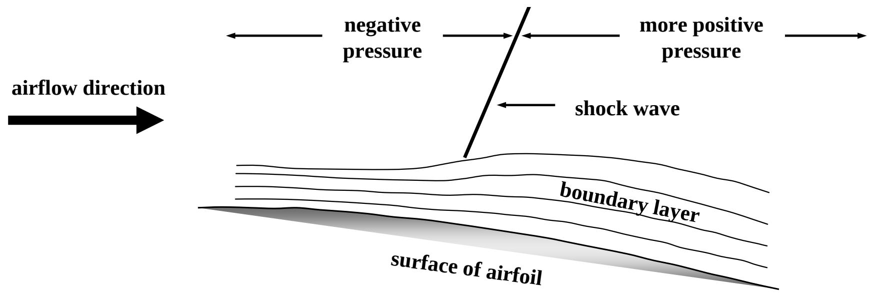


Figure 10-14

high-speed buffet

Shock wave formation on a wing and the flow separation behind the shock that results lead to a degree of turbulence in the airflow downstream of the shock wave. This turbulence increases with increasing speed. Impingement of this turbulent flow on the airplane's fuselage and tail surfaces can cause noticeable airframe buffet, referred to as *high-speed buffet.*

High-speed buffet is an entirely different phenomenon from the low-speed buffet condition that results from flight at speeds approaching the stalling speed, but it's characterized in the same way.

We'll be discussing high-speed buffet at greater length in the chapters entitled "Altitude Selection" and "Normal Cruise" where we explain the concept of a *buffet boundary* and its effects on altitude and speed selection.

effect of wing sweepback angle

Earlier in this chapter we defined the sweepback angle of a wing, called Λ (upper case Greek character lambda). Wings on airplanes designed for lower speeds usually have unswept wings; the wings on airplanes designed for higher speeds, however, use sweepback angle as a means of reducing drag at high speeds.

Refer to the illustration to the right: sweeping the wing has an effect which can be looked at in two different ways:

- Sweeping the wing effectively increases the chord length of the wing without increasing its thickness, thus making the wing appear thinner to the airflow. This has the effect of increasing the value of critical Mach number; or,
- Sweeping the wing introduces two components of airflow over the wing: chordwise and spanwise. The chordwise component is $V\cos\Lambda$, and the spanwise component is $V\sin\Lambda$. Both components are less than the free stream velocity, and hence the chordwise flow velocity is decreased by the sweepback.

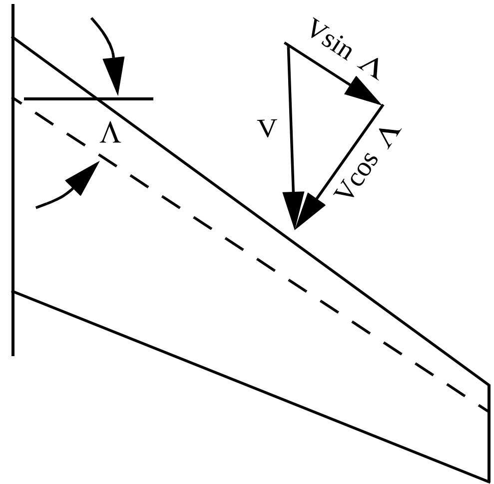


Figure 10-15

Both of these two different viewpoints are really one and the same. If you look at the flow over the wing parallel to the direction of flight then the first viewpoint – sweeping the wing makes it seem thinner – explains why sweepback is helpful. On the other hand, if you view the flow over the wing as being composed of two components, then the second viewpoint explains why sweepback is helpful. Take your pick, either way the answer's the same: sweeping the wings back improves the wing's drag characteristics at higher speeds.

So sweepback increases the critical Mach number. Refer to the illustration to the right. Looking at the drag coefficient, at a constant lift coefficient: at the lower Mach numbers, the drag coefficient is, for practical purposes, constant. When the velocity reaches M_{CR} however the drag coefficient begins to rise.

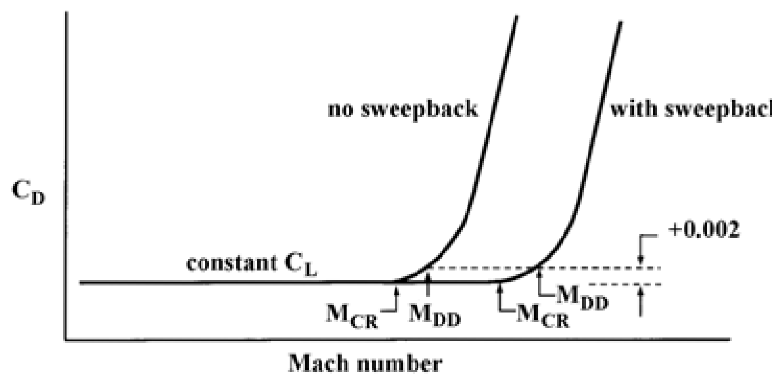


Figure 10-16

Note that initially the drag increases slowly as the Mach number increases above M_{CR}. In the illustration above, the Mach number at which the drag coefficient has increased by 0.002 above its baseline value is referred to as M_{DD}, the Mach number for drag divergence.[1]

The drag rises more rapidly after the region of supersonic flow has grown to occupy a substantial percentage of the chord and the shock wave is well developed.

1. You may encounter elsewhere a definition of critical Mach number that is different from the one given here. When some aerodynamicists refer to critical Mach number, they are referring to what we are here calling M_{DD}, the Mach number at which the drag has increased by 0.002 above its baseline value.

The illustration to the right demonstrates that wing sweepback actually has <u>two</u> beneficial effects: it not only delays the drag divergence, but it also reduces the <u>magnitude</u> of the drag increase. We see from the above discussion that wing sweepback has substantial benefits. You should be aware, however, that those benefits are not achieved without some penalty: wings having greater sweepback angles are less efficient at low speeds than wings having smaller sweepback angles. Like many other parameters of the design of any airplane, the angle of wing sweepback must be chosen carefully with the airplane's mission well in mind.

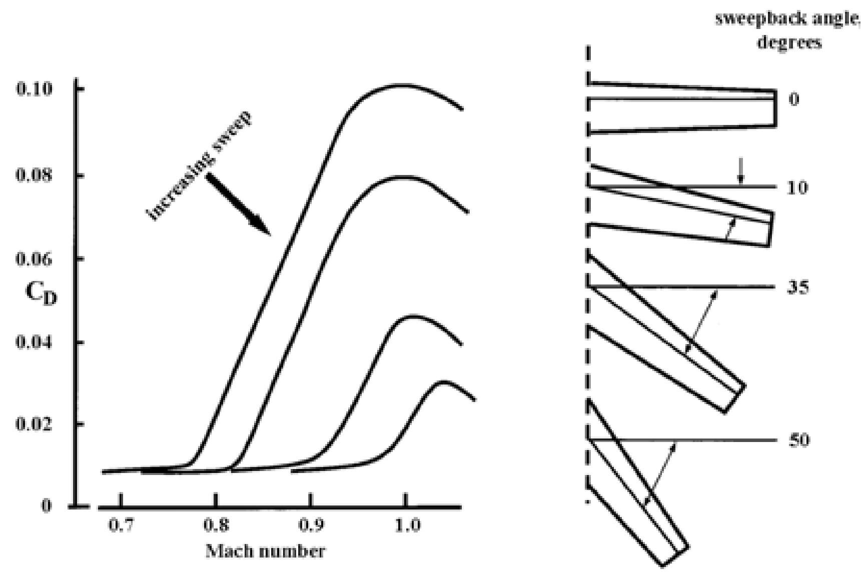


Figure 10-17

The Boeing 737, for example, is primarily intended for shorter routes and shorter runways. In order to achieve acceptable short runway performance, a quarter-chord sweepback angle of 25 degrees was chosen. This selection affords better takeoff distances, but the penalty is relatively poorer high-speed performance: the 737's cruising speed for good fuel efficiency is only about Mach 0.72 to 0.73 on the older versions of the airplane to Mach 0.78 to 0.79 for the newer versions. These were considered to be acceptable for the 737 in view of the shorter routes it flies.

The Boeing 747, on the other hand, is primarily intended for longer flights. For those missions, better high-speed performance is needed in order to keep flight times within reasonable limits. This airplane was designed with a quarter-chord sweepback angle of 37.5 degrees. This gives the 747 a much higher cruise speed: Mach 0.84 to 0.86 is the usual range of cruise speed for the 747 for good fuel efficiency.

At typical cruise altitudes, therefore, a 747 will cruise some 40 to 50 knots faster than a contemporary 737, with a much greater differential when compared to the older 737s. This speed advantage for the 747 is important given its typically much longer trip distances. The 747's takeoff distances, however, are substantially longer than those of the 737.

effect of Mach number on the drag polar

Earlier in this chapter you've seen the effect that higher Mach numbers have on the drag of the airplane. You've seen that airplane drag is essentially constant, for a given lift coefficient, regardless of speed – <u>up to the critical Mach number</u>. Above that speed the drag begins to rise, slowly at first and then more rapidly as the shock wave becomes larger and stronger.

You'll recall that in the preceding chapter, we introduced you to "the drag polar – a performance engineer's best friend", a plot of drag coefficient versus lift coefficient. Clearly, the high speeds will affect the drag polar.

Current practice at Boeing is to provide both "low-speed" and "high-speed" drag polars for the different airplanes. The high-speed polars look something like the illustration to the right.

In these polars, as you see, one line will serve all of the Mach numbers up to M_{CRIT}. Above that value, more lines are added for higher Mach numbers. Bear in mind that the value of M_{CRIT} varies from airplane to airplane – here we're showing an M_{CRIT} of 0.6.

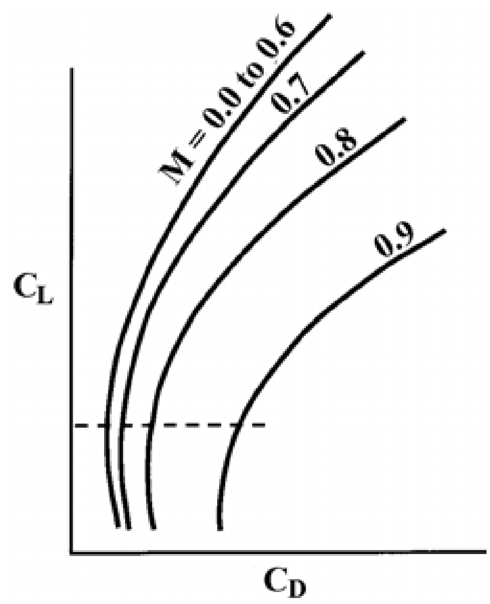


Figure 10-18

Chapter 11: Drag Analysis

Introduction

Any airplane design, whether by Boeing or one of its competitors, succeeds or fails depending on drag. Airlines make a profit or a loss, depending on drag.

No, these are not exaggerations. If Boeing were to offer a drag-inefficient airplane for sale, no airline would want to buy it since it would cost too much to operate: too much fuel consumption, not enough takeoff weight. If an airline operates drag-inefficient airplanes because of inadequate maintenance practices, that unnecessary drag could cause otherwise profitable operations to become unprofitable ones.

Because an airplane's drag is fundamental to so many of a performance engineer's duties, it deserves detailed discussion. In this chapter, we'll describe the components of an airplane's drag, where to find drag information, and where and how this information is typically used.

Pressure Drag and Skin Friction Drag

As discussed in a previous chapter, an airplane's motion through the air produces aerodynamic force. That force can be divided into two components: *lift* and *drag*.

In its broadest definition, "drag" is simply that component of aerodynamic force which acts to impede an airplane's motion through the air. Drag is distinct from lift in that drag is an aftward longitudinal force acting parallel to the path of flight in opposition to the engines' thrust force; lift is an upward vertical force, perpendicular to the path of flight in opposition to the airplane's weight, having no retarding effect on the airplane's motion.

Aerodynamicists will tell you that there are really only two sources of drag on an airplane: *pressure drag* and *skin friction drag*. That is, an airplane can only "feel" the aerodynamic retarding forces through air pressure (perpendicular to the surface of the airplane) or through skin friction, shearing force acting tangentially to the surface of the airplane.

Aerodynamicists will also tell you that several physical mechanisms in the flow field contribute to the pressure drag. Dividing the pressure drag into "components", according to how much of the drag is caused by each mechanism, is something we do all the time in practice. However, we should keep in mind that this division into components is not exact. Because the different flow-field mechanisms interact in complicated ways, it is not possible to rigorously define how much of the pressure drag is attributable to which mechanism. We work around this in practice by using theoretical idealizations to make estimates of the separate contributions ("components") of the drag.

With that in mind, let's list and then discuss the components of the drag.

Components of Drag

Here's a complete list of the components of an airplane's total drag, in alphabetical order, without showing the estimated contribution of each component to the total drag:

- compressibility drag
- excrescence drag
- form drag
- induced drag
- interference drag
- "other" drag
- skin friction drag
- trim drag

Let's discuss each of these in turn.

compressibility drag

As discussed in the chapter entitled "Wings", *compressibility drag* is encountered only at Mach numbers sufficiently high that a region of supersonic flow, and its resultant shock wave, have formed on the wings.

Compressibility drag can be avoided by flying at speeds below M_{CRIT}, however this will not be the optimum speed for fuel consumption, as you'll see in following chapters.

excrescence drag

Excrescence drag is the component of drag caused by the sum of all deviations from a smooth, sealed external surface. Such deviations include:

- discrete items, such as antennas, masts, external lights,
- mismatches and gaps, such as external patches, steps and gaps in skin joints and around windows, doors, access panels, and the like,
- internal airflow and leakage, for example leaks of air from higher pressure to lower pressure surfaces due to deteriorated seals,
- surface roughness, for example non-flush fasteners, rough paint and surface finish, dirty airplane exterior surfaces,
- control surface mis-rigging.

These can mostly be minimized by good maintenance practices, with the exception of the unavoidable discrete items such as antennas.

form drag

One component of “pressure drag”, discussed below, is *form drag*. Form drag is that portion of the pressure drag caused by the growth and/or separation of boundary layers.

induced drag

As discussed in the previous chapter, induced drag is the inevitable result of the difference in pressures between the lower and upper wing surfaces. The magnitude of a wing’s induced drag is largely influenced by its design.

Ideally, a wing will have a large span in order to minimize this component of drag. In practice, however, large spans are problematic for commercial jet transport airplanes because they may result in airport gate compatibility problems.

In the chapter entitled “Aerodynamic Devices” we’ll look at various wingtip treatments which may be used to reduce this component of drag.

interference drag

Interference drag is the increase in drag caused by the change in the aerodynamic flow pattern that results from the placement of two bodies in close proximity. That is, the total drag of the two bodies together will be greater than the sum of their individual drags.

A good example is the wing-body intersection which can cause interference drag; this drag, however, can be reduced by using fillets. Another good example is the integration of an engine nacelle with the wing to which it’s attached, which is a particularly difficult challenge for minimizing interference drag.

other drag

This component of drag includes a number of different elements. Examples of additional sources of drag that can occur at various times during flight will include:

<u>drag of the landing gear when extended.</u>
This drag item is substantial and must be accounted for when computing performance in any gear-down configuration.

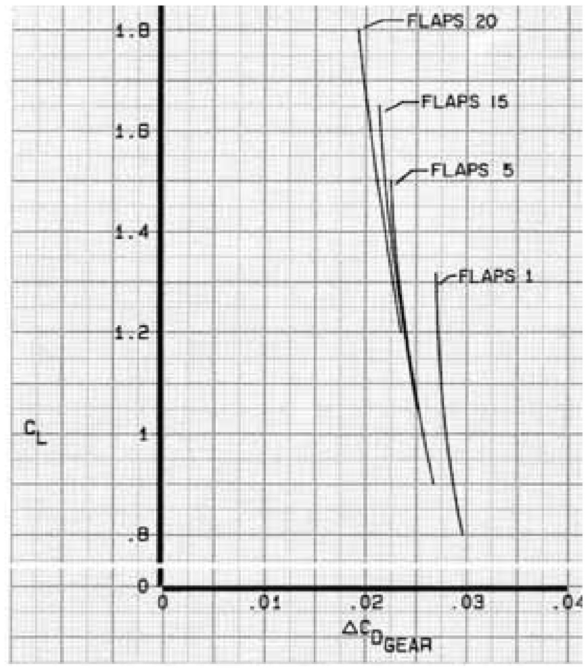


Figure 11-1

Boeing provides performance engineers with landing gear drag in a number of different ways, depending on the airplane model. It may be provided as an increment, $\Delta C_{D_{GEAR}}$ as shown in the illustration to the right, or it may be provided in low-speed gear-down drag polars.

<u>drag of the spoilers when extended</u>
Spoilers, also in some contexts referred to as speed brakes or speedbrakes, are described in detail in the chapter entitled "Aerodynamic Devices". Briefly, their function is to decrease lift and increase drag, thereby increasing the drag-to-lift ratio.

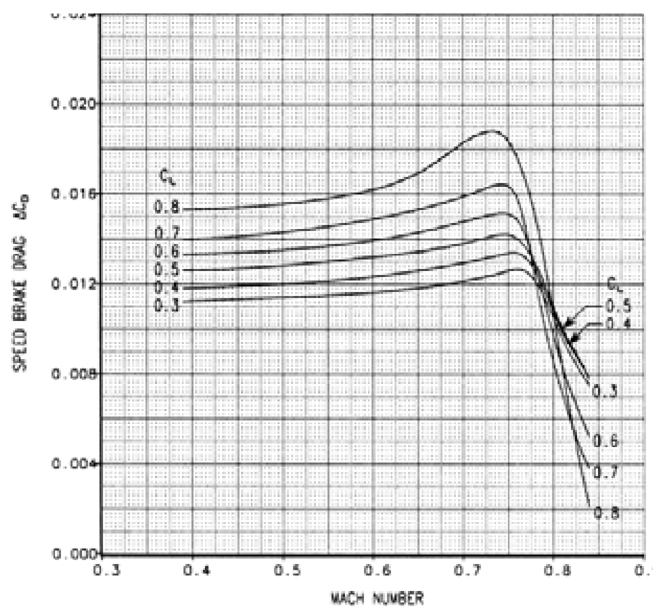


Figure 11-2

The principal use of spoilers is on the ground when decelerating in a rejected takeoff or during a landing. Use of spoilers at these times adds drag, but what's more important is that the reduction of lift caused by extension of the spoilers causes more of the airplane's weight to be placed on the landing gear, resulting in greater wheel braking effectiveness. Spoilers are also used in certain flight maneuvers, to allow steeper descent angles without an unacceptable increase of speed. Spoilers are also used <u>asymmetrically</u> when needed to augment the ailerons to achieve higher roll rates.

You'll observe that as the Mach number increases, the drag increment initially increases, then – strangely – decreases. This behavior is due to a condition known as "blowdown".

At higher Mach numbers, the aerodynamic forces pushing backward on the spoiler panels simply overpower the hydraulic pistons which raise them; the spoiler panels "blow down" to reduced angles of deflection resulting in a decrease of the drag they cause.

drag of an inoperative engine - windmilling drag

Whenever operating in an engine-inoperative condition, the drag increment resulting from the failed engine which is then "windmilling" – rotating because of the airflow into the inlet – must be accounted for. This drag increment is usually referred to as *windmilling drag* but sometimes as *windmilling and spillage drag*. In the illustration seen to the right, the drag increment is referred to as $\Delta C_{D_{ENGINOP}}$

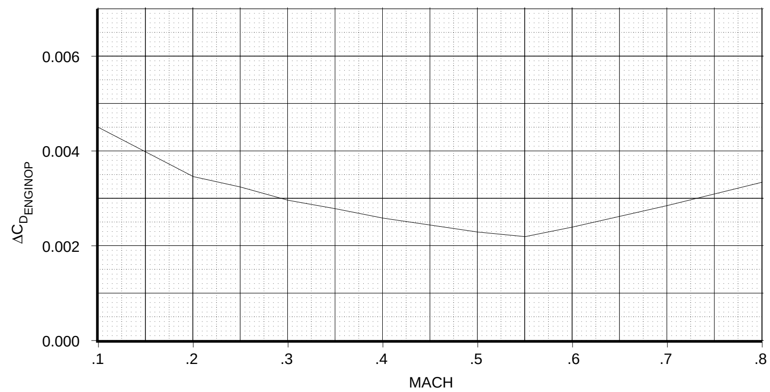


Figure 11-3

drag of deflected flight controls

Any flight control, when deflected, will cause some increase in the airplane's drag. And while most deflections of flight controls are momentary, that's not always the case.

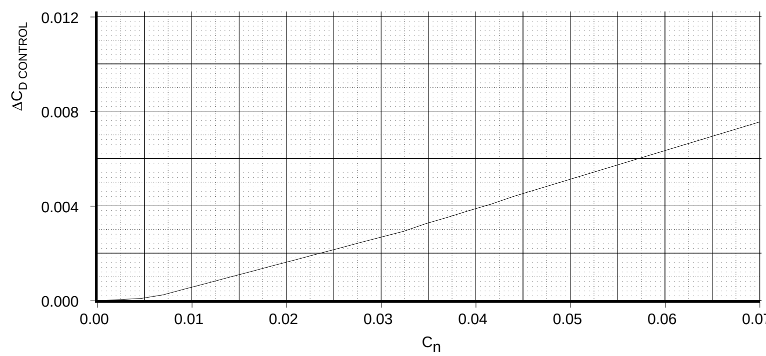


Figure 11-4

In some cases, the added drag resulting from deflected flight controls must be accounted for when calculating airplane performance. Two examples of that would include emergency descent, during which the spoilers are extended, and engine-inoperative flight such as climbout after take-off with an engine failure, for which there exist regulatory requirements for minimum gradient capability.

In engine-inoperative flight, the asymmetric condition of the thrust produces a yawing moment which must be counteracted with rudder deflection. The deflected rudder in turn causes a rolling moment which must be countered with aileron deflection. The combined increase in drag due to the deflection of the rudder and ailerons is referred to as *control drag* or *yaw drag*.

You'll notice that in the chart shown above the control drag $\Delta C_{D_{CONTROL}}$ is shown as a function of the yawing moment coefficient C_N. Moment coefficients in general are discussed in the chapter entitled "Lift and Drag" and the equations for the yawing moment coefficient C_N can be found in Appendix 1, "Summary of Useful Information For Performance Engineers".

Calculations of engine-out climb gradients are necessary for compliance with regulatory performance requirements for takeoff and landing.

skin friction drag

This component of drag is caused by the viscous shear forces generated when air passes over the external surfaces of the airplane. Skin friction drag may be reduced by minimizing the exposed surface area and by keeping the surface smooth.

Maintaining laminar flow on the forward portions of the airplane surfaces can also help to reduce skin friction drag, but this is difficult to achieve on large commercial jet transport airplanes. Research into methods for enhancing laminar flow is ongoing.

trim drag

In flight, the airplane's center of gravity will always be forward of the wing's center of lift. This couple causes a nose-down pitch moment. That nose-down moment is countered by a nose-up moment produced by the horizontal tail of the airplane.

Trim drag is that drag resulting from the deflection of the horizontal stabilizer and elevator needed to produce the downward force which in turn produces the required amount of nose-up pitch moment. Since the horizontal tail is producing a downward force, the wing must necessarily produce an extra element of lift to offset it;. As a result, there is a slight increment in the induced drag of the wing. This increment of induced drag may be considered to be part of the trim drag, or it may be considered to be part of the wing's induced drag since it really is additional drag due to lift.

effect of CG location on trim drag

We have just explained that trim drag is that component of drag resulting from the deflection of the horizontal stabilizer and elevator necessary to keep the airplane in longitudinal balance; we said also that the wing must create some amount of extra lift to offset the download on the horizontal tail.

The horizontal tail download force, however, is not a constant – it's a function of the position of the airplane's center of gravity relative to the center of lift. You'll see, then, that the location of the CG will have some effect on the total airplane drag force.

This topic is discussed at length in the chapter entitled "Alternate Forward CG". Please refer to that chapter if you want more information.

Parasitic Drag

This is a term sometimes used to denote all drag that does <u>not</u> result from either lift or compressibility. Thus, *parasitic drag* consists of the following:

- skin friction drag
- excrescence drag

- interference drag
- form drag

Pressure Drag

The component of drag caused by the pressure distribution over the three-dimensional shape of the airplane is referred to as pressure drag. This term includes several different elements: induced drag, shock drag, and form drag. Each of those is described in this chapter.

Pressure drag can be reduced by careful shaping of critical areas such as the cockpit and aft body closure.

Total Drag

In summary, the total drag of an airplane can be considered to be the sum of all of the items discussed above. Thus, total drag equals

	skin friction drag	"parasitic drag"
plus	excrescence drag	
plus	interference drag	
plus	form drag	
plus	trim drag	sometimes grouped together as "induced and trim drag"
plus	induced drag	
plus	compressibility drag	
plus	other drag	

The Drag Polar

The "drag polar" is discussed in simple terms in the chapter entitled "Lift and Drag". Briefly, a drag polar is a graph of the lift coefficient for any given condition plotted against the drag coefficient for the same condition.

You've seen earlier in this chapter that an airplane's total drag consists of a number of different components. In a drag polar, most of those are summed together as one single drag number. The drag polars include all the elements of drag except for the "other" drag: engine-inoperative ("windmilling") drag, flight control ("yaw") drag, gear-down drag, and spoiler drag.

The low-speed drag polars, by virtue of the lower speed regimes to which they apply, do not include any compressibility drag.

Shown to the right is a typical low speed drag polar; in this instance, it is the polar for an airplane with the flaps at a takeoff setting.

Observe that this polar shows the relationship of drag to lift at several different positions of the airplane center of gravity (CG). By doing so, it is accounting for the difference in trim drag at the different CG positions.

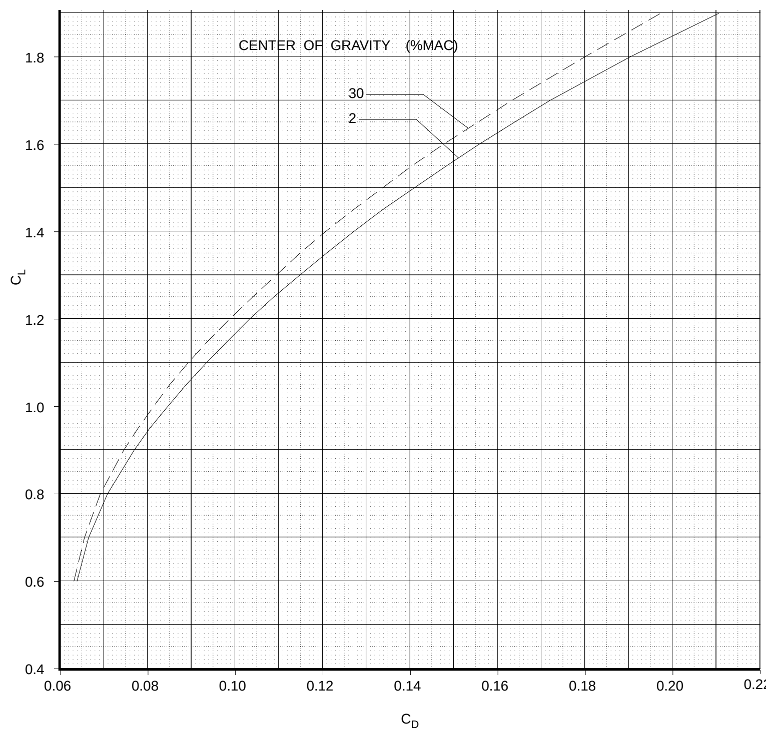


Figure 11-5

The high-speed drag polars, such as the one seen to the right, do show the effect of compressibility by showing the drag at varying Mach numbers.

The high-speed drag polars are usually shown for a center of gravity which is approximately in the middle of its range.

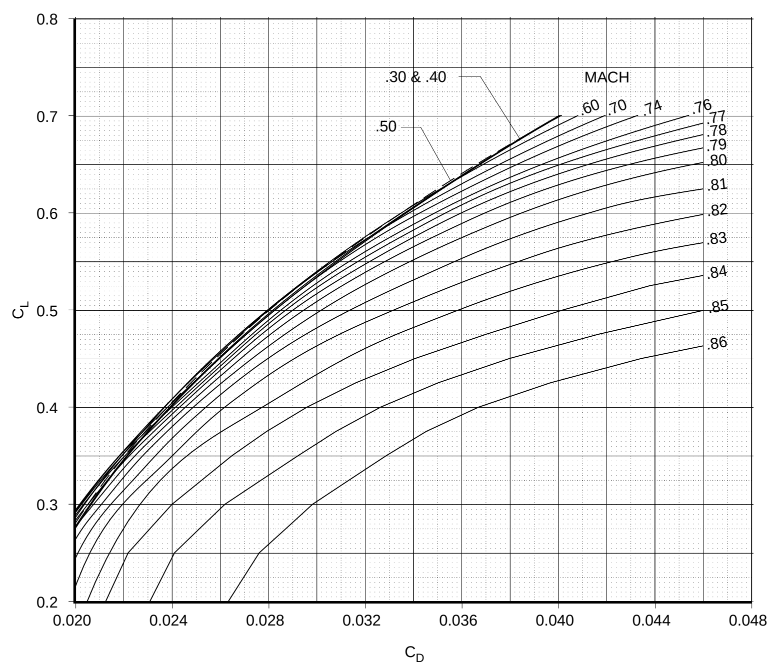


Figure 11-6

Reynolds Number Effect On Drag

In an earlier chapter entitled "Flow Near a Surface" we discussed the effect of viscosity on aerodynamic flow over a surface. We mentioned that this new parameter we call the Reynolds number is related to viscosity effects, and that the Reynolds number will therefore have an effect on the airplane drag.

The theory behind the calculation of the Reynolds number effect on drag is complex and far beyond the scope of this chapter. Suffice it to say that:

- The drag polars published in the Boeing Performance Engineer's Manuals are based on nominal values of air temperature and pressure.
- Any time the air temperature and/or pressure are different from the nominal conditions, the published drag coefficient will be slightly incorrect, the difference being a drag coefficient correction called $\Delta C_{D_{RE}}$ the Reynolds number correction.
- The values of $\Delta C_{D_{RE}}$ are relatively small but when striving for accurate flight planning the Reynolds number effects should be checked.
- The certificated performance contained in the Airplane Flight Manual and AFM-DPI does not include Reynolds number corrections, which are considered to be negligible for the conditions existing for takeoff and landing.

The "nominal" Reynolds number conditions that are the basis for the published drag polars are these:

- The nominal temperature is the standard day value for the reference altitude;
- The nominal air pressure is based on a "nominal altitude" which is defined as a function of the parameter $\frac{W}{\delta}$, where W is the airplane's weight and δ is the pressure ratio at the airplane's altitude.

The value of the Reynolds number correction is given by:

$$\Delta C_{D_{RE}} = B \times 10^{-4} \log_{10}\left[\frac{\frac{1}{M}\left(\frac{RE}{ft}\right)}{\frac{1}{M}\left(\frac{RE}{ft}\right)_{nom}}\right]$$

where B is a constant that is different for each airplane model
M is the Mach number

$\frac{1}{M}\left(\frac{RE}{ft}\right)$ is the Reynolds number given by the following equation:

$$\left(\frac{1}{M}\frac{RE}{ft}\right) = 5.13384\times10^{6}\left(\frac{\theta + 0.38312}{\theta^{2}}\right)\times\delta$$

where δ is the air pressure ratio
θ is the air temperature ratio

You see that $\frac{1}{M}\left(\frac{RE}{ft}\right)$ is evaluated twice: once for the actual values of δ and θ according to the conditions of flight, and once again for the nominal values of δ and θ.

Let's illustrate this with an example. Here are the example conditions:

- 757-200
- cruise at 35,000 feet
- weight = 220,000 pounds
- ISA and ISA+20°C

For the 757-200, the value of B is -49.434.

At 35,000 feet, δ is equal to 0.2353, and therefore $\frac{W}{\delta} = \frac{220000}{0.2353} = 934{,}977\ pounds$

At that value of $\frac{W}{\delta}$, we can. find from the PEM that the reference altitude is 36,937 feet.

At the reference altitude of 36,937 feet, the pressure ratio δ is equal to 0.214431 and the standard day OAT is equal to -56.5°C or 216.65°C. For that temperature, θ is 0.751865.

At the actual altitude and temperature, δ is equal to 0.235305 and θ is equal to 0.759355.

therefore, for the nominal conditions:

$$\frac{1}{M}\left(\frac{RE}{ft}\right)_{nom} = 5.13384\times10^{6}\left(\frac{0.751865 + 0.38312}{0.751865^{2}}\right)\times 0.214430 = 2{,}210{,}244$$

and for the given example conditions:

$$\frac{1}{M}\left(\frac{RE}{ft}\right) = 5.13384\times10^{6}\left(\frac{0.759355 + 0.38312}{0.759355^{2}}\right)\times 0.235305 = 2{,}393{,}485$$

From the two preceding values, then:

$$\Delta C_{D_{RE}} = -49.434 \times 10^{-4} \log_{10}\left[\frac{2{,}393{,}485}{2{,}210{,}244}\right] = -0.00017$$

Repeating the calculations, this time for ISA+20°C: the value of $\frac{1}{M}\left(\frac{RE}{ft}\right)_{nom}$ doesn't change. The value of $\frac{1}{M}\left(\frac{RE}{ft}\right)$ does change – it becomes 2,131,442. $\Delta C_{D_{RE}}$ becomes equal to +0.00008, so the air temperature increase from ISA to ISA+20°C causes the airplane drag coefficient to increase by 0.00025.

Control of Drag Through Maintenance Practices

Much of the science and art of airplane maintenance is focused on the control of drag by good maintenance practices including:

- elimination of rough or peeling paint;
- keeping the airplane clean;
- elimination of seals extruding into the airstream;
- avoiding mismatches and gaps such as steps and gaps in skin joints and around windows;
- ensuring that flight controls are correctly rigged;
- avoiding when possible the use of external patches, non-flush fasteners, etcetera;
- maintenance of seals to eliminate internal airflow and leakage.

This is not a complete list. Reference to the Boeing documents entitled "Maintenance For Fuel Conservation" is highly recommended.

Drag and the Airline Performance Engineer

Much of the science and art of performance engineering requires a knowledge of airplane drag, how it may be controlled through good maintenance practices, and how to calculate and use it as needed for performance-related calculations.

The duties of a typical airline Performance Engineer may include:

- collecting and/or analyzing airplane fuel consumption data recorded in flight;
- maintaining airplane fuel efficiency records;
- coordinating with the Maintenance organizations on fleet fuel efficiency matters;
- calculation of airplane flight planning data such as trip fuel loads and fuel reserves;
- coordinating with the Dispatch organizations on fleet fuel efficiency data enabling flight planning specifically tailored to each airplane ("tail number flight planning");
- calculation of climb gradients, obstacle clearance profiles, driftdown profiles;

- and much more.

Drag information for Boeing airplanes is made available to authorized persons in several formats and several different locations.

The Performance Engineer's Manual (PEM) chapter entitled "Aerodynamic Data" contains:

- the low-speed (flaps up and down) drag polars;
- the high-speed drag polars;
- the engine-inoperative ("windmilling") drag increment;
- the yaw drag ("control drag") increment;
- the spoiler drag increments;
- the landing gear drag increments;
- the Reynolds number correction to the drag coefficient.

The PEM chapter entitled "configuration tables" contains tabular data:

- the high-speed (flaps up) drag polar;
- the low-speed (flaps up) drag polar;
- the engine-inoperative drag increment;
- the engine-inoperative ("windmilling") drag increment;
- the yaw drag ("control drag") increment;
- the spoiler drag increments;
- the landing gear drag increments;
- the Reynolds number correction to the drag coefficient.

The databases for the INFLT (Inflight) and APM (Airplane Performance Monitoring) computer programs include the same information as the PEM Configuration Tables chapter.

Chapter 12: Aerodynamic Devices

Introduction

The term *aerodynamic device* as it's used in this chapter includes any device on an airplane which alters its standard flaps-up straight-and-level aerodynamic flight condition or is intended to improve its aerodynamic qualities.

Thus, "aerodynamic devices" includes the following:

- primary and secondary flight controls
- leading edge and trailing edge high-lift devices
- speedbrakes/spoilers
- flow-improvement devices
- drag reduction devices

This chapter will discuss each of these categories.

Primary Flight Controls

The airplane's *primary flight controls* are movable surfaces which allow the pilots to change the airplane's roll, pitch and yaw attitude.

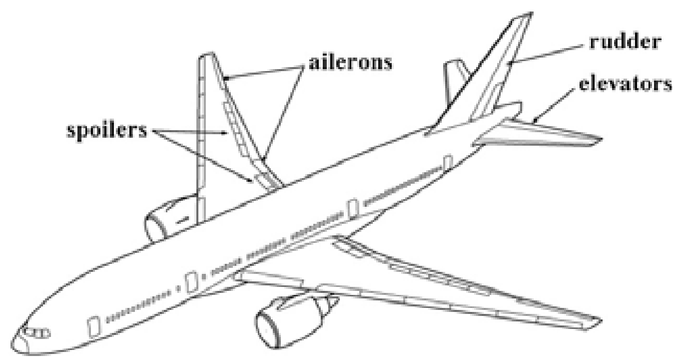


Figure 12-1

Roll control provides the ability to turn the airplane by causing the airplane to bank; pitch control allows the pilots to alter the airplane's vertical flight path by increasing or decreasing its pitch attitude; yaw control is needed to counter an asymmetric thrust condition, such as occurs in the event of an engine failure, and is also used to keep turns "coordinated" – preventing a slipping condition in a turn.

roll control

Roll control is provided by the *ailerons* and *flight spoilers*.

For a slow rate of roll to the left, for example, turning the control yoke in the cockpit slightly to the left will raise the aileron on the left wing and will lower the aileron on the right wing. As a result, the right wing will create slightly more lift than the left wing, and the airplane will roll to the left.

Turning the control wheel to the left by a larger amount will not only raise the left aileron and lower the right aileron, it will also raise the flight spoilers on the left wing. The rolling moment created by the ailerons is thus augmented by a rolling moment to the left caused by the spoiler deployment on the left wing resulting in a faster rate of roll.

Some Boeing airplanes have only one aileron on each wing. The others have two ailerons on each wing; one is on the outer portion of the wing, the other is on the inner portion of the wing. On these latter airplanes, the outboard ailerons are locked into their “faired” (that is, undeflected) position whenever the flaps are retracted. At the greater airspeeds associated with flaps-up flight, the inboard ailerons are adequate to produce the needed rates of roll.

On the 777 airplane, the inboard ailerons are more correctly referred to as *flaperons*, since they function not only as ailerons but also, in flaps-down flight, deflect symmetrically downward producing extra lift force.

pitch control

Pitch control is provided by the *elevators*. Moving the cockpit control columns forward causes the elevators to deflect downward, slightly decreasing the downward force produced by the horizontal tail surfaces. This results in a nose-down pitching moment. Deflecting the control columns rearward causes the elevators to deflect upward, increasing the download on the horizontal tail, and producing a nose-up pitching moment.

yaw control

An airplane’s *rudder* operates in a manner identical to that of the rudder on a boat. The rudder can be deflected to the right or left by pressing on the pilot’s right or left rudder pedal respectively. That rudder deflection causes the vertical tail to develop a lateral force which creates a yawing moment.

The primary use for the rudder is for *yaw control* for steering on the runway during takeoff and landing, and to counteract the thrust asymmetry in the event of an engine failure. Since the rudder has relatively little yaw effect at slow airspeeds, rudder pedal steering at the slower airspeeds during takeoff and landing is augmented with nose landing gear steering also controlled by the rudder pedals.

A *yaw damper* function of the autoflight system deflects the rudder slightly in flight as needed to control yawing in turbulence and to eliminate the coupled roll/yaw dynamic condition known as *Dutch roll*. It also works automatically to keep all turns properly coordinated between the roll and yaw modes.

Secondary Flight Controls

The term “secondary flight controls” includes the roll, pitch and yaw trim controls, and the spoilers/speedbrakes.

Let’s first define two terms:

- *Faired*: when we say that a flight control is in its faired position, we mean that it is not deflected from its neutral position – the aileron is aligned with the wing, the rudder is aligned with the vertical tail, the elevator is aligned with the stabilizer.
- “Trim”, “trimmed”, “trimming”: in general, “*trimming*” a flight control means adjusting that control, by one means or another, so that no force need be applied by the pilot to the rudder pedals or control column to maintain steady-state flight. In other words, a “trimmed” airplane will fly “hands-off” in a constant attitude.

Clearly, any deflection of any flight control will increase an airplane’s drag, thus it’s essential for fuel conservation to maintain airplanes in their optimum aerodynamic condition so that trim isn’t required for hands-off flight.

roll trim

To counter any tendency of an airplane to roll when the control wheel is released, there is an *aileron trim* system. It allows the pilots to apply small amounts of constant aileron deflection without force on the control wheel. Then, although the wheel will be slightly offset from its undeflected position, the pilot will not need to exert any force on the control column to maintain a wings-level attitude or a steady bank angle.

yaw trim

In a manner similar to that of the aileron trim system, the *rudder trim* system allows the pilots to apply a small amount of rudder deflection without requiring force on the rudder pedals.

Rudder and aileron trim are usually quite small deflections, as may be necessary for slight yaw and/or roll tendencies. They are principally useful for trimming an airplane for an engine-inoperative condition.

pitch trim

Although rudder and aileron trim are used only to small degrees, except for the engine-inoperative condition, *elevator trim* is used all the time. The airplane’s weight, center of gravity and velocity are constantly changing throughout the duration of any flight, necessitating frequent adjustments to the pitch trim.

The down-force that the horizontal tail must maintain for longitudinal balance varies widely with changing weight, CG and speed. For that reason, it would not be practical to trim the airplane in pitch in the same way it’s trimmed in roll and yaw.

Think of the airplane’s horizontal tail as being made up of two principal parts. The forward portion of the horizontal tail, that portion forward of the elevator hinge line, is referred to as the *stabilizer.* That portion of the horizontal tail aft of the hinge line is referred to as the *elevator.* It is considerably smaller than the stabilizer.

The stabilizer is mounted on a lateral axis which allows the stabilizer to be rotated nose-up or nose-down about its center of rotation. This rotation of the stabilizer is accomplished by a mechanical actuator controlled either from the flight deck or by the autoflight system.

The elevator is deflected upward or downward about its hinge line by movement of the control columns in the cockpit, as described above. The elevator alone, however, would be incapable of controlling the airplane in pitch throughout its range of weight, CG location and speed. For that reason, pitch trim is accomplished by rotating the stabilizer nose-up or nose-down about its axis. Thus, pitch trim is usually referred to as "stabilizer trim" or, more colloquially, "stab trim".

The stabilizer trim must be set before every takeoff to a pre-determined position which depends on the takeoff weight and CG location. By doing so, the pilots are assured that the force on the control column required for rotation to the liftoff attitude will be approximately the same for every takeoff, and that the airplane will be approximately "in trim" at the initiation of its after-takeoff climbout.

spoilers and speedbrakes

Spoiler panels are flat panels mounted on the wings, hinged at their forward edges, which may be rotated about their hinge lines so as to rise up into the airflow over the wing's upper surface. The function of these spoiler panels is to increase drag and to decrease lift by interfering with the airflow.

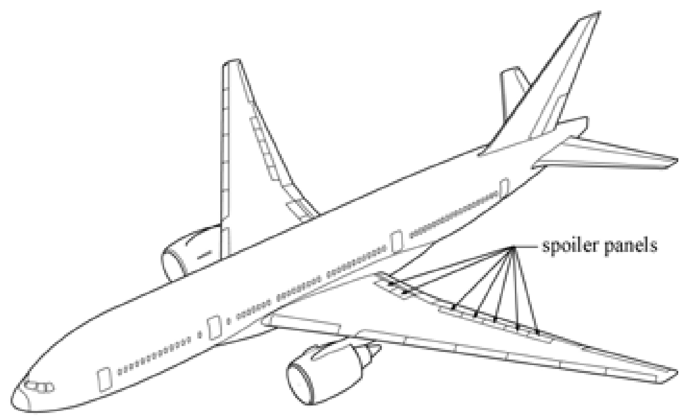


Figure 12-2

Spoilers are used to increase rates of descent and to improve deceleration during landing or a rejected takeoff.

During a landing or a rejected takeoff, the spoilers not only add drag but also, by reducing lift, place a greater amount of weight on the landing gear, enhancing braking effectiveness.

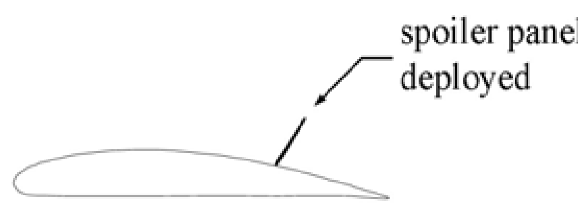


Figure 12-3

The spoiler panels serve one additional purpose: they augment the ailerons when a more rapid rate of roll is required. In this application, the spoiler panels are raised asymmetrically, only on the downgoing wing.

The terminology used concerning spoilers and speedbrakes is somewhat imprecise and needs a bit of explaining:

The flight control surfaces mounted on the wings are referred to as spoilers, since they "spoil" some of the lift and add drag when they're deployed into the airflow over the wing's upper surface. Each wing has a number of separate spoiler panels, each panel having its own actuator.

The arrangement and use of the spoiler panels varies from airplane to airplane. On the Boeing 737 and 747 airplanes, for example, distinction is made between *flight spoilers* and *ground spoilers*; only the flight spoilers will deploy when commanded in flight; all spoilers including the ground spoilers will deploy when commanded on the ground. On the 757, 767 and 777 airplanes, no such distinction is made; all spoiler panels will deploy when commanded.

When being used to augment the ailerons for greater rates of roll, the spoilers will deploy on one wing only (the down-going wing). In this instance, the spoiler deployment is commanded by deflection of the control wheel. For small angles of the control wheel, there will be aileron deflection only, with no spoiler panel actuation; when passing some specific angle of the control wheel, the spoiler panels will be commanded to deploy in addition to the ailerons.

When the spoiler panels are commanded to deploy in flight for the purpose of increasing drag and decreasing lift, as would be the case when an increased rate of descent is required without an accompanying increase of speed, spoiler panels will deploy symmetrically; they are then collectively referred to as *speedbrakes*. For such operation, the deployment of the spoiler panels is commanded by the pilot's use of the speedbrake handle in the cockpit.

When using the speedbrakes for descent control together with deflection of the control column to command a turn, the spoiler panels will deflect asymmetrically, thus accomplishing both functions at the same time.

For information on the arrangement and utilization of the spoilers on a given airplane, refer to that airplanes's documentation since there is variation between the models.

High-Lift Devices

The term *high-lift devices* includes those aerodynamic de-vices whose job it is to increase the lifting capability of the wing thus enabling slower flight speeds for takeoff and landing. There are two categories of high-lift devices: l*eading edge flaps or slats,* and *trailing edge flaps*.

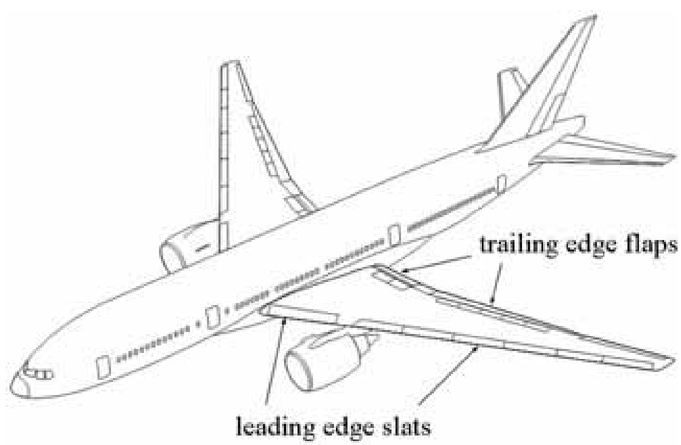


Figure 12-4

leading edge flaps or slats

In the chapter entitled "Lift and Drag" we discussed the airflow characteristics over an airfoil at increasing angles of attack. We mentioned that at high angles of attack the flow would begin to separate from the wing, and at even higher angles of attack the wing would no longer be capable of generating adequate lift force for flight.

For a given angle of attack, the leading edge devices do not increase the lift coefficient. Instead, the function of wing leading edge devices is to delay airflow separation until a higher angle of attack thus helping the wing to achieve higher lift coefficients than would be otherwise possible.

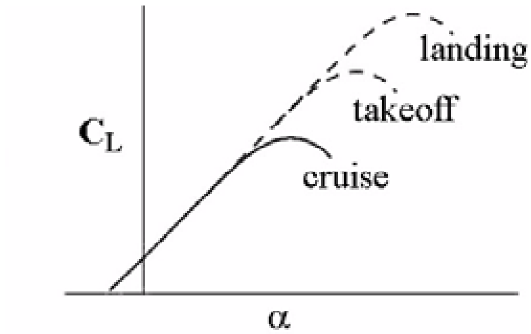


Figure 12-5

There are several different types of leading edge devices used on Boeing airplanes: slats, fixed camber Krueger flaps, and variable camber Krueger flaps.

The *leading edge slat*, shown to the right is, in effect, a portion of the wing's leading edge, which can be moved forward and downward by hydraulic actuators. When retracted, the slat is flush with the wing forming a smooth airfoil.

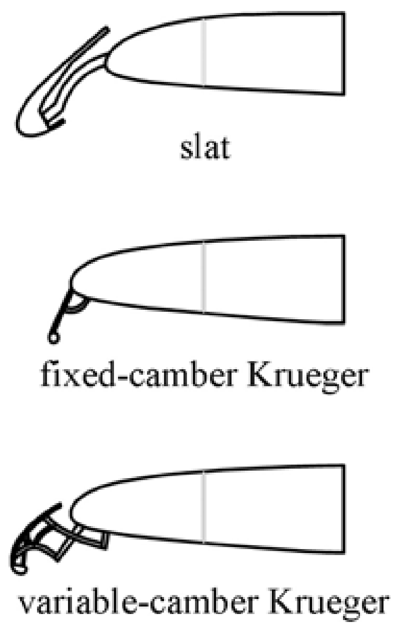


Figure 12-6

The leading edge fixed camber *Krueger flap* is a flat surface hinged at its forward edge, which can be rotated about its hinge line to form an extension to the leading edge. When retracted, the Krueger flap is flush with the lower surface of the wing leading edge. The 707 leading edge high-lift devices were exclusively of this design. On most current Boeing airplanes, the leading edge flaps inboard of the nacelles are fixed-camber Krueger flaps.

The leading edge *variable camber Krueger flap* is a modification of the fixed camber Krueger flap. The leading edge devices are made of a flexible material. When retracted, these leading edge flaps are flat and lie flush with the lower surface of the wing leading edge. As the variable camber Krueger flap is extended, it is forced into a curved profile by the actuating mechanism. Although mechanically more complex than the fixed camber Krueger flap, it is more efficient aerodynamically. This design is used only on the 747.

Some Boeing airplanes use a combination of these types.

trailing edge flaps

The principal function of trailing edge high lift devices is to increase the camber – curvature – of the wing enabling it to produce more lift, at the expense of increased drag. Trailing edge flap designs range from extremely simple to extremely complex. While the simpler designs are easier to manufacture, are lighter and also easier to maintain, they offer relatively less improvement to the wing's lifting capability. The more advanced trailing edge flaps, on the other hand, such as slotted Fowler flaps, are mechanically much more complex, heavier, and need more maintenance.

The *plain flap* shown to the right is a simple hinged surface which deflects downward about its hinge line. It's extremely simple mechanically, lightweight and easy to maintain, but relatively inefficient aerodynamically.

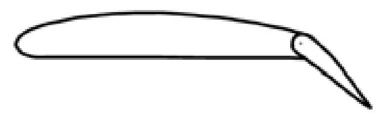


Figure 12-7a

The *split flap* shown here is also mechanically simple and easy to maintain. It is relatively inefficient, and incurs a large drag penalty. The principal advantage of split flaps is that they are very structurally strong and as a result can be extended at high speeds.

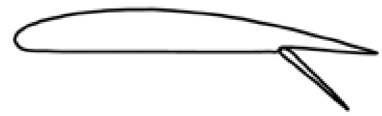


Figure 12-7b

The *slotted flap* shown is an improvement over the plain flap. Although more complex mechanically, the slot allows higher-energy air from below the wing to flow through the slot toward the lower pressure above the wing; this flow of air has the effect of adding energy to the boundary layer over the flaps and thus delaying the flow separation from the flap upper surface, improving the lifting capability. It is much more efficient aerodynamically than the plain flap.

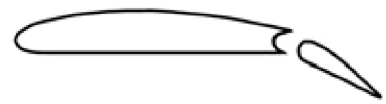


Figure 12-7c

The *Fowler flap* is also a slotted flap. In this design, however, the flap is characterized by an aft movement of the flap coupled with downward deflection. A Fowler flap thus increases wing area as well as wing camber. It is more complex design mechanically and is heavier, but is considerably more efficient aerodynamically.

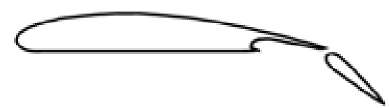


Figure 12-7d

Shown to the right is a *double-slotted Fowler flap*. Once again, a substantial increase in aerodynamic efficiency is achieved by this design, but again at the cost of greater weight and mechanical complexity.

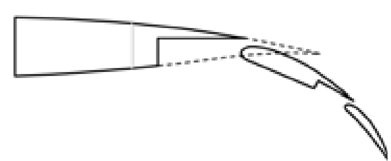


Figure 12-7e

The *triple-slotted Fowler flap* shown here is by far the most complex, but it offers the highest lift coefficients of all of these designs.

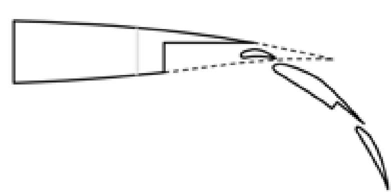


Figure 12-7f

The drag polars to the right show the relative effects on lift and drag of the different trailing edge flap designs. Note that the split and plain flaps offer relatively modest gains in lift coefficient when compared to no flaps, but incur substantial drag penalties. The slotted and Fowler flaps offer the greatest amount of lift increase without an attendant increase in drag.

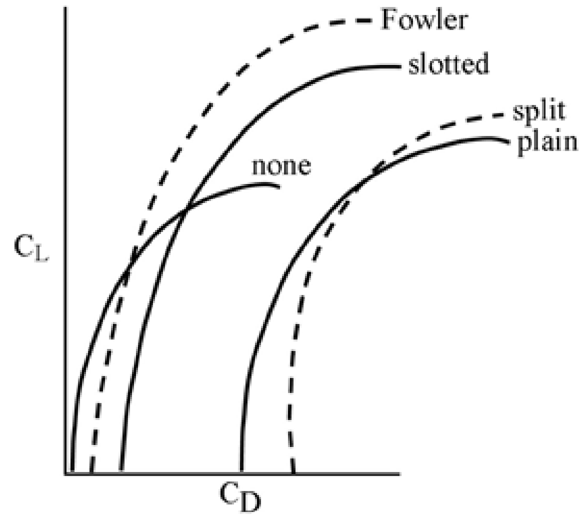


Figure 12-8

The Boeing 727 and 747 use exclusively triple-slotted Fowler flaps. When combined with efficient leading edge high-lift devices (variable camber Krueger flaps on the 747 and leading edge slats on the 727) these flaps enable the airplanes to takeoff and land within reasonable runway lengths for their missions.

The Boeing 737 and 757, on the other hand, combine double-slotted Fowler flaps with leading edge slats. The 767 and 777 airplanes take a slightly different approach – they use double-slotted flaps inboard of the engines and single-slotted flaps outboard of the engines; both airplanes utilize slats on the leading edges.

Clearly, there’s no single way to achieve higher lift coefficients. Designers have a number of choices and they select the wing high-lift devices carefully in order to optimize the design for the airplane’s intended mission.

Flow-Improving Devices

One class of aerodynamic devices is that type of device which can be used to improve the local airflow over various parts of the airplane. Examples of flow-improving devices used on Boeing airplanes include:

- Leading edge “fences”
- Vortex generators
- Vortilons
- Nacelle “chines”

leading edge fences

Only one Boeing airplane, the 727, utilizes a *leading edge fence*. There is one fence on each wing.

On that airplane, as shown in the illustration to the right, the fence consists of a flat plate approximately one inch thick, perpendicular to the surface of the leading edge slat, aligned in a chordwise direction and wrapped around the nose of the slat.

The function of the fence is to produce a chordwise flow of high-energy air which will act as a "fence", reducing the amount of airflow in the spanwise direction.

Figure 12-9

vortex generators

A *vortex generator* is an extremely simple aerodynamic device which is used on many airplanes, including all Boeing models, to improve airflow.

A vortex generator can be described as a small wing of very low aspect ratio. In typical applications, vortex generators are only a few inches high

On Boeing airplanes, vortex generators can be found on the body, engine nacelles, empennage, and wing.

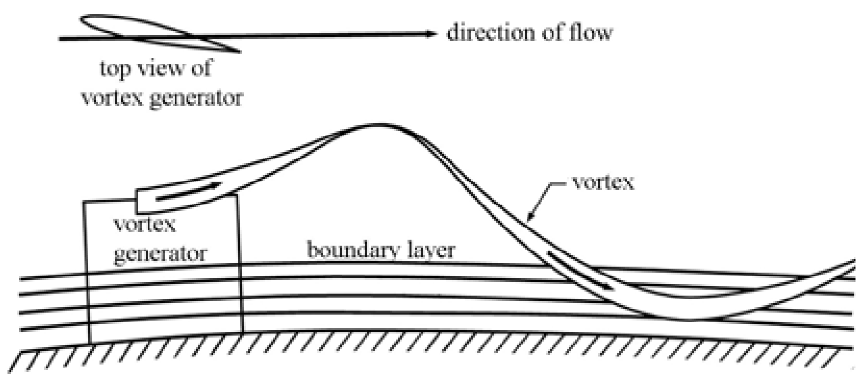


Figure 12-10

Since a vortex generator is placed at an angle to the airflow – refer to the drawing above – and since it has an extremely low aspect ratio it will generate quite a strong vortex from its tip. This vortex can – surprising as it may sound – be used to advantage in a number of ways. First, such a vortex will have the effect of stirring up the boundary layer downstream of the vortex generator and adding energy to it. This has the end effect of delaying boundary layer separation. Second, the vortex will act like a small chordwise fence downstream of the vortex generator, reducing spanwise flow and improving wing lift distribution. Because of these qualities:

- vortex generators can improve airplane performance by reducing the overall airplane drag;

- they can improve control authority thus enhancing the airplane's handling qualities, they can be used to reduce stall speeds and airframe vibration, they are commonly used on swept wings to alleviate "pitch-up" characteristics inherent in those wings;
- they can be used wherever airflow separation is a problem or could potentially be a problem.

On Boeing airplanes, vortex generators will be found in the following locations:

- 707: wing upper surface and under the horizontal tail;
- 727: vertical tail, wing leading edge and center engine inlet[1];
- 737-200: wing upper surface and the aft body adjacent to the horizontal tail;
- 737-300 and on: wing upper surface and engine nacelles;
- 757: wing upper surface;
- 767: wing upper surface and engine nacelles; outboard flap leading edges (-300/-400LR)
- 777: wing leading edges, outboard flap leading edges

Because the vortex generators in many instances play an important role in the handling qualities of the airplane, release of the airplane for flight may be prohibited or restricted if some or all of the vortex generators are missing from the airplane. The Configuration Deviation List (CDL) appendix to an airplane's Airplane Flight Manual will specify the number of vortex generators, if any, which may be missing for flight.

vortilons

A *vortilon* is simply a specialized form of vortex generator. On the Boeing 737NG, for example, there are three vortilons on each wing, located on the leading edge slat below its forward tip. Refer to the photograph to the right.

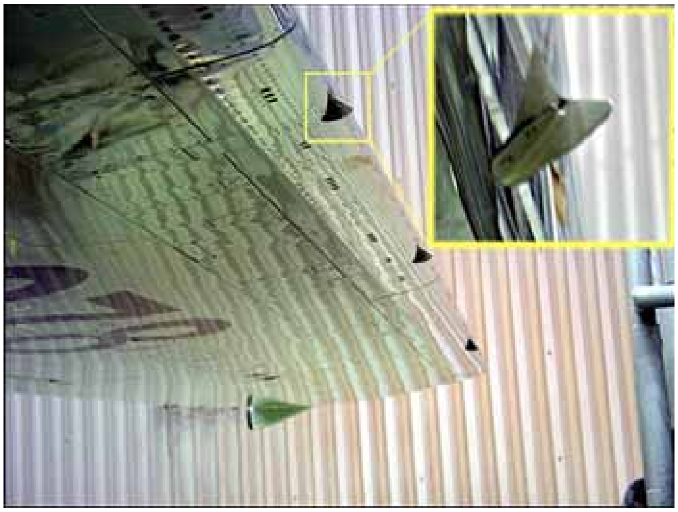


Figure 12-11

Because of its location well below the leading edge of the wing, at low angles of attack its wake will stream from the vortilon along the lower surface of the wing. The vortilon is principally useful at higher angles of attack, at which the vortilon will produce a vortex streaming over the wing's upper surface. In this

1. This is certainly the most unusual use of vortex generators on Boeing airplanes. The 727's center engine air intake duct is referred to as the "S-duct" due to the double-bend S shape needed to bring air to the engine intake from the duct's inlet above the fuselage. Due to the bends in the duct, flow separation was occurring resulting in a loss of engine performance. Installation of two rings of vortex generators in the duct restored acceptable airflow.

case, it helps to keep the boundary layer attached to the wing, reducing spanwise flow of air and providing improved roll control.

nacelle chines

The *nacelle chine* is another form of vortex generator, in this case a large one somewhat resembling a shark fin, mounted on the inboard side of the engine nacelle.

The chine generates a powerful vortex which streams up and over the wing, as seen in the photograph to the right.

Figure 12-12

Wingtip Treatments

As we discuss in the chapter entitled “Wings”, any real wing (as opposed to “two-dimensional wings” as may be used in wind tunnel testing of airfoil characteristics) will inevitably have some loss of efficiency because of the airflow pattern at the wingtips. The wingtip vortex condition represents a substantial loss of energy, equivalent to an increase of drag; as you now know, this is referred to as “induced drag”. Clearly, anything which can be done to reduce the intensity of the wingtip losses will ultimately translate into improved fuel efficiency.

Probably the most frequently seen wingtip treatment is the addition of a *winglet*. These are standard on Boeing Business jets, the 747-400, the MD-11, and they are optional on the 737-700/800/900. The 767-400 has taken a different approach: instead of winglets, that airplane utilizes a *raked wingtip* which we’ll discuss also.

winglets

A winglet is, in simple terms, a small vertical airfoil attached to the tip of an airplane’s wing, and oriented at an angle to the longitudinal axis of the airplane so that it presents a slight lateral angle of attack to the airflow. The function of a winglet is to reduce the strength of the wingtip vortex, to redistribute the lift across the wing, and thereby to decrease the wing’s induced drag component.

Since induced drag accounts for approximately 40 to 45 percent of total cruise drag, any reduction can obviously produce substantial fuel savings. The addition of a winglet to an existing wing, however, is not without some penalty. First of all, the winglet adds weight, and it also adds skin surface area.

Also, an airplane’s mission needs to be considered before deciding whether or not to add a winglet. To illustrate that point: although the 747-400 has a standard winglet, there is one special version of the 747-400, called the -400D, which does <u>not</u> have a winglet. The -400D is the “domestic” model delivered to one operator for use on that operator’s short domestic routes. The winglet doesn't provide any benefits to takeoff performance, and the cruise segments are too short for the drag benefit of the -400 winglet to overcome the weight penalty.

The work done in overcoming induced drag shows up as kinetic energy in a very spread-out flow field downstream of the wing that looks something like the sketch in Figure 10-11. A winglet reduces induced drag by altering this pattern in a subtle way that reduces the total kinetic energy. There is no simple intuitive way to visualize the form that these changes take, and the forces on the wing and winglet can be even more counterintuitive.

For example, when a well-designed winglet is added to a swept wing, there is a large thrust force on the winglet itself, and there is a large increase in induced drag on the outboard half of the wing that offsets nearly all of the thrust on the winglet. These large changes outboard add up to only about one half of the net drag reduction due to the winglet. The rest comes from a reduction in induced drag on the inboard half of the wing. So the effects of a winglet are subtle and involve the entire flow field.

The flow field changes by which wingtip devices produce reductions in induced drag are not easy to visualize intuitively, but the changes in spanloading and the resulting drag reductions can be computed by Computational Fluid Dynamics (CFD) software with sufficient accuracy to support design decisions.

Considerable research has been and continues to be expended on the optimum way to decrease induced drag. As yet there is no consensus on the ideal wingtip treatment, so you'll see a number of different treatments used on different airplanes.

Here are a few of them as used on Boeing airplanes:

<u>737 Business Jet and -700/800/900:</u>
These airplanes utilize a "blended winglet" design, clearly seen in the photograph to the right. In a blended winglet, the wing transitions to the eight-foot vertical winglet through a smooth radius.

On these airplanes, a cruise drag reduction of approximately four percent has been demonstrated.

Figure 12-13

747-400: This airplane, derived from the earlier 747-300, modifies that airplane's wing by extending the span approximately eight feet on each wingtip, and adding a vertical winglet approximately six feet high. This is not a blended winglet, however.

On the -400, the wingtip extension and winglet together offer approximately three and a half percent drag reduction in cruise.

Figure 12-14

MD-11: The winglet on this airplane is similar to that seen on the 747-400, but with the addition of a much smaller winglet extending downward near the leading edge of the wingtip.

This winglet gives approximately three and a half percent benefit in cruise drag levels.

Figure 12-15

winglet retrofits to older airplanes not built with winglets

You may occasionally see winglets on some Boeing airplanes not listed above. These have been added as retrofits, with the design and parts manufacturing by several different vendors. They are not supported by Boeing and no drag data is available to us.

raked wingtips

On the 767-400 airplane, Boeing designed a different type of wingtip treatment for drag reduction purposes. It can be seen clearly in the photograph to the right.

In this context, *raked* means that the wingtip is extended but the extension is raked back at a sweepback angle greater than that of the wing. This is not a winglet, simply a wingtip extension with a greater angle, but the goal is the same: reduction of the wingtip vortex and a redistribution of lift across the wing.

Figure 12-16

The structural weight implications of a raked wingtip are less than for a winglet. The wingtips extend the airplane's wingspan by approximately fifteen feet. The raked wingtip has been demonstrated to provide a cruise drag reduction of approximately five and a half percent.

Chapter 13: Evolution of the Jet Engine

Introduction

When Boeing entered the jet age in late 1958 with the Model 707-100, the Pratt and Whitney JT3C engines on that airplane were each capable of producing approximately 12,500 pounds of thrust at their takeoff setting.

In the approximately forty years between the 707-100's JT3C engines and those entering service on the Boeing 777, the advance in turbojet engine technology has been astounding – as exemplified by the increase in takeoff thrust available from less than 13,000 pounds to more than 115,000 pounds. Equally amazing are the increase in engine fuel efficiency and the decreases in noise and emissions.

In this chapter, we'll describe the evolution of the present-day turbojet engine from its modest beginning in 1937 with the first successful test run of the engine designed by Frank Whittle. We do this not for its historic interest but because it's quite educational. The stages through which this evolution has progressed demonstrate clearly how the principal features of today's engines enable them to achieve their remarkable performance.

The courteous assistance of the Pratt & Whitney Company and of Rolls-Royce PLC in the development of this chapter is gratefully acknowledged.

First, How Does a Jet Engine Work?

Before we see how the jet engine has evolved since 1937, it will be instructive to describe – in a most elementary way – just how a jet engine produces thrust.

One source defines the word *jet* as "a stream of a liquid, gas, or small solid particles forcefully shooting forth from a nozzle, orifice, etc.".

Isaac Newton's third law of motion predicts such a jet will produce an equal and opposite force. An example frequently used is the high-pressure water hose used by firefighters to deliver large volumes of water to battle a blaze. When opening the valve on such a firehose, the person holding it must be braced against the thrust of the hose because the large mass of water being ejected from the hose's nozzle produces an equal and opposite reaction that can easily push an unprepared person right off his feet. To a lesser degree, the same effect can be felt when using a simple garden hose for tasks around the home.

A *jet engine* is nothing more than a device designed to create large volumes of high energy gases and then exhaust them through a nozzle, thus creating the equal-and-opposite reaction force that we refer to as *thrust*.

An analogy is sometimes drawn between a jet engine and another kind of engine more familiar to all of us, the internal combustion reciprocating engine which powers our automobiles. Both of these engines do work by drawing in outside air, compressing it, adding fuel and then igniting the fuel-air mixture, and, after extracting energy from the hot gases, exhausting them back to the atmosphere.

This analogy is imperfect, though, in that the automotive engine uses the energy created by the combustion of the fuel-air mixture to move the pistons and thereby rotate the engine's shaft. When the gases are exhausted from the engine, they contain little energy.

A jet engine uses the majority of the energy created by the air-fuel combustion to rotate the turbines and thus power the compressor stages. *Some three-quarters of the energy produced at the combustion stage will be needed to power the compressors.*[1] The remaining energy exits the engine in the form of a high-velocity exhaust – a "jet" of exhaust gas – to produce thrust.

So What's a Turbojet?

Any engine that achieves thrust by exhausting high-energy gases is a jet engine, in the spirit of the definition of jet given above. The turbojet is simply one specific type of jet engine.

The turbojet is so called because the energy needed to drive the first stage of the engine – which is the compression section – is derived from a *turbine* which is driven by the hot exhaust gases, extracting some portion of their energy.

The First Aviation Jet Engine

In 1937, an English engineer named Frank Whittle, later knighted for his accomplishments, achieved the first successful sustained test run of a jet engine in a laboratory[2]. His engine's basic design differed from modern-day engines in two regards. The more noteworthy of those two is that the compression stage of the engine was of a centrifugal design rather than the axial design that is characteristic of all engines in current use on commercial passenger airplanes.

1. In "The Jet Engine", by Rolls-Royce PLC, it is stated that driving the compressor stages of a modern jet engine can demand up to approximately 200,000 horsepower, more than 149 million watts.
2. It should be noted, though, that Whittle's was not the first jet engine to power an airplane in flight. That honor went to an engine developed by Dr. Hans von Ohain in Germany, which when fitted to the Heinkel He178 flew for the first time on August 27, 1939. Whittle's engine first flew on May 15, 1941.

The illustration to the right shows the design of the Whittle-type turbojet engine.

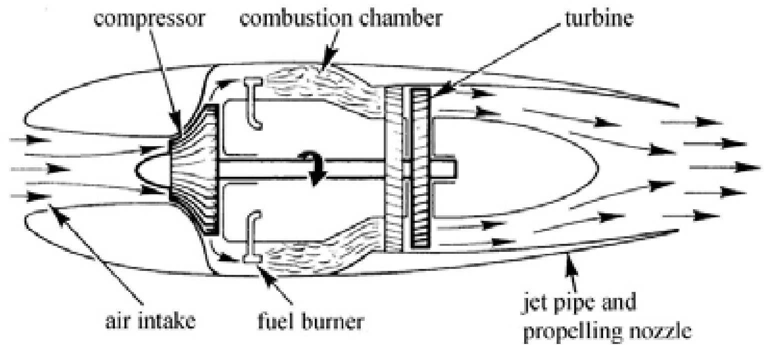


illustration courtesy of Rolls-Royce PLC

Figure 13-1

A centrifugal compressor utilizes an *impeller* which is rotated at high speed by the engine's turbine. Air is continuously brought from the air intake of the engine to the center of the impeller. Centrifugal action causes it to flow radially outwards along the vanes to the impeller tip, accelerating the air and causing a pressure increase.

Fuel is then added to the air and the mixture is ignited in the combustion chamber, increasing its energy. The flow proceeds downstream through the *turbine,* causing it to rotate. This extracts some portion of the flow's energy in order to turn the compressor.

The flow is then exhausted through the jet pipe and propelling nozzle where the energy remaining in the flow becomes, by virtue of Newton's third law, the propulsive force, or *thrust,* of the engine.

While centrifugal compressors are still in use in many engines and are almost exclusively used on Auxiliary Power Units (APUs), for turbojet airplanes this design has a significant disadvantage: to achieve the needed amount of air at the pressure required, the centrifugal compressor will have a relatively large cross-sectional area. In flight, a large cross-sectional area causes undesirable levels of drag. Additionally, centrifugal compressors aren't capable of developing the degrees of compression needed by modern engines, which require the incoming air to be compressed by as much as a factor of 50. Hence, a better design of the compressor is needed.

The Axial Compressor

The illustration to the right shows an axial compressor in use to provide high-pressure air to the combustion section. In this context, "axial" simply means that the air is compressed as it flows along the axis of the engine.

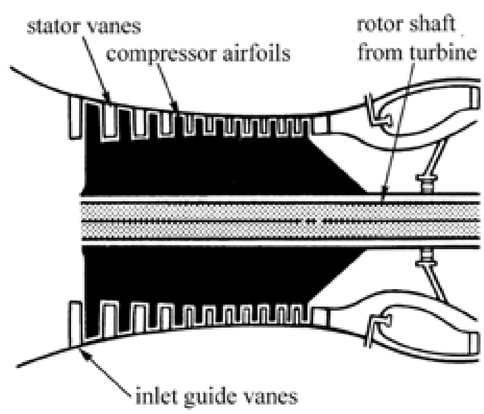


illustration courtesy of Pratt & Whitney

Figure 13-2

An axial compressor consists of a series of "stages", each stage consisting of a rotating disk of compressor airfoils and a ring of stationary stator vanes. Some engines also have a ring of inlet guide vanes at the front of the compressor.

After being delivered to the face of the compressor by the intake duct, the air passes through the inlet guide vanes. Their function is simply to guide the air on to the first row of rotor blades at an optimum angle. On most modern engines, the angle of the inlet guide vanes can be controlled to suit the airflow requirements at various operating conditions.

As the air enters the compressor, it's accelerated by the rotating blades and swept rearwards on to the adjacent row of stator blades. The pressure rise in the airflow results from the diffusion process in the rotor blade passages and from a similar process in the stator blade passages; the latter also serve to correct the deflection given to the air by the rotor blades and to present the air at the correct angle to the next stage of rotor blades.

Although the increase in the air pressure across each stage is quite small, any number of stages can be included according to the amount of compression required.

In the illustration above, you see that all of the compressor rotor disks are turning at the same number of revolutions per minute (RPM). This was typical of the early turbojet engines used on the first generation of commercial jet airplanes such as the 707-100.

The Single-Spool Axial-Flow Turbojet Engine

The first generation of engines used on commercial jet airplanes were *single spool axial flow turbojet* engines. The term "spool" is used to refer to a compressor and a turbine joined to each other by a shaft. The early engines had only one spool and thus all stages of the compressor and all stages of the turbine were operating at the same RPM. In the illustration, you see the design of an early single-spool turbojet engine having 12 compressor stages and three turbine stages. All of them are joined by a single rotor.

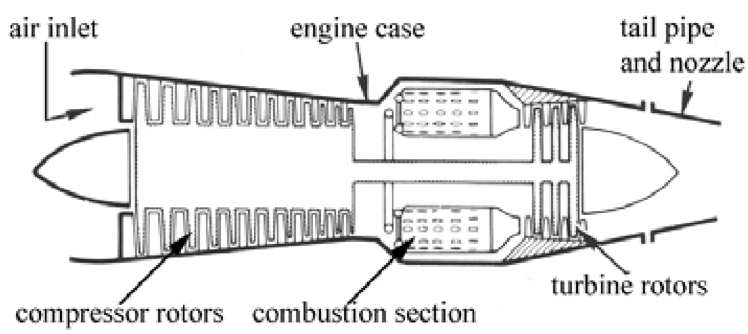


illustration courtesy of Pratt & Whitney

Figure 13-3

The Twin-Spool Axial-Flow Turbojet Engine

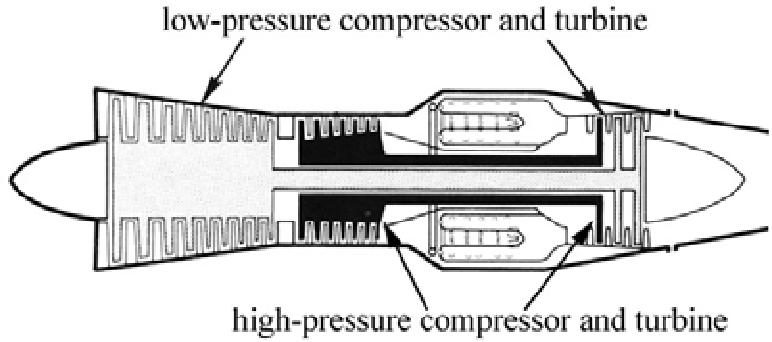


illustration courtesy of Pratt & Whitney

Figure 13-4

The first significant development to the turbojet engine after the introduction of the early single-spool engines was the introduction of a second shaft.

The "low-pressure compressor" is joined to the "low-pressure turbine" by a shaft. Concentric with that shaft is a second, outer, shaft which joins the "high-pressure compressor" with the "high-pressure turbine". These two compressor-shaft-turbine systems operate independently of each other.

The fact that the second shaft allowed the engine to have two independent stages of compression and two independent turbines meant that they could now be more accurately matched to the characteristics of the airflow through the engine, creating a significant improvement in efficiency.

The spool pairing the low-pressure compressor and turbine is sometimes called the "low pressure rotor". The spool pairing the high-pressure compressor and turbine is called the "high pressure rotor". These terms refer to the relative rates at which the two spools rotate. The high-speed rotor turns at an RPM which is significantly higher than that of the low pressure rotor.

The low pressure rotor is frequently referred to as the N1 rotor, and the high pressure rotor is called the N2 rotor. N1 and N2 are the RPMs of the two rotors. As you'll see later, their RPMs are usually expressed in terms of "percent N1" and "percent N2".

The Turbofan Engine

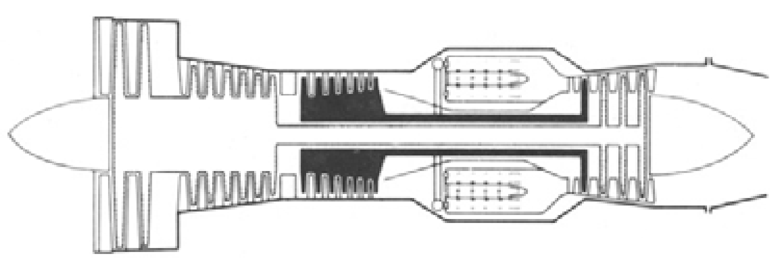


illustration courtesy of Pratt & Whitney

Figure 13-5

The next significant development in the evolution of the turbojet engine was the introduction of "fans". The first of these in commercial service were the Pratt & Whitney JT3D engines used on later 707s.

The "fan" consists of one or more stages of rotating airfoil blades at the front of the compressor section. These fan stages have a greater diameter than the rest of the compressor stages. For that reason, the fan stages perform two functions: they serve to compress the incoming air and pass it along to the next stage of compression, and they also accelerate a mass of air which will not pass through the core of the engine but will rather pass rearward along the outside of the engine case. In this latter function, the fan stages work rather like a many-bladed propeller.

The fan accelerates a large amount of air mass by a relatively small amount. This is more efficient than accelerating a smaller volume of air by a larger amount and thus enables an engine to produces thrust more efficiently than the earlier engines.

bypass ratio

The air which doesn't pass through the engine core but rather bypasses it and flows around the outside of the engine case is called – logically – the bypass air. The ratio of the amount of bypass airflow to the amount of air which goes through the engine core (the "primary flow") is termed the "bypass ratio".

The early turbofan engines had a low bypass ratio, on the order of one to one. Approximately half of the thrust was produced by the fan and the other half by the primary flow. As you'll see in the following paragraphs, for modern engines the bypass ratio can be ten or more, with more than 80% of the thrust produced by the fan.

short-duct and long-duct fan engines

The JT3D engine shown above is sometimes referred to as a "short-duct" fan engine. That is simply because the fan airflow exits from the fan section and exhausts immediately into the atmosphere, merging with the air flowing along the airplane's path of flight.

A "long-duct" fan engine is slightly different in that the fan air is contained within a duct which is concentric with the case of the engine core. In this design, the fan air merges with the primary flow at the tailpipe and nozzle of the engine. This type of engine first saw commercial service in the Pratt & Whitney JT8D series of engines powering the Boeing 727 and 737 airplanes.

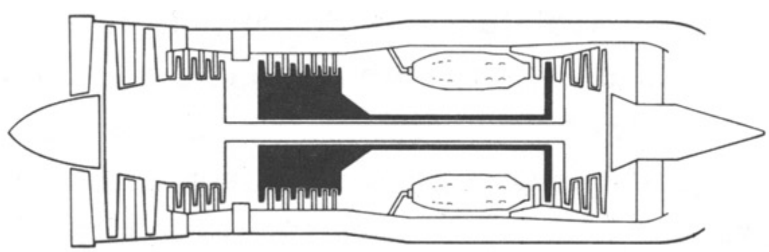


illustration courtesy of Pratt & Whitney

Figure 13-6

The High-Bypass Turbofan Engine

The next stage in the turbojet engine's evolution occurred beginning in 1969 with the Boeing 747. This was the introduction of engines having much higher bypass ratios. The first of these was the Pratt & Whitney JT9D engine, having a bypass ratio of approximately 5.

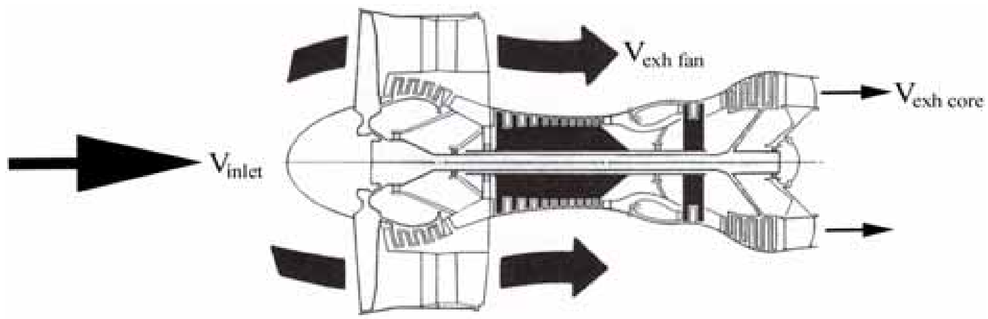


illustration courtesy of Pratt & Whitney

Figure 13-7

Why is an engine having a higher bypass ratio superior to its predecessor having a lower bypass ratio? The answer is *fuel efficiency.*

The overall propulsive efficiency of an engine can be expressed in terms of the inlet and exhaust velocities of the flow passing through it:

$$\eta_p = \frac{2 \times V_{inlet}}{(V_{inlet} + V_{exhaust})} \qquad \text{(eq. 1)}$$

where η_p is the propulsive efficiency

The overall thrust of an engine can be expressed in terms of the core and fan inlet and exhaust velocities:

$$F = \dot{m}_{fan}(V_{exh\ fan} - V_{inlet}) + \dot{m}_{core}(V_{exh\ core} - V_{inlet}) \qquad \text{(eq. 2)}$$

where $\dot{m}_{fan}$ is the mass flow rate of gas through the fan

$\dot{m}_{core}$ is the mass flow rate of gas through the core of the engine

$V_{exh\ fan}$ is the velocity of the gas exiting the fan

$V_{exh\ core}$ is the velocity of the gas exiting the core of the engine

V_{inlet} is the velocity of the air at the engine inlet

Equation 1 predicts that an efficiency of 100% would be achieved if the exhaust velocity were equal to the inlet velocity. However, Equation 2 shows that for this to occur the mass flow through the engine would need to be infinite for thrust to be produced. Obviously, infinite mass flow isn't achievable in the real world, but equation 1 does indicate that greater efficiency is obtained when a large mass of air is accelerated by a lesser amount compared to a small mass of air being accelerated by a greater amount.

The high-bypass ratio engine is the way to achieve the acceleration of a large mass of air by a small amount, thus helping the engine achieve greater efficiency than an engine having a low-bypass ratio.

As high-bypass engines have continued to evolve since their introduction, the bypass ratios have steadily increased. The Rolls-Royce turbofan engine being installed on the Boeing 787 airplane has a bypass ratio of ten to eleven.

The Triple-Spool High Bypass Turbofan Engine

One manufacturer of engines, Rolls-Royce PLC, produces three-spool engines. While more mechanically complex due to its three-shaft design, it enables better optimization of the rotation rates of the three stages of the compressor.

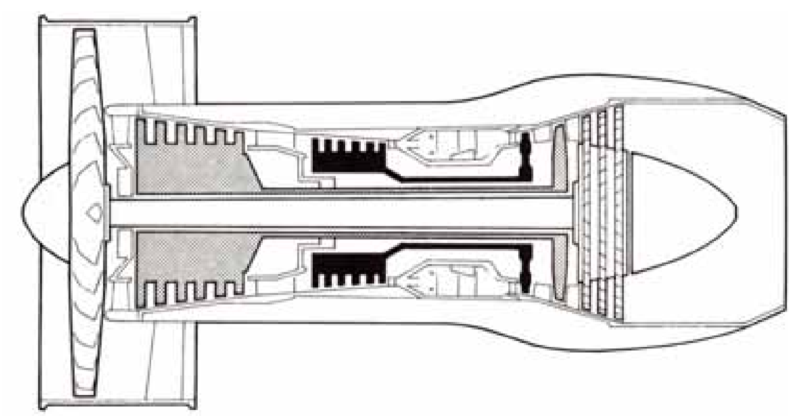


illustration courtesy of Rolls-Royce PLC

Figure 13-8

Electronically Controlled Engines

Evolutionary changes have taken place not only to the engine but also to the systems controlling the engines' thrust.

The engines mounted on earlier airplanes are controlled *mechanically.* On those airplanes, there is a mechanical connection between the thrust levers in the cockpit and the fuel control unit at the engine, a system of cables and pulleys. These mechanical systems components are heavy and require careful maintenance. The fuel control unit, the device that regulates the amount of fuel flowing to the combustors, is also a mechanical device. Although these were well designed, mechanical control systems have limitations on which parameters can be used in the function of metering fuel to the engines, limitations on the accuracy they can achieve, limitations on the performance they could deliver (e.g. uneven accelerations), and they were limited in the types of sub-functions they could support.

The next type of engine control, a *supervisory* control, was introduced in the early 1980s. This configuration still had a complete independent hydro-mechanical unit, but in addition had a separate electronic unit. The supervisory Electronic Engine Control (EEC) was not full authority, but had limited trim adjustment. Although this configuration improved accuracy and performance, it still retained the weight of the cable system and hydromechanical unit and the performance limitations.

All of the engines on recent airplanes are *FADEC* controlled. Full Authority Digital Engine Controls (FADECs) have complete control of the fuel metering valve. The complex mechanical control system has been replaced with a simpler fuel metering unit. Cables and pulleys have been replaced with simpler lighter electrical wiring that transmit the thrust lever position to the EEC. A FADEC also controls numerous other engine functions such as variable stator vanes (VSVs), various bleed valves, active clearance control, cooling and starting. Additionally, FADECs support improved cockpit displays, and improved dispatch and maintenance information.

The FADEC computing capability allows increased flexibility in the number and types of parameters that can be processed and used in the logic to start and control the engine, to tailor the logic for desired functionality, and to adjust ratings for specific customer needs. To illustrate this: all engines on the 737NG series of airplanes (the 737-600, -700, -800 and -900) are mechanically identical, and the purchaser of the airplane can select the thrust rating it needs for its operational requirements. For example, the 737-700 can be purchased with either 20,000 or 22,000 or 24,000 pounds takeoff thrust ratings – because the thrust rating is software-selectable.

Today's Engines – and Tomorrow's

You can see that the engines used on commercial jet transport airplanes have evolved extensively since they were first introduced. Not only has the thrust they're capable of producing increased by a factor of almost ten, the newer engines also produce much less noise and emissions.

To meet the never-ending demand for even more fuel-efficient engines that will make less noise and have less impact on the environment, engine manufacturers are constantly seeking new design features and manufacturing techniques. On the General Electric GEnx engine that powers the Boeing 787, for example, GE has introduced counter-rotating shafts, improved combustors, and complex highly advanced fan blades made of composite materials with titanium leading edges. Some compressor stages are now made of a single piece, rather than being a disk with a series of attached blades – these are called "blisks".

Keep an eye on the engine manufacturers: it's going to be fascinating to see what the next steps will be in the ongoing evolution of the jet engine.

Chapter 14: Jet Engine Fundamentals

Introduction

In the preceding chapter, you saw that the jet engines used to power airplanes are simply machines designed to produce large volumes of high-velocity exhaust gases. These gases create – thanks to Newton's Third Law of Motion – the thrust which accelerates the airplane and overcomes its aerodynamic drag.

You watched the evolution of the modern turbofan engine, and in the process you learned how it works: intake of the air, compression, addition of fuel, combustion of the fuel-air mix, extraction of some energy in the turbine to power the compressor, then exhausting the gases through a nozzle to produce thrust.

Today's jet engines are exquisitely precise machines capable of operating at over 14,000 RPM at takeoff power, experiencing internal temperatures exceeding 1600 degrees Celsius in the turbine inlet and internal pressures greater than fifty atmospheres in the compressor.

In this chapter, we'll introduce you to some of the fundamental factors in jet engine performance. We'll discuss the thrust equation, a number of factors affecting thrust, and we'll introduce you to some new terminology relating to these engines.

The Thrust Equation

Newton's third law of motion says that for every *action* there is an equal and opposite *reaction*. In the jet engine, the action is accelerating a mass of gas and exhausting it out through a nozzle. The equal and opposite reaction is what we call *thrust*.

Newton's second law says that, for constant mass:

$$F = ma \qquad \text{(eq. 1)}$$

where F is the force
m is the mass on which the force acts mass
a is the resulting acceleration

"Momentum" is defined as "a measure of the motion of a body equal to the product of its mass and velocity." Keeping this in mind, we can re-write equation 1:

$$F = \frac{d(mv)}{dt} \qquad \text{(eq. 2)}$$

This more general equation states that force is equal to the rate of change of *momentum*.

Let's re-write the equation once again, this time in terms relating to a jet engine:

$$F = \dot{m}\ (V_2 - V_1) \qquad \text{(eq. 3)}$$

where $\dot{m}$ is the gas flow rate, mass per unit time
V_1 is the initial velocity of the gas
V_2 is the final velocity of the gas

Let's re-write equation 3 in a more complete form:

$$F_{net} = [(\dot{m}_{core\ air} + \dot{m}_{fuel}) \times V_{core\ exhaust}] + (\dot{m}_{fan\ air} \times V_{fan\ exhaust}) - (\dot{m}_{air} \times V_{airplane}) \qquad \text{(eq. 4)}$$

Note that the force is now called F_{net}, for net thrust. What are those terms in parentheses? The term $(\dot{m}_{air} \times V_{airplane})$ is the momentum per unit of time of the air being taken into the engine. The term $[(\dot{m}_{core\ air} + \dot{m}_{fuel}) \times V_{jet\ exhaust}]$ is the momentum per unit of time of the gas leaving the core of the engine. Notice that we are now accounting for the fact that the mass flow of the exhaust is greater than the mass flow of the inlet, because fuel has been added to the intake air. Finally, the term $(\dot{m}_{fan\ air} \times V_{fan\ exhaust})$ is the momentum per unit of time of the air leaving the fan stage of the engine.

The *net force*, which is the thrust produced by the engine, is the algebraic sum of these three terms, which is the *rate of change of momentum* of the gas as it passes through the engine.

If we considered only the first two terms, the exhaust flow, we would see only the force created at the fan and the exhaust nozzle. But this is only half of the story, because we must remember that the flow of the air entering the engine also has momentum and the useful work that the engine is doing is the difference between the two.

Factors Affecting Thrust

A number of factors will influence the amount of thrust which an engine can produce. Let's examine them one at a time.

air density effect on thrust

You'll recall from an earlier chapter that air density is a function of temperature and pressure. The density of the air passing through an engine will have a very significant effect on thrust, because density has a direct effect on the mass flow rate that we call $\dot{m}$ in the thrust equation. Thus any change in temperature or pressure will affect the engine's thrust.

In commercial jet airplane operation, how much variation of air density can we expect?

For any given altitude, density will decrease with both increasing temperature and increasing altitude, as predicted by the relationship:

$$\rho = \frac{\rho_0 T_0}{p_0} \times \frac{p}{T} \qquad \text{(eq. 5)}$$

where ρ is the air density
p is the air pressure
T is the air temperature, in absolute units
ρ_0, T_0 and p_0 are the sea level standard day values of ρ, T and p respectively

(You'll recognize Equation 6 as a development of the basic equation $\delta = \sigma\theta$.)

First, let's examine the effect of air temperature on density, and then we'll look at the effect of altitude on density.

If, for example, we look at the effect of temperature on air density at sea level, it will be as shown to the right.

Here we're showing only temperatures warmer than the standard day value at sea level of 59 °F.

At temperatures below the standard day value, the density will increase above its standard day value of 0.002377 slugs per cubic foot.

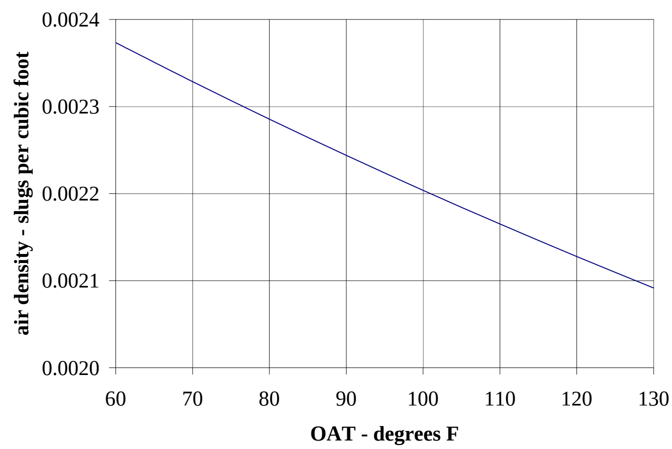


Figure 14-1

The next figure, shown at the right, shows how air density varies with altitude. In this chart, we have assumed a standard day temperature at all altitudes.

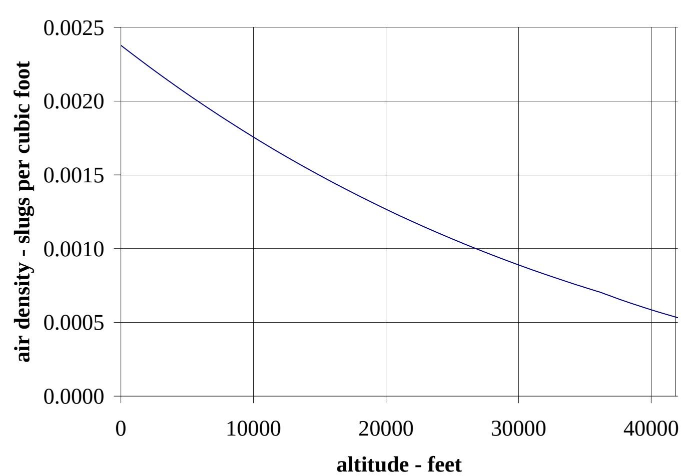


Figure 14-2

Even just within the envelope of allowable temperature and altitude for takeoff, called the *takeoff environmental envelope*, the range of air density is quite large.

If you checked maximum and minimum altitudes and temperatures for takeoff on your airplane, you would find that the air density in the takeoff temperature-altitude envelope varies all the way from approximately 32 percent greater than standard (on a sea level cold day) to more than 30 percent less than standard (on a high altitude hot day).

Thus, you can see that temperature and altitude will have a significant effect on the allowable takeoff weight because of the wide variation in air density within the takeoff environmental envelope.

velocity effect on thrust

The speed of the airplane also has a marked effect on the thrust produced by the engine. In fact, there are really two separate effects:

- As the airplane velocity increases, the air at the engine's inlet is increasingly compressed, resulting in higher air densities. This is sometimes referred to as the *ram effect*. This increases the thrust by increasing the mass flow of gas through the engine.
- As the airplane velocity increases, the momentum of the air at the inlet (its mass times its velocity) will increase. Remember that the useful work done by the engine – its thrust – is the difference between the exhaust momentum and the intake momentum – so increasing the momentum at the intake has the effect of decreasing the net thrust.

If we were to advance the thrust levers to the takeoff setting before beginning the takeoff roll, how would the thrust vary as we accelerated, at constant thrust lever position?

The resulting net effect of the airplane's velocity is shown in the figure to the right.

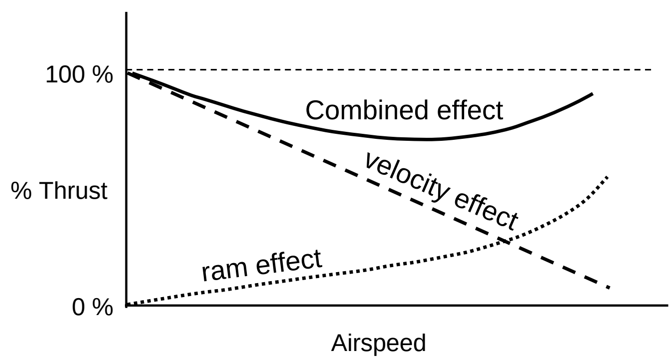


Figure 14-3

As you see, the effect of velocity is first to decrease the thrust, at the lower speeds, and then as the ram effect becomes increasingly significant, to increase the thrust.

This illustration is for example only, the magnitudes are not to scale.

humidity

The humidity of the air has a negligible effect on thrust.

installation losses

When an engine is installed on an airplane, it's used for more than just thrust. For example, when installed on the airplane the engine will also:

- provide energy to run some of the airplane's hydraulic pumps;
- provide energy to run the generators that power the airplane's electrical system;
- provide high pressure air to be used in the airplane's environmental control systems[1];
- provide hot air, when needed, for anti-icing/de-icing purposes, to protect the wing leading edges and engines from accumulating ice when flying through icing conditions.

All of these will decrease the amount of thrust available because they are extracting energy from the engine, with the end result of less momentum in the exhaust gases.

Also, when run on the test stand the shapes of the air inlet and exhaust may differ substantially from what they will be on the airplane, and this too will have an effect on thrust.

Because of these differences between the test stand and the real operational world, when engines are run on the manufacturer's test stand they will produce more thrust than they will when installed on the airplane.

For this reason, the engine thrust charts you will use for performance work will be revised from the manufacturer's test stand data by including these thrust losses. The thrust shown will be designated as "installed net thrust", meaning simply that it's the thrust available *on the airplane*, as opposed to the thrust available from the same engine on the manufacturer's test stand.

Average Engine Thrust and Minimum Engine Thrust

Not all engines of the same type will perform in exactly the same manner. When tested, it will be found that some engines will produce slightly more thrust than some other engines being run at the same power setting. This variation between engines is simply due to small differences resulting from manufacturing tolerances.

Because of this normal engine-to-engine variation between engines of the same type, and considering environmental variations like temperature and humidity, Boeing uses two different definitions of thrust: *average* engine thrust and *minimum* engine thrust. The first of these two is the <u>average</u> thrust of the engines tested at Boeing; the latter of the two is that <u>lowest</u> level of thrust that is guaranteed by the manufacturer.

This difference is significant when computing airplane performance. When calculating takeoff data, for example, for conservatism Boeing is required to use the minimum engine thrust level of engine performance. An operator is thus assured that even if his airplane, by chance, has two

1. This is true on all Boeing models up through the 777. On the 787, however, for the first time, the air conditioning and pressurization systems will not be provided with air from the engines.

engines that were both on the low side of the thrust range, they can still be confident that their takeoff performance will meet all Boeing and regulatory requirements.

Where performance is not critical – during enroute climb or cruise, for example, Boeing uses average engine thrust to compute the published data.

Lower Heating Value (LHV)

Combustion engines, both reciprocating engines and jet engines, do their work by converting the energy contained in the fuel they consume into propulsive energy. In automotive uses, the reciprocating engine combusts the fuel in cylinders, causing the engine's crankshaft to rotate which in turn rotates the vehicle's wheels through a system of gears. For airplanes, the jet engine burns fuel in the engine's combustion section and exhausts the combustion gases through the engine's turbine and tailpipe, producing thrust.

The fuel used, whether automobile fuel or jet fuel, contains energy that is released by the process of combustion with air. And different fuels may contain slightly different amounts of energy. That energy content is called the fuel's *lower heating value* or *LHV*.[1]

The result of this fact is that in the process of producing thrust, the amount of fuel that will be consumed is a direct function of the amount of energy contained in the fuel per unit of volume or weight. An engine burning fuel containing a higher amount of energy per unit will consume fewer units of fuel; an engine burning lower-energy fuel will consume more units of fuel to produce the needed thrust.

The energy content of fuels depend on a number of factors, including the source of the crude petroleum that's refined into vehicle fuel, the refining process used, the density of the fuel, and other factors.

At Boeing, we use units of British Thermal Units (BTUs) per pound as a measure of fuel energy content (the metric unit is megajoules per kilogram, MJ/kg). A BTU is defined as the amount of heat required to raise the temperature of one pound of liquid water by one degree from 60° to 61°F at a constant pressure of one atmosphere.

The standard value of jet engine fuel LHV used by Boeing in producing performance data is 18,580 BTUs per pound of fuel. When conducting test flights for the determination of performance characteristics, we collect a sample of the fuel being used for the flight and measure the energy content of that fuel. We then correct the data collected in flight for the effect of variations of fuel energy, so the resulting data is standardized to the Boeing reference value of 18,580 BTUs per pound.

1. Yes, there also exists a slightly different measurement of fuel energy content, called the *Higher Heating Value*, but a discussion of the difference is of no practical value since we can not use HHV in assessing airplane performance. Discussions of this subject are available from Wikipedia (search for *heat of combustion*) and other sources.

Commercially available jet fuel may vary by approximately plus or minus one percent from the standard value of 18,580.

LHV varies, in an approximate manner, as a function of fuel specific gravity. Boeing assumes a linear variation of LHV with specific gravity at a fuel temperature of 60°F, following the equation:

$$LHV = 22777 - 5220 \times SG_{60} \qquad \text{(eq. 6)}$$

where *LHV* is the fuel energy content in BTUs per pound
SG_{60} is the specific gravity of the fuel at a temperature of 60°F

If the fuel specific gravity is measured at a fuel temperature other than 60°F, it may be corrected to 60°F knowing its temperature, thus:

$$SG_{60} = SG + 0.0063(T - 15.56) \qquad \text{(eq. 7)}$$

where *SG* is the measured specific gravity
T is the temperature of the sample measured in °C

Did you notice that LHV decreases as fuel specific gravity increases? It might seem logical to assume that fuel that has a higher specific gravity would also produce more energy per unit of weight than a fuel having a lower specific gravity, and yet you see from the equation above that the opposite is true: <u>lighter</u> fuel has more energy per unit of weight.

This fact arises because denser fuels have a higher ratio of carbon atoms to hydrogen atoms. The energy of combustion of carbon is less than that of hydrogen. Denser fuel, having relatively less hydrogen, therefore produces less energy per unit of <u>weight</u>.

Interestingly, however, denser fuels have higher energy content per unit of <u>volume</u>. This is due to the fact that the higher density fuels, due to their molecular structure, can pack more molecules into a given volume such as a liter or a gallon. Although each molecule has slightly less energy, the number of molecules per unit of volume increases with density more than the energy content of each molecule decreases. The net effect is a greater amount of energy per unit of volume.

Engine Station Designations

For reference and convenience, engine manufacturers assign "station numbers" to different locations within an engine. These station numbers increase from the front of the engine toward the rear.

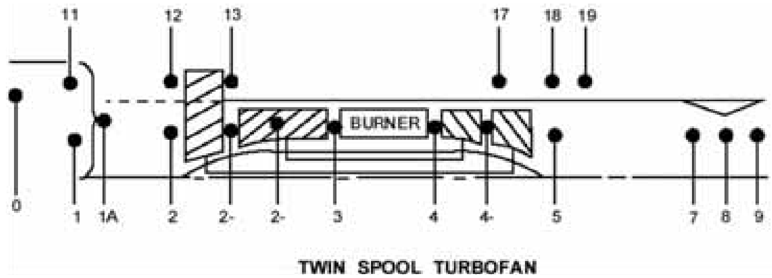


Figure 14-4

The illustration shows the station designations for a typical two-spool turbofan engine. For example, station 0 is the free stream ahead of the engine; station 2 is immediately ahead of the fan stage, of the air which will pass through the compressor, where station 12 is the station just ahead of the fan stage, but of the air which will only pass through the fan; station 5 is at the exit from the low-pressure turbine, and so on.

For a triple-spool engine, the station designations are similar but not exactly the same. Refer to the drawing to the right.

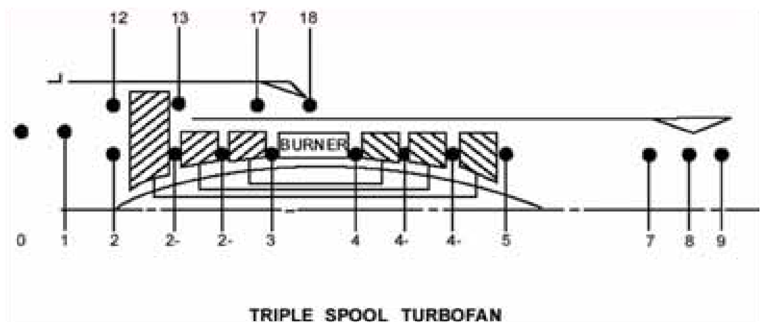


Figure 14-5

A parameter measured at a specific engine station will be referred to by that station number. For example, the total pressure of the air flow at the inlet to the compressor will be called p_{t2} and the temperature at the turbine exhaust will be called T_5. You will see an example of the use of these designations below where we discuss "EPR".

Other Jet Engine Terms

A number of other terms are frequently encountered in jet airplane performance work. The following is a list of those terms:

bleed (airbleed)

Air that is extracted from an engine, through a valve which may be either automatically controlled or manually controlled from the flight deck, is called *bleed air.* Bleed air is extracted from the compressor section of a engine and thus is at elevated pressure and temperature compared to the ambient air outside the airplane.

Bleed air is used for a number of different purposes: air conditioning and pressurization, ice protection, operation of air-driven hydraulic pumps, and a few other functions.

compressor stall

The compressor of any axial flow turbojet engine consists of a ring of compressor blades mounted on a rotating disc, alternating with a ring of stationary blades connected to the engine case. Since these blades are aerodynamic devices, they are shaped somewhat like an airfoil. As airfoils, they can stall when operating at excessively high angles of attack relative to the airflow through the compressor.

Compressor stall is a condition within an engine at which time the airflow over the compressor blades is stalled. Stall is uncommon when the engine is operated within its design parameters.

See the discussion below under “surge”.

EGT

This stands for *Exhaust Gas Temperature*. It is the temperature of the exhaust gas leaving the turbine section. This temperature is displayed on the flight instrumentation in the cockpit.

EPR

This stands for *engine pressure ratio*. It’s the ratio of the gas flow total pressures measured at two specific points: at the entrance to the exhaust nozzle (station 7) and at the inlet to the fan and compressor (station 2).

EPR is one of the two commonly used parameter for setting engine thrust. It’s the primary thrust setting parameter on both Pratt and Whitney and Rolls-Royce engines.

EPR is defined simply as:

$$EPR = \frac{p_{t7}}{p_{t2}} \qquad \text{(eq. 8)}$$

where p_{t2} is the total pressure measured at station 2

p_{t7} is the total pressure measured at station 7

On the Rolls-Royce RB211-535 engine, EPR is defined differently

$$EPR = \frac{p_{t13}}{p_{t12}} \qquad \text{(eq. 9)}$$

where p_{t12} is the fan stage entrance total pressure

p_{t13} is the fan stage exhaust total pressure

On the Rolls-Royce RB211-524 G and H engines, it's different again. Here, the fan stage exhaust total pressure p_{t13} is combined with the core exhaust total pressure p_{t7}, and compared to P_{t2}:

$$EPR = \frac{\frac{p_{t7} + p_{t13}}{2}}{p_{t2}} \qquad \text{(eq. 10)}$$

flameout

Once an engine is started and is up to normal running parameters, the igniters which start the ignition of the fuel in the combustion chambers are turned off, and the flame sustains itself. In some extreme conditions, it is possible for the flame to be lost, resulting in the condition called *flameout,* necessitating an inflight restart of the engine.

Since flameout can result from flight through severe turbulence, for earlier engines standard operating procedures call for turning the engine ignition system on during flight through those conditions. More modern engine control systems will detect a flameout and automatically turn on the ignitors, so pilot action to turn on the engine ignition system during these conditions is not necessary.

N1, percent N1, corrected N1

N1 is the term used to refer to the rotation rate of the low pressure spool of a multiple-spool engine. Its units are revolutions per minute (RPM).

The term *percent N1* (%N1) is still referring to the rotation rate of the low pressure spool, but in this case the RPM are expressed as a percentage of a nominal reference value of N1. On the Rolls-Royce RB211 engine used on the 747-400, for example, the reference N1 is 3900 RPM. Thus, if the low pressure spool is rotating at 3900 RPM, that would be expressed as 100% N1.

The reference RPM is usually selected close to the typical RPM at the takeoff thrust setting. Thus, takeoff thrust settings are typically close to 100%. On some engines, the takeoff N1 may be slightly greater than 100%; this should not be construed as exceeding any engine limitation.

N1 is the second of the two commonly used thrust setting parameters, EPR being the other one. It is the primary thrust setting parameter on the General Electric and CFMI engines which power many Boeing airplanes.

In some engine charts, you'll encounter a parameter called *corrected N1*. Here's what that means: if you were to make graphs of thrust versus N1, the charts would be different for different temperatures and different altitudes. For convenience, instead of using N1 for this sort of work, we can use *corrected* N1, which allows the N1 values for different conditions all to be plotted on a single chart. This is sometimes referred to as "generalizing" the data.

Corrected N1 is defined as:

$$corrected\ N1 = \frac{\%N1}{\theta_T^x} \qquad \text{(eq. 11)}$$

where θ_T is the total air temperature ratio $\frac{T_T}{T_0}$

x is the power to which the total temperature ratio θ_T is raised

The value of *x* is determined by the engine manufacturer; using that value of *x* in the equation for corrected N1 allows the data to be generalized. It depends on the manufacturer and engine type.

For many engines, the value of x is 0.5; for those engines, then:

$$corrected\ N1 = \frac{\%N1}{\sqrt{\theta_T}} \qquad \text{(eq. 12)}$$

For some other engines, a different value of *x* is used. Some engines have values of *x* near 0.6.

If you need to compute corrected N1, or to find actual N1 from corrected N1, you will need to know the value of x. This can be obtained from the engine manufacturer, and it's also shown in the Boeing Performance Engineer's Manual.

For a detailed discussion of "generalizing" data, see Discussion 1 in the Additional Discussions section at the end of this chapter.

corrected fuel flow

Just above, we discussed "corrected N1" and said that the reason for using corrected N1 instead of N1 is that it allows us to "generalize" the data, making a single chart usable for a range of conditions rather than requiring a separate chart for each different set of conditions.

For the same reason, you will usually see fuel flow data from the engine manufacturer provided in the form of *corrected fuel flow.*

Corrected fuel flow is defined as:

$$corrected\ fuel\ flow = \frac{fuel\ flow}{\delta_T \theta_T^x} \qquad \text{(eq. 13)}$$

where δ_T is the total pressure ratio $\frac{p_{total}}{p_0}$, equal to $\delta_{ambient}(1 + 0.2M^2)^{3.5}$

θ_T is the total temperature ratio $\frac{T_{total}}{T_0}$, equal to $\theta_{ambient}(1 + 0.2M^2)$

x is a power to which the temperature ratio is raised

Another way to express this equation, which is perhaps a little easier to use, is:

$$\textit{corrected fuel flow} = \frac{\textit{fuel flow}}{\delta_{ambient}\,\theta_{ambient}\,(1 + 0.2\,M^2)^{3.5 + x}} \qquad \text{(eq. 14)}$$

where $\delta_{ambient}$ is the air pressure ratio $\dfrac{p}{p_0}$

$\theta_{ambient}$ is the air temperature ratio $\dfrac{T}{T_0}$

M is the Mach number

<u>The power *x* here is not necessarily the same as the value of *x* used to generalize the N1 data.</u>

N2 and N3

You know now that N1 denotes the rate of rotation of the low pressure spool (sometimes called the low-pressure rotor). It follows then that N2 denotes the rate of rotation of the second, high pressure spool of a two-spool engine (the high-pressure rotor).

On current Rolls-Royce engines, there are <u>three</u> separate rotors rather than two. They're called the *low pressure rotor,* the *intermediate pressure rotor* and the *high pressure rotor.* The low pressure rotor is the slowest turning rotor, rotating at N1; the intermediate pressure rotor rotates faster, at N2, and the high-pressure rotor operates at N3 RPM, which is faster yet.

surge

This term relates to an abnormal transient condition of unsteady airflow through an engine. *Surge* may result from a condition of compressor stall. In an engine surge, the flow through the compressor may reverse violently, placing heavy transient loads on compressor blades. A surge may cause a loud "bang" sound. If an engine is already damaged, an engine surge can cause additional damage.

Surge can result from strong crosswinds at low airspeeds, for example during takeoff, or other conditions such as very rapid acceleration or deceleration of the engine. Conditions such as wake vortex, in which pressures and airflow direction at the engine inlets may fluctuate rapidly, can induce engine surge.

Modern engines are fitted with *surge bleed valves* (Rolls-Royce calls them *handling bleed valves)* which operate automatically. When opened, the valves dump air from the compressor into the fan duct or overboard, thus "unloading" the compressor by reducing the pressure ratio across it. Bleeding air from the compressor in this manner helps to restore steady flow in the engine. These valves also will open at conditions of low thrust.

Today's engines are also fitted with *variable-angle compressor stator vanes* at some stages of the compressor, and, in some instances, *variable-angle compressor inlet guide vanes*. These adjust-able-angle blades facilitate airflow control, and reduce the need for the gross action of opening bleed valves. Many engines have some combination of the two strategies.

Additional Discussion

discussion 1: generalizing data

It is sometimes useful in engineering work to simplify the presentation of data in such a way as to make a single chart show the desired data instead of a number of charts. That single chart is sometimes referred to as a *generalized* chart. Examples that come to mind are the generalized thrust required chart and the generalized fuel flow tables.

Also, in this chapter you have seen us refer to corrected N1 and corrected fuel flow. That simply means generalized N1 and generalized fuel flow – data that has been modified in such a way as to make one chart or table serve over a range of variables rather than for just a single value of that variable.

To illustrate our meaning, here's an example. This example will be an aerodynamics example rather than a propulsion example simply because it will be easier to understand.

Let's suppose that we wish to have a graph of an airplane's lift coefficient in cruise as a function of its Mach number and weight. That's easy enough, we know the equation:

$$C_L = \frac{W}{1481.4\, M^2 \delta S}$$

where *W* is the weight in pounds, a variable
M is the Mach number, a variable
S is the wing reference area in square feet, a constant
δ is the air pressure ratio, a variable

You see that the lift coefficient is a function of three variables: the weight, the Mach number, and the air pressure ratio δ. Because it's a function of three variables, it would not be possible to make a single chart that would be valid for all possible conditions. For example, we could make a single chart that is valid for all weights and all Mach numbers, but then we would need a chart for each value of δ.

If, for example, we were cruising at 35,000 feet the chart of lift coefficient would look like this:

But this chart is valid <u>only</u> for a cruise altitude of 35,000 feet. We'll need another chart for each other altitude.

That's not very convenient...

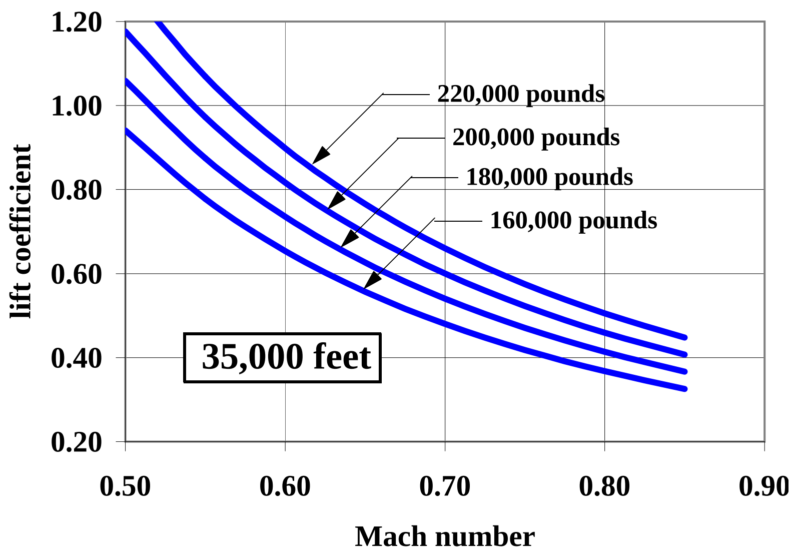


Figure 14-6

Here's a way we could "generalize" the data so that a single chart could be valid for all altitudes: let's define something called "generalized lift coefficient":

$$\textit{generalized lift coefficient} = C_L\delta = \frac{W}{1481.4\,M^2\,S}$$

You see that now the "generalized lift coefficient" $C_L\delta$ is a function of only two things: weight and Mach number. We could therefore make a single chart that would be valid for all altitudes!

Here's a chart of generalized cruise lift coefficient for our airplane. Notice that it's valid for all altitudes.

Does it work? Let's do a check case to compare these two charts. Using the generalized chart, let's find the cruise lift coefficient for a weight of 200,000 pounds at Mach 0.80.

From the generalized chart, I read a generalized lift coefficient of 0.108. For an altitude of 35,000 feet, the value of δ is:

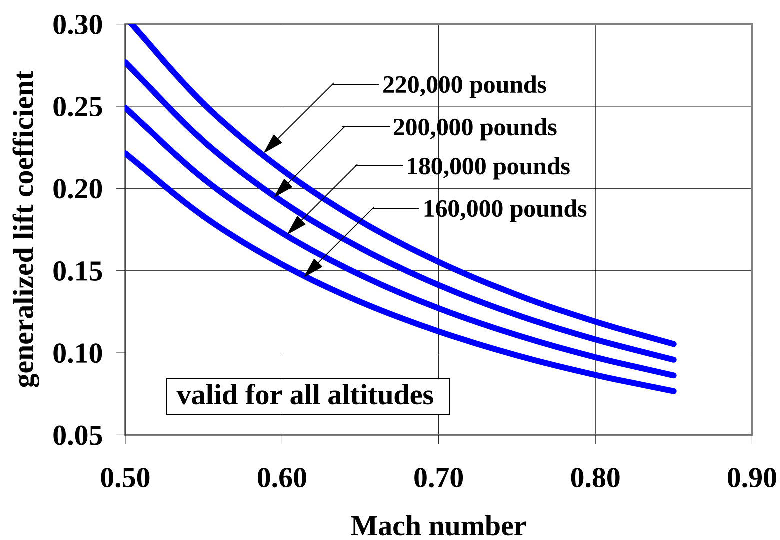


Figure 14-7

$$\delta = \theta_{ISA}^{5.25588} = \left(\frac{288.15 - 0019812 \times 35000}{288.15}\right)^{5.25588} = 0.2353$$

Knowing the generalized lift coefficient $C_L\delta$ of 0.108 and the δ of 0.2353, we find that:

$$C_L = \frac{C_L\delta}{} = \frac{0.108}{0.2353} = 0.459$$

Comparing that value for δ with the value found in the first graph of lift coefficient, we find they agree. I have thus generated one chart for lift coefficient that serves all altitudes.

You’ll probably never hear of “generalized lift coefficient' – we just made that up for this example. But you will hear about “generalized thrust curves” of thrust required versus Mach for lines of weight. Here, the generalized thrust is defined as $\frac{thrust}{\delta}$.

A generalized thrust curve can be useful because it allows us to find quite simply the generalized thrust for any combination of altitude, weight and speed. Knowing that makes it possible to find (for example) the fuel flow and the EPR or N1 thrust setting.

Chapter 15: Thrust Ratings

Introduction

In this chapter, we need to discuss the factors which limit the amount of thrust that an engine can safely produce, and how that affects the way the engine will be operated in airline service. You'll see that several factors limit the thrust available, and that the operating limits vary depending on the phase of the flight: takeoff, enroute climb, and the like.

Building a thrust rating is a very complex task wherein the engine manufacturer must satisfy a myriad of conflicting requirements in terms of airplane thrust requirements, engine aerodynamics, materials technology, nacelle sizing, manufacturing cost, maintenance and weight.

You have already learned that the two primary thrust setting parameters used on today's commercial jet airplanes are EPR and percent N1. Now you'll learn about the five different levels of thrust setting, and you'll see how EPR and percent N1 charts look quite different.

Thrust Ratings

So what is a thrust rating? Simply put, a *thrust rating* is a maximum level of engine thrust that is permitted for a specific phase of flight. For example, takeoff has its own specific thrust level, called *Maximum Takeoff Thrust*. That level of thrust can not be used at any other time during the flight. Also, as you'll see, it can be used only for a specified number of minutes when taking off.

Normal climb to altitude has a different thrust level called *Maximum Climb Thrust*, or MClT. Similarly, cruise has its own rating, called *Maximum Cruise Thrust*, or MCrT.

For emergency purposes such as driftdown following an engine failure in cruise, there is a specific thrust rating called *Maximum Continuous Thrust*, or MCT.

A fifth rating exists, called *Go-around Thrust*, sometimes also referred to as Maximum Inflight Takeoff Thrust. This is a special rating used only at lower speeds and altitudes such as during a missed approach when maximum possible performance may be required.

Factors That Limit Thrust

Today's jet engines are engineering marvels. But they do have limitations on the amount of thrust that can be commanded by the pilots.

First of all, it's necessary to consider the pressures within the engine. More precisely, we must consider the pressure differential across the engine case – that is, the internal pressure minus the ambient pressure outside the engine.

The compressors of today's engines routinely produce internal pressures of up to forty atmospheres or more; newer engines are now approaching fifty atmospheres. Think of it: that's more than seven hundred pounds per square inch.

For any engine, thrust is controlled, whether manually or automatically, by using the thrust levers. As the thrust levers are advanced, the fuel flow is increased. That causes the turbine RPM and thrust to increase. Along with the thrust increase will come an increase in the engine's internal pressures, because the compressor stages, driven by the turbine stages, will turn faster and will cause more compression of the incoming air.

Clearly, exceeding some specified level of thrust could cause the engine case pressure differential to become unsafe. The engine case which contains all these high-pressure gases has structural limitations, after all – if you keep raising the pressure within the case, it will eventually fail. Think of what can happen if you overinflate a balloon.

So the first of the limiting factors is the engine's case differential pressure. This is referred to as the *pressure-limited condition.* Greater thrust means greater internal pressures – how much is allowable?

Second, as the thrust levers are advanced and more fuel is injected into the combustors, the temperature of the gas entering the turbine stages increases. It's interesting that at high thrust settings, the gas temperatures within the engine, specifically at the exhaust from the combustion chamber where the flow enters the turbine, are in excess of fifteen hundred degrees Celsius on today's engines. In fact, this temperature far exceeds the melting point of the metal used to make the turbine blades, necessitating <u>very</u> good cooling systems within the engines.

Also, the rate of rotation of the high-pressure turbines are in excess of ten thousand RPM at takeoff thrust. Given the combined effect of the high temperatures and the high centrifugal loads on the turbine blades, you may well wonder how they survive in that environment at all.

So the second of our factors limiting the thrust available is the internal temperature which will be experienced at the critical locations in the engine. This is called the *temperature-limited condition*.

There's a third limitation to engine thrust, which is an RPM limit in the fan stage. Knowing that the fan stages of today's engines are more than nine feet in diameter, you can imagine the velocity at the tips of the fan blades. Using a nine-foot fan diameter as an example, for each one thousand RPM the fan blade tip velocity will be approximately 470 feet per second. Given that typical takeoff N1 RPMs are on the order of 3000, this means that the fan blade tips are operating at or a little above the speed of sound.

In addition to fan blade tip speed considerations, the centrifugal force on the blades must also be allowed for in the design. Rolls-Royce calculates that the centrifugal forces on a fan blade can be on the order of 100 tons.[1]

1. Rolls-Royce PLC: "The Jet Engine", page 104. Quoted with their kind permission.

Let's discuss each of these limitations in turn. We will be showing the takeoff EPR limitations for the Pratt & Whitney PW4056 engine. In the following charts, the term "Takeoff OAT" means the Outside Air Temperature at the time of the takeoff.

the pressure limit

For a given pressure altitude, the thrust at the pressure-limited condition is the same regardless of temperature, and so is the EPR.

With increasing pressure altitude, the atmospheric pressure outside the engine case will decrease. To avoid exceeding the case differential pressure limit, the pressure-limited thrust must be slightly less than at the lower altitude.

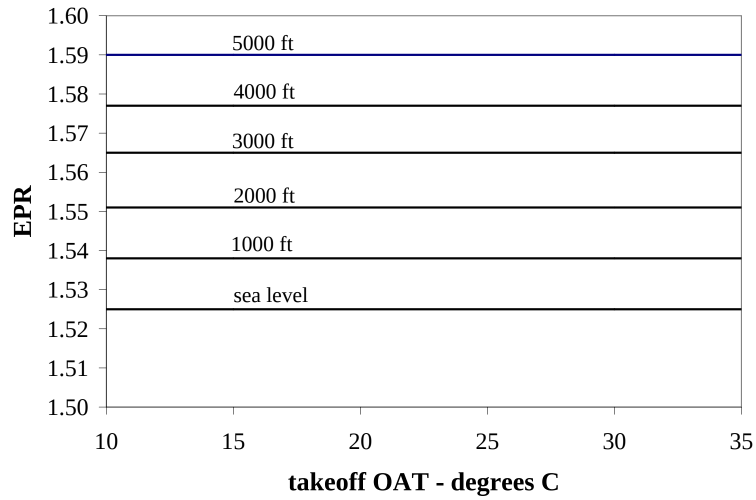


Figure 15-1

When you look at an EPR chart, however, you'll see that the EPR for the pressure-limited thrust <u>increases</u> with increasing altitude. We are frequently asked why this is the case.

<u>For the same EPR or N1</u> thrust will decrease with increasing altitude. The engine manufacturers, recognizing that good takeoff performance may be critically important on the more challenging routes, allow some increase in the pressure-limited takeoff thrust setting at altitudes above sea level, in order to regain some of that lost thrust. Thus the EPR or N1 will increase with altitude to retain as much thrust as possible. That increase will be a compromise between the airplane's need for thrust versus the engine's design constraints and cost considerations.

the temperature limit

Fortunately, this one is a little more intuitively obvious.

The object of this limitation on thrust is to keep the engine at acceptable internal temperatures. Exceeding the temperature limitation will result in overheating the turbine, possible blade failure, and engine damage.

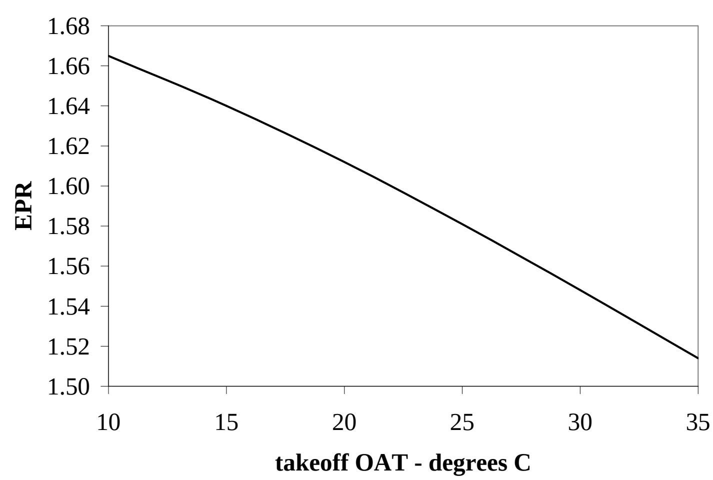


Figure 15-2

For some engines, such as the PW4056 shown here, the temperature limit is a single line regardless of altitude. For some other engines, you will find that the temperature limit line will be different for different altitudes.

Regardless of whether there is only one temperature limit line or more than one, it will be seen that as the air temperature increases, the EPR decreases. That's logical. It's a fact that for each one degree the air temperature increases, the engine's internal temperature increases on the order of ten degrees because of the compression that the air undergoes. So as the outside air temperature increases, the thrust must decrease in order to avoid exceeding the design temperature limit.

N1 RPM limit

As described above, it might be necessary to limit the engine RPM in order to avoid overloading the fan blades in centrifugal force, and to control the fan blade tip speeds. This is purely an N1 limit and is not a function of altitude.

The N1 limit is rarely more restrictive than the pressure and temperature limits.

the limits combined

The illustration on the right shows how the two limitations are combined into one.

You'll observe that at the colder temperatures, for any given altitude the EPR limit is initially "flat", or constant. For example, at sea level the limit EPR is constant at 1.525 up to a temperature of approximately 34 degrees Celsius.

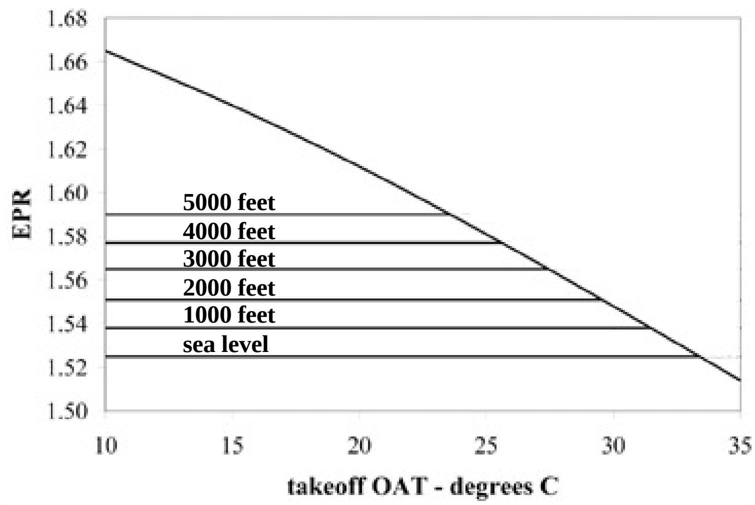


Figure 15-3

If we were to continue to operate the engine at an EPR of 1.525 at a temperature greater than 34 °C, we would exceed the temperature limitation and could damage the turbine blades. Thus, sea level temperatures greater than 34 °C require that the EPR be set to a value <u>less</u> than its "flat" value of 1.525, a value which decreases with increasing OAT.

Flat Rated Thrust

You'll frequently hear performance engineers speaking about *flat rated* thrust.

Flat rated refers to the thrust which is within the pressure-limited region – that is, the <u>flat</u> portion of the EPR versus temperature line.

Engine manufacturers will frequently say "our engine is sea level flat-rated to XX degrees" as a selling point. That simply means that the engine does not become temperature limited at sea level until the outside air temperature is equal to XX degrees. For airlines that operate in hot environments, being flat rated to a high temperature is desirable since it means that the airplane may be able to takeoff at higher weights on the hotter days. because more thrust will be available.

N1 as the Primary Thrust Setting Parameter

In the above discussions, we have been showing EPR as the primary parameter for setting engine thrust. If you operate Pratt and Whitney or Rolls-Royce engines, that's fine. But if you operate airplanes with engines manufactured by General Electric, you'll need to be familiar with the use of N1 as the primary thrust setting parameter. N1 is different from EPR as a thrust setting parameter in one major way.

The figure on the right, shown for one altitude, illustrates this.

An engine operating in its flat rated thrust region would expect to use a constant EPR with increasing temperature until reaching the temperature at which the flat rate thrust just becomes equal to the temperature limit thrust.

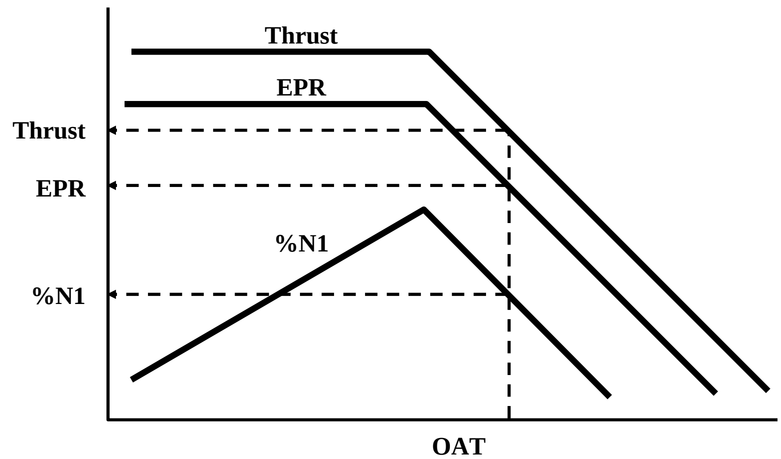


Figure 15-4

Engines using N1 as the thrust setting parameter, on the other hand, will not use a constant %N1 in the flat rate region of thrust. Instead, you will see an increase of %N1 with increasing temperature, up until the temperature at which flat rated thrust equals temperature limited thrust; after that, the %N1 will decrease with a further increase in air temperature.

N1 compared to corrected N1

You just saw that the thrust setting N1, in the flat rated region of thrust, increases with increasing temperature up to the flat rated thrust region maximum temperature, after which the N1 decreases with further increase in temperature.

The figure below illustrates this. You'll see on the right, however, that a plot of corrected N1 versus temperature shows a relationship just like EPR versus temperature. If necessary, refer back to the preceding chapter for the definition of corrected N1.

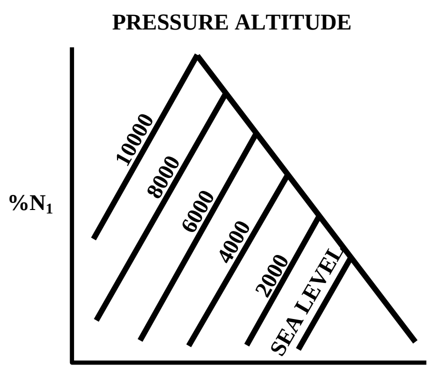


Figure 15-5a

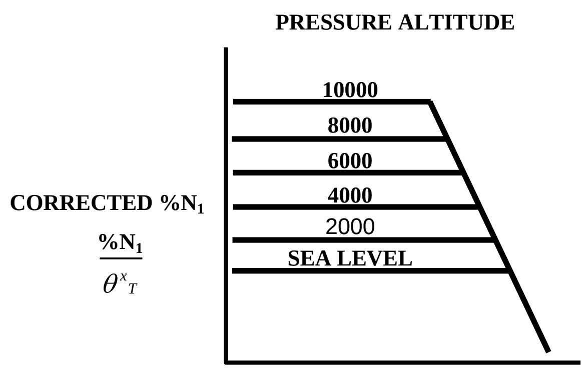


Figure 15-5b

Effects of Bleeds on Thrust Settings

In the previous chapter, we defined "airbleeds". We said that high pressure air can be extracted from the engines for a number of uses, principally air conditioning and pressurization, also for icing protection.

You would expect that opening a valve somewhere in the engine case surrounding the compressor section and extracting air would have the effect of reducing the amount of energy in the exhaust gases, and such is the case. It will also have the effect of decreasing, slightly, the total pressure at the engine exhaust and therefore will affect EPR or N1. The engine control system will try to avoid the drop in EPR or N1 by dumping more energy (i.e. fuel) into the combustor, which results in the engine running hotter and at higher rotor speeds. The higher temperatures and speeds may cause the engine to wear out more quickly, and may even exceed the certified redlines for the engine. If the engine is operating at rated thrust, it is typical for the engine control to minimize the increases in temperature and rotor speed by reducing EPR or N1 slightly when bleed is extracted. Therefore, we should expect to see adjustments to the rated thrust setting EPR or N1 when airbleeds are being used and that, with one notable exception, is the case.

The one notable exception to requiring an EPR or N1 adjustment is the use of engine bleed air for engine anti-ice protection. At takeoff thrust, the takeoff EPR or N1 is the same for engine anti-ice on or off. If you were in the cockpit with the engines running and you turned engine anti-ice ON, the engine control would automatically dump in more fuel to counteract the bleed effect. This would be handled within the engine control, so you would not see any movement of the thrust lever. You would not see a drop in EPR or N1 but rather a slight increase in fuel flow. You're getting a little bit of extra performance in this case, but since engine anti-ice is only used at low temperatures, it does not represent any harm to the engine.

When doing performance calculations, it's important to include the effects of airbleeds where appropriate. Also, keep in mind that there are times when using a non-normal bleed configuration will improve the allowable takeoff weights. As an example, takeoffs are normally made with the engines providing bleed air to the air conditioning and pressurization systems, and the published thrust setting charts and takeoff performance charts are based on the thrust available in that configuration. However a "packs-off" takeoff procedure is permitted. Takeoff with the airbleed valves closed instead of open will permit higher takeoff thrust settings and therefore greater takeoff weights. The takeoff EPR or N1 charts will show the correction to be applied when operating with the air conditioning bleeds turned off.

The Five Thrust Ratings

At the beginning of the chapter, we said that there are five separate thrust ratings, each applicable to a particular phase of flight.

Three of these are called *certified* thrust ratings, meaning that they are the basis of airplane performance data which is governed by aviation regulations such as the United States Federal Aviation Regulations and thus are published in the Airplane Flight Manual (AFM) and have the force of law. Compliance with these certified thrust ratings is mandatory.

Let's examine these one at a time.

maximum takeoff thrust

The maximum takeoff thrust rating is the highest level of thrust available from an engine. It is used only during takeoff. It can not be used at altitudes above the maximum altitude certified for takeoff and landing.

This is the thrust level which is the basis for all of the takeoff performance charts. Because aviation regulations mandate the required levels of performance for takeoff (for example, the minimum allowable climb gradients, takeoff distances, and so on), the takeoff thrust rating is one of the "certified" thrust levels and data for the takeoff thrust setting appears in the AFM.

Takeoff thrust may be used only for a maximum of five minutes, with one exception: the time limit is extended to ten minutes for airlines that have purchased a special Airplane Flight Manual appendix called the "ten-minute appendix". The Certificate Limitations section of that appendix states specifically:

> *"The time limit on the use of takeoff thrust is increased to 10 minutes provided this use is limited to situations where an engine failure actually occurs and there is an obstacle in the takeoff flight path."*

The ten-minute appendix therefore is useful only at those airports which have distant obstacles that might severely limit the allowable takeoff weight if takeoff thrust were limited to five minutes. Relatively few airports are so limited, but for those that are, the ten-minute appendix can be well worth its purchase price.

When you look at a takeoff thrust setting chart, you'll usually see a note saying something like "40 to 80 knots". This is a typical speed range for the takeoff power setting and final adjustment. For higher speeds, EPR or N1 is adjusted because of various influences such as intake air compression, thus influencing the displayed EPR or N1 at a fixed thrust lever setting.

You'll also note that the takeoff EPRs and N1s are based on the use of normal airbleeds, meaning that the effect of the bleed for air conditioning and pressurization is included, but the effect of the bleed for wing anti-icing is typically <u>not</u> included.

go-around thrust

Go-around thrust is sometimes referred to as *inflight takeoff thrust*, which is an appropriate name since <u>go-around thrust is usually the same as takeoff thrust</u>[1]. Although the thrust is the same, the EPRs or N1s are different from the takeoff thrust values because they are corrected for the effect of the speeds at which the airplane flies during the approach and landing. (You'll recall from the discussion above that the takeoff thrust settings do not include any correction for speed and therefore are valid only when used at speeds below approximately eighty knots.)

Go-around thrust is the basis for the calculation of the "approach climb" and "landing climb gradient" charts that are published in the AFM and for which the regulations mandate minimum performance levels. Therefore, like the takeoff thrust rating, go-around thrust is also a "certified" thrust rating and the thrust setting charts therefore appear in the AFM.

There is a five minute limit on the use of go-around thrust, as there is for takeoff thrust. Also, as is the case for takeoff thrust, the go-around thrust rating may not be used above the maximum altitude certified for takeoff and landing.

maximum continuous thrust

This is the third of the "certified" thrust levels.

Maximum Continuous Thrust, or MCT, is a special thrust rating that is only usable in the event of some emergency situation. <u>It may not be used in normal operation</u>.[2] MCT is the greatest amount of thrust that can be used in flight, with the exception of takeoff and landing.

1. There are exceptions for some high-thrust ratings for the 747, for which go-around thrust is approximately 10% less than takeoff thrust. This helps maintain airplane controllability when two engines have failed on the same side of the airplane.
2. For some engines, the maximum continuous thrust rating is the same as the maximum climb thrust rating; in those cases, this statement does not apply.

One of the most important uses of maximum continuous thrust is for continued operation after an engine failure during takeoff, when the time limit on takeoff thrust has expired. If the airplane must still climb to clear a distant obstacle, its performance is based on MCT.

Another situation in which maximum continuous thrust is important is terrain clearance following an engine failure during the flight. An operator whose routes take the flight across high terrain must pay attention to the altitude capability of the airplane with an engine inoperative (and with two engines inoperative on a 3- or 4-engine airplane, under certain conditions specified by the regulations).

MCT is the basis of the “enroute climb” charts that appear in the Airplane Flight Manual. A word of explanation is needed here: in the AFM, “enroute climb” does NOT mean normal climb to the cruise altitude or the step climb between cruise altitudes. Instead, the “enroute climb” charts appearing in the AFM show the gradients available with an engine inoperative (and also with two engines inoperative, in the case of 3- or 4-engine airplanes) with the operating engines set to maximum continuous thrust.

These enroute climb charts are used to calculate the airplane’s performance during the driftdown and leveloff following an engine failure in cruise. The aviation regulations mandate minimum acceptable levels of performance for driftdown and leveloff, and so this thrust rating is a certified rating. The MCT thrust setting charts thus appear in the Airplane Flight Manual, They appear also in the Performance Engineer’s Manual (PEM) and in the documents made available to flight crews, such as the Quick Reference Handbook (QRH).

maximum climb thrust

Neither maximum climb thrust nor maximum cruise thrust, which will be discussed next, is a true limitation on the engine’s performance. The engine control system will not prevent a pilot from exceeding either of these thrust ratings, unless doing so would require exceedance of the highest certified thrust level for that phase of flight, i.e. takeoff or maximum continuous thrust.

This thrust rating does not relate to climb during the takeoff phase of flight – that climb is performed at maximum takeoff thrust, up to the five-minute time limit, and then at maximum continuous thrust in the event of an engine failure during takeoff. Instead, the maximum climb thrust rating, MCLT, is the thrust rating which applies to normal climb to altitude after takeoff, or when performing step-climb from one cruise altitude to the next.

There is no time limit on the use of maximum climb thrust.

Because enroute climb performance is not discussed in aviation regulations such as the Federal Aviation Regulations (FARs), MCLT is not considered to be a “certified” thrust level. The maximum climb thrust settings are not published in the Airplane Flight Manual. They appear instead in the PEM and QRH.

maximum cruise thrust

This thrust rating is used as a reference for the thrust available for cruise. It is not a true limit on cruise thrust, as the engines can be advanced to maximum continuous thrust in an emergency situation. It is usually designated as MCRT. It's usually a slightly lower thrust rating than maximum climb thrust, although in some instances the two are the same. There is no time limit on the use of maximum cruise thrust.

Cruise is normally conducted at a thrust setting somewhat less than maximum cruise thrust. MCRT is the upper cruise thrust threshold for normal operation. Should the pilot wish to cruise at the fastest speed possible, he may advance the thrust levers to the MCRT setting and accept the resulting speed provided, of course, that it doesn't exceed the maximum certified speed V_{MO}/ M_{MO}. These are discussed in the chapter entitled "Speeds".

Because maximum cruise thrust is not the basis for any performance level mandated by the aviation regulations, the data for MCRT doesn't appear in the Flight Manual, but rather in the PEM and the QRH.

Derates

For most of today's engines, the manufacturer makes available to operators one or more alternative sets of thrust ratings. There are two types of derates. Some earlier engines were provided with derates based on the performance of a different (lower thrust) engine: for example, the AFM for the 747-200 powered with Pratt & Whitney JT9D-7Q engines has an available appendix providing the performance based on the thrust of the -7Q engine when operated to the (lower) takeoff EPRs of the earlier -7 engine.

The other form of derate is a "fixed percentage" derate. For example, 747 AFM performance appendices are available for the General Electric CF6-50E2 engine at 4% thrust reduction and at 10% thrust reduction.

In general, performance at derate thrust is available through an Airplane Flight Manual appendix, or through AFM-DPI.

Thrust Bumps

Sometimes, an engine manufacturer will offer an increased level of thrust, on a specified engine, in order to provide enhanced performance capability. These increases are not available over the entire altitude-temperature envelope, rather they're confined to a specific region of the altitude-temperature envelope. These increases are known as *thrust bumps*.

Bumps can take a number of forms, as shown in the illustration to the right.

The manufacturer may choose to keep the same level of temperature-limited thrust but increase the pressure-limited thrust somewhat, as shown in the illustration as Bump 1.

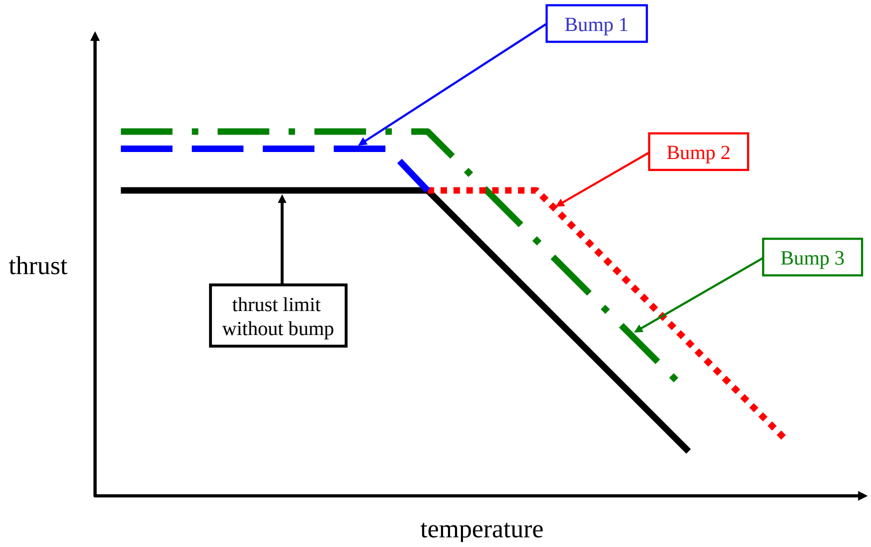


Figure 15-6

On the other hand, the manufacturer may choose to retain the same pressure limit but increase the temperature limit, shown as Bump 2.

The manufacturer may also elect to increase both the temperature and pressure limits, shown as Bump 3.
Some bump ratings are available only under specific conditions. One such is the “Denver bump” which is usable only within a narrow altitude range that includes the airport at Denver, Colorado, at which takeoff is frequently performance-limited due to the airport elevation of approximately 5300 feet. For a similar reason, one of the Rolls-Royce Trent engines has a “Johannesburg bump”.

Where a bump rating is available for a given engine type, performance data for operation with at that bump rating is available for purchase as an AFM appendix or inclusion in AFM-DPI.

Chapter 16: The Flight – a Performance Overview

Introduction

In the preceding chapters, we have discussed those basic elements of aerodynamics and propulsion that will be needed when computing the performance of a commercial jet transport airplane. It's time now to apply these fundamentals to real performance problems.

In this chapter, we'll look at an overview of a typical flight. We're going to discuss the documents which will be needed in performance work, then we'll talk about the various regulatory agencies and the bodies of regulations which apply to airplane operations. We'll then go through an entire flight, segment by segment, to show you what regulations are relevant and what sort of performance calculations will be necessary.

In the chapters following this one, we will go much deeper into detail on the actual methods for calculating the performance for each segment of the flight.

Keep in mind, please, that everything you see in this and the following chapters will be relevant to Boeing airplanes, and that documents and methods may be somewhat different for the airplanes manufactured by other companies.

First Of All, What Is Performance?

One source defines *performance* as "the manner in which or the efficiency with which something fulfills its intended purpose." That's a good starting point.

Let's put it this way: airplane performance is the description of the way the airplane acts – "performs" – under a specified set of conditions, in order to achieve its intended purpose of flying from point A to point B.

Obviously, then, *airplane performance* includes many elements:

- the physics of flight;
- the parameters used to describe airplane performance, such as lift and drag coefficients, tire-to-ground friction, thrust, and the like;
- calculation methods, such as step integration, iterations, and so on;
- methods for presenting data, such as charts and tables;
- regulatory requirements mandating certain levels of flight capability.

To begin, let's see what documents are available to help us calculate an airplane's performance.

Documents

Many different documents are relevant to airplane performance. Let's look at each of them.

the Airplane Flight Manual

Every single airplane produced by Boeing has an Airplane Flight Manual (AFM). That AFM, approved by the United States Federal Aviation Administration (FAA), or the equivalent AFM approved by another regulatory body such as the European Aviation Safety Agency (EASA), is the primary document controlling the airplane's use for flight.

The AFM, prepared and produced by Boeing to FAA (or other regulatory agency) specifications, is the definitive source of information on performance of the airplane *where that performance is mandated by specific regulations*. For example: takeoff and landing are the most strictly regulated segments of a flight. The AFM contains data which show very precisely the maximum weights at which an airplane will comply with the regulations controlling takeoff and landing distances, climb and obstacle clearance, brake energy, and so on.

Other segments of a flight are <u>not</u> regulated by law. For example, data showing the performance during climb to altitude doesn't appear in the flight manual. The AFM <u>does</u> regulate the allowable ranges of altitudes and temperatures for flight, but has no data for things such as cruise speed, holding fuel flow, and so on.

The AFM is a complex document, and while very thorough and detailed it's not really user-friendly. Performance engineers must exert every reasonable effort to become familiar with it, because it legally governs an airplane's performance during takeoff and landing and a few other segments of a flight.

<u>Every AFM contains a list of the manufacturer's serial numbers of the airplanes to which it applies</u>. An operator may have a number of different AFMs for the same model of airplane, depending on the exact characteristics of each airplane in that fleet. <u>When using an AFM to compute airplane performance, it's essential to check that the correct AFM is being used</u>.

The AFM is a regulatory document and it has the force of law. For that reason, compliance with the weight limits and all other limitations that appear in the AFM is mandatory.

The strong statement made above deserves a little discussion. While the statement itself is true, there are certain exceptions to it that are legally permitted.

An *overweight landing* is a good example. It happens occasionally that a pilot will consider it safer to land at a weight exceeding the certified landing weight limitation, rather than to continue the flight. This might be the case in the event, for example, of an engine fire occurring during the takeoff. In an event such as this, <u>it is within the authority of a pilot in command of an airplane to disregard an AFM limitation if he deems it necessary for safety</u>.

Since every limitation exists for a valid reason, whether structural, aerodynamic, airplane handling qualities or whatever, violating a limitation is not to be done without considering possible consequences. In the case of an overweight landing, for example, the risk of structural damage is very small due to the conservatism in the way certified structural limit to landing weight is defined. The Maintenance Manual does call for an overweight landing inspection in such an instance, however.

Another exception is the allowance of *dispatch deviations*. These are legally authorized deviations from published AFM performance levels, as might be necessary when dispatching an airplane with some piece of equipment inoperative, or with some airplane external component such as a vortex generator missing from the airplane. Dispatch deviations will be discussed further a little later in this chapter.

The AFM is composed of four sections, plus one or more appendixes, as follows:

Section 1 – Certificate Limitations. This section contains the limitations which must be observed. The following are the certificate limitations of particular relevance to performance:

- Certified Weight Limitations. These are the certificate limitations to the allowable taxi weight, takeoff and landing weights, and zero fuel weight. Bear in mind, of course, that many factors may limit the allowable weights to values less than those shown in this chapter. For example, if taking off from a relatively short runway the runway length-limited takeoff weight will probably be less than the certified takeoff weight limit, and that performance weight limit will take precedence over the certified value.
- Operational Limits. These include the runway slope limitation, the tailwind limitation for takeoff and landing, the maximum operating altitude, and the maximum altitudes for takeoff, flight, and landing.
- Center of Gravity Limits. These are the *certified* limits to the airplane's center of gravity, based on criteria of structural strength, airplane longitudinal stability, and handling qualities, and they may never be knowingly exceeded during normal airplane operation. These certified limits are not practical for use in day-to-day operation however because they don't include any allowances for operational variables such as passenger seating, fuel density and fuel load, fuel usage, inflight movement of passengers and crew, and so on. It is the function of the airplane's takeoff weight and balance system – its "loading system" – to ensure compliance with the certified CG limits while accounting for all operational variables.
- Airplane Operation Limits. These specify the kinds of flight operation that the airplane is approved for, including VMC (VFR) and IMC (IFR) operation, extended overwater operation, and flight in icing conditions.
- Fuel System Limitations. These include the fuel density limits, the allowable lateral fuel imbalance, and limitations to the way the fuel is used during the flight.
- Engine Limitations. The engine limitations include the maximum allowable time for operation at the takeoff thrust exhaust gas temperature (EGT), and the requirements for the use of engine anti-icing protection.
- Speed Limitations. These limits include the maximum speeds for flight with the flaps extended, the maximum speed for inflight maximum maneuvering, and the maximum operat-

ing speeds V_{MO} and M_{MO}. They also include the maximum speed for flight with the landing gear extended and during landing gear extension or retraction.

Sections 2 and 3 – Procedures. Although Chapters 1 and 4 are the two most frequently used in performance work, performance engineers should also develop some familiarity with Sections 2 and 3, both of which relate to procedures. Although Sections 2 and 3 do not contain any performance data, some of the procedures in these chapters are the basis of performance information published in Section 4 and elsewhere.

First of all, let's make it clear that pilots don't normally refer to the AFM for procedures information. That sort of information is published in other documents such as the Quick Reference Handbook (QRH) which is part of the Flight Crew Operations Manual, the FCOM. The regulatory agencies don't try to establish all flight crew procedures. However, there are certain "ormal and non-normal procedures which the regulatory agencies do wish to have incorporated into the AFM because they feel that it's necessary to publish guidance information for those specific circumstances.

Section 2 is called *Non-Normal Procedures*. The introduction to this section says, in part,

> *"...This section provides the abbreviated non-normal procedures which arise out of system malfunction or failure, and/or which involve the use of special systems or the alternative use of regular systems to protect crew and passengers from serious harm, and to maintain the airworthiness of the airplane..."*

Appearing in this section are procedures such as rapid decompression and emergency descent, ditching, inflight engine restart, and some others. These are all procedures for circumstances which are out of the ordinary.

Section 3 is entitled *Normal Procedures*. These procedures are those which the regulatory agency regards as having particular importance. Quoting from the introduction to Section 3:

> *"...Normal procedures in this section include those procedures where an action or sequence of actions, or prohibition of incorrect actions, which if not observed, could result in an adverse effect on the airworthiness of the airplane or the safety of the passengers and crew..."*

Included in Section 3 are procedures such as turbulent air penetration, automatic landing system demonstrated crosswinds, demonstrated Required Navigation Performance (RNP) values, and a few more.

Section 4 – Performance. This chapter will take one of two very different forms, depending on the airplane to which it applies.

For earlier airplanes, Section 4 will contain all of the charts detailing the takeoff and landing performance, and the enroute data used to calculate the performance of the airplane following an engine failure in flight. These charts are complex and not intuitively obvious – training on their structure and use is highly recommended. Knowledge of the Airplane Flight Manual presentation of performance data can provide an effective mental model leading to a more thorough understanding of performance concepts that will serve a performance engineer well.

AFM-DPI

For the 777 and later airplanes including the 737NG series, a software application called the "Airplane Flight Manual - Digital Performance Information" (AFM-DPI) takes the place of the performance charts contained in Section 4 of the earlier AFMs. The AFM-DPI performs a precise calculation of the airplane's takeoff performance for specified conditions. It can perform a single-point calculation for a single set of conditions, or it can do a series of calculations for a range of conditions.

Whether an older style of AFM with charts in Section 4 or a newer airplane having AFM-DPI, the primary functions of either of those is to:

- enable the computation of all performance-limited takeoff weights, including field length limited weight, climb-limited weight, obstacle-limited weight, brake energy-limited weight;
- compute the takeoff speeds V_1, V_R and V_2 for takeoff;
- compute the "enroute climb" performance, which includes the engine-inoperative gradients and altitude capability. These are used for determining the driftdown flight path following engine failure in flight;
- compute the climb-limited and field length-limited landing weights

The AFM-DPI called a "first principles" application. It is more accurate than the previous method of presenting the data as a series of charts in Section 4. This is simply because there is usually some loss of accuracy when graphing complex data that is computed for a range of conditions, and that loss of accuracy is necessarily taken in the conservative direction. AFM-DPI, on the other hand, computes its data directly from tables of basic performance data such as the drag polars, thrust data, and so on. Thus, higher allowable takeoff weights may be achieved by using AFM-DPI instead of the corresponding AFM charts. For this reason AFM-DPI has been made available for purchase on a number of earlier airplanes such as the 747-400, which may be able to achieve substantial payload increases by using it.

An AFM-DPI may periodically be revised, just as an AFM Section 4 may be periodically revised. It's the duty of the user to be sure that he uses up-to-date information. The AFM Log of Pages provides the current revision number of the AFM and the correct dates for each page, as well as the AFM-DPI serial number.

The AFM-DPI is an FAA-approved software application, and has the same regulatory status as the AFM.

Airplane Flight Manual appendixes. The AFM for any airplane will contain one or more appendixes. These provide additional information not contained in the four sections of the AFM. Appendixes are also the means used for providing special certification data for specific airplanes.

A standard appendix provided in every AFM is called the Configuration Deviation List (CDL). That will be discussed later in this chapter. Another standard appendix is provided to authorize takeoff at reduced thrust levels using the "assumed temperature" method, which will be described later in this chapter.

Other appendixes may be included in an AFM by purchase, for example an appendix authorizing takeoff and landing at airport pressure altitudes exceeding that altitude shown in Section 1 of the basic AFM. For some Boeing models, other purchased appendixes might provide performance data at alternative thrust setting levels known as "derates", or operation with ten minutes of takeoff thrust instead of the usual five. There are many other examples.

An AFM user can find out what appendixes are issued to that AFM by looking in the front of the AFM on the page entitled "Appendix Applicability". If an appendix is not shown in the appendix applicability list for the airplanes whose serial numbers are shown for that AFM, it may not legally be used for those airplanes. Also, the appendix applicability page will show which appendixes may be used at the same time, in conjunction with each other.

the Flight Crew Operations Manual.

For the 707, 727, 737-100 and the 747-100/-200/-300, the Flight Crew Operations Manual (FCOM) contains two chapters relating to performance. These two chapters are entitled "Performance Dispatch" and "Performance Inflight".

The Performance Dispatch chapter provides simplified data intended primarily for use on the ground for planning purposes prior to flight. This includes simplified presentations of takeoff and landing weights and flight planning data such as climb time, fuel and distance, cruise speeds and fuel flows, descent data, driftdown data, and more.

As its name suggests, the Performance Inflight chapter contains data intended primarily for use by pilots in flight. This includes thrust settings, takeoff speeds, cruise speeds, cruise thrust settings and fuel flow information, abnormal operation data such as driftdown and engine-inoperative cruise, and some other data.

the Flight Planning and Performance Manual

For the 737-200 through -900, the 747-400 and all later airplanes through the 777, the operators are provided with the Flight Planning and Performance Manual (FPPM) for their airplanes to assist in the calculation of performance for all segments of flight. Beginning with the 787 airplane, the FPPM is replaced with a software package called the "Performance Engineer's Tool" (PET). That will be discussed later.

The FPPM is not regulatory in nature, although there is considerable data presented in it using simplified formats for segments of flight that are controlled by regulations: takeoff and landing, the flight path following engine failure in flight ("driftdown"), reserve fuel quantities, and a few others. The FPPM is used primarily by performance engineers, as it's more detailed and complex than the FCOM performance chapters. It's intended as a planning tool, rather than for the conduct of a flight.

Many charts and tables appearing in the FPPM are simplified presentations of AFM charts or AFM-DPI data. In the process of simplifying the data, some conservatism may be necessary. For example, the charts in the FPPM for runway length limited takeoff weight, obstacle limited takeoff weight and some others are less precise and will yield slightly lower results than can be obtained by using the AFM charts. If takeoff weight is not critical, using the FPPM charts is acceptable but when takeoff weight must be maximized it will be necessary to prepare a more detailed analysis using the AFM or AFM-DPI.

In the event of any discrepancy between the FPPM and the Airplane Flight Manual, the AFM always takes precedence.

Dispatch Deviations and the Master Minimum Equipment List

The Master Minimum Equipment List (MMEL) is one of three documents dealing with dispatch deviations. The word "*dispatch*, in its broader sense, means the assignment of an airplane for a revenue operation and the preparation and delivery to the pilots of all of the information needed for that flight. The term "dispatch deviation" refers to the use in revenue service of an airplane that is not in the as-certified condition because it has one or more <u>inoperative</u> or <u>missing</u> systems or components.

The MMEL is a document published by the FAA that has regulatory authority over the use of an airplane having any system or component <u>inoperative</u>, if that system or component has relevance to airworthiness and/or the safety of flight. The MMEL is a list of items that may be inoperative for a revenue flight.

Many different items of equipment might become inoperative. They could range from something small such as a cockpit warning light to something large like an air conditioning pack. <u>Any item that is incapable of fulfilling its intended function, or is incapable of operating within normal limits, is considered to be inoperative</u>.

Some items may not have safety of flight implications, such as a coffee maker in the galley – these items are not included in the MMEL. Some items might be <u>obviously</u> necessary for safe flight, such as wings or engines, and they aren't included in the MMEL either.

For some items, it's not <u>obvious</u> whether or not they should be required to be operative for flight, and they're included in the MMEL to eliminate the chance of any uncertainty about them. Sometimes the MMEL requires them to be operative, sometimes it allows dispatch with them inoperative.

As a general rule: if an item is inoperative that might reasonably be considered necessary for the safe conduct of the flight and if that item is not included in the MMEL, then it <u>must</u> be operative for flight.

Dispatch of an airplane having a system inoperative may entail special restrictions such as altitude or speed, may require the operator to establish special maintenance and/or operations procedures for such dispatch, and might also involve special procedures for the flight crew.

It must be emphasized that there are several very important principles underlying the MMEL:

- the MMEL isn't intended to be a means for helping airlines save money on maintenance or spare parts, but rather as a way of helping them achieve good levels of dispatch reliability despite occasional component or system failures, without compromising safety.
- the MMEL will never authorize revenue service with a component or system inoperative that could conceivably compromise flight safety.

In the past, some airlines reduced their maintenance costs by continuing to operate with one or more items inoperative for extended periods of time. While that practice wasn't unsafe or technically illegal, it violated the intent of the MMEL. In order to eliminate such practices, the MMEL now imposes limitations on the period of time that an airplane may operate with a specific component or system inoperative before taking corrective maintenance action.

All candidates for inclusion in the MMEL are carefully examined for safety implications by a panel of experts including engineers, pilots, and even flight attendants. If equivalent safety can't be maintained with a particular component or system inoperative, then it won't be included in the MMEL.

For every MMEL candidate item, analysis is made of possible safety implications of failure of another component or system on a flight dispatched with that MMEL item inoperative. For example: may we dispatch with the passenger oxygen system inoperative? Yes, but only under one condition: that the cruise segment of the flight be conducted at or below 10,000 feet, at which altitude all passengers can breathe normally without supplemental oxygen. If an airplane having an inoperative passenger oxygen system were to operate <u>above</u> 10,000 feet, then one single failure – a cabin pressurization control valve failing to the full open position, for example – would leave the passengers without the necessary supplemental oxygen for the duration of the descent to a lower altitude at which it's no longer needed.

Where it's relevant to AFM-controlled performance levels, the Flight Manual may contain data pertaining to inoperative systems. For example, for most models the MMEL authorizes operation with the brake anti-skid system inoperative, provided that appropriate performance penalties are applied. The AFM contains data allowing the user to compute those penalties. Similarly, for some airplanes the AFM may contain data on operation with a wheel brake deactivated.

The MMEL as approved by the FAA is called the "master" minimum equipment list because it contains entries relating to all the different variants of a given airplane model. For example, the 737 MMEL contains information for the 737-100 on up through the 737-900. It's the responsibility of each airline to create their own MEL by customizing the MMEL to its own fleet. This is done by deleting everything that isn't applicable to its airplanes or their equipment, and by adding the operator's own procedures and guidance material as needed for operation with specified items inoperative.

An airline's customized MEL may be <u>more</u> restrictive than the master MEL if the airline wishes, but it may never be <u>less</u> restrictive.

the Configuration Deviation List

The Configuration Deviation List (CDL) is the second of the three dispatch deviations documents.

The CDL is an appendix to the Airplane Flight Manual that authorizes dispatch with certain specified airplane components physically <u>missing</u> from the airplane (as opposed to *inoperative*). For example, if a vortex generator is missing from one wing, the CDL will specify whether or not the airplane may be flown in revenue service and what, if any, penalties, restrictions, or special requirements might apply. A vortex generator, being a single piece of metal, can't be <u>inoperative</u> but it can be <u>missing</u>. Other examples might include winglets, flap track fairings, landing gear doors, and the like.

Some missing items authorized under the CDL can have MMEL implications as well. For example, if the lens covering a rotating beacon light becomes broken and must be removed, the maintenance procedure calls for removing the light bulb of the beacon light. This renders the beacon light inoperative, and the provisions of the MMEL for that inoperative item must be observed.

Since the CDL is a Flight Manual appendix, it has the full authority of law. There are no time restrictions for CDL items because the airplane with items missing that are authorized by the CDL is still considered to be in a certified configuration.

the Dispatch Deviations Guide

The Dispatch Deviations Guide (DDG) is the third of the three dispatch deviations documents. It's a Boeing document and is not FAA-approved. It's provided to operators to assist them in dispatch under the terms of the MMEL and/or the CDL.

The MMEL and the CDL don't provide everything that an operator may need to operate under their provisions. For example, many MMEL items require the operator to have in place special maintenance or operations procedures when dispatching with those items inoperative. The MMEL does <u>not</u>, however, provide those procedures. Many smaller airlines that lack the expertise to develop them can rely on the DDG to provide suggested procedures. The DDG also provides guidance on missing items named in the CDL by identifying them with drawings or photographs thus avoiding any uncertainty that might exist about the identity of the part being named in the CDL.

the Performance Engineer's Manual

The Performance Engineer's Manual (PEM) is a Boeing document, not FAA-approved. It's intended to assist in the preparation of airplane performance data by providing basic aerodynamics and propulsion parameters. For example, the PEM contains the drag polars, thrust and fuel flow data, lift curves, and other basic information.

With the proliferation of computer programs available to handle virtually all performance calculations, use of the PEM by airlines has been greatly reduced but it remains an excellent reference source of the basic data used by those programs, as well as for the occasional need to hand-calculate some sort of performance problem not handled by software.

Software

A number of computer applications are available from Boeing to assist in performance calculations. These include:

takeoff and landing software

Boeing provides its customers with software capable of computing the allowable takeoff and/or landing weights for specified conditions of altitude, temperature, runway length, and so on.

For the earlier airplanes, the software utilizes a database which consists of the Airplane Flight Manual performance charts “digitized” into computer-readable tables. For the later airplanes that are delivered with AFM-DPI as described above, the software consists of the computing “engine” that is the core of AFM-DPI, plus a database of aerodynamic, propulsion, and operational data.

Boeing also provides on request a software package called STAS. That software is a SCAP-compliant takeoff analysis application. There is a similar SCAP-compliant software application called LAND.

SCAP is the *Standard Computerized Airplane Performance Interface Specification* published by the International Air Transport Association (IATA). There are two of these, one for takeoff and another for landing. They establish sets of protocols for computerized takeoff and landing weight analysis software that may be adopted by manufacturers and airlines. The intent is to facilitate consistency between the data and software provided by different manufacturers or vendors, making it more understandable and usable.

A single SCAP-compliant software package such as Boeing’s Standard Takeoff Analysis Software (STAS) can be used to analyze the data for any manufacturer’s airplane, provided that the manufacturer’s takeoff analysis methods are also SCAP-compliant.

The SCAP specification also enables users to write computer code for printing takeoff analysis outputs in their own preferred formats if desired.

Inflight (INFLT)

This is also a first principles system, drawing on a database of basic aerodynamic, propulsion, and operational data. It enables the user to calculate performance data relating to climb, cruise, descent, driftdown, and flight planning. It works with a companion program called REPORT which enables the software to print out the data in a number of different formats.

Airplane Performance Monitoring (APM)

This software application allows the user to analyze actual airplane cruise data collected in flight to assess the fuel mileage level at which an airplane is performing relative to new airplane performance. By monitoring the airplanes in this way, an operator may be able to optimize fleet efficiency and thereby reduce fuel costs. The APM software can work with a companion program called HISTRY which facilitates the process of keeping time history records of airplane fuel mileage.

Boeing Performance Software (BPS)

This is a Windows-based graphical user interface which simplifies the task of running performance calculations on the takeoff and landing software, as well on INFLT and APM. It is an interactive program which creates the necessary input files to the user's specifications, then calls the computing engine of the appropriate program to do the calculations, then finally formats the results for viewing or printout.

Boeing Climbout Program (BCOP)

The BCOP application can analyze the performance of SIDs, STARS, go-around and engine-out procedures. For a unique airframe/engine combination and user-specified aircraft configuration, BCOP uses specific airport characteristics and user specified vertical and lateral profiles to produce three dimensional flight path information. A subset of the 70 BCOP output parameters available includes latitude, longitude, altitude, speed, climb gradient, rate of climb, time, fuel, ground track distance, and aircraft heading.

Unlike other takeoff and landing software tools, BCOP does not solve for a limiting weight; it's used to generate the flight path corresponding to the selected inputs, including weight.

Performance Engineer's Tool (PET)

The Performance Engineers Tool is a ground-based software application designed for use by airline performance engineers to conduct detailed airplane performance studies. PET is being developed to support the successful introduction and in-service operation of the 787 Dreamliner and is scheduled for initial release in March 2008. Plans are in place for PET to encompass all Boeing models soon after the 787 introduction.

Capabilities of the application include takeoff analysis, landing analysis, enroute performance, mission studies, flight path analysis, noise calculations, airplane performance monitoring, and integrated data display. As such, PET will ultimately replace the following Boeing publications and software applications:

- Boeing Performance Software (BPS)
- Boeing Climbout Program (BCOP)
- Airplane Flight Manual - Digital Performance Information (AFM-DPI) user interface
- Flight Planning and Performance Manual (FPPM) - 787 only
- Performance Engineers Manual (PEM) - 787 only

Regulatory Agencies and Regulations

previously

At the beginning of the airline jet era around 1958, the regulatory agencies and bodies of regulations were more numerous than they are now. In addition to the Federal Aviation Administration of the United States (FAA), there was the United Kingdom Civil Aviation Authority – the UKCAA – as well as the Australian CAA, the Hong Kong Civil Aviation Department (the HKCAD), the Luftfahrt-Bundesamt of Germany (the LBA), the Direction Générale de l'Aviation Civile of France (the DGAC), the Civil Aviation Bureau of Japan (the JCAB), Transport Canada, the Civil Aviation Administration of China (CAAC), and a few more.

Rather than develop their own sets of certification and operations requirements, many countries, although maintaining their own aviation regulatory authorities, accepted either the FAA or UKCAA rules. They also accepted FAA or UKCAA approval of the airplane flight manuals to be used by airlines licensed in their countries.

In the late 1980s, beginning with the 747-400, the Joint Aviation Authorities (JAA) was established, a shared operation of Great Britain, France and Germany.

By the early ‘90s, the regulatory rules had shaken down to only those of the FAA and the JAA, with some minor national deviations. By 2005, a new agency, the European Aviation Safety Administration (EASA), supplanted the JAA, assuming responsibility for all of the nations of the European Union.

at present

At the time of this writing, we can say that airlines are mostly regulated by the rules of either the FAA or the EASA. In a few instances, such as the airlines operating under the authority of Transport Canada, there are some small differences from the FAA or EASA standards.

The rules that the FAA enforces for operators under its control are called the Federal Aviation Regulations, or “FARs”. The FARs are included in Part 14, “Aeronautics and Space” of the United States *Code of Federal Regulations.* The portion of the FARs relevant to airline operations are principally FAR Part 25, “Airworthiness Standards: Transport Category Airplanes”, and FAR Part 121, “Operating Requirements: Domestic, Flag, and Supplemental Operations”.

The rules followed by airlines under the control of the EASA are known as the *Joint Aviation Requirements* (JAR). More specifically, that portion of the JAR relating to airworthiness is called JAR Part 25, and that portion relating to operations is known as “JAR-OPS”.

The JAR-OPS body of regulations includes some minor “national authority unique data”, allowing for small deviations from JAR-OPS as required by the aviation regulatory bodies of different countries for operators licensed in their countries.

Preparation For Flight

Preparation for a flight begins long before the scheduled time of departure. Some elements of flight preparation take place literally years before the flight. Other tasks such as route analysis will be needed in advance of any flight over a new route; some tasks are done on an ongoing basis, such as airplane condition monitoring, described below. Some tasks are done on a flight-by-flight basis specifically for the airplane to be used over the route to be flown, in the process known as “dispatch”.

the strategic goals of the flight

Before any specific departure can be planned and prepared for, an operator must develop its strategic goals for its flights, considering the market it plans to serve and how the airline intends to serve it.

These strategic goals will influence a number of decisions that must be made. Initially, an airline’s strategic goals will influence the decision on the airplane type or types to be purchased. What airplane will be able to carry the desired amount of payload over the expected route distances, with the greatest efficiency and passenger appeal?

A “payload-range” chart such as the one shown to the right is a useful tool in deciding the suitability of a particular airplane model to a particular operation.

This chart for the 747-400 demonstrates clearly the relationship that exists between takeoff weight, range (route length), and payload capability.

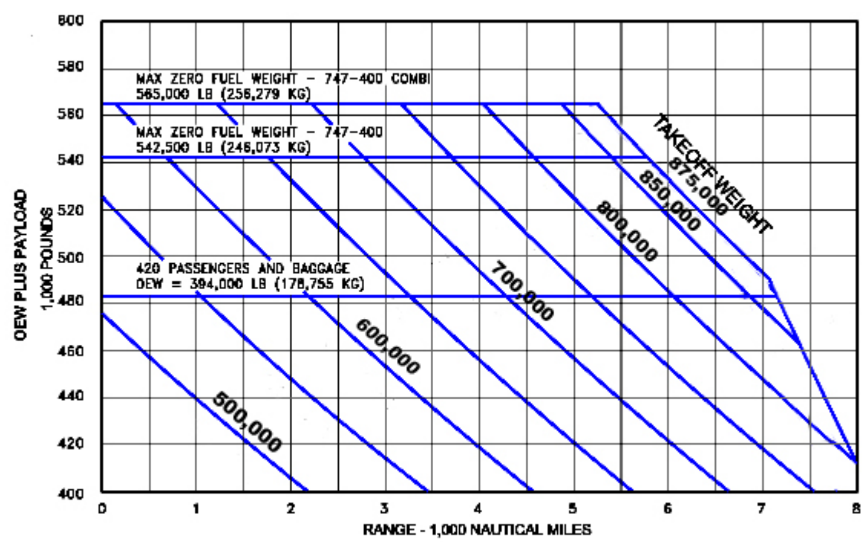


Figure 16-1

It shows that this version of the airplane can carry 420 passengers and their baggage at some assumed average weights over a route length of 7200 nautical miles, assuming a typical operating empty weight and reserve fuel, but it will need a takeoff weight capability of more than 850,000 pounds; that fact could dictate what airports and runways could or could not be used for such a flight.

The strategic goals may also influence other decisions such as the number and/or frequency of flights on a route, and the selection of the flight speeds. On a highly competitive route, for example, it might be desirable to use a high cruising speed in order to offer shorter flight times as an inducement to passengers. A high cruising speed is not the most economical speed for flight in most cases, however.

Operators need to look carefully at their cost structure if minimum cost is the goal. In order to know what speeds to fly, the operator needs to know (1) the cost of the fuel that's being used, and (2) the time-related costs of operating the flight. The choice of flight speeds depends on these two factors. Should they fly faster, to save time? Fly slower, to save fuel? Which is more desirable for this route? And how can we know the optimum speed for minimum trip cost?

In a later chapter entitled "Cost Index", we'll be looking in much more detail at this subject. A simple definition is that "cost index" is a number representing the comparison – the ratio – of the cost of *time* per hour to the cost of *fuel* per 100 pounds, both expressed in the same cost units such as dollars. It will range from zero to some maximum number depending on model. Here's a brief description:

- A cost index of zero will mean, in effect, that "time is of no importance at all, saving fuel is the only goal" in which case the flight will be operated at the most fuel-efficient speeds, regardless of the impact on the flight time;
- A maximum cost index will mean that "time is the most important goal, saving fuel is of no importance." In this case, the flight will be operated at much higher speeds, thus saving time without consideration of the adverse effect of the higher speed on fuel consumption;
- For typical operators, their cost index will lie somewhere between these two extremes, and the operator will need to conduct a careful study of the cost environment in which it operates in order to determine the cost index which applies to them. This cost index can then be provided to the dispatchers and flight crews, allowing them to plan for and conduct the flight at the most suitable speeds.

The speeds for climb, cruise and descent which yield minimum cost are referred to as ECON speeds.

route analysis

Before scheduling any airplane to follow a particular path over the ground, that path must be examined carefully. A "route" will consist of a series of geographic points of specified latitude and longitude ("waypoints"), connected by straight lines[1]. The route will begin with a specified departure path to be followed after takeoff; it will terminate with some specified approach path as the airplane nears the destination airport's landing runway.

A route may also include one or more alternate airports to which the airplane can divert in case of problems encountered enroute and to protect against the possibility that landing at the destination airport may be impossible due to adverse changes in weather or some other condition.

While the vast majority of routes are not very demanding from a performance standpoint, every new route under consideration must be examined to see if it will impose any constraints on a flight utilizing that route. These will be discussed later in this chapter.

1. More precisely, the waypoints are connected by great circle segments. For short distances, a great circle segment is virtually the same as a straight line.

airplane performance monitoring

A new airplane doesn't stay new very long. The thumps and bumps of daily operations take their toll on an airplane and its engines. As a result, an airplane will experience some increase in fuel consumption over time. Good maintenance practices will minimize that increase but can't eliminate it.

This increase in fuel consumption should be accounted for when planning each flight. Failure to account for any fuel consumption increase will mean that a flight will consume more trip fuel than planned and that extra burnoff will come out of the reserve fuel quantity.

Additionally, over time the variability in individual airplanes' fuel consumption, <u>if not tracked and accounted for</u>, can undermine confidence in the adequacy of dispatch reserve fuel quantities, possibly leading to increasing loading of additional fuel at the pilots' discretion.

Efficient airlines monitor carefully the condition of the airplanes in their fleet. This entails the collection and analysis for each airplane of data recorded during flight, a process sometimes referred to as "airplane performance monitoring". Using the results of these analyses, the airlines strive, by coordinating the performance experts with the maintenance experts, to keep the airplanes at the highest practicable fuel efficiency through good maintenance practices. The financial benefits of such a program are substantial.

The flight planning organizations should be kept informed of the fuel consumption condition of each airplane, enabling it to tailor flight planning to the exact fuel burn characteristics of the airplane to be used. This is sometimes referred to as "tail number flight planning".

Dispatch

The dispatch process involves the preparation and delivery to the pilots of all of the information needed to conduct a flight safely and legally using a specific airplane over a specific route on a specific day.

Flight Dispatchers are specially trained personnel who are qualified to prepare and compile the flight information and discuss it with the pilots. In the United States, dispatchers are required to hold an FAA Flight Dispatcher license if they are to operate in that role, and they share legal responsibility with the flight crew for the safe conduct of the flight.

Flight dispatch includes a number of responsibilities. Those responsibilities may include:

- preparation of the "flight plan", which will be discussed briefly below;
- computation of the allowable takeoff weight for the airplane considering the runway to be used for takeoff;
- computation of the allowable payload (passengers and/or cargo);
- calculation of the airplane's expected weight and center of gravity location (CG) at the time of takeoff. The dispatcher must confirm that the weight and CG are within the limitations for that airplane on that flight;

- compilation and delivery to the pilots of the weather information relevant to the takeoff airport, the destination airport, and the route to be flown;
- compilation of any specific guidance information such as Notices to Airmen (NOTAMs) which might describe inoperative navigation aids, runway closures, and other information which the pilots must be made aware of. There may also be pilot reports of enroute conditions (PIREPs) which may guide the pilots in their conduct of the flight;
- consideration of any conditions which might impose constraints on the flight.

flight constraints

Minimum Equipment List (MEL) constraints. The MEL, described previously, is the FAA-approved document which authorizes operation of an airplane having one or more components or systems inoperative, subject to specified restrictions. As an example, suppose that one air conditioning pack of a 777 is inoperative. This places two constraints on the flight: first, the cruise altitude is limited to 35,000 feet for reasons of smoke evacuation in the event of a cargo fire; second, there will be penalties imposed on the allowable takeoff weight and the enroute climb limit weight because of a change to the thrust required.

Configuration Deviation List (CDL) constraints. The CDL, also described previously in this chapter, is the appendix to the AFM which authorizes dispatch with pieces of the airplane missing, subject to restrictions in some cases. For example, if the raked wingtip of a 777 is missing, dispatch is permitted but there is a penalty on the takeoff weight, and the flight plan fuel load must be increased to compensate for an increase in the airplane's drag.

Enroute weather constraints. Weather conditions along the route of flight might necessitate, for example, a detour around a known area of turbulence. This could affect the flight's time and fuel load. Also, the anticipated weather conditions at the destination airport may place a constraint on a flight depending on the equipment in the airplane and the pilot qualifications for approaches and landings in marginal conditions of ceiling and visibility.

Weather often plays a role in ATC delays, and drives requirements for reserve and/or alternate fuel.

the flight plan

Performance-related activity will begin usually one to two hours before flight. Some activity such as payload planning may begin even earlier. The principal performance activity will be the preparation of the "flight plan" for the upcoming flight. The flight plan is a document required for dispatch that provides the following information:

- the exact route of the flight, waypoint by waypoint;
- the altitudes and speeds for the flight;
- the expected total time and fuel consumed for the trip;
- the winds and temperatures anticipated along each leg of the flight;
- the expected flight time to each waypoint;

- the anticipated fuel remaining at each waypoint;
- the route, time, and fuel to the alternate airport(s)

A copy of the flight plan is filed with Air Traffic Control. Before departure, the pilots obtain clearance to fly the route “as filed” or with necessary amendments from ATC. This is required for operations under FAR Part 121.

The flight plan must account for the anticipated enroute conditions of weather and possible delays or any dispatch deviations which would affect the planned fuel load. This requires the dispatch organization to work in close coordination with the meteorology, maintenance, and air traffic control personnel so that the flight plan presented to the crew will be an accurate representation of the flight they’re about to conduct.

The flight plan fuel load will include some additional “reserve” fuel as required by the relevant regulations and any specific company policies. At a minimum, the reserve fuel usually allows for an approach and missed approach at the destination airport followed by flight from the destination airport to the alternate airport; it will include some allowance for variations in enroute conditions, such as winds greater than forecast; it may include an allowance for some period of time in a holding pattern at the destination airport. By FAA rules, the reserve fuel requirements are different for domestic and international flights, but the International Civil Aviation Organization (ICAO) rules don’t make that distinction.

The flight plan is used by the crew throughout the flight. It provides them with all the navigation data necessary for following the route; it provides them with flight progress information such as the planned time and fuel to each waypoint, so that they can identify any substantial deviation from the plan; it also provides them with information on the alternate airports which may be used in case of some abnormal situation.

Release For Flight

A flight is “released” when all dispatch conditions described above have been met and the pilots have accepted all of the data. They will have been briefed on the conditions of the flight, weather, and any noteworthy items such as PIREPs and NOTAMs. They will have been provided with the flight plan.

They will have been informed of the maintenance status of the airplane, particularly any dispatch deviations which could have implications on crew workload or special procedures to be followed.

They will have been informed of the anticipated takeoff weight and the airplane center of gravity position. They will have confirmed that the anticipated weight for both takeoff and landing will be within the structural and performance limitations, and that the center of gravity is within its operational limits.

Under FAA regulations, both the captain and the dispatcher must formally sign the flight release, indicating that both agree that the flight can be conducted safely and legally.

Takeoff

Takeoff and landing are by far the most complex parts of a flight. Takeoff has the greatest number of variables that must be considered. It demands a high level of attention to detail throughout the procedure and can be very unforgiving of flight crew error.

the environmental envelope for takeoff

The ranges of allowable altitude and temperature for takeoff and landing are subject to limitations that are together known as the *environmental envelope*. The “envelope” is a set of altitude and temperature limits plotted on axes of pressure altitude versus temperature. All flights must be conducted within the temperature/altitude boundaries shown on the chart.

A typical environmental envelope is shown to the right.

You’ll see that the altitude limitations for takeoff and landing (the shaded area) for this airplane are shown as 8400 feet as the upper limit and minus 1000 feet as the lower limit.

The temperature limits for takeoff and landing are ISA+39.4 °C up to 54 °C as a maximum, and -54 °C as the minimum. Other airplane models may have different limits.

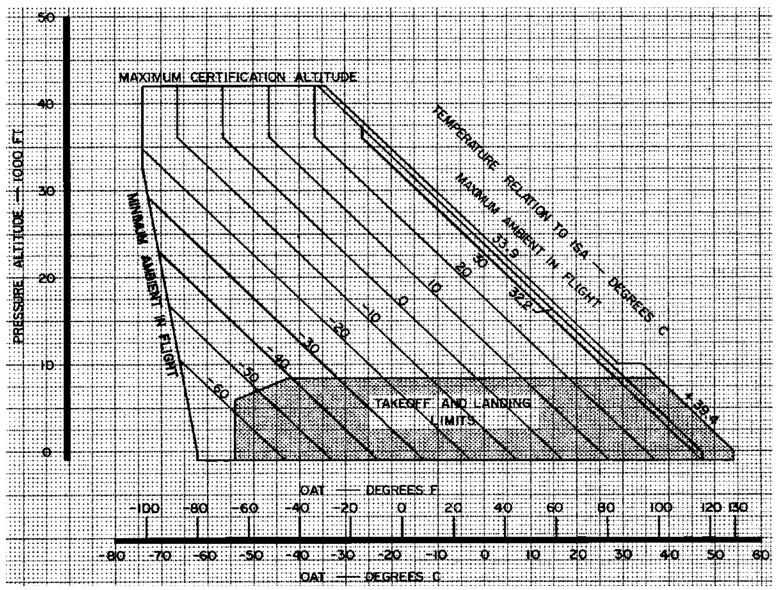


Figure 16-2

For most Boeing airplanes, Airplane Fight Manual appendixes are available that extend the upper altitude limit for airlines desiring to operate into airports located at elevations above the usual limit.

the allowable takeoff weight

Takeoff is the topic which by itself represents the greatest amount of workload a performance engineer is likely to have. When routes are shorter and runways are longer, time spent trying to maximize the allowable takeoff weight may have no real value. On the other hand, under more demanding conditions every pound of allowable takeoff weight increase will yield almost one more pound of payload that can be carried (think of the payload-range chart, for a constant range). This can make the difference between a profitable flight and an unprofitable one.

The allowable weight for any given takeoff is a function of:

- pressure altitude,
- temperature,

- wind velocity and direction,
- runway length,
- clearway and stopway,
- runway slope,
- runway condition (i.e. dry, wet, icy, or contaminated with slush or standing water),
- pavement strength,
- obstacle heights and distances, if any,
- the use of or requirement for an engine-inoperative turn procedure after takeoff,
- engine bleed configuration,
- flap setting,
- airplane condition (MEL or CDL items),
- speed limit of the tires on the airplane,
- type of wheel brakes on the airplane,
- specialized takeoff techniques such as improved climb.

For the given takeoff conditions, the allowable takeoff weight will be the smallest of:

- the certified limit weight,
- the field length-limited weight,
- the climb-limited weight,
- the obstacle-limited weight including the effects of any turn procedures,
- the tire speed-limited weight,
- the brake energy-limited weight,
- the “return-to-land” limit weight,
- the landing-limited weight,
- the enroute limit weight,
- the pavement strength limit weight, where appropriate,
- the noise limit weight, where applicable.

Let’s look briefly at each of these in turn. They’ll all be discussed in detail in following chapters.

<u>The certified weight limit</u>. This is the maximum value specified in Section 1, the limitations section, of the AFM. In some cases it is a true structural limit, in other cases it might be less than the structural capability of the airplane, but either way it’s still the maximum weight allowed for takeoff under any circumstances.

The field length weight limit. This is the maximum weight that will allow the airplane to meet all four of the following requirements:

- The distance for takeoff to a height of 35 feet with all engines operating, multiplied by a factor of 1.15, must not exceed the takeoff distance available;
- The distance for takeoff to a height of 35 feet allowing for an engine failure, assuming that the engine failure occurs at speed V_{EF} one second before V_1, must not exceed the takeoff distance available;
- The distance to accelerate to V_1 and then decelerate to a full stop, with all engines operating[1], assuming that the brakes are applied two seconds after reaching V_1, must not exceed the accelerate-stop distance available.
- The distance to accelerate to V_{EF} at which an engine is assumed to fail, and then to decelerate to a full stop assuming brake application beginning two seconds after V1, must not exceed the accelerate-stop distance available.

Aside from the weather conditions at the time of takeoff, the field length limit weight will primarily be influenced by the characteristics of the runway to be used: its length, its “clearway” and “stopway” if any (these will be defined in detail in a later chapter), its slope, and its condition, whether dry, wet, or “contaminated” – that is, covered with ice, snow, slush, or standing water.

The climb weight limit. FAR Part 25 mandates minimum levels of performance in climb after takeoff, defining a number of climb “segments” and the corresponding minimum climb gradients in still air required for each segment. Those minima depend on the number of engines on the airplane.

The “climb limit weight” is the weight which will allow the airplane to meet, ***with one engine inoperative***, the specified minimum climb gradients. In most instances, the “second segment” is the most restrictive. For a four-engine airplane, the minimum second segment gradient is 3%; it’s 2.7% for a three-engine airplane, and 2.4% for a two-engine airplane. Second segment begins at the point at which the landing gear is fully retracted after takeoff and continues to the point at which the airplane levels off to accelerate and retract the flaps. The calculation of second segment gradient is done at the altitude and temperature existing at the gear-up point, even though the gradient will decrease slightly beyond that point as the airplane’s altitude increases.

This weight limit must not be confused with the obstacle limit weight. It is simply the weight which will allow the airplane to meet the minimum climb gradient required with one engine inoperative, regardless of whether obstacles are present or not. The climb gradient requirement is

1. This requirement for a rejected takeoff due to an event other than engine failure came into the regulations relatively recently, in realistic recognition of the fact that although rejected takeoffs are uncommon, the cause for many of them is something other than engine failure. Since a rejected takeoff with all engines operating requires more distance than a rejected takeoff with a failed engine, this new requirement is more conservative – but not unreasonably so. Examples of reasons for a rejected takeoff other than engine failure might include the failure of a tire or a cockpit warning of some system malfunction, among others.

really a means for mandating a margin of airplane performance capability with an engine inoperative, rather than a requirement for a specific flight path angle.

<u>The obstacle weight limit</u>. If obstacles such as rising terrain, trees, buildings, towers or the like are present and are located within a specified obstacle clearance area beyond the end of the takeoff runway, the weight of the airplane must allow its climb path after takeoff, *with an engine inoperative,* to clear all of those obstacles by a specified margin of height.

Some parameters of the obstacle clearance flight path may be optimized to achieve the greatest possible obstacle-limited weight. For example, the altitude for flap retraction and acceleration may be modified depending on the location and the height of the obstacles.

In many instances, an obstacle-clearance flight path will include a turn procedure, either to avoid an obstacle or to place the flight path over the least restrictive obstacles. The angle of bank in the turn must be accounted for in the climb gradient calculation, since the additional drag in the turn will decrease the available gradient.

Obstacle clearance limits and different scenarios for obstacle clearance will be discussed in greater detail in a later chapter.

<u>The tire speed weight limit</u>. An airplane tire experiences centrifugal force within the tire body that increases with the wheel's rotation rate as an airplane accelerates for takeoff. The structural design of the tire limits it to some maximum rolling speed on the ground, and this limit speed is provided by the tire manufacturer. Exceeding the tire's speed restriction could result in failure of the tire during takeoff or landing, with possible safety implications. The highest speed limit for tires in present-day use is 235 statute miles per hour, or 204 knots.

A tire's greatest rotation rate occurs at the point of liftoff from the runway. Liftoff speeds can be very fast, particularly at high weights and small flap settings, and under high altitude hot day conditions in which the airplane's true speed is considerably greater than its indicated speed. The airplane's weight must be such that liftoff will occur before its ground speed reaches the limit value. This requirement can impose a weight limitation on the takeoff.

<u>The brake energy weight limit</u>. Airplane brakes perform their intended function by turning an airplane's kinetic energy into heat energy. This heat results from the friction that is created when brakes are applied to retard an airplane's speed on the ground. Should that heat energy exceed a safe level, brake and tire failure are possible, and landing gear fires can occur.

For this reason, it isn't permitted to make a rejected takeoff under conditions in which the brake application speed would exceed a safe value, called V_{MBE}, the speed for maximum brake energy.

As in the case of tire speed limits, discussed above, takeoff or landing conditions in which an airplane's rolling speed on the ground will be greater (higher altitudes, higher temperatures, tailwinds, lower flap angles) could necessitate restrictions on the airplane's takeoff weight.

The takeoff weight limited by landing. It is very possible, particularly on shorter flights, to take off at a weight great enough that the weight of the airplane, when arriving at the destination, could exceed the allowable landing weight. This must necessarily be avoided.

The takeoff weight limited by landing will be simply the maximum landing weight at the destination airport plus the anticipated fuel burnoff between takeoff and landing.

The return-to-land limit weight. For the McDonnell-Douglas DC-9, MD-80 series and MD-90 series, and the Boeing 717, in order to avoid the need for a fuel dump system it's required to check that the takeoff weight does not exceed a value which will allow the airplane to take off, return to the departure airport for landing within 15 minutes, and meet the approach and landing climb gradient requirements. For all Boeing models other than the 717, Boeing satisfactorily demonstrated to the FAA that a weight restriction was not necessary.

The takeoff weight limited by enroute restrictions. As mentioned earlier, an airplane's weight along its intended path of flight may be restricted by considerations of driftdown or loss of pressurization, depending on the height of the terrain along the route.

In such cases, it may be necessary to restrict the allowable takeoff weight so that the airplane's inflight weight at all points along the route, allowing for the fuel consumed after takeoff, complies with these restrictions.

The weight limit due to runway pavement restrictions. Some runways, because of their construction parameters such as pavement thickness and sub-base load-bearing strength, may impose a constraint on the weight of an airplane using it.

One system in use to designate the load-bearing strength of a runway is known as the ACN/PCN system. The pavement's strength is denoted by its Pavement Classification Number, or PCN. The load exerted on a pavement by the landing gear of an airplane is denoted as its ACN, or Airplane Classification Number. The ACN is not permitted to exceed the PCN of the runway to be used, in order to prolong pavement life and prevent possible pavement damage.

The ACN of an airplane is a function of not only its weight but also the design parameters of its landing gear such as the distances between the wheels of a multiple-wheel landing gear assembly.

The takeoff weight limited by noise restrictions. Many airports are located in urban or suburban areas that are sensitive to the noise emitted by airplanes on takeoff or approach and that impose restrictions on the allowable noise emissions.

The amount of airplane-generated noise perceived by a person on the ground is a function of the person's distance from the airplane and his location relative to the departure path of the airplane, whether beneath it or to the side of it. Heavier airplanes will be perceived as noisier since they will be at a lower altitude along the departure path. Thus noise restrictions may necessitate special approach or departure procedures for noise abatement and, in some instances, will restrict the allowable takeoff weight.

thrust for takeoff

A number of options exist relating to the amount of engine thrust used for takeoff.

Full takeoff thrust. The performance-related takeoff weight limitations such as the field length limit weight and the climb limit weight are predicated on the use of full rated thrust for takeoff. The maximum allowable takeoff thrust is specified in the Airplane Flight Manual.

When an airplane will be taking off at the performance-limited weight found from the AFM or AFM-DPI, it will be necessary to use the maximum allowable thrust for that takeoff so that it will achieve its certified performance. However, ***when the actual takeoff weight will be less than its performance-limited value, it is permitted to use less than the maximum thrust for takeoff.***

Why would an airline want to use such a procedure? Simply because engines that are operated at less than the maximum allowable thrust will experience less deterioration and thus will cost less money to maintain in the long run. Engine reliability may also be improved through a program of thrust reduction.

There are two methods certified for use when the takeoff weight permits reduced thrust. These will be discussed in detail in a later chapter entitled “Reduced Thrust for Takeoff”. Briefly, they are:

Reduced takeoff thrust using the Assumed Temperature Method. Performance-limited takeoff weights are a function of temperature. This is the case simply because engine thrust depends on the air density and the allowable thrust setting, both of which are functions of temperature. Also, the airplane’s aerodynamic forces depend on the air density.

When the actual takeoff weight will be less than the performance-limited value, the AFM permits the pilots to use a level of thrust for takeoff which is less than the full rated takeoff thrust level. This is done by determining the takeoff temperature *at which the performance-limited weight would just equal the expected takeoff weight,* and then using the takeoff thrust setting appropriate to that “assumed temperature”.

The assumed temperature method is inherently somewhat conservative, and it has full regulatory approval.

“Derates”. Engine manufacturers determine and publish the maximum allowable thrust settings which can be permitted without exceeding any of the engine’s limitations. Derates are discussed in a previous chapter entitled “Thrust Ratings”. Derates are also discussed in the chapter entitled “Reduced Thrust For Takeoff”.These maximum allowable thrust settings are certified and published in the Airplane Flight Manual thus imparting to them the force of law.

However, engine manufacturers may also publish for any given engine a “derate” thrust level. A derate thrust level is, in effect, another certified thrust rating which is somewhat less than its maximum capability. Thus, an engine may be used to make a takeoff at its maximum takeoff thrust or at one of its derates, depending on the takeoff weight.

Derate thrust ratings are also certified and the derate thrust settings are published in the Flight Manual.

May derates and the assumed temperature method be combined? Yes, when the anticipated takeoff weight permits, it's allowed to apply the assumed temperature method to a derated thrust takeoff analysis.

takeoff speeds

In order to achieve the certified levels of takeoff performance, it is essential that the flight crew use great care to compute and use the correct speeds for the takeoff. It's no exaggeration to say that failure to observe the correct speeds for takeoff can result in a catastrophic accident. This has indeed happened a number of times.

There are three primary speeds which are used for the takeoff, and they are referred to as V_1, called "V-one", V_R, called "V-R" and V_2, called "V-two". These will be discussed in detail in an upcoming chapter, but here's what they are in simple terms:

$\mathbf{V_1}$ has historically been known as the "decision speed". This is a misnomer, as V_1 should be more properly called something like "the rejected takeoff action speed". The current FAA definition of V_1 says, in part: "V_1 means the maximum speed in the takeoff at which the pilot must take the first action (e.g., apply brakes, reduce thrust, deploy speed brakes) to stop the airplane within the accelerate-stop distance."

Initiating a rejected takeoff after passing V_1 may result in an over-run incident or accident with possible airplane damage or destruction and injury or loss of life.

Pilots should always observe the published V_1 for the given takeoff conditions. It's true that an RTO is possible from some speed greater than V_1 when an airplane is taking off at less than the field length limit weight, but unless the pilot has specific information authorizing that, any exceedance of V_1 is potentially hazardous.

$\mathbf{V_R}$ is the "rotation speed" at which the airplane's pitch attitude will be increased by the pilot from its on-ground "three-point" attitude to an angle which is sufficient to create the lift needed for takeoff. Smooth rotation initiated at V_R and conducted at the recommended rotation rate will result in the proper transition to the initial climb segment of the takeoff.

$\mathbf{V_2}$ is sometimes referred to as the "takeoff safety speed". More properly: *V_2 is the speed that will be attained by the airplane at a height of 35 feet following rotation initiated at V_R with one engine inoperative*. In the case of an engine-inoperative takeoff, the pilot will maintain V_2 until ready to accelerate and retract the flaps.

These takeoff speeds are functions of weight, pressure altitude, temperature, and flap setting. Under some conditions, the takeoff speeds may be increased above their baseline values in order to achieve climb or obstacle clearance performance benefits. This is a takeoff technique called "improved climb". Improved climb will be discussed in a later chapter.

For a normal all-engine takeoff (rather than the conservative assumption of takeoff with an engine failure, which is the basis for the certified weight limits) initiating rotation at V_R will result in an airspeed faster than V_2 when passing through 35 feet after liftoff. The difference will be approximately 10 to 20 knots for a four-engine airplane, or 15 to 25 knots for a two-engine airplane.

takeoff stabilizer trim

Beginning with the takeoff, and continuing throughout the flight, an airplane's center of gravity will be constantly changing as the fuel is consumed and passengers and crew move around the airplane. Also, throughout the flight the distribution of the aerodynamic forces on an airplane will vary depending on speed, weight, center of gravity location, flap setting, and other factors. The continual variation of these two parameters and the ranges over which they vary means that the airplane's flight controls must be capable of handling a broad range of pitch characteristics.

As discussed in the chapter entitled "Aerodynamic Devices" in which we described the pitch controls – that is, the elevators and the stabilizer trim system – we said that the stabilizer trim system is used to keep the airplane "in trim". Being *in trim* is important for two reasons: first, it eliminates the need for the pilot to be constantly exerting force on the flight controls, and second, it reduces the drag increase that results when the elevators are deflected from their faired position.

It is particularly important that the airplane be in trim for takeoff. It's desirable that the control forces that the pilot making the takeoff must apply to the flight controls should be approximately the same for every takeoff, so the pilot knows what to expect and how to control the airplane consistently. It's also desirable that the forces on the control column should be very small during the initial climbout after liftoff so that if control force is inadvertently reduced or released the airplane will not suddenly and unexpectedly pitch nose up or nose down.

As part of the flight release process, the pilots will be provided with the computed takeoff weight and center of gravity location. This allows them to determine the proper setting of the stabilizer trim for takeoff. Following the takeoff, as the pitch characteristics change, the necessary amount of pitch trim will change.

techniques for increasing takeoff weight

When the allowable weight for a given takeoff is somewhat less than desired, there are several techniques that may, conditions permitting, be utilized to increase it.

"Improved climb", When the takeoff weight is limited by the second segment requirement or obstacles, this technique may be utilized. It is published in the AFM as an approved method for increasing the climb limit weight.

When the takeoff weight will be less than the field length-limited value, it is possible to increase the takeoff speeds V_1, V_R and V_2 above their baseline values without causing the takeoff or RTO to exceed the available distances. For reasons that will be seen in a later chapter, increasing the takeoff speeds will improve the climb-limited takeoff weight and, depending on the location of a critical obstacle, if any, may improve an obstacle-limited weight.

In effect, improved climb is simply a way of exchanging surplus runway length for a better climb capability.

<u>Alternate forward CG</u>. Takeoff performance in either the AFM charts or AFM-DPI is based on the assumption that the airplane's takeoff center of gravity will be at its forward limit. For performance, a CG at its most forward position is the most adverse case, therefore this assumption will be conservative whenever the CG is actually farther aft.

In a forward CG condition, the airplane's horizontal tail must generate a greater amount of downward force, when compared to a more aft CG, in order to keep the airplane aerodynamically balanced. The wings must then produce a greater amount of lift in order to offset the greater download on the horizontal tail. Greater wing lift requires greater speeds and thus faster takeoff speeds; it also means greater aerodynamic drag. A more aft CG means less wing lift, lower speeds and less drag. These will offer improvements to the field length limit weight and, in some but not all instances, to the climb limit weight.

Provided that it will ensure that the takeoff CG will be kept aft of the forward limit, an airline may be permitted to take credit for the performance improvements. Data for these performance benefits is available for purchase in the form of an AFM appendix or inclusion in AFM-DPI. This subject is discussed at length in the chapter entitled "Alternate Forward CG".

takeoff on wet or contaminated runways

A runway is called *wet* when a quantity of rain has fallen that exceeds the ability of the pavement macrostructure to absorb it. There is therefore some depth of water, however small, standing on the surface of the pavement. The pavement will show reflections. However, the term "wet" is used to refer only to conditions of standing water having a depth of three millimeters or less.

A runway is called *contaminated* when it is covered over more than 25 percent of its surface area with ice or compact snow, or more than three millimeters of standing water or slush,

Takeoff and landing are prohibited when the slush or standing water depth exceeds 0.5 inches (FAR rules) or 15 millimeters (JAR rules).

In general, contamination may cause a loss of acceleration due to increased drag, and may also cause a loss of deceleration when stopping, due to a reduction of brake effectiveness. Wet or icy runways, or runways contaminated with compact snow, will cause a loss of brake effectiveness without causing an increase in drag. The end result is an increase to stopping distances without any change to acceleration distances.

Runways contaminated with standing water or slush, on the other hand, will experience both increased drag and loss of brake effectiveness. Both the acceleration and stopping distances will be affected.

The effects of standing water or slush are particularly complex. The amount of drag increase and the amount of brake effectiveness reduction depend on the airplane's speed and the depth of the contaminant. Further, when the airplane's speed is high, the contamination will have the effect of lifting the tire completely off the runway surface, reducing the tire-to-ground coefficient of friction to almost zero. This is referred to as "dynamic hydroplaning".

Because the effects described above may alter the balance of "go" performance relative to "stop" performance, adjustments may be necessary to the runway length-limited weight and the V_1 speed when taking off from a contaminated runway.

Contaminated runway performance will be discussed in detail in a later chapter.

Climb To Cruise

Once clear of all obstacles and in a "clean" configuration (landing gear and flaps retracted), the airplane will accelerate to its initial enroute climb speed and initiate its departure climb. In the absence of any air traffic control restraints, the climb will be a continuous one from the departure altitude right on up to the assigned cruise altitude.

climb thrust setting

During enroute climb, the engines may be set to the maximum climb thrust setting, MCLT. Since there is no time limit on the use of maximum climb thrust, it will normally be maintained all the way to the cruise altitude. You'll recall that MCLT is not a thrust level used in the calculation of performance levels that are controlled by regulatory requirements, and thus it doesn't appear in the AFM.

When possible, many operators use derated climb thrust settings provided by the engine manufacturers, for the same reason as for using thrust reduction for takeoff: improvements in maintenance cost and reliability. Reduced enroute climb thrust has its greatest benefits at the lower altitudes, so some airlines may use reduced climb thrust at the lower altitudes, transitioning to full climb thrust at the higher altitudes. The thrust management function of flight management computers typically include a "washout" feature that restores full climb thrust following a specific thrust-versus-altitude schedule during the climb.

climb speed

There are a number of different possible enroute climb speeds. Some of them are for general planning, and others might be required for specific reasons.

Under the heading of planning:

- <u>Speed for a cost index of 0</u>. We discussed earlier in this chapter the concept of cost index as a measure of the relative value of time and fuel. Climbing at the speed corresponding to a cost index of zero would yield the climb time, fuel and distance for minimum trip *fuel consumption*, assuming that zero is also used to compute the cruise and descent speeds;

- Speed for the maximum cost index. This is the opposite case, in which the climb speeds are being used to provide minimum trip *time*, assuming that the maximum index is also used to compute the cruise and descent speeds. There will be a substantial increase in the trip fuel burnoff at these speeds;
- Speed for a specified intermediate cost index. When the cost index is computed for the airline's time and fuel cost structure, climb at this speed will result in the minimum overall trip *cost*, assuming that the same index is also used to compute the cruise and descent speeds.

Special climb speeds that might be required on occasion:

- Speed for maximum angle of climb. Air Traffic Control (ATC) constraints or enroute obstacles might make it necessary to maximize the enroute climb path angle. For example, this might be the case if ATC requested the airplane to be at or above some specified altitude at a down-route waypoint. In such a case, the steepest possible climb path is the goal. This speed is sometimes referred to as V_x.
- Speed for maximum rate of climb. If it becomes necessary for the airplane to minimize the time it will require to reach the cruising altitude, this speed could be used. It is sometimes referred to as V_y. For aerodynamic reasons, the speed for maximum angle of climb is different from, slower than, the speed for the maximum rate of climb.
- Speed for a specific waypoint time constraint. Under some ATC conditions requiring the airplane to cross a specified waypoint at a specified time, a function known as Required Time of Arrival, or RTA, it might be necessary to adjust the climb speed and possibly the cruise speed in order to comply with that constraint.

In the absence of ongoing precision climb speed guidance from a flight management computer, a flight crew may elect to control the airplane's pitch attitude – and therefore its speed – to follow a simplified climb speed schedule. Such a climb speed schedule is customarily shown as a constant calibrated airspeed after takeoff, up to an altitude (the "crossover altitude") at which the CAS becomes equivalent to a selected Mach number, and thereafter to adjust pitch to maintain that Mach number until reaching the cruise altitude.

Such a speed schedule would be shown as, for example, **280/.8**, meaning that the crew should climb at 280 knots calibrated airspeed until that speed becomes equal to Mach 0.8, and thereafter maintain Mach 0.8 until reaching the cruise altitude.

In some instances, a climb speed schedule might be, for example, **250/280/.8**. This says that 250 knots should be maintained below 10,000 feet (or some other locally-defined altitude), then 280 knots CAS, then Mach 0.8. The 250-knot maximum target speed is a regulatory requirement based on the concept that airplanes flying at slower speeds in the lower (more crowded) airspace regions will be better able to avoid a mid-air collision because the slower speed affords the pilots and air traffic controllers more time to react after identifying a potential collision threat.

Cruise

After the enroute climb to the assigned cruise altitude is complete, the airplane will level off and begin cruise along the route toward the destination. Except for short flights, cruise is the longest part of a flight and there are substantial benefits for the airline that selects the cruise altitudes and cruise speeds carefully.

cruise altitude

There are a number of different elements to the selection of the best cruise altitude:

- the altitudes that are available from Air Traffic Control
- the altitude for maximum air fuel mileage (ignoring the effects of winds aloft)
- the altitude for minimum trip fuel consumption (considering winds aloft)
- the altitude for minimum trip cost
- the altitude capability of the airplane - the “thrust-limited” altitude
- the altitudes that will provide acceptable buffet protection, that is, the ability to experience load factors of satisfactory magnitude without incurring buffet

Let’s look at each of these in turn.

<u>Altitudes that are available from ATC</u>. Depending on the general direction of the flight and the route that is to be followed, ATC will make available only certain altitudes. The dispatcher must choose from among these altitudes the one he believes will best meet the other criteria.

The altitudes that are available will, in some areas, depend on the airplane that will be flown on the route. Airplanes which meet the requirements of the Reduced Vertical Separation Minimum (RVSM) standards have a greater choice of altitudes. Let’s talk about that for a moment.

RVSM is a relatively recent development, which came about because of the increased accuracy of altimeters and autopilot altitude-keeping capability seen in more recent airplane models. In view of these improvements, and in light of increasing airspace congestion, reducing the vertical separation between airplanes in flight was seen to be feasible and also desirable because it would allow more airplanes to operate within a given airspace environment.

Most of the world’s major air routes are now designated for RVSM operations, but there still remain some routes that are not.

In airspace not designated for RVSM operations, airplanes flying in the opposite directions along the same route above 29000 feet are required to keep a minimum of 2000 feet of vertical spacing between them. Airplanes flying in the same direction along a route are therefore separated by 4000 feet. The available altitudes eastbound are 29000, 33000, 37000 and 41000 feet. The westbound altitudes are 31000, 35000 and 39000 feet.

RVSM reduces the vertical separation between opposite-direction tracks from 2000 to 1000 feet, and separations between same-direction tracks from 4000 to 2000 feet. It makes six additional altitudes above 29000 feet available for operation. The additional altitudes enable more aircraft to fly more time/fuel efficient profiles and provide the potential for enhanced airspace capacity.

RVSM operators must receive authorization from the appropriate civil aviation authority. RVSM airplanes must meet required equipment and altitude-keeping performance standards. Operators must operate in accordance with RVSM policies/procedures applicable to the airspace where they are flying. Airplanes that don't meet the RVSM requirements must fly below or above the RVSM airspace, or seek special exemption from the requirements. Non-RVSM airplanes forced to fly below the altitudes reserved for RVSM airplanes may incur a substantial fuel burn penalty for doing so.

Altitude for maximum fuel mileage. Cruise fuel consumption depends on altitude. Rather than look at just how much fuel is burned per hour, though, it's necessary to do the same thing that we do in our cars: we look at *fuel mileage*. Let's discuss that topic for a moment.

Fuel mileage for an automobile can be defined as:

$$\textit{fuel mileage} = \frac{\textit{distance driven}}{\textit{quantity of fuel consumed}}$$

If, for example, an automobile can travel 100 miles while consuming only four gallons of fuel, that's a fuel mileage of 25 miles per gallon. It follows that the fuel used to drive a car over a specified distance is equal to that distance divided by the car's fuel mileage. For example, 200 miles at 25 miles per gallon means eight gallons will be consumed.

For an airplane, fuel mileage is almost the same, except in aviation work we measure fuel mileage this way:

$$\textit{fuel mileage} = \frac{\textit{nautical air miles flown}}{\textit{quantity of fuel consumed}}$$

Notice that the numerator of that term is nautical AIR miles (NAM) flown. The fuel mileage thus computed is referred to as *still air fuel mileage*. The importance of this distinction will be made clear soon.

If you follow through on this equation, it turns out that:

$$\textit{fuel mileage} = \frac{\textit{nautical air miles}}{\textit{hour}} \times \frac{\textit{hour}}{\textit{fuel consumed}} = \frac{\textit{true airspeed}}{\textit{total fuel flow rate}}$$

At lower cruise altitudes, fuel mileage is relatively poor because the fuel flow rate is relatively high. As the cruise altitude increases, the fuel mileage will increase because of decreasing fuel flow rates – up to a point. Above that point, the fuel mileage will begin to decrease. The altitude which offers the highest fuel mileage for the given weight and speed is referred to as the "optimum altitude". Flight at any altitude above or below optimum will result in a fuel consumption penalty (assuming little or no difference in the winds between altitudes. See the following discussion on altitude selection for minimum trip fuel consumption).

To illustrate briefly, taking a 747-400 at 800,000 pounds cruising at Mach 0.86: at 29,000 feet the true airspeed (TAS) is 509 knots and the fuel flow per engine is 7138 pounds per hour (fuel mileage 17.8 NAM per 1000 pounds of fuel); at 31,000 feet the TAS is 505 knots and the fuel flow is 7038 pounds per hour per engine (17.9 NAM per 1000 pounds of fuel) and at 33,000 feet the TAS is 500 knots and the fuel flow is 7114 pounds per hour per engine (17.6 NAM per 1000 pounds of fuel). The optimum altitude would be slightly under 31,000 feet.

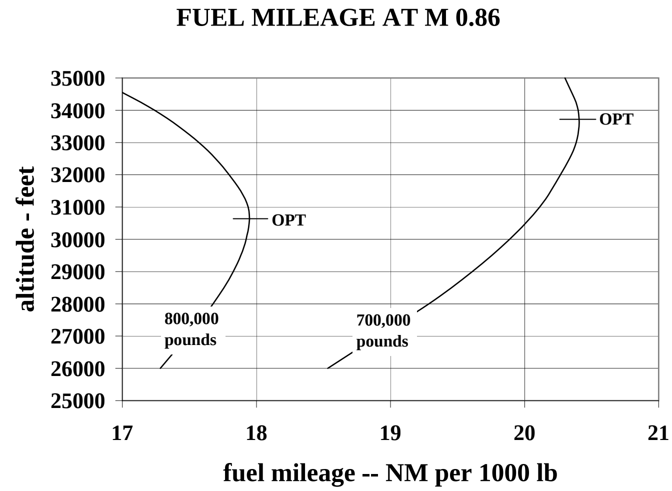


Figure 16-3

Optimum altitude depends on weight. At heavier weights, optimum altitude will be lower, and it will be higher for lighter weights. In the illustration above, you can see that for a lighter weight of 700,000 pounds, the fuel mileage is much better, and the optimum altitude is higher – at this weight, almost 34,000 feet. For that reason, on long flights an airplane will usually increase its cruise altitude one or more times in an attempt to stay close to the optimum altitude as it increases due to the weight decrease resulting from fuel burnoff. This is a procedure known as "step climb" since the flight vertical profile then looks something like a flight of stairs.

Altitude for minimum fuel consumption. One might think that the altitude which offers the best still air fuel mileage – optimum altitude – would also be the altitude that offers the minimum fuel consumption. On closer examination, though, we can see that this might or might not be the case. The variable that makes the difference is the enroute wind that the airplane is experiencing in cruise.

Winds can vary greatly between different cruise altitudes. The most noteworthy example of this is the wind speed variation experienced in the area of a *jet stream*.

A jet stream, of which there are a number existing over the earth's surface, is a high-velocity current of air flowing through the surrounding lower-velocity air surrounding it. It can be likened to a river of air flowing rapidly through the atmosphere.

Jet streams usually occur at higher altitudes, typically near the tropopause at approximately 36,000 feet, although at some times they can be encountered at lower altitudes. Jet stream velocities in excess of 200 knots have been measured.

Because of jet streams, velocity differences between altitudes in excess of one hundred knots are commonplace. Significant velocity differences can <u>also</u> occur for other reasons and in locations other than jet stream areas.

How do wind differences between altitudes affect the airplane's fuel consumption? Heres's an example: let's say that you're cruising at optimum altitude, expecting that your fuel mileage would be at its minimum. But let's say that at optimum altitude you're experiencing a 50-knot headwind, while airplanes cruising 4000 feet lower are experiencing only 25-knot headwinds.

Yes, you're getting the best possible nautical AIR miles per pound – fuel mileage – at optimum altitude, but you're losing out on ground speed, hence your flying time over a specified distance will be longer. What you should really be considering is how many GROUND miles per pound of fuel you're achieving. Just as NAM per pound is equal to true airspeed divided by fuel flow, ground miles per pound is equal to ground speed divided by fuel flow. The fuel flow doesn't change as a function of wind: for a given altitude, speed and weight that's a constant. But the ground speed <u>does</u> change.

Going back to the fuel mileage chart we showed above: let's say our weight is 800,000 pounds and we're cruising at Mach 0.86. At 31,000 feet cruise altitude (FL310) we said that the true airspeed is 505 knots and the fuel flow is 7038 pounds per hour per engine. FL310 is approximately the optimum altitude. Four thousand feet lower, at a 27,000 foot cruise altitude, the true airspeed is 513 knots and the fuel flow is 7334 pounds per hour per engine.

If the headwind at FL310 is 50 knots, then the ground miles per 1000 pounds would be

$$\frac{(505 - 50)}{(4 \times 7038)} \times 1000 = 16.16$$

At 27,000 feet, with a 25-knot headwind, the ground miles per 1000 pounds would be

$$\frac{(513 - 25)}{(4 \times 7334)} \times 1000 = 16.63$$

This isn't a big difference but it's not insignificant when considering long cruise distances. For example, to fly 100 NM at 31000 feet in this example would require 6187 pounds of fuel, to fly the same 100 NM at 27000 feet would require 6011 pounds. That's not a big difference – but now imagine the difference over a flight of thousands of miles.

You can see that – particularly for longer flights – optimum altitude may not be the best altitude to fly. The flight plan should consider the winds that prevail at other altitudes with an eye to choosing an altitude below optimum when the winds warrant that.

The altitude that offers the minimum fuel consumption for the given weight, speed and wind profile is sometimes called the "recommended" altitude.

Present-day Flight Management Computers (FMCs) can calculate and display both the optimum altitude for maximum still air fuel mileage, and also the recommended altitude which optimizes the trip fuel consumption by considering the effect of the forecast winds aloft.

Altitude for minimum trip cost. We've already discussed the concept of cost index as a measure of the relative value of time and fuel. If a flight is being planned for minimum cost, then the cost index will affect the cruise altitude. In this case, the term "optimum altitude" is defined as the altitude yielding the least total cost per unit of distance flown, without accounting for the wind-altitude profile.

The total cost per unit of distance is the sum of the fuel cost per unit of distance plus the cost of time per unit of distance.

If, for example, the cost index is zero, then you already know what to expect: zero cost index means "think only of fuel consumption, time has no importance" so the most economical cruise altitude would be the same as that used for maximum fuel mileage – that is, optimum altitude.

If, on the other hand, one were to assume a high cost index, meaning "think much more about time, fuel consumption is much less important", then the most economical cruise altitude would be lower, to take advantage of the higher true airspeeds (that is, less trip time) despite the higher fuel flow. The higher the cost index, the lower the optimum altitude.

Similar to the "recommended" altitude for minimum fuel consumption previously discussed, the "recommended" altitude for flight at ECON speed would be the altitude yielding minimum cost when winds are considered.

Altitude capability. Engine thrust isn't unlimited. You know that there are limitations on the amount of thrust that can be demanded during climb (maximum climb thrust) and during cruise (maximum cruise thrust), called MCLT and MCRT respectively. These thrust limits depend on altitude and temperature.

It is possible that an airplane might not have adequate thrust available, at its maximum thrust setting, to either climb to or to maintain cruise speed at the selected altitude. This is less a problem with current engines, but in the past some airplanes with older engines had very limited altitude capability on hotter days.

Maneuver capability. If you have read the chapters earlier in this book dealing with aerodynamics, you'll recall that an airplane flown at a very low speed will encounter a condition called "buffet", – shaking – because the airflow over the wing is beginning to break down at the high angle of attack needed for that slow flight. You will also recall that buffet will also occur at very high speeds due to shock wave formation on the wings and the separated turbulent flow downstream of the shock wave. The former is called *low speed buffet* and the latter is called *high speed buffet*. Either condition should be avoided, and in particular the low speed buffet as it's a natural aerodynamic warning to the pilot of an impending stall.

The buffet speeds depend on the airplane's altitude. As an airplane climbs to higher altitudes, the speed below which low speed buffet will occur increases, and the speed above which high speed buffet will occur decreases. If the buffet speeds are plotted as a function of altitude, the resulting curve is referred to as a "buffet boundary". Flight within that boundary is acceptable from a buffet standpoint, flight outside the boundary is unacceptable.

The boundary depends on weight as well as altitude, being more restrictive at heavier weights.

Here's another factor to consider: making a turn in an airplane increases the load on the wing, just as a weight increase does, because to turn it's necessary to *bank* the airplane. This will be discussed in a later chapter, but in brief:

$$airplane\ lift\ required\ =\ weight \times \frac{1}{cos(bank\ angle)}$$

By "airplane lift required" in this equation we mean the lift necessary to maintain altitude in the turn. Thus you see that maneuvering an airplane in flight will impose aerodynamic loads that are equivalent to a heavier airplane in wings-level flight. For example, a 25 degree bank is the same as a ten percent increase in weight, in terms of aerodynamic load. Thus, the buffet boundary depends on not only altitude and weight, but also on maneuvering.

What else can impose an effective weight increase? Turbulence can impose effective weight increases of 30% or in some cases even more, although they're usually quite short in duration.

The higher an airplane flies, the less becomes its ability to experience increased gee forces from maneuvering or from turbulence without encountering buffet. This is referred to as its *maneuver capability.* While the FAA has no regulatory minimum on the allowable maneuver capability, the JAR rules do mandate a minimum of 1.3 g to buffet.[1] Many airlines that follow the FAA rules also adopt a 1.3 g minimum buffet margin as their standard practice.

Depending on an airplane's wing design, maintaining at least a 1.3 g maneuver capability could possibly restrict the airplane to somewhat lower altitudes.

From the above discussion, you can see that an airplane can encounter buffet in any of four ways:

- deceleration to the speed at which the onset of low-speed buffet occurs
- acceleration to the speed at which the onset of high-speed buffet occurs
- climb to the altitude at which buffet onset occurs
- experiencing an effective weight increase due to maneuvering or turbulence

For a much more detailed discussion of this topic, refer to the chapter entitled "Cruise Altitude Selection".

1. The expression "1.3 g to buffet" means simply that the airplane has the capability to be subjected to a 30% increase in lift from maneuvering or turbulence before encountering buffet or artificial stall warning.

cruise thrust

The maximum thrust available for use in cruise is called, logically, Maximum Cruise Thrust. There is no time limit on its use. However, most flights won't need to use that much thrust.

As with maximum climb thrust, the maximum cruise thrust limit is not regulatory and does not appear in the AFM. However, adherence to this engine manufacturer's limit is standard practice; Boeing does not publish any cruise data that would violate the limit. Further, as with maximum climb thrust, engine warranties could be affected if an operator exceeds this limit.

Unless an airplane is at the thrust-limited altitude corresponding to maximum cruise thrust, then the cruise thrust setting will simply be as required to maintain the desired cruise speed.

cruise speed

Selecting the best cruise speed is just as important as choosing the best cruise altitude. and just as fuel mileage depends on altitude, it also depends on speed.

In the illustration on the right, you see the fuel mileage as a function of Mach number at an altitude of 31,000 feet. This chart is for the same 747-400 that we showed you earlier when we discussed optimum altitude.

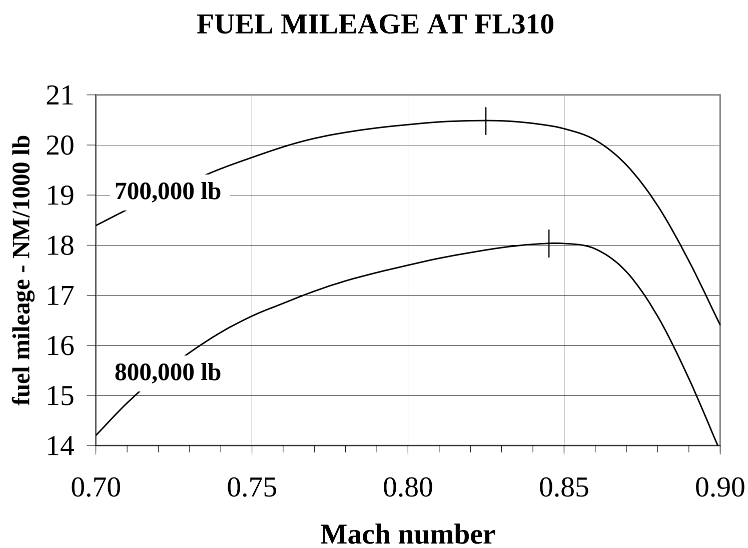


Figure 16-4

As you saw when we discussed the climb speeds, there are a number of choices of cruise speed:

Speed for a cost index of zero. You'll recall that a cost index of zero demands the speed for the minimum *fuel consumption for a given distance*, regardless of the time required to travel the distance. In the case of cruise, this is the speed for the maximum fuel mileage, which is called Maximum Range Cruise, or MRC. It's the speed that will result in the greatest distance flown for the quantity of fuel burned, or conversely the minimum fuel burned over a specified distance. In the illustration above, you'll see that MRC at 31,000 feet at a weight of 800,000 pounds is approximately M 0.845 and at a weight of 700,000 pounds it would be M 0.825.

You'll recall that when we discussed cruise altitudes, we talked about the difference between the best altitude for air fuel mileage and the best altitude for fuel consumption. The first of those two didn't consider enroute winds, the second one did. Similarly, the speed for best fuel mileage and the speed for minimum fuel consumption may be different. The effect is relatively small, however. Using the 747-400 example again, we'd find that the difference in cruise speed for minimum fuel consumption is only about six knots between a 100 knot headwind and a 100 knot tailwind. Flight Management Computers adjust the ECON speeds for the effect of cruise wind.

Speed for the maximum cost index. Using the maximum cost index means, in effect, "time is everything, fuel consumption isn't a consideration". This selection will yield the fastest possible cruise *speed*.

In cruise, the maximum possible cruise speed is limited by one of three possible factors: the maximum allowable flight speed, called M_{MO} (maximum operating Mach number) or, at lower altitudes, V_{MO} (maximum operating airspeed), and the thrust available. For some airplanes, the onset of high-speed buffet, as discussed above, may limit the maximum allowable speed.

The maximum operating Mach number M_{MO} depends on the airplane, being as low as 0.82 for some 737s, and as high as 0.92 for the 747-400.

Speed for a specified intermediate cost index. As was the case for climb, the cruise speed can be optimized for minimum trip *cost*. Now, instead of looking for the minimum fuel burn per mile, as we do for a cost index of zero, we're looking for the minimum cost per mile. This depends again on the airline's ratio of time cost to fuel cost.

An operator having high time-related costs but low fuel costs will want to fly faster, for a shorter trip time. Conversely, an airline having lower time-related costs but higher fuel costs will want to fly slower, to conserve fuel at the expense of a longer trip time.

Long Range Cruise. This speed, referred to as LRC, is neither the speed for minimum fuel consumption nor the speed for minimum trip time but instead is a compromise speed somewhere in between. It offers good fuel mileage but is faster than the maximum range cruise speed.

In today's world of automated flight control systems and flight management computers it's a simple matter to fly at any cost index – just enter it into the FMC and away you go – so the use of LRC has decreased. On the older airplanes that weren't computer-driven, LRC was easy to use but still gave good economics. If flying a 727, for example, as long as you were close to optimum altitude you knew that LRC would be Mach 0.79, but if you were flying a 747-200 you knew it would be Mach 0.845.

Speed for a specific waypoint time constraint. When requested by Air Traffic Control to pass over an enroute waypoint at a specified time, it may be necessary to adjust the cruise speed accordingly. Today's FMCs can do this sort of calculation easily – it's called a Required Time of Arrival (RTA) calculation – but in earlier days it was necessary to know the ground speed that would be necessary to comply with the constraint and then convert that to a calibrated airspeed – not a simple task.

engine failure during cruise

Although engine failure is uncommon these days, it's both prudent and also legally required to allow for a possible failure at any point during a flight. Should an engine fail during cruise, that failure might, depending on weight and its altitude at the time of the failure, force the airplane to descend to a lower altitude.

Along the route there may be terrain sufficiently high to affect the flight. Terrain can, under some conditions, necessitate restrictions to the airplane's takeoff weight and its fuel load. The regulations require flight planners to consider the following possibilities:

- It must be possible for any airplane to suffer the failure of one engine at any point along a route. In the event of such an engine failure, the airplane must be able to either (a) maintain level flight at least 1000 feet above all of the terrain lying within a specified distance on either side of the route, or (b) to descend to a lower altitude if necessary ("driftdown"), clearing all terrain along the route by 2000 feet during this descent and then maintain at least 1000 feet above obstacles after leveling off at the engine-inoperative altitude. The airplane must also be capable of "a positive net gradient" – which is just legal wording meaning "level flight or better" – 1500 feet above the airport where it intends to land.
- For an airplane having three or four engines, if any point along the route is more than 90 minutes flying time from an airport where it could land in case of an emergency, it must be possible for the airplane to suffer the failure of <u>two</u> engines and then perform a driftdown procedure clearing all terrain by a specified margin of altitude.

It's important to mention here that the driftdown requirements just discussed are based on *net flight paths*. Actual driftdown flight paths computed from the airplane data are called *gross flight paths*. To allow for operational variables, however, the regulations require some amount of conservatism to be applied to the gross flight paths when determining terrain or obstacle clearance during driftdown. After applying the specified conservatism, the new lower flight paths are called the net paths. It is the net driftdown path that must clear the terrain or obstacles by the specified height margins.

While most routes <u>won't</u> necessitate weight restrictions in order to meet the requirements described above, some routes are more challenging. Some of the most difficult ones are those that fly between Asia and Europe, which may take an airplane over the Himalaya Mountains. These mountains stretch across six nations: Bhutan, China, India, Nepal, Pakistan and Afghanistan. This area contains more than 100 peaks surpassing 7200 meters (23,622 feet) in elevation, with the highest peak exceeding 29,000 feet. Another region containing challenging terrain is South America, with routes across the Andes mountain range where the highest of the peaks approaches 23,000 feet. Although the Andes can't match the Himalayas in height, they do so in width and the range is more than twice as long.

It's the responsibility of an airline to analyze all of its routes considering possible engine failure, and to establish weight restrictions if necessary to ensure compliance with the rules. In some instances, special procedures may be required for terrain clearance following engine failure, and the flight crews must be apprised of such special procedures where required. On some routes, an airline will establish an "escape route" avoiding the higher terrain following an engine failure. In some instances, an airline may find it necessary to conduct the flight along a path containing less challenging terrain, in order to be able to fly the route at the desired weights.

loss of pressurization during cruise

The pressurization system of an airplane maintains the air inside the cabin at a pressure which will provide an amount of oxygen sufficient to sustain life. Typically, pressurization systems are capable of maintaining a cabin pressure that would correspond to a height above sea level of not more than about 8000 feet, even when cruising at the maximum certified altitude.

Clearly, if the pressurization system fails and the cabin pressure drops to the ambient pressure at the cruise altitude, the passengers and crew must have sources of oxygen to sustain life at least long enough for the crew to perform an emergency descent to an altitude low enough that supplemental oxygen is no longer required. For this reason, all airplanes that are capable of high-altitude cruise must be equipped with supplemental oxygen systems for the passengers and the crew.

Passenger supplemental oxygen systems take one of two forms on Boeing airplanes, depending on the model:

gaseous systems

A *gaseous* system consisting of a number of large high-pressure cylinders of compressed oxygen gas that can be delivered to passengers when needed. Delivery of oxygen is done through oxygen masks that deploy automatically when the cabin pressure altitude exceeds a specified level, usually 14,000 feet. The number of oxygen cylinders is a customer airline option. Airlines that will be operating over routes having considerable high terrain enroute will usually opt for more cylinders, allowing longer periods of flight at higher altitudes before needing to descend to an altitude where oxygen is no longer required. By adding cylinders, the gaseous system can generally be configured to allow flight over any route terrain. Some gaseous systems include the ability to stop the flow of oxygen to unused masks, extending the duration even further.

chemical systems

A *chemical* system consisting of many small canisters containing two chemicals that are normally separated from each other but that will create oxygen gas when allowed to mix. That mixing begins when the oxygen masks are deployed and pulled downward by the passengers to activate the generation of oxygen. Each group of immediately adjacent seats, such as row 15 seats A, B and C, will have a single oxygen canister providing gas to individual masks for each seat in that group. Once triggered, a chemical oxygen generator will continue to produce oxygen until the supply of chemicals is exhausted.

Chemical oxygen canisters are limited in the amount of time that they can create oxygen. The standard system consists of cylinders that create oxygen for only about 12 minutes[1]. There is an optional 22-minute system which will be more desirable for operators having high terrain enroute.

1. On some airplane type, you may also see these same canisters rated for 15 or 18 minutes.

The flow of oxygen coming from a chemical system is tailored to meet a predetermined descent profile. Flight can't be planned over a route along containing terrain that would require flight above this profile.
It is the responsibility of an airline to examine all of its routes for compliance with the pertinent regulations concerning loss of pressurization enroute. Special procedures may be required for terrain avoidance during the necessary emergency descent – sometimes called "an escape route" – which must be published for crew guidance.

The computation of driftdown and emergency descent profiles will be discussed in detail in a later chapter.

ETOPS

FAR Section 121.161 states that two-engine airplanes are not permitted to fly along a route having any point more than 60 minutes from an acceptable landing airport. An exception to this rule is granted provided that such flights comply with a special body of rules known as Extended Operations (ETOPS). These ETOPS rules are also required for 3- or 4-engine airplanes whose routes take them more than three hours at any point from an acceptable landing airport.

ETOPS rules require that

- the airplane must be certified as ETOPS-compliant in terms of the requirements such as extended cargo compartment fire suppression time requirements and so on; also
- the airline must be ETOPS-approved as having met the ETOPS requirements for unique flight dispatch, flight crew and maintenance procedures.

ETOPS is a complex subject that is treated extensively in other documents such as the ETOPS Guide volume 3 "Operational Guidelines and Methods", published by Boeing Flight Operations Engineering. Discussion of ETOPS rules and requirements is beyond the scope of this document.

Descent From Cruise

Since we have already discussed two special types of descent – driftdown and emergency descent – we'll limit this discussion to normal descent following the completion of the cruise segment of the flight.

In order to conserve fuel, it's generally desirable to conduct descent as much as possible at idle thrust, until thrust is required to offset the additional drag of landing gear and flaps when the airplane is approaching the destination airport. Any use of thrust above idle during enroute descent will mean an increase in overall trip fuel consumption. One of the most complex calculations in the FMC is the determination of the idle thrust descent.

As in the cases of climb and cruise, there are three choices of ECON descent speed:

- <u>speed for a cost index of zero</u>. Again, this selection taken together with the use of zero cost index for climb and cruise will ensure the least possible <u>fuel consumption</u> for the flight

- speed for the maximum cost index. As discussed above, this cost index used in conjunction with the same cost index for climb and cruise will ensure the least possible time for the flight
- speed for an intermediate cost index. Here again, this selection of speed is intended for use when minimum overall cost for the flight is desired.

In addition to the three regular speed schedules, it may on occasion be necessary to modify the descent speed in order to satisfy an Air Traffic Control requirement for a specified altitude or airspeed at a particular waypoint along the descent flight path.

It is also possible that ATC will request slower or faster speeds for the purposes of maintaining the necessary spacing between airplanes flying the same approach routes.

Calculating the parameters such as time, fuel and distance for normal enroute descent will be discussed in more detail in the chapter entitled “Enroute Descent”.

Approach and Landing

Just as there were quite a number of regulatory requirements relating to the takeoff and initial climbout, there are also a number relating to the approach and landing. These take two forms: gradient requirements in the event that an approach must be discontinued at any time, and landing distance requirements.

Under the heading of climb requirements, there are two distinct rules.

approach climb

It might be necessary for an airplane to break off an approach for a number of reasons: weather below landing minima, conflicting traffic on the approach, equipment malfunctions, ATC requirements, or other possibilities.

It is assumed that this event could happen during the initial approach phase, and for that reason the landing gear is assumed to be retracted and the flaps are at the approach position, not yet fully extended for landing. One engine is assumed to fail when the go-around is initiated, and the remaining engine(s) is/are at the go-around thrust rating.

The regulatory agencies mandate a minimum acceptable climb gradient in still air in this configuration. The weight at which the airplane can just meet the specified minimum gradient is called the approach climb landing weight limit.

landing climb

Should the necessity for a missed approach occur during the final approach when the airplane is fully configured for landing, there is another requirement that the airplane must be able to meet a specified minimum gradient, called the landing climb requirement.

In this case, all engines are assumed to be operating, at the thrust which will be available eight seconds after advancing the throttles for the go-around. The landing gear is considered to be extended, and the flaps are at the fully extended landing position. The speed is assumed to not exceed the normal approach reference speed V_{REF}, for the configuration.

The weight at which the airplane can just meet the specified minimum gradient is called the landing climb landing weight limit.

landing distance

The landing distance, like the takeoff distance, consists of a number of segments:

The flare segment. This is the distance traveled after crossing the runway threshold at fifty feet height, until the airplane touches down. It is assumed that the airplane's speed over the threshold will be the landing reference speed, called V_{REF}. Please refer to the chapter entitled "Speeds" for a complete definition of the reference speed.

The transition segment. This is the distance traveled along the runway between the time of touchdown until the airplane is at its on-ground attitude with all stopping devices deployed – that is, speedbrakes and wheel brakes. During the transition, the pilot will follow a procedure which is intended to minimize the stopping distance. First, at some point late in the flare or immediately after touchdown, the throttles will be retarded to idle. Second, the speedbrakes will be extended. Third, the wheel brakes will be applied. Finally, the thrust reversers will be deployed.

The stopping segment. This is the distance beginning at the end of the transition and continuing until the airplane has come to a complete stop. The calculation of this distance is based on maximum manual braking and full deployment of the speedbrakes when producing certified landing performance data.

Present-day Boeing airplanes are equipped with a system that automatically deploys the speedbrakes on touchdown. In the calculation of the certified landing distances we are allowed to take credit for that automatic deployment. <u>Deployment of the speedbrakes is essential in achieving the best possible stopping performance</u>. This is because speedbrake extension causes the wings to cease creating substantial lifting force, meaning that almost all of the airplane's weight will be placed on the landing gear. This gives the greatest possible effectiveness to the wheel brakes. Because of the importance of speedbrake deployment, pilots are trained to confirm proper deployment after touchdown and to deploy them manually in the event automatic deployment doesn't occur.

Current airplanes are also equipped with an "autobrake" brake system. This system is designed to apply hydraulic pressure automatically to the wheel brakes after the airplane has touched down and the wheels have begun to rotate. The autobrake system has a pilot-selectable level of brake application, from more gentle braking up to a maximum level of autobrake application. For reasons of passenger comfort, pilots will typically select a lower degree of autobraking, commensurate with the distance available for the deceleration.

The sum of the three segment distances discussed above is the actual landing distance.

Based on flight test results, we can calculate an airplane's actual landing distance under any conditions.

certified dry runway landing distance required

The certified landing distances– those landing distances published in the AFM – must be used for dispatch purposes, rather than the actual landing distances. One requirement of dispatch is that the airplane must be capable of landing within the certified distance when arriving at the destination (or alternate) airport. This requirement can result in restrictions to the allowable takeoff weight if the runways are relatively short at the destination or alternate airports. The maximum weight at which the airplane can meet the landing distance requirements for legal dispatch is termed the "runway limited landing weight".

The certified landing distances take no credit for autobrake operation. They are based on maximum manual brake application by the pilot. (In practice, however, pilots don't use maximum braking except when necessary, for reasons of passenger comfort.) Also, the certified landing distances take no credit for the use of reverse thrust.

The calculated actual landing distances are divided by a factor of 0.6 to arrive at the certified distances. Thus an airplane must be capable of landing to a full stop within 60% of the available runway length.

In the event that a landing must be conducted with an inoperative anti-skid system, the AFM contains certified landing distances for that condition. Landing with anti-skid inoperative will entail considerably longer landing distances, as the pilot will necessarily be cautious in the amount of pressure applied to the brake pedals in order to avoid skidding.

certified wet runway landing distance required

A runway which is wet will not offer the same degree of braking effectiveness as a dry runway. When landing will be necessary on a wet runway, therefore, the regulations require that an additional allowance must be included in the certified landing distance.

Where the certified dry runway landing distance required was equal to the calculated actual landing distance divided by 0.6, the certified wet runway distance is equal to the certified dry runway distance plus an additional 15 percent to allow for the reduction of braking effectiveness. The actual dry runway landing distance is thus multiplied by 1.6667 and again by 1.15. An airplane that can actually land within a total distance from the threshold of 5000 feet on a dry runway will therefore legally require a runway length of 9583 feet on a wet runway.

quick turnaround on the ground

Under the heading of takeoff earlier in this chapter, we discussed the necessity to consider heating of the wheel brakes in the event of a rejected takeoff. We pointed out that brakes will become hot because during a stop the airplane's kinetic energy is converted by the brakes into heat energy.

The same is true during a landing. The rollout on the ground while the brakes are being applied will result in considerable brake heating, even though during the typical landing less than maximum effort braking will usually be applied.

Later airplanes have brake temperature monitoring systems. However, the temperature sensors are necessarily at some distance from the brake stack, and there is some delay time in the temperatures shown by the monitors.

The ability of the wheel brakes to dissipate heat energy gained during either an RTO or a landing is very limited. If you've ever looked at a wheel brake installation on a commercial jet airplane, you'll remember that the brakes are contained within the wheels. The brake installation provides very little clearance between the brakes and the inside diameter of the wheel and because of this, natural cooling of the brakes is very slow, especially when sitting stationary on the ground.

Should the wheel brakes exceed a known energy limit, it is possible that the wheels and tires will become overheated by heat energy conducted from the very hot brakes through the metal mass of the wheels. It's possible that extreme heat could cause an explosion of the tire due to the tire pressure increase resulting from the heating. This could be, in the extreme case, hazardous.

To prevent the possibility of tire explosions, the wheels are fitted with thermal plugs known as "fuse plugs". These plugs are made of a soft metal having a low melting point, and their function is to melt when the wheel is overheated, releasing the pressure within the tire.

An unfortunate fact about brakes, wheels, and fuse plugs is that due to the considerable mass of the brakes and wheels, much time can elapse between brake application and fuse plug melting. In fact, it's possible that fuse plug melt could occur as much as forty-five minutes to an hour after brake application! In the extreme case of brake heating, a high-energy rejected takeoff, the fuse plugs can be expected to melt within a very few minutes. In the less extreme case, however, the brakes will be less hot and the time required to heat the fuse plugs to their melting temperature can be very long.

Look at the chart to the right, which illustrates the facts we've been discussing above.

You see that the brake temperatures rise almost immediately upon brake application. While the chart shows a brake temperature of about 1300 °F, in an extreme condition of use such as a takeoff RTO, the temperatures may exceed 2000 degrees.

Notice that there is a substantial delay between the time at which

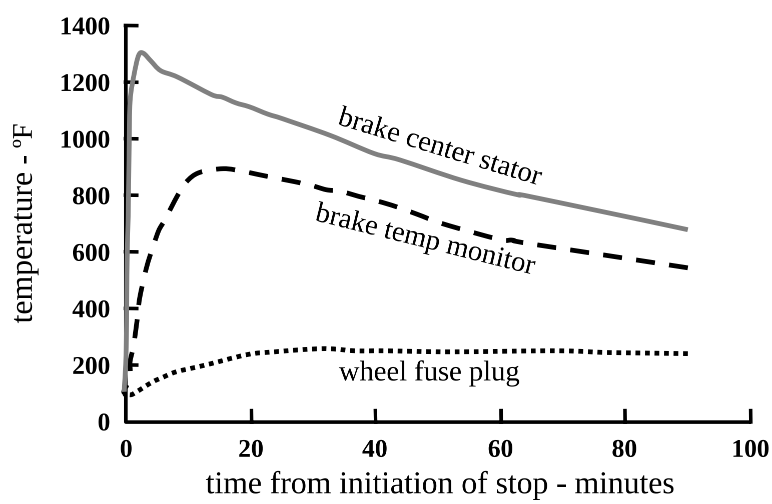


Figure 16-5

the brakes reach their peak temperatures and the time at which the brake temperature monitors reach their peak reading.

Notice also the very long delay between the temperature peak of the brakes and the time at which the wheel fuse plug reaches its maximum.

Finally, note the relatively slow rate of heat dissipation. After 90 minutes, in this example, the temperature at the wheel fuse plugs is still very close to that peak.

Suppose that an airplane were to make a normal landing, exerting just enough brake energy to result in fuse plug melting. Suppose too that the ground time is scheduled to be short, as is quite often the case for short-haul operators who make many takeoffs and landings in a single day in domestic service. It's possible that the fuse plugs could melt after leaving the destination airport terminal for taxi-out and takeoff. A flat tire can not be detected from the cockpit, with the exception of 777 airplanes fitted with tire pressure monitoring equipment. This could mean that the crew might make the next takeoff, or possibly the next landing, on flat tires. While we don't know of a case in which this has caused a hazardous situation, it's obviously to be avoided at all costs.[1]

Deflation of one tire of a landing gear imposes additional energy requirements on the brakes of the other wheels increasing the likelihood of their deflation in turn.

For this reason, the AFM contains a chart entitled "Maximum Quick Turnaround Weight". The purpose of this chart is to provide a weight above which a landing could possibly result in fuse plug melt and tire deflation. When this weight is exceeded during a landing, the airplane is required to remain on the ground for a specified period of time, after which time, if no tires have deflated, it can safely be assumed that no fuse plugs <u>will</u> melt and the next leg of the flight may be initiated.

The calculation of the maximum quick turnaround weight is conservative in that it assumes maximum manual braking with no credit for reverse thrust; still, this weight can be unconservative because as the chart above shows, even after complying with the quick turnaround time, the brakes will still be quite hot.

brake heating precautions

The calculation of a maximum quick turnaround weight described above is a potential trap for the pilot or planner who fails to recognize two very important facts: *(1) it's very easy to make the brakes hot but difficult to make them cool; and (2) brake energy is cumulative.*

Short-haul domestic operators are usually well aware of these facts, because an airplane following a schedule characterized by short flight times and short ground times is the most likely to encounter brake cooling problems (although they can occur on long-haul flights as well).

1. In fact, we're aware of at least one instance of an airplane landing with all main landing gear tires deflated. While this caused damage to the landing gear, the landing was otherwise uneventful.

Consider this scenario: an airplane makes a flight into an airport having a shorter runway. Its landing weight does not exceed the quick turnaround weight. It remains on the ground just long enough to discharge and take on payload, then taxis out and takes off toward the next destination. The flight is a short one; the airplane lands again at a weight less than the AFM quick turnaround weight, The ground time is again short, following which the airplane taxis out for the next sector. Can you see the hazard?

The pilot in such a scenario hasn't violated any limitations. But they have perhaps forgotten rule 1 of brakes, that *it's easy to make them hot but difficult to make them cool*. Thus, the second takeoff of the schedule was made with the brakes already containing a substantial amount of heat energy.

Rule number 2 of brakes, *brake energy is cumulative,* means that the energy of the second landing will add to that amount of energy that remains in the brakes from the first landing. If the second flight was short, there was little cooling between the second takeoff and the second landing. Thus the brakes are considerably hotter after the second landing than they were after the first landing.

Repeat this pattern a few times, and it's clear that fuse plug melting is possible and even probable, due to the accumulation of brake energy coupled with the fact that brake cooling on the ground and with the gear retracted in flight is slow. Sooner or later, one or more tires could deflate.

How does the smart short-haul operator avoid that? By having the pilot extend the landing gear a few minutes early on each approach, and/or leave the landing gear extended for a few minutes after each takeoff. Gear down cooling in flight is as much as ten times faster than it is when the airplane is parked or in flight with the gear retracted.

Data on recommended brake cooling times is available to operators through the Operations Manual or the Flight Planning and Performance Manual.

landing on contaminated runways

During the discussion of takeoff on contaminated runways, we discussed the effect that contamination can have on both acceleration and deceleration. This will be further discussed in the chapter entitled "Takeoff On Contaminated Runways".

During landing, any contamination will have the effect of reducing the braking effectiveness. Landing distances on contaminated runways are not published in the FAA Airplane Flight Manual, although that data does appear in the AFMs of some other regulatory agencies.

We do, however, provide guidance information for pilots in the Operations Manual or Performance Inflight chapter of the Quick Reference Handbook, based on the braking conditions reported by other pilots.

Chapter 17: Speeds

Introduction

Many tasks we do relating to airplane performance require us to select the airplane's speed. Pilots fly to specific speeds in every segment of a flight. Some speeds are forgiving – a certain amount of deviation won't matter much – but other speeds are critical. Accidents have happened when, for example, pilots rejected a takeoff at a speed exceeding V_1 or landed with excessive speed.

We teach pilots to "fly by the numbers", meaning that they should always observe closely the correct speeds for a given maneuver. So let's examine all of these "numbers" so that we can have a clear understanding of their exact meaning and how they're applied.

Low-Speed Buffet and Stall

Early in his flight training, every student pilot learns about stalls. He is taught how to recognize a stall, how to recover, and how to avoid one. He learns that in well-designed airplanes a stall is not inherently a dangerous maneuver – unless it occurs at an altitude too low to allow a complete recovery. He learns too that his stalling speed is affected by a number of variables, including the airplane's weight, its flap setting, and any load factor imposed on the airplane by turns or other flight maneuvers.

The stalling speed is used as a basic criterion for computing many different flight speeds. For example, during a takeoff including the failure of one engine, the airplane speed at 35 feet after liftoff is the speed called V_2. This must be a minimum of 13% above the stalling speed (20% above the stalling speed, as determined using different stall speed definition criteria, for earlier airplane models). Thus we need to look very carefully at stalling speeds.

An airplane's lift is a direct function of both its angle of attack and its speed. For a given weight, decreasing speed, at a constant angle of attack, means decreasing lift. Increasing angle of attack, at a constant speed, means increasing lift. As an airplane decelerates (decreasing lift), it's necessary to increase its angle of attack (increasing lift) in order to continue to generate the lift needed to support the weight.

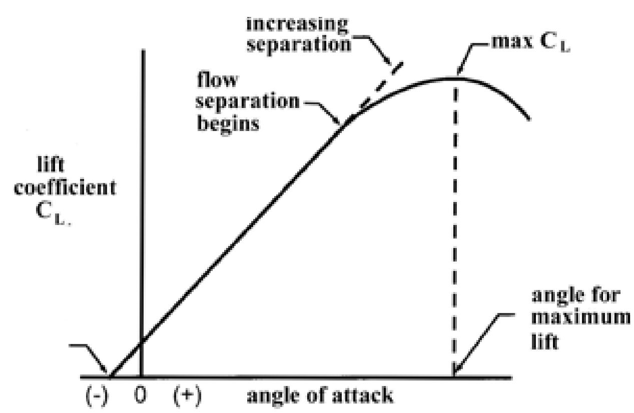


Figure 17-1

As discussed in the chapter entitled "Lift and Drag", at a lower angle of attack the pattern of airflow over the upper surface of a wing remains smooth, but as the angle of attack increases it becomes harder and harder for the wing to maintain smooth flow. At some angle the wing will begin to experience airflow separation. This separation creates a turbulent wake, and that wake can impinge on the horizontal tail or the fuselage creating a noticeable shaking of the airplane known as *buffet*. If pressed to even higher angles of attack, the airflow will continue to deteriorate and finally will become completely separated, rendering the wing incapable of creating adequate lift for flight. Complete loss of lift is referred to as *stall*.

The first aerodynamic warning of an impending stall, then, is the condition known as *buffet*. Some airplanes exhibit very strong buffet characteristics, others may have very little buffet before stalling. The amount of buffet experienced prior to stall is also a function of the flap setting.

Some amount of buffet is really a beneficial characteristic of an airplane, since it serves as natural warning of an impending stall.

V_{IB} – initial buffet speed defined

The speed at which the buffet first becomes noticeable is called V_{IB}, the "initial buffet speed", and this speed must be determined by flight test. Of course, the term "noticeable" is rather imprecise, so it's necessary to put a detailed definition to the term "initial buffet". The FAA defines it this way: <u>initial buffet speed is that speed at which, as an airplane approaches stall, an accelerometer mounted at the pilot's seat track would measure 0.1 gees of peak-to-peak vertical acceleration</u>. That would be similar to driving an automobile over a road that has a moderately corrugated[1] or "washboard" surface causing a series of vertical oscillations.

$V_{S\ FAR}$ – the FAR stalling speed defined

If we were to make a flight test to determine the airplane's stalling speed, we would need to record a number of parameters. While recording, we would reduce power and allow the airplane to decelerate. As the airplane decelerated, we would necessarily increase the angle of attack to maintain the same amount of lift.

At some speed we would experience initial buffet. Continuing the deceleration, we would ultimately lose lift. At some point, the pilot would consider the airplane to be fully stalled and would then reduce the airplane's pitch attitude to effect a recovery to normal flight as the airflow reattaches to the wing upper surface.

The plots of speed and the computed corresponding values of C_L would look like the figure below.

1. *corrugated*: characterized by a series of alternating grooves and ridges

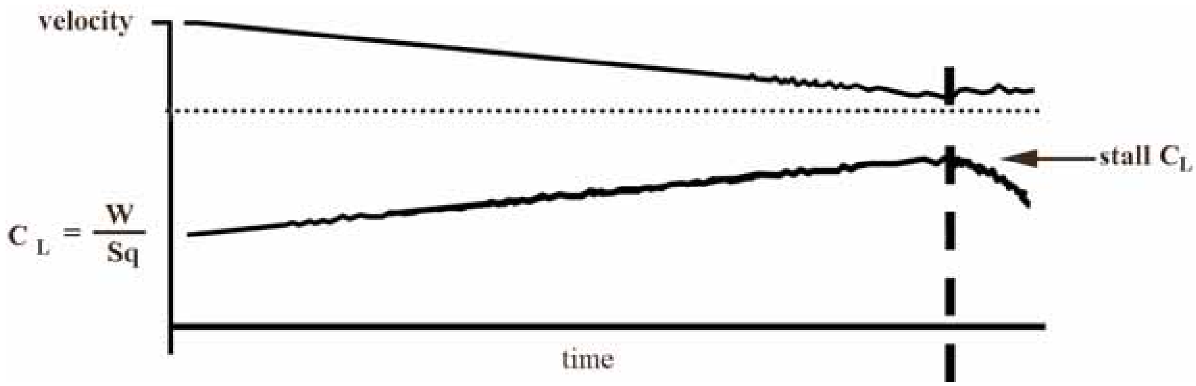


Figure 17-2

In the chart, we are using the simple equation for lift coefficient that you saw in the chapter entitled "Lift and Drag":

$$C_L = \frac{W}{Sq}$$

where *W* is the airplane weight
S is the wing area
q is the dynamic pressure

As you see in the graph, the value of C_L increases steadily to compensate as speed decreases because the dynamic pressure q is decreasing. At the point of stall, the speed reaches the minimum value obtained during the stall maneuver. Then during the recovery from the stall, the speed will again increase.

What is termed the "FAR stalling speed" ("FAR" being the Federal Aviation Regulations) is what's seen in this example. The slowest speed recorded during the stall test is declared to be the stalling speed. The corresponding C_L is termed the "FAR stall C_L" and is used to calculate stall speeds for other weights.

One significant problem with using this method of determining stall speed was that it was strongly dependent on pilot technique. An "aggressive" pilot could fly the airplane to a lower speed than a less aggressive pilot, thereby producing lower stall speeds.

FAR part 25.201 defines stall recognition this way:

- A nose-down pitch that cannot be readily arrested;
- Buffeting, of a magnitude and severity that is a strong and effective deterrent to further speed reduction;
- The pitch control reaches the aft stop and no further increase in pitch attitude occurs when the control is held full aft for a short time before recovery is initiated.

Even so, pilot technique and judgement is still a factor in determining the stall speeds in this manner.

The other significant problem with the FAR stall speed definition is the assumption that lift equals weight throughout the maneuver, when in fact the lift at the FAR stall speed may be significantly less than the weight.

$V_{S\ 1\text{-}g}$ – the 1-g stalling speed defined

In the mid-1980s beginning with the 767-300, a new more precise definition of stalling speed evolved. This definition provided for a testing protocol which defined stall speed in a manner that is both independent of test pilot technique and more representative of the actual speed at which lift is lost on the wing.

In testing to the new definition, the recovery from the stall is begun sooner, rather than pressing on into a condition in which the airplane has actually been flown to speeds lower than that which will sustain 1-g flight – that is, past the aerodynamic point of stall. The data recorded in these tests now includes the vertical load factor, called *n*. This is measured by a vertical accelerometer.

The C_L now has a slightly different more precise definition:

$$C_L = \frac{nW}{Sq}$$

We no longer assume that lift equals weight during a stall (that is, that n =1). Now we're accounting for actual vertical acceleration in computing C_L. This is now called a "1-g" stall C_L. Here's what the flight test speed and load factor would look like, with the corresponding 1-g C_L:

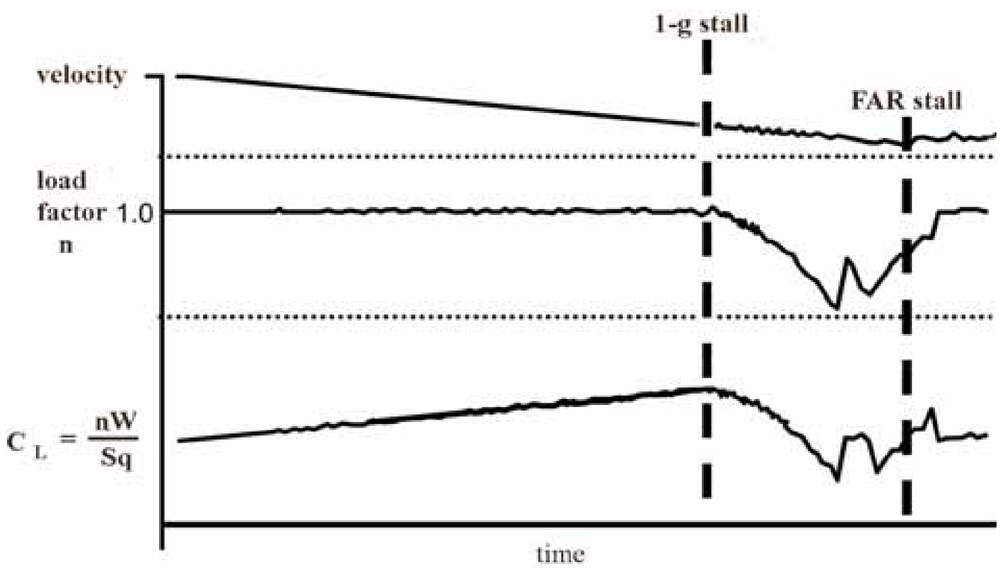


Figure 17-3

What you see here is quite different from the earlier graph. Here we see that as the airplane approaches the stall, the wing is no longer supporting the full weight of the airplane, as exhibited by the decrease in the load factor n. The lift is no longer equal to the weight. The lift coefficient begins to decrease at the point at which the load factor suddenly begins to decrease. The load factor may go as low as 0.70 to 0.75 before recovery is begun.

The maximum lift coefficient now occurs at a considerably faster speed than it did under the FAR stall definition. The speed corresponding to this maximum lift coefficient is termed “the 1-g stalling speed”. <u>The 1-g stalling speed is six to eight percent faster than the FAR stall speed</u>.

operational speed factors

This fact has a bearing on performance calculations. For example, the takeoff initial engine-inoperative climb speed V_2 of the earlier airplanes was required to be a minimum of 20% above the FAR stalling speed. When the stalling speeds were re-defined in the mid-eighties, it was clear that this “operational speed factor” of 20 percent, if applied to the new higher stalling speeds, would cause V_2 to become faster than it would have been following the FAR stall speed definition.

Years of experience had shown that the V_2 speeds used by commercial airplanes were safe and efficient and did not need to be increased, even if stall speeds were re-defined to higher values.

For that reason, a new speed additive of 13% is used for the newer “1-g stall” airplanes. The minimum margin from stalling speed for the landing reference speed V_{ref}, which was 30% for the earlier airplanes, becomes 23% for the 1-g stall airplanes.

It’s important to understand that although the speed factors are less when applied to 1-g stalling speeds, <u>this does not imply any reduction of safety margins</u>. The lower speed factor is offset by the higher 1-g stall speed, resulting in essentially the safety margins in both cases.

factors that affect stalling speed

For an airplane in a given configuration, the stalling speed for any weight may be computed from the published stall C_L. But that value of stall lift coefficient, whether the earlier FAR stall or the 1-g stall, isn’t simply a matter of weight.

In fact, the following factors will affect all stall speeds:

- the center of gravity;
- the thrust;
- altitude;
- aeroelastic effects;
- rate of entry;
- load factor.

Let’s examine each of these in turn.

Center of Gravity effect. It's not intuitively obvious, but it's a fact that the airplane's center of gravity affects the amount of lift the wing must create for a given airplane weight, and this in turn affects the stalling speed. Thus it's necessary to determine the stall coefficients of lift over the airplane's usable range of CG.

In flight, an airplane is being acted upon by four forces: weight acting vertically, lift acting perpendicular to the path of flight, and drag and thrust acting parallel to the path of flight. When the airplane is in *steady-state flight* – that is to say, at a constant attitude and constant speed – the lift will be equal to the weight, and the thrust will be equal to the drag.

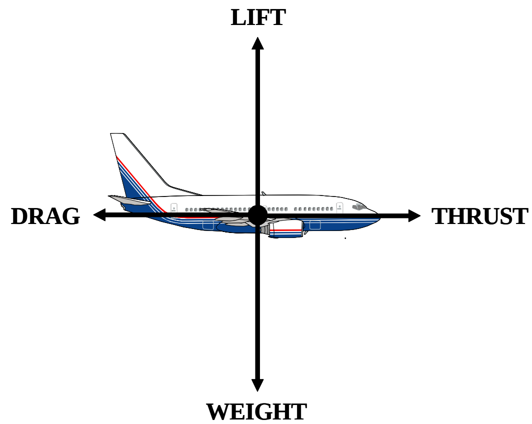


Figure 17-4

For the following discussion of CG effect on stall speed, we're going to consider only the vertical forces of weight and lift.

For equilibrium, the lift and the weight are equal and opposite, both acting through the airplane's center of gravity. But the illustration above is simplified, because in fact the airplane's total lift force is really made up of two main components: the wing lift acting upward and the tail lift, a much smaller force, acting downward.

For simplicity, we are ignoring the very small amount of lift produced by the fuselage.

Consider an airplane at a forward center of gravity position, as shown to the right. The lift produced by the wings acts through the "center of lift". The weight acts through the airplane's center of gravity. The CG is always forward of the wing's center of lift, otherwise the airplane would be unstable in the pitch axis.

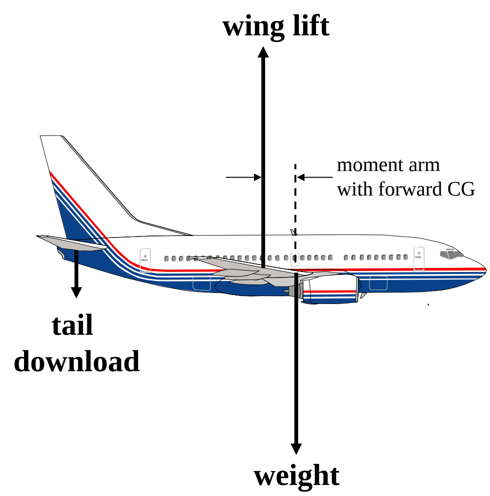


Figure 17-5

Let's discuss that instability for a minute:

For an airplane with a conventional aft-mounted horizontal tail, it's typically necessary to maintain the center of gravity ahead of the center of lift. Why?

A system is considered to be *stable* when it tends to return to its original state after it's submitted to an external disturbance that moves it away from its original state. *Pitch stability* can thus be defined as the tendency of an airplane, after being moved away from its original pitch attitude by some external disturbance, to return to its original attitude without requiring corrective action by the pilot. This stability is affected by the design and location of the horizontal tail, and by the location of the airplane's center of gravity.

Consider an airplane in level flight, when subjected to a vertical gust. The wing angle of attack, and therefore the lift, will momentarily increase. With the CG forward of the center of lift, this will induce a nose-down pitching moment about the CG, tending to restore the airplane to its original equilibrium attitude. A center of gravity <u>behind</u> the center of lift would result in a <u>nose-up</u> pitching moment, which would tend to drive the airplane <u>farther</u> from its equilibrium attitude rather than back <u>toward</u> it; this would be an unstable condition.

With a very forward CG position, there is a substantial moment arm between the location of the CG and the wing's center of lift, hence there's a strong nose-down pitching moment about the CG, produced by the wing's lift force. To keep the airplane in longitudinal balance, there must be an equal amount of moment in the opposite direction about the CG, generated by the horizontal tail which produces a force acting in the downward direction. This is sometimes referred to as a "download".

Since we are assuming that the airplane is in steady-state unaccelerated flight, the <u>sum</u> of (wing lift upward + tail force downward) is equal to and opposite in direction to the weight. Further, the <u>moment</u> sum of (nose-down wing lift moment about the CG + nose-up tail download moment about the CG) is zero. You can see that the airplane's wings must produce an upward force that is considerably greater than the airplane's weight.

Now consider the same airplane, at the same weight, but with a more aft CG.

At a more aft CG, while the CG is still forward of the wing's center of lift, the moment arm between them is substantially less and hence the nose-down pitch moment is less and the compensating tail download required will be less also.

As a result, for the same weight <u>in the aft CG condition the wing doesn't need to provide quite as much lift force as it does with a forward CG</u>. But again, the sum of the wing lift plus the tail download is equal to the weight, and that sum is the airplane lift.

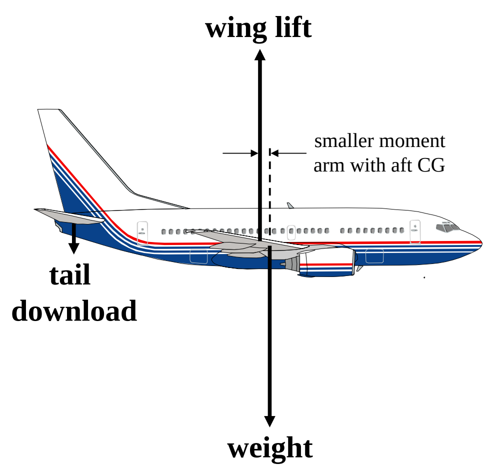


Figure 17-6

Let's graph that fact this way: weight and the lift required to support the weight are linearly related to each other. But we have seen that for the same weight, the lift required from the wing at a more forward CG must be greater than the lift at a more aft CG. If the CG were located right at the wing's center of lift, then the wing lift and the weight would be equal. But this will never happen – the CG must always be forward of the center of lift of the wing, for reasons of pitch stability.

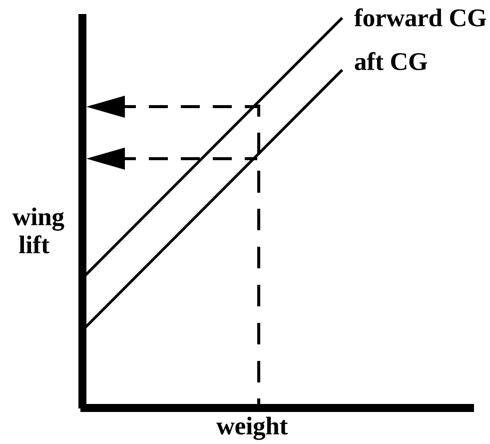


Figure 17-7

To summarize this discussion in simple terms: for any given value of airplane weight, there will be different values of wing lift required depending upon the CG position.

We know from the earlier chapter entitled "Lift and Drag" that the wing lift and the wing coefficient of lift are related by:

$$wing\ lift\ =\ C_{L_{wing}} \times \frac{1}{2}\rho V^2 S$$

We also know from the discussion in the chapter entitled "Lift and Drag" that the wing lift coefficient depends on the angle of attack, as shown to the right. This curve is not affected by the position of the CG.

For any given weight and CG, we can determine the wing lift required. Given an altitude and temperature we can then calculate the C_L for a range of velocities at that constant weight. For each value of C_L thus calculated we can find the corresponding airplane angle of attack required.

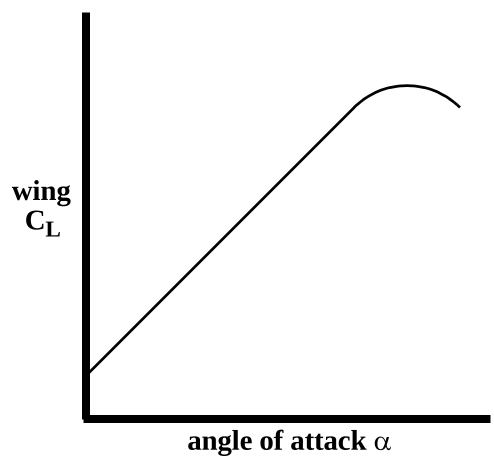


Figure 17-8

Finally, we can make a graph of angle of attack versus velocity, for constant airplane weight, as shown to the right.

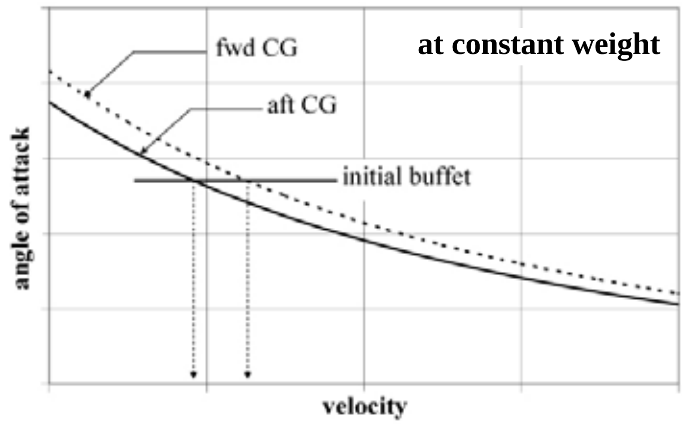


Figure 17-9

Since the airplane wing lift depends on the location of the CG, as we saw just a minute ago, we will have different lines of angle of attack versus velocity for different CG locations, as you see in the graph. This is because more wing lift would be required at a given weight at a forward CG than at an aft CG. To produce that greater lift at a given velocity, angle of attack must be increased to increase the wing coefficient of lift.

In the above discussion, we're talking about wing lift coefficient and wing angle of attack. For practical purposes, that's the same as the airplane's lift coefficient and angle of attack. The contribution of the horizontal tail is very small when compared to that of the wings. Further, the horizontal tail's angle of attack in steady-state flight is a negative value in order to produce the download necessary for equilibrium in pitch. It can't be expected to enter buffet and stall itself, as it will never reach such high angles of attack. Thus buffet and stall are characteristics of the wing only.
Initial buffet, and stall will occur at the same angle of attack regardless of the CG location. Thus, it's clearly seen that airplane buffet and stall will occur at faster speeds at a more forward CG than they will at a more aft CG.

This fact has a bearing on airplane performance in some areas. For example, as described above the minimum value of V_2 is 13% above the stall speed (1-g stall criteria). That stall speed is required to be based on the most adverse CG position. What would be the most adverse position in this instance? The most forward allowable CG would be the most adverse, since the stall speed would be higher and thus the V_2 would be faster and hence more conservative.

You'll see how we can take advantage of the effect of CG position on stall speed at a later point in this book, where we will discuss the subject of "alternate forward CG limits" for improved takeoff performance.

Thrust effect. As an airplane's angle of attack increases approaching a stall, the thrust of the engines will have a slight vertical component, very slightly offsetting the weight and thereby reducing the amount of lift that the wing must generate. The effect of this will be to reduce the stalling speed.

Stall testing is, by regulation, conducted at idle thrust or, at the most, not more than zero thrust. The stall speeds thus obtained are conservative in that they are very slightly slower than they would be at typical inflight conditions.

Altitude effect. Over the range of altitudes certified for takeoff and landing, altitude has a small effect on stall C_L, but it's not negligible. The cause of the altitude effect lies in the reduced viscosity of the air at the less dense and colder atmospheric conditions at the higher altitudes.

On the earlier Boeing airplanes, stall speeds were established by flight tests conducted at high altitudes, from nine to fourteen thousand feet. These speeds were then used for all altitudes, and were therefore conservative for the lower altitudes.

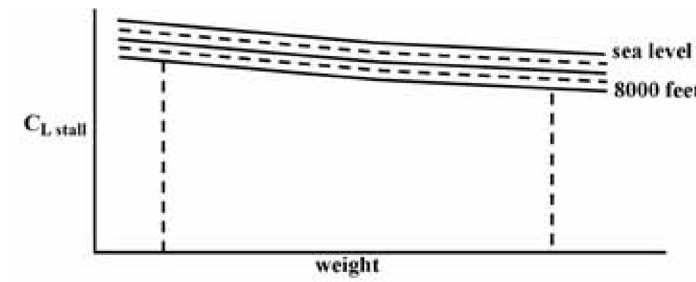


Figure 17-10

On the later Boeing airplanes, beginning with the 737-500 and MD-11, stall tests have been flown at, and stall speeds published for, a range of altitudes. The published stall speeds are thus more accurate. If comparing the stall speed at ten thousand feet to that at sea level, typically a two to three knot difference will be seen. This slightly slower stall speed at the lower altitudes allows us to use slightly slower values for the minimum V_2, for example, which offers a small improvement in field length weight limits.

Weight effect. This is sometimes referred to as an "aeroelastic" effect.

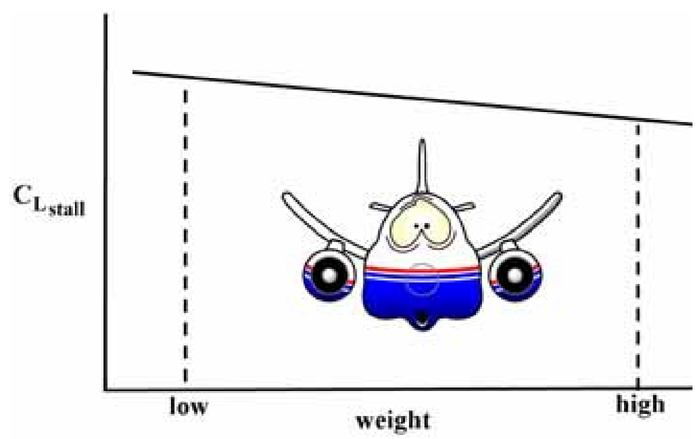


Figure 17-11

The "elastic" part of that term refers to the fact that the airplane's wings are, to a degree, flexible. At higher weights, there will be a greater amount of wing flexing than at lower weights. This changes the distribution of lift along the wing as it bends and twists, resulting in a slightly lower stall coefficient of lift at the high weights.

That lower stall coefficient, of course, results in a faster stalling speed at heavy airplane weights.

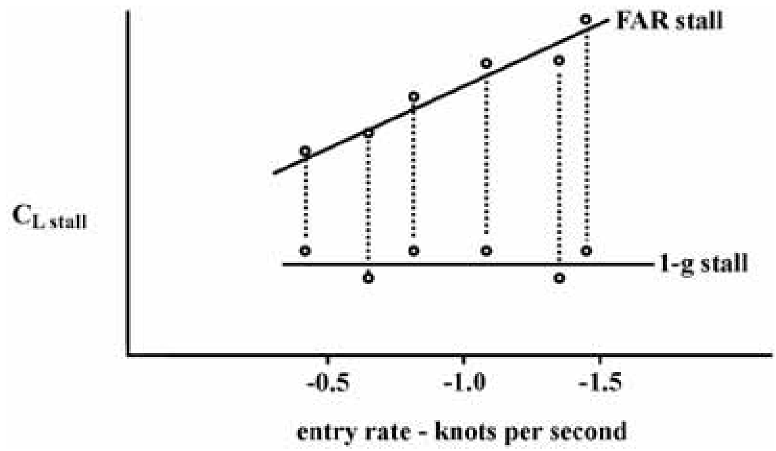


Figure 17-12

Rate of entry effect. The airplane's deceleration, that is, the rate at which it is slowed to the stall during the flight test maneuver, has an effect on FAR stalling speeds, although not on the 1-g stall speeds.

For the certification flight tests, the airplane is trimmed at 1.20 times the estimated $V_{S\ FAR}$ or 1.13 times the estimated $V_{S\ 1\text{-}g}$. Stall tests are conducted at a variety of entry rates, and the corresponding coefficients of lift are plotted against entry rate.

The regulations require that the FAR stalling speed be taken based on an entry rate during the test maneuver of one knot per second. This plot enables us to determine the coefficient of lift without actually flying at that exact entry rate.

Stall tests conducted to the 1-g rules are negligibly affected by entry rate, since the stall is considered to occur at the "g-break". This reduces the dynamic effect of entry rate to the point that test data show essentially no effect of entry rate on 1-g stalling speeds. This illustrates another benefit of using 1-g stall speeds compared to FAR stall speeds.

Load factor. The term "load factor" means simply that, for any one of several possible reasons, an airplane may be necessarily generating lift greater than its weight.

The load factor can be stated as:

$$\text{load factor } n = \frac{\text{airplane lift}}{\text{weight}}$$

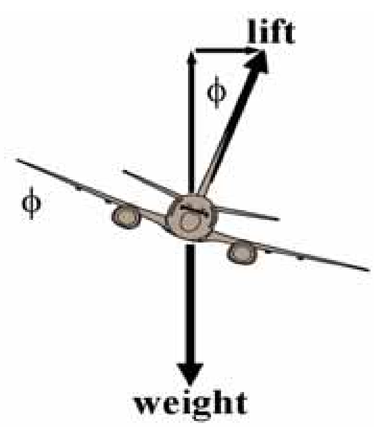


Figure 17-13

One of the most frequent sources of a load factor is when an airplane enters an angle of bank for the purpose of turning. An airplane making a turn must generate more lift than the airplane's weight, as shown in the illustration to the right. The vertical component of the airplane's lift must be equal to the weight. This means that the lift is greater than the weight, and is given by:

$$lift = nW = \frac{weight}{\cos\phi}$$

Since load factor is lift divided by weight it follows that, in a turn, the load factor can be expressed as:

$$\text{load factor } n = \frac{1}{\cos\phi}$$

Thus, to compute the stall speed in a turn, it’s necessary to use the airplane lift, rather than the weight, in the calculation. For example, in a turn having a bank angle of 60°, you would need to calculate the stall speed using a value of lift equal to twice the airplane weight, since the cosine of 60° is 0.5 yielding a load factor of 2.0.

The wing will stall at the same angle of attack, whether in a turn or in straight and level flight. However, the speed at which the stall occurs will be higher during a turn than in straight and level flight.

What are some other sources of load factor?

One is turbulence. What we call “turbulence” is one or more sudden vertical movements of an airplane that are caused by rapid changes in the direction and/or velocity of airflow encountered by the wing that result in changes to the wing lift force.

On one flight or another, we’ve all probably experienced the transitory changes of load factor in turbulence. While unpleasant, nowadays it’s very rarely dangerous, given present-day weather radar that enables us to avoid it, advanced meteorological skills that allow us to predict when and where it’s going to happen, and the strength of modern airplanes. Still, high load factors can and do occur without warning. Load factors ranging from zero to plus two, while uncommon, still occur on commercial flights. In the extreme, load factors far greater than that have occurred. In 1966, for example, a Boeing 707 accidentally encountered severe clear-air turbulence during a flight passing near Mount Fujiyama in Japan. The load factors from the turbulence exceeded the structural capability of the airplane and it broke up in mid-air. A US Navy fighter sent up to look for wreckage also encountered the turbulence and recorded load factors as high as plus nine and minus four.

Another source of load factor can be maneuvers such as the pull-out from a dive. In fact, pilots undergoing stall training are cautioned about recovering too aggressively from a stall, because an aggressive recovery, attempting to stop the rate of descent too rapidly, can cause an “accelerated stall” condition in which the recovery from a stall may result in another stall due to high load factors in the recovery.

stall warning

For safety reasons, whenever flying at relatively low speeds such as during takeoff and approach it is essential that pilots have adequate warning of an impending stall. FAR part 25.207 states it this way:

> *Stall warning with sufficient margin to prevent inadvertent stalling with the flaps and landing gear in any normal position must be clear and distinctive to the pilot in straight and turning flight....The warning may be furnished either through the inherent aerodynamic qualities of the airplane or by a device that will give clearly distinguishable indications under expected conditions of flight...*

Stall warning can take several different forms. One of those is already familiar to you: initial buffet. That is what the FAR quoted above refers to as "the inherent aerodynamic qualities of the airplane". This initial buffet speed, you'll remember, is defined as the speed at which a vertical accelerometer mounted at the pilot's seat track will measure 0.1 gees peak-to-peak vibration.

The other form of stall warning is termed an "artificial stall warning", usually referred to as a "stick shaker". A stick shaker is a mechanical device added to the airplane to warn the pilot of an impending stall as the airplane's angle of attack approaches the stall angle of attack. When actuated by the system responsible for stall warning, the stick shaker causes the pilots' control columns to shake very noticeably, simulating the way the control columns would behave in a real initial buffet thus providing the required "clearly distinguishable" warning.

The regulations require that the stall warning, whether natural or artificial, must provide a warning at least 7% for the FAR stall, and 3% or 3 knots of calibrated airspeed for 1-g stall, whichever is greater, above the certified stalling speed. Typically, a transport airplane may meet this requirement on the basis of natural warning in <u>some</u> configurations and conditions but not in all, thus necessitating the installation of an artificial stall warning system. This is the case with all Boeing jet transport airplanes.

V_{MCG} – the Ground Minimum Control Speed

One of the most fundamental rules in takeoff performance is that the allowable takeoff weights <u>must</u> allow for the possible failure of an engine at any time during takeoff. This requirement influences a number of elements which together make up takeoff performance, including the takeoff *speeds*.

The speeds used by the pilot in conducting a takeoff must consider the possibility of engine failure. A necessary constraint on the takeoff speeds is that they must provide safe handling qualities if an engine fails at any point during the takeoff – and one of the major considerations is the ability to maintain directional control with one engine inoperative.

Assume that an engine fails during the takeoff acceleration. The immediate effect of the failure is the creation of a large moment due to the thrust asymmetry, causing the airplane to yaw in the direction of the failed engine. If, for example, the left engine of a twin-engine airplane fails, the airplane's nose will immediately begin to turn to the left because of the asymmetry of the thrust: the left engine is producing little or no thrust or even drag, when windmilling, while the right engine is at takeoff thrust.

When the yaw begins, the pilot responds by applying opposite rudder in an attempt to keep the airplane from deviating from the runway centerline. Under some conditions of CG and runway surface condition the effectiveness of nose landing gear steering can be very small so we do not consider nose gear steering as part of the procedure. The result of this fact is that the only means of controlling the airplane directionally, for the purposes of determining the minimum control speed, is the rudder.

But the rudder is an aerodynamic device, and the yawing moment it's capable of generating is a function of the velocity of the air flowing over the vertical tail. At low velocities during the take-off acceleration, it can't generate enough yawing moment to counteract the asymmetric thrust. The result: the engine thrust must be reduced and the takeoff rejected, or else the airplane will go off the side of the runway.

At high velocities during the takeoff acceleration, the rudder will be capable of generating more than enough yawing moment to counteract the thrust asymmetry. At these speeds, then, the airplane could suffer an engine failure and remain controllable.

V_{MCG} – the ground minimum control speed defined

The minimum speed at which the airplane can just maintain adequate directional control on the ground with the critical engine inoperative, using only the rudder, is called the ground minimum control speed, V_{MCG}. It is subject to the following constraints:

- the "critical engine" has failed
- the operating engine(s) are at the takeoff thrust setting;
- the airplane is controllable using the rudder only;
- the rudder pedal force required can't exceed 150 pounds;
- the airplane must be controllable using normal piloting skills;
- during the recovery maneuver, the maximum deviation from the runway centerline is 30 feet.

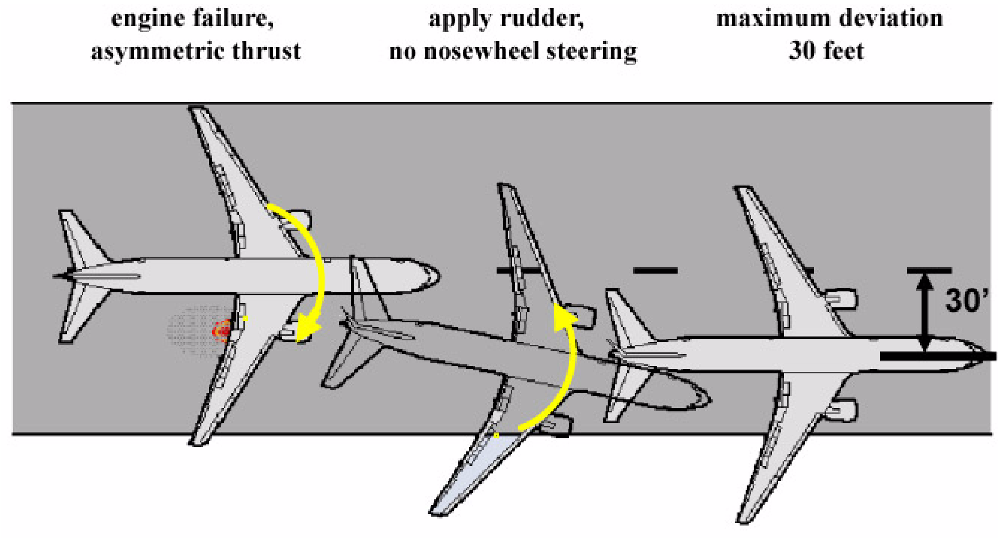


Figure 17-14

In the illustration, the right engine fails and the airplane yaws to the right and deviates from the centerline. The pilot applies opposite rudder and recovers from the yaw. The airplane's path never exceeds 30 feet from the centerline during the recovery, using rudder control only.

Reference is made above to the failure of the "critical engine" as part of the definition of the minimum control speed. For most of today's two-engine airplanes, neither engine's failure will create a situation more adverse than would result from the failure of the opposite engine. Obviously, on a four-engine airplane, failure of an outboard engine will be more critical, but whether the left outboard or the right outboard engine makes no difference.[1]

It's worth mentioning here that the V_{MCG} constraints shown above do <u>not</u> include consideration of a crosswind. However, a crosswind from the direction of the failed engine would significantly increase the minimum control speed. In the illustration above, a crosswind from the right would be adverse, because it would cause the airplane to "weathervane" – that is, turn its nose toward the direction from which the wind is coming. A crosswind coming from the side <u>opposite</u> the engine failure would be favorable.

Control of yaw, such as that demanded by the V_{MCG} requirements discussed above, is one of the determinants in designing an airplane's rudder size and deflection limit.

1. On the 707 airplane, however, there truly <u>is</u> a critical engine: the right outboard (number 4) engine. This is because the 707's left outboard engine produces slightly more takeoff thrust than the right outboard, due to a difference in the airbleed requirements. Failure of the right outboard engine thus results in a slightly greater yawing moment than does failure of the left outboard engine.

V_{MCA} – the Air Minimum Control Speed

When airborne, it is also necessary to consider the handling qualities of an airplane having an inoperative engine. Just as there is a minimum speed on the ground at which directional control can be maintained – V_{MCG} – there is also a minimum speed in the air at which directional control can be maintained. This is called V_{MCA}, the air minimum control speed.

The most significant difference between the ground and air minimum control speeds is that in the air, the pilot is allowed to hold a bank angle of up to five degrees in the direction away from the failed engine. This five degree bank assists the rudder in maintaining directional control.

V_{MCA} is subject to the following requirements:

- the engine fails suddenly;
- the airplane is in its most critical takeoff configuration with the landing gear retracted;
- the airplane is out of ground effect;
- the CG is at its most adverse position (aft);
- the remaining engine(s) are operating at maximum takeoff thrust;
- the rudder pedal force can't exceed 150 pounds;
- it is not allowed to consider thrust reduction on the operative engine(s);
- during recovery, the airplane can't assume any dangerous attitude;
- the recovery can't require exceptional piloting skill, alertness or strength to prevent a heading change of more than 20 degrees.

In the illustration, the right engine has failed, creating a tendency to turn to the right. By rolling the airplane to a five degree left bank a tendency to turn to the left is created, offsetting to some degree the effect of the thrust asymmetry.

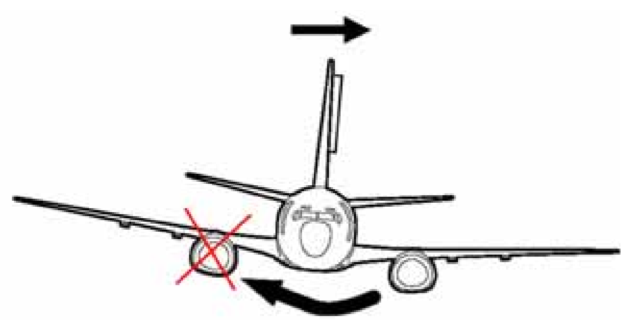


Figure 17-15

The air minimum control speed is not permitted to exceed the 1-g stall speed by more than 13%, or the FAR stall speed by more than 20%.

V_{MCA} with two engines inoperative

For airplanes having three or four engines, it's a further requirement of the regulations that during an approach and landing the failure of a second engine must be considered, and that the corresponding V_{MCA} must be determined. Again, the regulations allow a bank angle of up to five degrees to assist in directional control. The operating engine(s) is/are assumed to be at the go-around thrust setting.

These requirements for minimum control speeds V_{MCA} and V_{MCG} often have a role in the design of the rudders of commercial jet airplanes.

V_{MU} – the Minimum Unstick Speed

When the airplane first leaves the ground during takeoff, its airspeed is at the slowest extreme of its entire flight speed envelope. Never will it be flown more slowly than at this point of liftoff. As a result, it's necessary to ensure that the airplane exhibits acceptable handling characteristics at this low airspeed.

We determine by flight test the minimum possible speed at which the airplane is just capable of lifting off the runway – liftoff is sometimes called "unstick" – and demonstrate that the airplane can then be flown without exceptional piloting skills.

This "minimum unstick speed" is determined by accelerating down the runway with the airplane in its maximum possible nose-up attitude – that is, the attitude of the airplane when its main landing gear is on the ground and its tail is just touching the runway – the "geometry-limited" attitude. The speed at the point at which the tires just leave the ground is called the "unstick speed" and since the tests are conducted at the extreme nose-up attitude, these are the minimum unstick speeds, called V_{MU}.

The minimum unstick tests must be conducted over a wide range of thrust-to-weight (T/W) ratios. The T/W has a significant effect on the unstick speed because the thrust is helping the airplane to achieve liftoff at these high attitudes due to its vertical component, which reduces slightly the amount of lift needed from the wings.

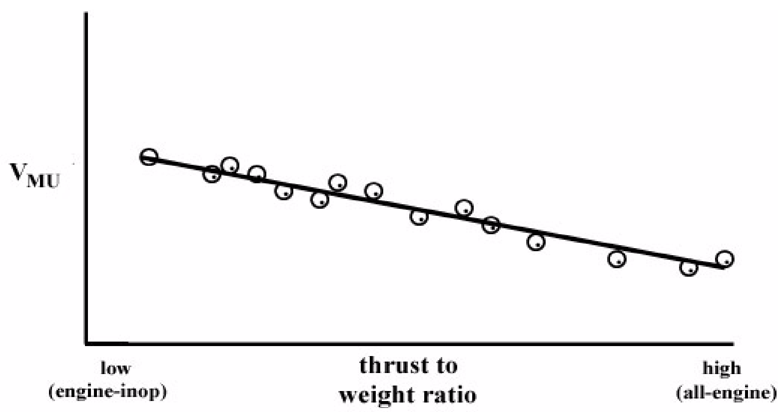


Figure 17-16

When the tests are completed, we have a chart similar to what you see in the illustration. A single line of V_{MU} versus T/W is drawn through the data points.

V_{MU} isn't important to the pilot during takeoff. He doesn't know, or care, what his liftoff speed will be. However, V_R, the speed at which the airplane will be rotated to its takeoff attitude, truly IS important to the pilot, and V_{MU} is one of the two main criteria which determine the rotation speeds. We're going to discuss V_R in just a moment, but first let's talk about V_{LOF} and V_2. You'll understand the reason for this sequence soon...

V_{LOF} – the Liftoff Speed

There's nothing special about V_{LOF} – it's simply the speed at which the airplane will leave the runway following rotation at V_R.

In order to ensure good flight handling qualities at liftoff, the regulations place two constraints on V_{LOF}:

- the liftoff speed must be not less than ten percent faster than the minimum unstick speed V_{MU}, in a normal all-engine takeoff;
- the liftoff speed must be not less than five percent faster than the minimum unstick speed V_{MU}, in the event a takeoff is continued following an engine failure.

These two requirements are based on rotation to the liftoff attitude at the most rapid practical rate.

V_2 – the Initial Engine-Inoperative Climb Speed

The initial climb speed V_2 is the speed that the airplane will attain at a height of 35 feet after lift-off, after being rotated at V_R with an engine inoperative. 35 feet is the value used for the usual dry runway case; for non-dry runway conditions, for the engine-inoperative takeoff case only, this is reduced from 35 feet to 15 feet.

In order to ensure good handling qualities at this early stage of the climbout, the regulations place the following requirements on V_2:

- The V_2 speed may not be less than 13% above the 1-g stall speed, or, for the earlier airplanes, 20 percent above the FAR stall speed;
- The V_2 speed may not be less than 10% above the air minimum control speed.

Obviously, in the course of an all-engine takeoff, due to the all-engine acceleration, the airplane's speed at 35 feet after rotation at V_R will be more than that required speed margin above the stall speed, but that's conservative. Typically, all-engine takeoff initial climb speeds will be ten to twenty knots faster than V_2.

V_R – the Rotation Speed

The speed at which the pilot will pull back on the control column to raise the nose of the airplane to its takeoff attitude is the speed called V_R, rotation speed. This speed has critical implications for takeoff safety, because it directly affects the liftoff speed, V_{LOF} and the initial takeoff climb speed V_2 as well as the takeoff distance.

You have seen above that both V_{LOF} and V_2 have minimum values imposed on them by the regulations. It's the airplane manufacturer's job, then, to publish for the pilots the rotation speed for takeoff for any conditions of weight, altitude, temperature, and flap setting that will satisfy the regulatory criteria.

Further, the rotation speed must be the same for either the all-engine or the engine-inoperative takeoff. It's not reasonable to expect the pilots to decide which of two rotation speeds to use, based on the number of engines operating, at this busy and critical time during the takeoff. Thus, the published rotation speeds must result in liftoff and initial climb speeds that meet or exceed the criteria we listed above <u>with or without an engine failure during the takeoff</u>.

takeoff dynamics

The liftoff speed and the initial climb speed are both directly related to the rotation speed: a slower V_R means a slower V_{LOF} and a slower V_2, a faster V_R means a faster V_{LOF} and a faster V_2. The dynamic characteristics of the airplane during rotation and liftoff are what determine the exact relationship between V_R and V_{LOF} and between V_R and V_2. These dynamic qualities must be established by flight tests. Only then can we know for sure what rotation speeds will be needed to meet the regulatory criteria.

Two different series of flight tests are conducted to determine the V_R – V_{LOF} – V_2 relationships. The difference between the two series of tests is the rate at which the airplane is rotated to its take-off attitude. One series of tests is run using a "rapid" rotation rate of, typically, four to six degrees per second. The second series of tests is run using a "normal" rotation rate, approximately two to three degrees per second.

In order to cover the entire range of possible takeoff conditions lying within the takeoff altitude-temperature limits envelope and up to the certified takeoff weight limit, the tests are run at a wide range of thrust-to-weight ratios. To accomplish this, the tests are conducted both with all engines operating and with a simulated engine failure.

The result of these tests is a precise knowledge of the airplane's takeoff dynamics for any allow-able takeoff conditions. The tests will determine the speed increments between V_R and V_{LOF} and V_2 at both rapid and normal rotation rates. At the rapid rotation rates, the speed increments will be less than for the normal rotation rate.

rotation speeds based on the V_{LOF} criterion

First, we're going to determine the rotation speeds that will result in liftoff speeds that just meet the minima listed above: 105 percent of V_{MU} for the engine-inoperative takeoff, or 110 percent of V_{MU} for the all-engine case. Since we know the speed increments between V_R and V_{LOF} from flight test, this is straightforward. <u>For this criterion, we'll use the speed increments between V_R and V_{LOF} based on the rapid rotation tests</u>. This practice ensures that even in the event of an excessive rotation rate, minimum speed margins will still be met at liftoff.

Refer to the illustration to the right. We have taken the plot of V_{MU} versus T/W that we showed you previously, and have added two lines: over the range of T/W that we would expect for the engine-inoperative takeoff, we have added a line of $1.05V_{MU}$ and for the higher range of T/W that we would anticipate for all-engine takeoffs, we have added a line of $1.10V_{MU}$.

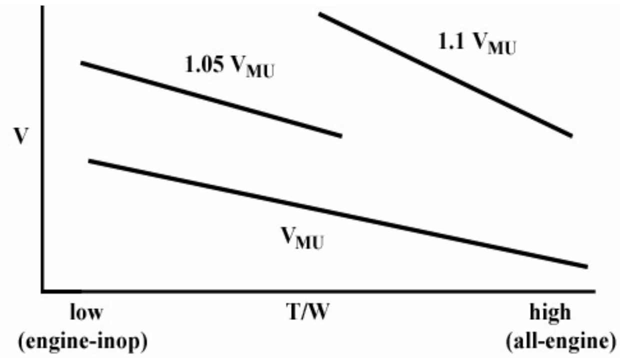


Figure 17-17

What we'll do now is apply to the two new lines the speed increment data we determined from flight test. That is, we're going to take the target liftoff speeds of $1.05V_{MU}$ and <u>subtract</u> the ΔV between V_R and$_{LOF}$ we obtained from the engine-inoperative tests at the rapid rotation rate. Then we're going to take the target liftoff speeds of $1.10V_{MU}$ and subtract the ΔV we obtained from the all-engine tests at the rapid rotation rate.

Here's what we get when that's done. We have added two dashed lines. The dashed line on the left represents the rotation speed with an engine inoperative ($V_{R\text{-}EI}$) that would produce a liftoff speed equal to or greater than $1.05V_{MU}$ with an engine inoperative when a rapid rotation rate is used.

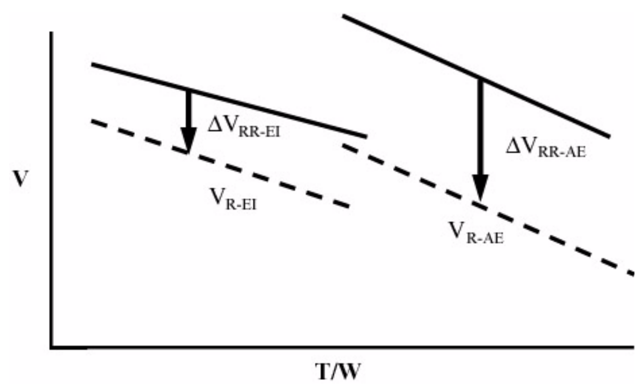


Figure 17-18

The right-hand dashed line is the rotation speed for the all-engine takeoff ($V_{R\text{-}AE}$) that would produce a liftoff speed equal to or greater than $1.10V_{MU}$ with all engines operating when a rapid rotation rate is used.

These two lines are derived by subtracting the ΔV for the rapid rotation engine-inop ($\Delta V_{RR\text{-}EI}$) from the 1.05 V_{MU} line, and subtracting the ΔV for the rapid rotation all-engine ($\Delta V_{RR\text{-}AE}$) from the 1.10 V_{MU} line.

Because the two new V_R lines are based on rapid rotation, they will be conservative for the case of normal rotation. As shown in the illustration to the right, initiating rotation from these V_R values at a normal rate would result in liftoff speeds above the required minimum lines of 1.05 and 1.10.

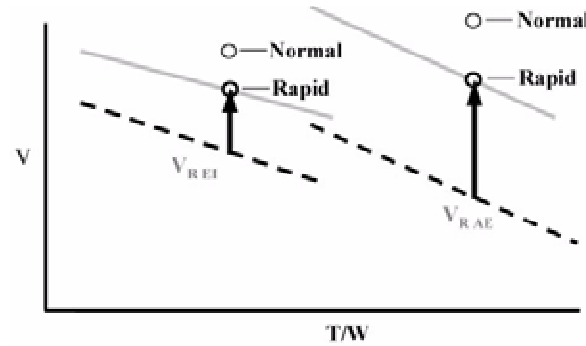


Figure 17-19

There's one final step in the determination of the V_{LOF} -limited rotation speeds: we must ensure that the published rotation speeds will result in liftoff speeds that meet the required speed margins above V_{MU} whether taking off with an engine inoperative or with all engines operating.

Let's say that the V_R lines we have established by flight test are A-B for the engine-inoperative case (lower T/W) and C-D for the all-engine case (higher T/W).

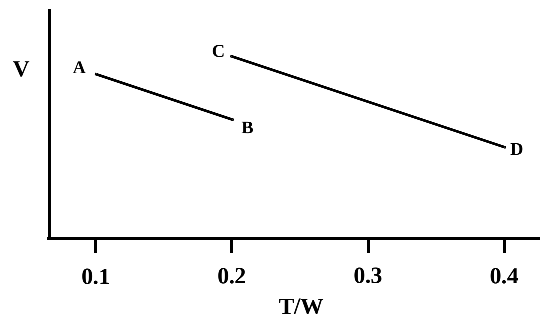


Figure 17-20a

We'll take line A-B and convert it to an equivalent all-engine line, simply by multiplying the engine-inoperative T/W values by two (for a two-engine airplane) to get the equivalent all-engine T/W values. The speed values remain the same, so points A and B simply move to the right by a factor of two on thrust-to-weight ratio.

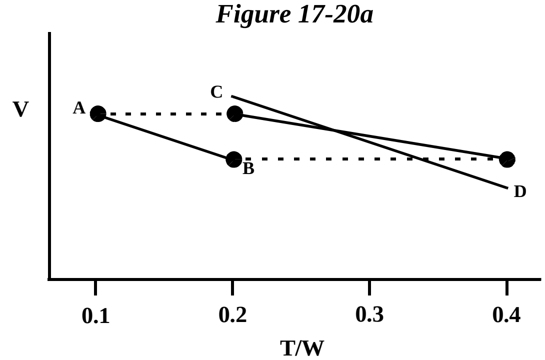


Figure 17-20b

We take the higher of the two lines; we now have a rotation speed schedule as shown by the heavy line in the illustration on the right.

We can now be confident that a single value of V_R calculated at the appropriate all-engine T/W for the given airplane weight, altitude and temperature will ensure that the liftoff speed will always meet the V_{MU} criteria, whether the takeoff is an all-engine takeoff or a takeoff with an engine failure.

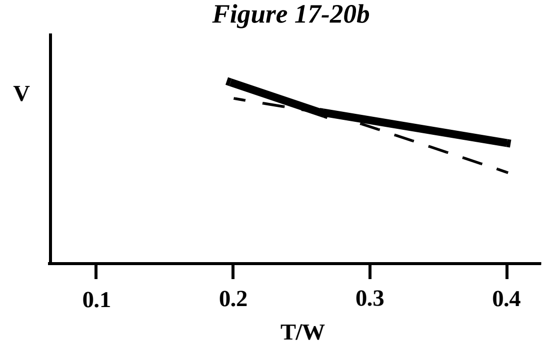


Figure 17-20c

But we have satisfied only one of the two criteria. We must also be sure that the V_R will meet the minimum V_2 requirement.

rotation speeds based on the minimum V_2 criterion

The second task is to determine the rotation speeds that will satisfy the requirement for minimum V_2 in terms of the stall speed. Here, we'll use the flare data coming from the tests conducted with a normal rotation rate.

You'll recall that the regulations require V_2 to be a minimum of 13 percent faster than the 1-g stall speed, or 20 percent faster than the FAR stall speed for the older airplanes.

The process for finding V_R here is similar to, although not exactly the same, as that used to determine the minimum V_R to meet the V_{LOF} requirements.

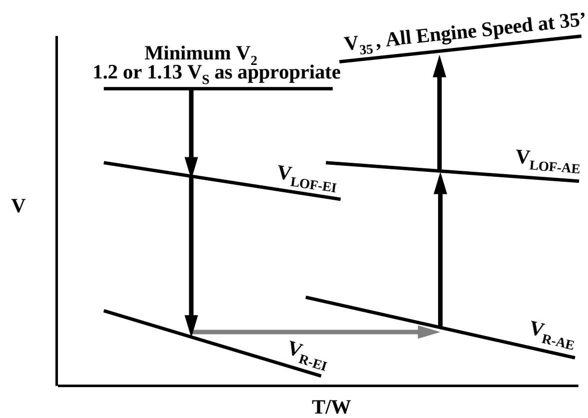


Figure 17-21

We begin with the line of minimum V_2, which will be either 113% or 120% of stall speed, depending on the airplane model.

Now, using the speed increments for a normal rotation rate with one engine inoperative, we move downward and find first the engine-inoperative liftoff speed V_{LOF-EI} based on the minimum V_2, and then the engine-inoperative rotation speed V_{R-EI} also based on the minimum V_2 criterion.

Now, moving to the right horizontally to a new point at the same speed but at double the T/W (assuming an airplane having two engines), we establish the corresponding point on the all-engine rotation speed line, V_{R-AE}.

Finally, using the all-engine normal rotation rate speed increments we can move upward to find the all-engine liftoff speed and the all-engine speed at 35 feet, sometimes called V_{35}.

We need to mention one more fact about the rotation speed based on the minimum V_2 criterion. What we have said in the preceding paragraphs about determining V_R based on minimum V_2 is correct – but for AFM-DPI airplanes, we don't establish the V_R value based on flight test. Since AFM-DPI calculates the entire takeoff process analytically, second by second, it begins with the V_R based on liftoff and then will increase it if necessary to achieve the minimum V_2 at 35 feet.

determination of the V_R for publication

We have now established two separate rotation speed schedules that will:

- ensure that the liftoff speeds will be at least 105% of V_{MU} (with an engine failure) or 110% of V_{MU} (for an all-engine takeoff) with a rapid rotation rate;
- ensure that V_2, the speed at 35 feet, will be at least 120% of the FAR stall speed (older airplanes) or 113% of the 1-g stall speed (newer airplanes) with a normal rotation rate.

The rotation speeds that we will publish for the pilots will be simply the greater of these two, thus ensuring that both criteria will be met or exceeded under all conditions, either with all engines operating or with an engine failure during the takeoff.

Some airplane models have dynamics through the rotation and liftoff phases such that the liftoff criterion on V_R is the more limiting. Other airplane types have different dynamic characteristics and for these airplanes the V_2 criterion will be the more limiting on V_R.

Airplanes having long aft bodies are typically limited by the liftoff criterion. Examples of this are the Boeing 727-200 and the 757. Those airplanes have relatively fast minimum unstick speeds simply because, due to their body length, they're unable to reach high rotation attitudes on the ground. After meeting the minimum unstick speed criterion, when they pass 35 feet their speeds exceed the minimum V2 requirements.

Airplanes having shorter aft bodies and consequently slower minimum unstick speeds are typically limited by the minimum V_2 criterion. An example of this is the Boeing 737-200. Most airplanes, however, are a mix of liftoff-limited and V_2-limited, depending on the flap setting.

an additional consideration in determining V_R and V_2

on some airplane models, such as the 757-300, the 737NG series, the 767-400, and the 777-300, there is one more factor which must be considered in determining the V_R and V_2 values to publish: the possibility of tail strike during rotation. This is most obvious for the very long-bodied airplanes such as the 757-300, but it can also be a consideration for other airplanes particularly at the smaller flap settings which require higher body attitudes at rotation to achieve the necessary lift. This is the case for the 737NG at flaps 1 and 5.

By increasing the V_R and V_2 to values greater than those based on V_{LOF} and minimum V_2, the liftoff body attitude is decreased, which has the desired effect of increasing the aft body height above the runway at liftoff, thereby reducing the likelihood of a takeoff tail strike.

V_{EF} – the Engine Failure Speed

Whenever we compute the allowable takeoff weight for a given runway length, we must assume the possibility of an engine failure during the takeoff roll. We must consider that the engine failure could occur at any time during the takeoff, and must design our takeoff procedures on that basis. What's more important, though, is that for performance purposes we must assume that the engine failure occurs at the most critical time.

When computing takeoff performance, as you'll see in a later chapter, we will assume different engine failure speeds to assess the effect of that variable on the distance to reject the takeoff and also on the distance to continue the takeoff. Thus, V_{EF} is a major factor in takeoff performance.

For now, let's simply say that <u>V_{EF} is the speed at which an engine is assumed to fail during a takeoff</u>.

V_{MBE} – the Maximum Brake Energy Speed

An airplane's wheel brakes are simply friction devices that are used by the pilot to decelerate an airplane. By pressing on the brake pedals in the cockpit, the pilot causes pressure to be applied to a ring of hydraulic pistons on the brakes. These pistons press together a "stack" of rotating brake disks ("rotors") and non-rotating brake disks ("stators"). The friction thus caused between the rotors and stators absorbs the airplane's kinetic energy, slowing it. But the kinetic energy doesn't simply disappear – instead, the brakes turn it into heat.

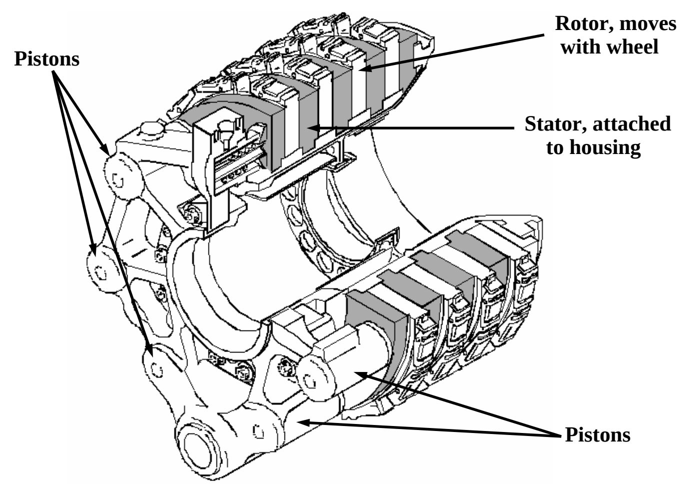


Figure 17-22

Brake manufacturers test their products on *dynamometers*. A dynamometer is a device capable of measuring torque – in this case, the braking torque produced by the brake under test. The dynamometer consists of a rotary mass having a large amount of rotational inertia. The dynamometer's mass is first spun up to by a motor to a high energy level; by applying the brake under test to stop the dynamometer's rotation and measuring the rate of rotational deceleration, the manufacturer can determine the torque. Additionally, dynamometer tests are used to establish the maximum amount of energy that a given brake design is capable of absorbing without becoming hot enough to cause the brakes to overheat. This is confirmed by flight test accelerate-stop testing.

As a brake is used repeatedly in service, its mass decreases because the brake rotors and stators are gradually worn away due to the friction. Reduction of brake mass due to wear means that a worn brake will become hotter, for a given amount of energy absorbed, than a newer brake. As a conservatism, the current rules require that the tests for maximum brake absorption capability must be performed on brakes worn to 100% of their allowable wear.

For models for which the certification flight tests were not conducted using fully worn brakes, an alternative means of accomplishing worn brake accountability was first to allow credit for the beneficial effect of operational reverse thrust in reducing RTO kinetic energy. Second, allowable in-service wear limits for brakes were adjusted until worn brake accountability was achieved on these older airplanes.

In a high-energy rejected takeoff, the brakes will become extremely hot – the brake temperatures will exceed 2000 degrees Fahrenheit. Earlier brakes made of steel could literally melt if they were overused. The newer “carbon” brakes won’t melt but may become literally white-hot. Overheated brakes can cause brake fires.

The speed V_{MBE} is simply the maximum takeoff speed, for a given amount of airplane mass, at which the brakes may be applied in the event of a rejected takeoff without exceeding the brake energy absorption limitations. It assumes that the pilot applies the maximum manual brake pressure.

V_{MBE} expressed in units of true groundspeed is a function only of the certified brake energy limit, the airplane’s mass and the runway slope. This latter factor must be considered since on an uphill slope a small amount of the airplane’s kinetic energy will be transformed into potential energy, thus requiring less effort from the brakes. Also, on an uphill slope a small component of the airplane’s weight acts as a retarding force.

On a downhill slope, however, the brakes must necessarily work harder to stop the airplane since the lost potential energy converts into additional kinetic energy that must also be absorbed by the brakes, and because a small component of the airplane’s weight acts as an accelerating force.

V_{MBE} in units of true groundspeed will not be affected by altitude and temperature. V_{MBE} in units of calibrated airspeed will depend on the takeoff altitude and temperature as well as the wind component along the runway.

V1 – the RTO Action Speed

For every single takeoff, there are three critically important speeds that the pilot must observe carefully. Two of those you already understand: V_R and V_2, the rotation and engine-inoperative initial climb speeds. But the third one needs very careful and detailed discussion because it’s widely misunderstood. That’s the speed we call V_1.

V_1 has historically been referred to as the *decision speed*, but that isn’t really a good name for it. If a pilot making a takeoff experiences some critical situation such as engine failure and he decides at V_1 to reject the takeoff rather than continue it, he would initiate the rejected takeoff (RTO) procedure some short time after V_1 since no pilot can react instantaneously to a decision to reject. Even the best pilots have some delay time between making the decision to reject and then initiating the rejected takeoff procedure by first applying the wheel brakes.

In FAR Part 1, the FAA defines V_1 as “the maximum speed in the takeoff at which the pilot must take the first action (e.g. apply brakes, reduce thrust, deploy speed brakes), to stop the airplane within the accelerate-stop distance. V_1 also means the minimum speed in the takeoff, following a failure of the critical engine at V_{EF} at which the pilot can continue the takeoff and achieve the required height above the takeoff surface within the takeoff distance.”

This definition is also misleading, since it tells us that a takeoff cannot safely be rejected after V_1, nor can it be continued from a speed less than V_1. As you'll see soon in the chapter entitled "Field Length Limited Takeoff Weight", the FAA definition is true only if the airplane is taking off at the maximum allowable weight for the runway length available.[1]

So let's draw the definition of V_1 a little more broadly for now, and we'll be more specific when we get to the chapter on field length limit weight. At that time, we'll discuss in detail the relationship of weight and V_1 to the runway requirements.

a broad definition of V_1

Before every single takeoff, a pilot will be provided with a value of the V_1 speed for that takeoff under the given set of conditions. That V_1 speed might come from the pilot's Quick Reference Handbook (QRH), it might come from an onboard computer such as the Boeing Operational Performance Tool, or it might come from a computer-generated table of allowable takeoff weight for the assigned runway.

The pilot must consider that value of V_1 to be his critical "go/no go" speed. In practical terms, here's what that means to him:

- If he elects to perform a rejected takeoff for any reason, he is expected to apply the wheel brakes to initiate the RTO at or before V_1. If he is unable to apply the brakes at or before V_1 then he must continue the takeoff. An RTO may be initiated after V_1 only as an absolute last resort if the pilot has reason to believe that the airplane is unsafe or unable to fly.
- If an engine failure occurs more than one second – a few knots – before reaching V_1 he must perform an RTO. Continuing a takeoff after an engine failure occurring more than one second before V_1 will cause the airplane to be at an unsatisfactory and potentially unsafe height when passing the end of the runway.

Failure to comply with the two rules listed above would be acceptable only if the pilot is provided with more detailed specific guidance on his takeoff speeds such as a reduced allowable speed for a continued takeoff, an increased or decreased allowable speed for an RTO, or something similar.

minimum V_1 – the earlier airplanes

V_1 has a minimum allowable value. That minimum depends on the airplane model, as the regulatory requirements changed in this regard some years ago.

For earlier airplanes, the V_1 speed was never allowed to be less than the ground minimum control speed. The intent here was to ensure that a continued takeoff would never be attempted from a speed less than the speed at which adequate directional control is assured with an engine inoperative.

1. It would also be true in the case of takeoff with a reduced thrust setting based on the field length limit weight. Refer to the chapter entitled "Reduced Thrust For Takeoff" for discussion of this subject.

minimum V_1 – the later airplanes

For later airplanes, the rule is a bit different. For these airplanes, the engine failure speed is not allowed to be less than the ground minimum control speed. That's different, isn't it? Remember that V_1 is, by definition, the speed occurring one second after engine failure. If the engine failure must not occur at less than the ground minimum control speed, then the V_1 for these later airplanes may not be less than the ground minimum control speed plus the increment of speed gained during the one second interval between V_{EF} and V_1. That new speed, one second of acceleration faster than V_{MCG} has the name V_{1MCG} and it is the minimum allowable value of V_1 on the later airplanes.

The minimum V_1 for the newer airplanes is thus one second of acceleration faster than the minimum V_1 for the older airplanes. That acceleration is, of course, with an engine inoperative. V_{1MCG} will be slightly affected by the airplane's weight since the amount of speed gained during the one second between engine failure and V_1 is a function of acceleration and that, in turn, depends on weight.

maximum V_1

V_1 also has a maximum allowable value. More accurately, it has two maxima, and we must use the lesser of the two:

The first of the maxima is the maximum brake energy speed, discussed above. Remember that V_1 is a brakes-on speed. We can't allow the brakes to be applied at a speed greater than the speed for maximum brake energy, hence V_{MBE} is the upper limit to the allowable range of V_1 values.

The second of the two maxima is the rotation speed. Once the airplane has reached the rotation speed the pilot is committed to continuing the takeoff – it's too late to attempt to stop. Following the initiation of rotation, any attempt to lower the airplane's nose back to the runway could be hazardous and is not permitted.

the V_1 limits: a summary

Here's how we can summarize the upper and lower limits on V_1:

- For the earlier airplanes, V_1 must be greater than or equal to V_{MCG} and must be less than or equal to V_{MBE} or V_R, whichever is less;
- For the later airplanes, V_1 must be greater than or equal to V_{1MCG} and must be less than or equal to V_{MBE} or V_R, whichever is less.

V_{REF} – the landing reference speed

The landing reference speed V_{REF} is used for several purposes.

First and foremost, when calculating the AFM landing distances it's the speed at which the airplane is assumed to be flying as it crosses the runway threshold. The "threshold" may be defined as the approach end of the runway. In some instances, a runway may have some portion of its length not authorized for landing ("displaced threshold"), in which case the threshold is considered to be the end of that portion of the runway that is authorized for landing, and the displaced threshold is clearly marked for the benefit of the pilot.

It's also the speed that the pilot will use for figuring his approach and landing speeds. For example, Boeing standard practice is to teach pilots that their speed across the threshold should be V_{REF} plus one half of the steady headwind component plus the gust intensity, not to exceed V_{REF}+20. In the absence of wind, it is standard practice to use V_{REF}+5 as the speed across the threshold.

The formal definition of V_{REF} is that it is equal to 130% of the FAR stall speed for the earlier airplanes or 123% of the 1-g stall speed for later airplanes. V_{REF} is based on is the stall speed for the landing flap position.

There is a V_{REF} for each certified landing flap setting. For the 747-400, for example, flaps 25 and flaps 30 are both certified for landing and there are corresponding $V_{REF\ 25}$ and $V_{REF\ 30}$ speeds published in the AFM.

For most Boeing airplanes, V_{REF} is also used as the basis for the flap retraction speed schedule and the flap extension speed schedule, as we show below. For this purpose, the V_{REF} for the largest flap setting will be used. It is Boeing standard practice, when no flap setting is indicated for a value of V_{REF}, that such a speed is based on the most deflected flap position – the largest landing flap setting.

Maneuvering Speeds

Maneuvering speeds are those minimum speeds recommended for maneuvering the airplane after takeoff and before landing. The intent of the maneuvering speeds is to ensure adequate margin from stick shaker or initial buffet.

"Adequate" margin in this context is considered to be a speed that will allow maneuvering at bank angle up to 25 degrees plus an unintentional overshoot of 15 degrees, for a total bank angle of 40 degrees. You'll recall that the load factor is given by $\frac{1}{cos(bank\ angle)}$ which, for a 40 degree bank, would correspond to a load factor of 1.305 gees.

Maneuver speeds are chosen to meet a number of criteria:

- simplicity of use (round easy-to-remember numbers);
- commonality with other models;

- maneuver margin to stick shaker equal to or greater than 1.3 gees;
- provide climb capability or acceleration that is close to optimum for that flap setting;
- never greater than the flap placard speeds;
- never less than the reference speed V_{REF} for the given flap setting;
- compatible with actual times required for flap retraction or extension;
- provide relatively constant pitch attitudes in level flight with minimal thrust changes required at the different flap settings.

For all Boeing airplanes subsequent to the 737-300/-400/-500, the maneuvering speeds are expressed as additives to V_{REF}. While it's true that V_{REF} is a landing speed, it can equally well be, and is, is used as the basis for the flap maneuvering speeds during takeoff as well, because it provides a simple and accurate means of adjusting maneuvering speeds for weight.

For example, for the 767 the flaps up maneuvering speed is V_{REF30}+80 knots (or simply V_{REF}+80), for flaps 1 it's V_{REF} +60 knots, for flaps 5 it's V_{REF}+40, for flaps 15 and 20 it's V_{REF}+30. For flaps 25, the maneuvering speed is V_{REF25}, and for flaps 30 it's V_{REF30}. These increments of 20, 40, 60 and 80 are easy to remember, and are consistent with the 747-400, the 757 and the 777 airplanes. The 737NG airplanes use additives of 10/30/50/70 instead of 20/40/60/80.

The subscript in the terms V_{REF30} and V_{REF25} above refer to the fact that for the 767 there are two landing flap settings of 25 and 30. The flaps 25 V_{REF} is used as the maneuvering speed for flaps 25, but for all other flap settings the maneuvering speeds are based on the reference speed for flaps 30.

Operators of older Boeing airplanes may observe that for their airplanes a different style of maneuvering speeds is used. These are referred to as "block" maneuvering speeds. Block maneuvering speeds are fixed speeds (for example, 150 knots/170 knots/190 knots/210 knots) that are chosen to be valid for a "block" of gross weights within a specified range.

Flap Retraction/Extension Speed Schedules

Following the takeoff, the airplane must accelerate from its initial climb speed and, during that acceleration, retract the flaps to their faired (fully retracted) position. Bear in mind that flap extension greatly increases an airplane's drag and reduces its climb performance, so it's desirable to retract the flaps as soon as it's safely possible.

During flap retraction after takeoff, the wing's lift coefficient is decreasing and the stall speed is increasing. Also, takeoff departure paths frequently require turns at low speed and altitude, and those turns will increase the stall speed at any given flap setting.

Retracting the flaps too soon after takeoff could compromise stall margins; delaying retraction until beyond the proper speed may result in approaching or exceeding the flap “placard” speeds. Placard speeds will be discussed in the following topic.

After takeoff, the pilot will follow a “flap retraction speed schedule” that follows the maneuvering speeds discussed above. The general rule is that as he accelerates the pilot will select the next smaller flap setting when he reaches the maneuvering speed for his current flap setting. Thus when taking off in a 767 at flaps 5, the pilot will select flaps 1 when accelerating through $V_{REF30}+40$ and will select flaps up when accelerating through $V_{REF30}+60$. The emphasis on the term “as he accelerates” simply means that when flying at constant speeds, such as may occur for short periods of time during maneuvering after takeoff, the pilot would maintain the maneuvering speed appropriate to his present flap setting, and would not select a smaller flap setting until he is again accelerating toward the flaps-up speed.

During the approach for landing, the airplane must gradually decelerate toward its landing reference speed and extend its flaps in order to cross the landing threshold at the proper speed to ensure the correct landing distance capability. The process here is the opposite of the flap retraction schedule: at $V_{REF30}+80$ the pilot will select flaps 1, flaps 5 will be selected at $V_{REF30}+60$, and so on. Once again, he is following the rule of selecting the next flap setting when decelerating through the maneuvering speed for his present flap setting.

Flap and Landing Gear Placard Speeds

For a number of reasons, it’s necessary to impose restrictions on the airplane’s speed when the flaps are extended. The speed is also restricted when the landing gear is extended or is in operation, whether in the process of extending or retracting. If the placard speeds are exceeded, damage to the flap or landing gear structure could possibly occur.

These are referred to as *placard* speeds because they are written on a placard near the landing gear lever in the cockpit in view of the pilots. These speeds are also published in Section 1 of the AFM, so compliance is legally required.

The flap placard speeds V_{FE} (the subscript FE stands for **F**laps **E**xtended) are a function of the flap setting. The criteria used in establishing the flap placard speeds are principally structural loads on the flaps and flap mechanisms due to aerodynamic forces. Smaller flap settings will have higher placard speeds than the larger flap settings.

There is a 20,000-foot altitude restriction on the use of flaps on Boeing airplanes, specified in the AFM. We are frequently asked why there is this restriction. The answer is simply that we don’t design the flaps for extended operation in flight, but rather to allow us to use slower speeds for takeoff and landing only. As a result, we don’t certify them for use at altitudes greater than 20,000 feet and don’t demonstrate their capabilities above that altitude by flight test.

In the case of the landing gear, there are actually two different placard speeds. One set of speeds are called V_{LO} and M_{LO}, where the V and M mean airspeed and Mach number respectively, and the subscript LO means "**L**anding gear **O**perating" – that is, in transition between retracted and fully extended. The other set of placard speeds are called V_{LE} and M_{LE}. The subscript LE means that the **L**anding gear is in the fully **E**xtended and locked position.

The criteria used in establishing the landing gear speed limitations are several and complex, and are beyond the scope of this discussion. Aerodynamic loads on the landing gear including their actuating mechanisms and the landing gear doors are the major consideration.

V_{MO} and M_{MO} – the Maximum Operating Speeds

V_{MO} is the maximum airspeed, and M_{MO} is the maximum Mach number, up to which the airplane is certificated to be flown intentionally. Because V_{MO} and M_{MO} are specified in Section 1 of the AFM, they may not legally be exceeded in operation.

V_{MO} and M_{MO} appear in the flight manual as shown to the right, as a function of altitude. At the lower altitudes, the airplane's maximum speed will be limited by the value of V_{MO}, in this instance 340 knots. At this higher altitudes, the speed will be restricted by the M_{MO} line. Since for a given Mach number the airspeed depends on the temperature, and hence on the altitude, the line of M_{MO} shows a decreasing airspeed limit at the higher altitudes. The altitude where V_{MO} and M_{MO} are equal is sometimes referred to as the "crossover altitude". Here you see that it occurs at about 26,000 feet.

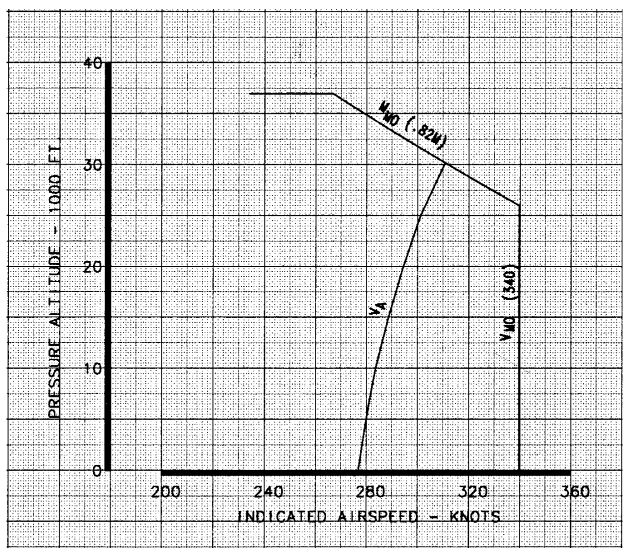


Figure 17-23

Why are there two different speed restrictions, one based on airspeed and the other on Mach number?

Aerodynamic loads are a direct function of equivalent airspeed, and equivalent airspeed is not much different from calibrated or indicated airspeed (the difference being ΔV_C, the compressibility correction. This is discussed in the chapter entitled "Measurement of Airspeed".)

But as altitude increases, for a given indicated airspeed the Mach number increases. Higher Mach numbers have a marked influence on the lift distribution over the wing and consequently on the handling characteristics of the airplane in pitch.

At the higher altitudes, therefore, the airplane's velocity must be more restricted due to flight handling characteristics than aerodynamic loads.

Does this mean that a flight is in danger if V_{MO} or M_{MO} is accidentally exceeded? No, certainly not. FAR Section 25.253 says, in part:

> *(a) Speed increase and recovery characteristics. The following speed increase and recovery characteristics must be met:*
> *(1) Operating conditions and characteristics likely to cause inadvertent speed increases (including upsets in pitch and roll) must be simulated with the airplane trimmed at any likely cruise speed up to V_{MO}/M_{MO}. These conditions and characteristics include gust upsets, inadvertent control movements, low stick force gradient in relation to control friction, passenger movement, leveling off from climb, and descent from Mach to airspeed limit altitudes.*
> *(2) Allowing for pilot reaction time after effective inherent or artificial speed warning occurs, it must be shown that the airplane can be recovered to a normal attitude, and its speed reduced to V_{MO}/M_{MO}, without –*
> *(i) Exceptional piloting strength or skill;*
> *(ii) Exceeding...V_{DF}/M_{DF} or the structural limitations; and*
> *(iii) Buffeting that would impair the pilot's ability to read the instruments or control the airplane for recovery...*

Before explaining this requirement, let's take a minute to explain the speeds V_{DF}/M_{DF} that appear in subparagraph (ii) above. These are called the "demonstrated flight diving speeds" and Boeing is required to demonstrate in flight test that each part of the airplane will be free from excessive vibration, sometimes referred to as "flutter", or structural damage under any appropriate speed and power conditions up to V_{DF}/M_{DF}.

What this regulation is requiring is both realistic and practical. It says simply that if flying at the maximum allowable airspeed V_{MO} or Mach number M_{MO}, a reasonable inadvertent speed increase must not cause the airplane to exceed the demonstrated diving speeds and it must not require unusual pilot skill or strength to recover the airplane back to V_{MO}/M_{MO}.

By protecting V_{MO} and M_{MO} as required, we can be confident that flight up to V_{MO} or M_{MO} is safe, even allowing for an unknown upset event.

Chapter 18: Calculating Takeoff Distances

Introduction

In the chapter entitled "The Flight - a Performance Overview", we listed the different FAA regulatory requirements that control the maximum allowable takeoff weight of an airplane.

There are many different requirements regulating an airplane's allowable takeoff weight, but the principal ones among that number are the requirements concerning takeoff distance, climb, and obstacle clearance. In this chapter we'll be discussing the takeoff distances; in the following chapter, we'll show you how the takeoff distances relate to the field length limit weight and its associated V_1 speed.

While it's unlikely that the typical performance engineer will ever need to calculate the takeoff distances required for a given takeoff weight, it's still very instructive to understand how it's done. This discussion will hopefully provide the reader with a more complete understanding of the subject of the allowable takeoff weight for a given runway with a given set of takeoff conditions. In a few places where proprietary methods are involved, we will offer discussion without providing complete details. This won't reduce the quality of the discussion, however.

What we're going to be showing you in this chapter are the methods for calculating the takeoff distances known as *point distances*. By that we mean that the distances we calculate will be the distances traveled by a single point on the airplane, as if the airplane were all concentrated in that one point. In the next chapter you'll see how to apply two length decrements known as a "lineup allowances" to the published runway lengths available, in order to account for the fact that an airplane is <u>not</u> a single point. The lineup allowances depend on the geometry of the airplane and its landing gear.

NOTE: In this and the following chapter we will be considering only takeoff on a dry runway. Takeoff performance on wet runways or runways covered with a contaminant such as slush will be discussed in detail in a later chapter.

Takeoff Distance Rules

Before we can show you how to calculate the takeoff distances, we need to show you the rules regulating the structure of those distances. We'll be showing you here the FAA rules, but the rules used for certification to other agencies' requirements are usually only slightly different. The FAA rules will serve well to illustrate the methods involved in calculating the distances.

The rules have changed a number of times over the years, particularly those rules relating to the rejected takeoff (RTO) accelerate-stop distance. We'll start by showing you the current rules and then we'll show you some of the earlier versions of the rules.

The current FAA regulations, in effect since 1998, require that we consider four cases when determining the minimum required takeoff distance for a given takeoff weight, or the allowable takeoff weight for a given runway length:

- the case of an all-engine takeoff, in which the airplane accelerates to the rotation speed V_R then rotates to the liftoff attitude and begins its initial climb, with all engines operating throughout the takeoff;
- the case of an engine failure during a takeoff following which the pilot elects to continue the takeoff rather than perform a rejected takeoff (RTO);
- the case of an engine failure during a takeoff following which the pilot elects to conduct an RTO;
- the case of a takeoff during which some event occurs that causes the pilot to elect to conduct an RTO even though all engines continue operating normally.

When we know how to compute the distances for those cases, we'll be able to relate the distances to the takeoff weight. Then, given a runway's characteristics including its length, we'll be able to calculate the allowable takeoff weight for that runway. That weight will always be based on the most conservative – the longest – of these distances.

Let's discuss each of them in turn.

the all-engine takeoff distance

More than 99% of all takeoffs are routine affairs with nothing unusual about them. The takeoff distance, then, is simply the distance from the point at which the airplane begins its takeoff roll to the point at which the lowest point on the airplane (usually but not always the main landing gear) achieves a height of 35 feet above the takeoff surface, where the takeoff is considered to end.

This assumes that the airplane accelerates to the rotation speed V_R appropriate to the weight, altitude, temperature and flap setting, rotates to the liftoff attitude, leaves the takeoff surface at the liftoff speed, and accelerates to the all-engine initial climb speed at 35 feet.

Regulatory agencies, however, traditionally take the view that routine variations exist in everyday operational factors such as pilot technique and airplane weight, which might make the actual takeoff distance somewhat greater than the computed value. Therefore, for the purposes of calculating the allowable takeoff weight, the regulations require us to add a 15% conservatism to the calculated all engine takeoff distance.

the engine-out accelerate-go distance

It's possible that an airplane will need to make a continued takeoff following an engine failure, rather than reject the takeoff and stop. This would most probably happen in the event of an engine failure after V_1, in which case the standard procedures call for a continued takeoff.

The structure of the one engine inoperative accelerate-go distance is:

- acceleration with all engines operating from the beginning of the takeoff roll to the point of an engine failure;
- one engine inoperative acceleration from the engine failure speed V_{EF} to the rotation speed V_R;
- one engine inoperative rotation, liftoff, and acceleration to the engine inoperative initial climb speed V_2 at 35 feet above the takeoff surface.

The accelerate-go distance ends at the 35-foot point. We do not include any additional measure of conservatism in this case as we did in the all-engine accelerate-go case.

the engine-out accelerate-stop distance

The structure of this case is as follows:

- all-engine acceleration to the engine failure speed V_{EF};
- one second of one engine inoperative acceleration to V_1;
- as a conservatism, two seconds of additional distance at the RTO initiation speed V_1;
- transition to the full stopping configuration which includes wheel brake application, then retardation of the thrust levers to the idle position, and then extension of the speedbrakes;
- deceleration to a full stop with one engine inoperative and the remaining engine(s) at idle thrust; credit is not taken for the use of reverse thrust on the operating engine(s).

the event-caused all-engine accelerate-stop distance

Engine failure during takeoff is rare these days, but other events can and occasionally do occur that may cause the pilot to elect to reject the takeoff.

What might constitute an *event* that would cause a pilot to decide to reject a takeoff with all engines operative? At low speeds, a number of conditions might cause this, including a tire failure or a system malfunction warning. At speeds approaching V_1, however, pilots are taught that the only reasons to reject a takeoff are either an engine failure or fire, or the perception that the airplane has become unsafe or unable to fly. <u>After</u> V1, a pilot might <u>in very rare circumstances</u> decide to reject a takeoff if he believes that the airplane is unsafe or unable to fly, but that case is not considered in our calculations. <u>Everything in a pilot's training forbids initiating an RTO after V_1 except in the most extreme circumstances</u>.

To account for the possibility of an all-engine RTO, this case assumes that all engines operate normally throughout the acceleration and RTO procedure:

- all-engine acceleration to a speed at which some event occurs;
- one second of all-engine acceleration to V_1;
- as a conservatism, two seconds of additional distance at the RTO initiation speed V_1;

- transition to the full stopping configuration which includes wheel brake application, followed by retardation of the thrust levers to the idle position and then extension of the speedbrakes;
- deceleration to a full stop with all engines operating at idle thrust; credit is not taken for the use of reverse thrust.

Notice that in both of the accelerate-stop cases above, the speed V_1 is defined as that speed occurring one second following the engine failure or the event. Thus <u>the only difference between the engine-out and event-caused RTO distances is the thrust used in the calculation following the engine failure or the event</u>.

Because any rejected takeoff is complex and potentially hazardous, the regulations have always required that we include a margin of conservatism in the calculated accelerate-stop distances. You'll observe the third bullet in both cases above: two seconds at V_1 speed. Assuming a V_1 of, say, 135 knots, that's 456 feet of additional distance as a conservatism. Bear in mind, though, that in normal line operations, when an engine failure or event isn't expected, two seconds can pass by very quickly.

which RTO distance is greater?

Consider the two accelerate-stop cases described above. We're required to use the more conservative of the two as our accelerate-stop distance. Which will be the more conservative – that is, the longer distance – of the two?

Typically, the event-caused all-engine accelerate-stop distance is longer due to the greater amount of idle thrust during the deceleration segment. However, there are exceptions to this:

For the same takeoff conditions, the acceleration distance from V_{EF} to V_1 is greater in the engine-out case than it is in the all-engine case due to the lesser amount of takeoff thrust with an engine inoperative. The deceleration distance is greater in the all-engine case due to the greater amount of idle thrust with all engines operating. Under some conditions, the difference in the distance from V_{EF} to V_1 is more than the difference in the deceleration distance, resulting in a greater accelerate-stop distance for the engine-out case.

This is more evident when looking at a takeoff on a wet runway. As you'll see in the chapter entitled "Takeoff On Non-Dry Runways", credit is taken for reverse thrust when the runway is not dry. In that case, the all-engine deceleration distance is based on the use of all engines in reverse thrust, where the engine-out deceleration distance is based on one engine in reverse (or two symmetric engines in reverse, in the case of the 747) engines in reverse.

Whichever of the two distances is greater will be called the event/engine-out accelerate-stop distance.

For simplicity and clarity, in this chapter we will show the distance calculations for the all-engine RTO case.

defining the accelerate-stop procedure

After an airplane commences its takeoff roll and reaches the event speed (or engine failure speed), if the decision is made to reject the takeoff the regulations state how the transition to the stopping configuration is to be calculated.

Calculation of the transition to the stopping configuration follows a specific schedule of time intervals between the steps of the transition. They can be shown as you see in the illustration to the right. The horizontal scale of the diagram represents time.

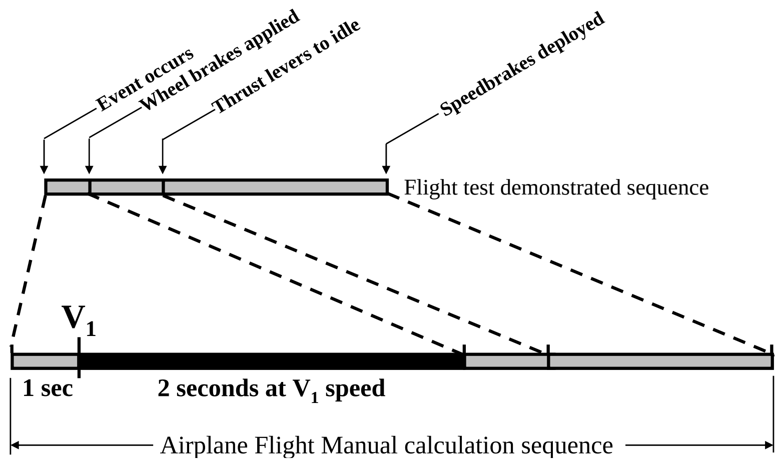


Figure 18-1

The upper gray bar represents the sequence of events in the transition from occurrence of the event up to the time the airplane is in the full stopping configuration as demonstrated during flight testing. In flight test, things happen pretty fast: the brakes are quickly applied after the simulated occurrence of the event, the thrust levers are almost as quickly retarded to idle, and then the speed brakes are deployed a short time after that.

The lower bar shows the sequence of events and their timing as used in the calculation of the certified accelerate-stop distance. As certified, the sequence is longer (and the distance correspondingly greater) in two ways: one second is assumed between the occurrence of the event and V_1, and two seconds at V_1 speed are added. Following that, the time intervals are as demonstrated in flight test. The brakes are assumed to be applied at the end of the two-second interval, then the thrust levers are retarded to idle, and the speedbrakes are deployed.

Clearly the certification sequence is conservative compared to what can be demonstrated in flight test. That conservatism isn't unreasonable. The tests, after all, are conducted by experienced flight test pilots in a controlled test environment in which the occurrence is anticipated. It's unlikely that even a well-trained and experienced line pilot on a routine flight would be able to respond to an unexpected event and conduct an RTO within a comparable time and distance.

The accelerate-stop rule discussed above is taken from FAR Part 25.111 Amendment 25-92, and is known as the *Amendment 25-92 rule*. It was used in the certification of the 737-600/-700/-800/-900, the 747-400F, the 757-300, the 767-400, and the 777-200LR and -300ER, and the 787.

While all of the airplane models listed above are certified to the same set of rules, it should be understood that the time intervals between brake application and thrust levers to idle, and between thrust levers to idle and speedbrakes deployed, vary from model to model.

prior to Amendment 25-92

The only significant difference between what we've described above and prior regulations lies in the accelerate-stop definition.

Prior to 1998, when the current rules were enacted, the event/engine-out accelerate-stop definition was more conservative. Here too, it required consideration of both an engine-out case and an event-caused all-engine case, whichever was more conservative:

- all-engine acceleration to a speed at which some event occurs that causes the pilot to decide to reject the takeoff, or at which engine failure occurs;
- all-engine/engine-out acceleration to V_1 at which speed the RTO procedure is initiated, then:
- two seconds of additional all-engine/engine-out acceleration distance, then:
- transition to the full stopping configuration which includes wheel brake application, retardation of the thrust levers to the idle position, and extension of the speedbrakes, then:
- deceleration to a full stop with all engines operating at idle thrust.

This is more conservative than the regulation now in force for several reasons: first, the distance from V_1 to the brakes-on speed is very slightly increased, but more significantly the brakes-on speed is faster so the stopping distance will be greater.

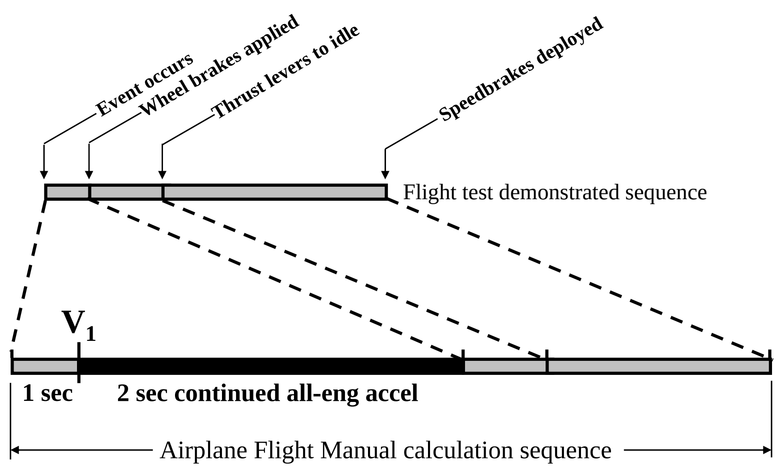


Figure 18-2

This definition was taken from FAR Part 25.111 Amendment 25-42, sometimes called the *Amendment 25-42* rule. It was only applied to the 777-200 and -300 airplanes (basic versions, not the models designated as ER or LR).

prior to amendment 25-42

Earlier yet, the accelerate-stop distance definition was different again, and in this earlier definition <u>no consideration was made of an event followed by an all-engine stop</u> – the only accelerate-stop requirement to consider was that with an engine inoperative:

- all-engine acceleration to the speed V_{EF} at which an engine failure occurs;
- one second of one engine inoperative acceleration to V_1 at which time the speed brakes are applied;
- transition to the full stopping configuration including retardation of the thrust levers to the idle position and then extension of the speedbrakes;

- as a conservatism, two seconds of distance at constant velocity at the speedbrakes-deployed speed;
- deceleration to a full stop with one engine inoperative.

This definition was used in the certification of the 737-300/-400/-500, the 757-200, the 767-200 and -300, and the 747-400.

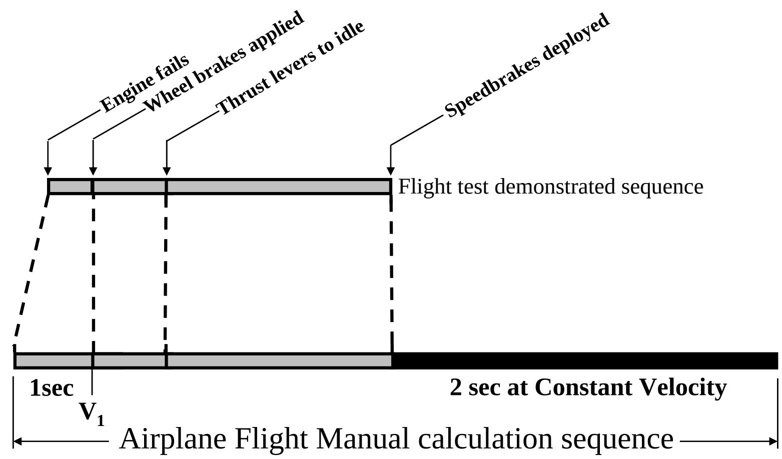


Figure 18-3

even earlier

The earlier Boeing commercial jet airplanes, from the 707 through the 747-300, were certified to several other slightly different sets of accelerate-stop distance requirements.

For the 707-300, for example, V_1 was considered to occur at the same time as engine failure; brake application occurred 0.39 seconds later, followed by throttles to idle and speedbrake deployment. The total demonstrated time was 1.73 seconds, the AFM calculation sequence was a total of 3.73 seconds.

For the earlier 727-100 and -200 airplanes, V_1 was considered to occur <u>three</u> seconds after engine failure, followed by brake application, throttles to idle and speedbrake application. – The demonstrated times were a total of 1.38 seconds, but the AFM calculations assumed a demonstrated time of 1.73 seconds, the same as the 707-300, for conservatism. The reason for such a long time delay before applying the brakes was that, due to the location of the engines on the aft fuselage, it was more difficult for the pilots to sense engine failure, hence a longer response time. The AFM calculation sequence was a total of 6.73 seconds: three seconds to V_1, 1.73 seconds demonstrated total time, plus an additional two seconds for pilot reaction delay times.

Later in the 727 program, in recognition of the difficulty in sensing engine failure, a light was installed on the instrument panel to indicate engine failure when it occurred; since this facilitated earlier recognition, the time from engine failure to V_1 was reduced to <u>one</u> second, and the total AFM calculation sequence was then 4.73 seconds.

For the 737-100 and -200, including the -200ADV (advanced) model, V_1 is defined as one second after engine failure, with brake application 0.29 seconds later. The demonstrated times were a total of 0.95 seconds, to which were added two seconds more for pilot reaction delay times, thus the AFM calculation sequence was a total of 3.95 seconds.

For the 747-100, -200, -300 and SP models, V_1 is for the first time defined as the brakes-on speed, occurring one second after engine failure. The total demonstrated time from engine failure to speedbrake deployment was 1.24 seconds; with one second from engine failure to brake application and another two seconds for pilot reaction delay times, the AFM calculation sequence was a total of 4.24 seconds.

summarizing the takeoff distance rules

You see from the above that there has been a bewildering assortment of different rules governing the way that the certified engine-out accelerate-stop distances should be calculated.

Although the accelerate-stop rules have changed over time, the rules governing the all-engine takeoff distance rule and the one engine inoperative accelerate-go distance rule have remained unchanged.

Further, it's worth noting that in all of the different versions of the accelerate-stop rules, there has always been two seconds of extra distance included as a conservatism to allow for reasonable pilot reaction times. The only difference has been in how the two seconds of distance has been calculated.

V_1 and the brakes-on speed

From the paragraphs above, you've seen that for the earlier airplanes, brake application is considered to occur shortly after V_1; for some later models, V_1 and brake application are considered to occur simultaneously; for the Amendment 25-42 and Amendment 25-92 airplanes, brake application is considered to occur two seconds after V_1.

It is essential to understand that, despite the fact that the accelerate-stop distances for some of the airplanes are <u>calculated</u> assuming brake application after V_1, <u>for practical purposes</u> Boeing considers V_1 in all cases to be the brakes-on speed. Any time interval between V_1 and brake application used in calculating the certified accelerate-stop distances is considered to be an "invisible" conservatism, included in the certified distances but not a part of the rejected takeoff procedures.

It is standard Boeing policy, when teaching rejected takeoff procedures, to teach that <u>if the pilot can't apply full brake pressure at or before the V_1 speed, the pilot must continue the takeoff rather than attempt the RTO</u>. The <u>only</u> exception to this rule is if the pilot considers the airplane to be unsafe or unable to fly.

This is consistent with the definition of V_1 contained in FAR Part 1:

> *V1 means the maximum speed in the takeoff at which the pilot must take the first action (e.g., apply brakes, reduce thrust, deploy speed brakes) to stop the airplane within the accelerate-stop distance. V1 also means the minimum speed in the take-*

off, following a failure of the critical engine at VEF , at which the pilot can continue the takeoff and achieve the required height above the takeoff surface within the takeoff distance.

Calculating the All-Engine Acceleration Distance

Now that you understand the structure of the certified takeoff distances, let's move on to see how they're calculated. This is where we go back to basic physics, because the acceleration distance is based very simply on the rule of F = ma, force equals mass times acceleration.

In the following calculations, we're going to assume an airplane having two engines, weighing 240,000 pounds, taking off on a sea level standard day on a runway having no slope, with no wind. In the engine-inoperative accelerate-go case we will assume that the engine fails at a speed of 150 knots. V_R is 160 knots for these conditions.

forces on the airplane during acceleration

The forces acting on an airplane on the ground during the takeoff acceleration are not constant and therefore must be calculated using a step-integration method.

On a dry pavement, the forces acting on the airplane parallel to the runway as it accelerates are as shown in the following illustration.

Causing the acceleration is the thrust of the engines.

The thrust is not constant, it varies with speed.

If there is a downhill slope to the takeoff runway, there will be a small contribution to the acceleration force from the component of the airplane's weight parallel to the runway.

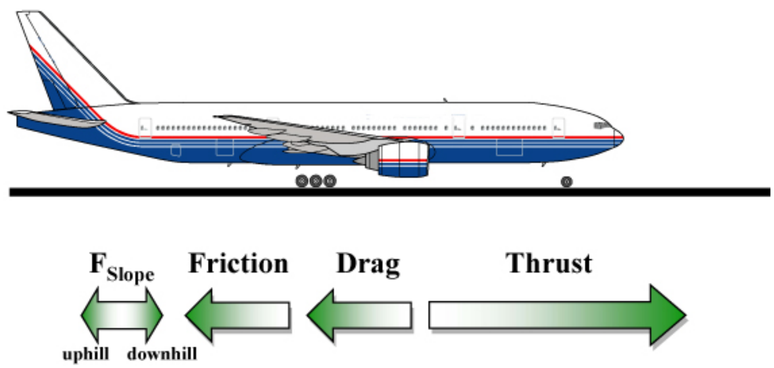


Figure 18-4

Acting in opposition to the acceleration are, first, the aerodynamic drag of the airplane due to its velocity; second, the "rolling friction" drag between the tires and the pavement; third, the component of the airplane's weight along the runway if the runway has an uphill slope. Let's look at these forces mathematically. The summation of the forces can be expressed as:

$$\Sigma forces = T - D - \mu(W - L) - W\sin\phi \qquad \text{(eq. 1)}$$

where T is the thrust of the operating engine(s)
D is the aerodynamic drag

μ is the rolling coefficient of friction retarding the acceleration
W is the airplane weight
L is the aerodynamic lift force
ϕ is the runway slope in degrees

The third term, $\mu(W-L)$ is the friction force, which is equal to the coefficient of friction μ multiplied by the net downward force acting on the tires.

The friction force and the rolling coefficient of friction μ deserve more discussion.

Imagine for a minute that you have an airplane raised above the ground on jacks. The brakes are off. You spin one of the wheels. The wheel, after being spun, gradually slows down and stops. Why? Because of the small but inevitable friction within the wheel bearings, causing rotational energy loss and thus a gradual slowing of the wheel rotation rate.

When a wheel is rotating <u>on the ground</u>, there's another rotational energy loss: the energy required to cause the tire to flex as it rotates. The tire, particularly the sidewalls, flexes as the tread comes into contact with the ground and causing that flexure requires energy.

So you see that energy is required to rotate an airplane wheel and tire due to internal friction and tire flexure. Where does this energy come from? It comes from the friction between the tire and the ground. That friction force, acting aftward, creates the torque necessary to overcome the energy losses due to the internal friction of the bearings plus the tire flexing.

The friction force, since it acts in the aftward direction, retards the airplane's forward motion on the runway. The force is a direct function of the downward force placed on the airplane's landing gear: that is, the airplane weight minus any lift it is generating. The coefficient of friction μ, when multiplied by the downward force on the landing gear, yields the total "rolling friction" drag. As you see, the value of μ includes both the internal friction and tire flexure losses.

Runway slope is designated as a positive number if it's uphill and a negative number if it's downhill. For practical purposes at very small angles, the sine of the angle of the runway slope is equal to the angle expressed in radians, which in turn is equal to the runway slope expressed as a decimal, such as 0.02 meaning a 2% slope.

the acceleration equation

We can say that:

$$\Sigma forces = \frac{W}{g} \times a \qquad \text{(eq. 2)}$$

where g is the acceleration of gravity
a is the acceleration along the runway

Thus:

$$\frac{W}{g}a = T - D - \mu(W - L) - W\sin\phi$$

or:

$$a = \frac{g}{W}[T - D - \mu(W - L) - W\sin\phi]$$

We'll combine lift and drag into one term, since they're both a function of dynamic pressure:

$$a = \frac{g}{W}[T - \mu W - (D - \mu L) - W\sin\phi]$$

Now, expressing D and L aerodynamically in terms of the lift and drag coefficients:

$$a = \frac{g}{W}[T - \mu W - (C_D - \mu C_L)qS - W\sin\phi] \qquad \text{(eq. 3)}$$

where C_D is the drag coefficient
C_L is the lift coefficient
q is the dynamic pressure
S is the reference wing area

From Equation 3, you can see that the acceleration is affected by the dynamic pressure q. The term $(C_D - \mu C_L)$ is always a positive value and thus, since q increases with airspeed, the acceleration decreases as velocity increases.

The thrust of the engines is also a function of the airplane's airspeed, generally decreasing as the airplane accelerates along the ground, as explained in the chapter entitled "Jet Engine Fundamentals". The thrust is shown in the following table as a function of airspeed.

Given Equation 3, let's calculate the acceleration *a* for a range of airspeed. For illustration purposes, let's assume an airplane weight of 240,000 pounds, a sea level standard day, and a level runway with no wind. We'll use the following values for the parameters: $(C_D - \mu C_L)$ = 0.08, μ = 0.0165, and S_{ref} = 1951.

Remember that, <u>for V in feet per second</u>, $q = 0.5\rho V^2$ and that ρ for a sea level standard day is 0.02377 slugs per cubic foot.

Using the variables listed above, and assuming the available thrust to be shown in the table as a function of speed, we can compute the acceleration values shown in this table:

GS - knots	TAS - knots	dynamic pressure q	F - thrust	F - slope	μW	$(C_D$-$\mu C_L)$ qS	accel ft/sec/sec	accel knots/sec
0	0	0.00	35532	0	3960	0	9.0	5.3
20	20	1.35	34653	0	3960	211	8.7	5.2
40	40	5.42	33775	0	3960	846	8.4	5.0
60	60	12.19	32896	0	3960	1903	8.0	4.8
80	80	21.67	32017	0	3960	3382	7.6	4.5
100	100	33.86	31139	0	3960	5285	7.1	4.2
120	120	48.75	30260	0	3960	7609	6.6	3.9
140	140	66.36	29381	0	3960	10357	6.0	3.5
150	150	76.18	28942	0	3960	11890	5.6	3.3
160	160	86.67	28503	0	3960	13527	5.3	3.1

Table 18-1

The acceleration versus velocity can be graphed as shown here.

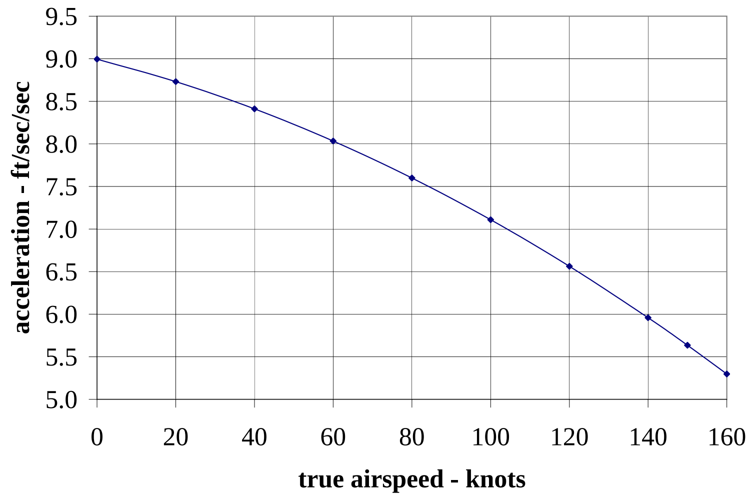


Figure 18-5

You can see that the initial acceleration is quite high – about nine feet per second per second – but this decreases to less than six feet per second per second by the time the airplane has accelerated to 160 knots.

So we know the acceleration capability, how do we get from that to distance?

The basic equation for distance says that distance traveled equals velocity multiplied by time. That would be fine if the velocity were constant, but it's not.

Let's look at the acceleration down the runway as the sum of a series of small steps of velocity change (that is, *acceleration*), for example, from zero to 20 knots, 20 to 40 knots, 40 to 60 knots, and so on. If we add up these incremental distances, the sum would be the total acceleration distance. This makes the total distance much easier to compute, because over a relatively small speed increment such as twenty knots, the acceleration is almost constant.

For a small velocity increment ΔV, we can find the average velocity from:

$$\bar{V} = \frac{\Delta s}{\Delta t}$$

where Δs is the incremental distance over the speed increment (this is what we're trying to calculate

$\bar{V}$ is the average velocity over the velocity increment

Δt is the incremental time over the velocity increment

Also, the average acceleration can be found from:

$$a = \frac{\Delta V}{\Delta t}$$

where a is the acceleration
ΔV is the speed increment

Solving for the incremental distance:

$$\Delta s = \frac{\bar{V}\Delta V}{a} \qquad \text{(eq. 4)}$$

Suppose, for example, that we want to find the incremental distance from zero to 20 knots airspeed, using the acceleration data we developed above. We can see from the data that the acceleration over that velocity increment would be approximately 8.85 feet per second per second. The average speed would be ten knots, and the speed increment is 20 knots, so (remembering to put in 1.6878 twice to convert knots to feet per second):

$$\Delta s = \frac{10 \times 20 \times 1.6878^2}{8.85} = 64\ feet$$

We could repeat this for the speed increment from 20 to 40 knots, 40 to 60 knots, and so on. Here's a sample table following this method:

GS - knots	TAS - knots	accel ft/sec/sec	s - feet	Σs - feet
0	0	9.0		0
20	20	8.7	64	64
40	40	8.4	199	264
60	60	8.0	346	610
80	80	7.6	510	1120
100	100	7.1	697	1817
120	120	6.6	917	2734
140	140	6.0	1183	3917
150	150	5.6	713	4630
160	160	5.3	808	5438

Table 18-2

You see that the second velocity increment would have a distance increment of 199 feet, so the total distance from zero to 40 knots would be 264 feet.

The total distance would be 610 feet to 60 knots, and so on.

The total distance to accelerate with both engines operating from zero to 160 knots would be 5438 feet.

Remember that in each of these velocity increments we have been treating both acceleration and velocity as constants. We know that this isn't exactly correct, so how accurate are the distances calculated in this table?

We could make it more precise by taking a greater number of smaller speed increments. For example, we could accelerate from zero to one knot, one knot to two knots, and so on. Would this yield significantly more accurate results? Logic says yes – correctly – but in fact if you were to step integrate using one knot increments from zero to 150 knots, the new distance number would be 4635 feet, a difference of only five feet.

This method is referred to as “step integration”, and it’s the way that Boeing calculates all takeoff acceleration distances in its computer programs. (Of course, it works equally well for deceleration distances, as you’ll see later.)

Summary

Earlier in this chapter we said that we would need to compute four different takeoff distances: the all-engine takeoff distance, the engine-out accelerate-go distance, the engine-out accelerate-stop distance, and the all-engine accelerate-stop distance. These four distances are required in order to find the maximum allowable takeoff weight for the available runway length.

In the paragraphs above, you’ve seen how to calculate acceleration distances with all engines operating. That acceleration segment is part of all four of the certified distances as follows:

- for the all-engine takeoff distance: use this method to compute the acceleration distance from the beginning of the takeoff roll, sometimes called “brake release”, up to the rotation speed V_R;
- for the engine-out accelerate-go and accelerate-stop distances: use this method to calculate the distance from brake release up to V_{EF} at which the engine is assumed to fail;
- for the event-caused all-engine accelerate-stop distance: use this method to calculate the distance from brake release up to the V_1 speed.

Engine Spindown

Before we can calculate the engine-inoperative accelerate-go distance and the all-engine accelerate-stop distance, we need to know how engine thrust behaves following either engine failure (or fuel cutoff) or rapid retardation of the thrust levers to idle, known as “throttle chop”.

When an engine fails, thrust doesn’t immediately drop to zero – it passes through a period of time during which the engine decelerates from its takeoff thrust setting. This period is known as the *spindown*. Similarly, when the thrust levers are retarded to the idle position, the thrust doesn’t immediately drop to idle thrust. There is some interval of time following an engine failure or a throttle chop during which the engine’s rotors will spin down (or *spool down)* from the takeoff RPMs, due to their rotational inertia. In order to calculate

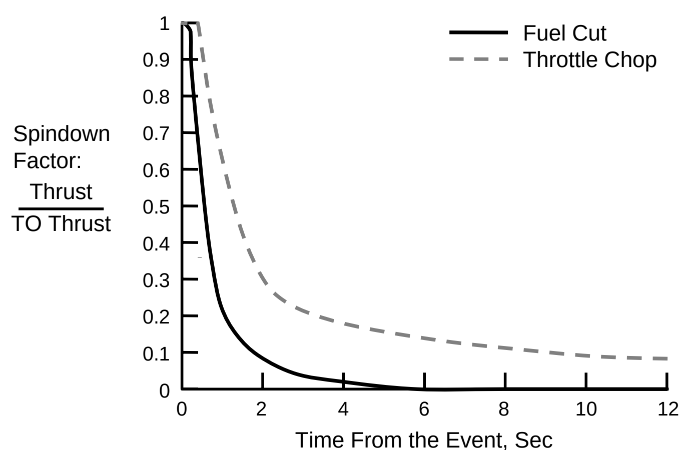


Figure 18-6

the acceleration distance following engine failure or throttle chop, we need to know the amount of thrust that will continue to be generated during the spindown period. This is a function of time.

A plot of an engine's residual thrust versus time following failure is sometimes termed a "spin-down" plot. The plot presents a ratio of the current thrust divided by the full thrust at the takeoff setting, as a function of time. This is shown in Figure 18-6 above.

You see that the thrust decay depends on whether the thrust lever was retarded to idle or the fuel cutoff lever was placed to the cutoff position. An engine failure is considered to be the same as a fuel cutoff in terms of the spindown factor.

We will use the more conservative of these two different spindown curves, depending on what distance is being calculated:

- For calculating the continued takeoff distance following an engine failure, we will use the "fuel cut" curve. We do this because that spindown curve yields slightly less thrust and thus a slightly longer acceleration distance;
- For calculating the rejected takeoff distance following an event, we will use the "throttle chop" curve. In this case, throttle chop is the conservative condition since it yields slightly more thrust and hence a slightly longer deceleration distance. Also, it is more representative of the case where there has been no engine failure.

Calculating the Engine-out Acceleration Distance

We need to be able to calculate the acceleration distance following an engine failure for one of the four certified distances. Specifically, for the engine-out accelerate-go case, we must calculate the segment of distance from engine failure to the rotation speed V_R. Now that we know how to account for engine spindown, we can do that.

As you've seen, during the spindown period the thrust of the failed engine decays very quickly as a function of time. To find the distance the airplane travels during this period requires a step integration process as a function of <u>time</u> rather than as a function of <u>speed</u> as we did previously. Doing so, however, creates a complication because, although we know the spindown factor as a function of time, we know the thrust only as a function of speed.

Here's the complication: for the very first increment of time after engine failure, one second, we need to calculate the average acceleration in order to find the airplane speed after one second. But we can't find the average acceleration because we don't know the thrust after one second. And we don't know the thrust after one second because we don't know the speed after one second. And we don't know the speed after one second because we don't know the average acceleration. In other words, it can't be uniquely solved in a single step. We'll have to do an *iterative* calculation, meaning that we'll have to make an initial guess and then refine it until it's acceptably accurate. If you're not familiar with iterative calculations, here's an illustrative example of what we mean, step by step.

Knowing the thrust at time zero after engine failure, and knowing also the velocity (150 knots) and the spindown factor (1.000), we can calculate the instantaneous acceleration at that instant of time. Using equation 3 would give us an instantaneous acceleration of 3.34 knots per second at the speed of 150 knots.

time	TAS	thrust op eng	spindown factor	thrust inop eng	total thrust	μW	$(C_D\text{-}\mu C_L)qS$	accel kt/sec	avg accel	av TAS	ΔV	ΔS	ΣS
0	150.00	28944	1	28944	57887	3960	11890	3.34					

Table 18-3a

Let's use that as our first guess at the average acceleration for the first second of time after engine failure. Then the speed after one second will be 153.34 knots. Following equation 3, that would yield an instantaneous acceleration at 153.34 knots of 1.52 knots per second.

time	TAS	thrust op eng	spindown factor	thrust inop eng	total thrust	μW	$(C_D\text{-}\mu C_L)qS$	accel kt/sec	avg accel	av TAS	ΔV	ΔS	ΣS
0	150.00	28944	1	28944	57887	3960	11890	3.34					
1	153.34	28796	0.233	6744	35540	3960	12425	1.52					

Table 18-3b

We know that the speed of 153.34 knots after one second isn't precise, so let's refine that calculation. We now have an approximate average acceleration: the average of the instantaneous values at 150 and 153.3 knots. That average is $\frac{3.34 + 1.52}{2} = 2.43$ knots per second.

time	TAS	thrust op eng	spindown factor	thrust inop eng	total thrust	μW	$(C_D\text{-}\mu C_L)qS$	accel kt/sec	avg accel	av TAS	ΔV	ΔS	ΣS
0	150.00	28944	1	28944	57887	3960	11890	3.34					
1	153.34	28796	0.233	6744	35540	3960	12425	1.52	2.43	151.67	3.340	352	352

Table 18-3c

We'll use that as our second estimate of the average acceleration for the first second, and thus the new value of speed after one second would be 152.43 knots, a bit slower than our first guess. Using that speed we can calculate a new instantaneous acceleration after one second, that would be 1.54 knots per second.

time	TAS	thrust op eng	spindown factor	thrust inop eng	total thrust	μW	$(C_D\text{-}\mu C_L)qS$	accel kt/sec	avg accel	av TAS	ΔV	ΔS	ΣS
0	150.00	28944	1	28944	57887	3960	11890	3.34					
1	152.43	28836	0.233	6744	35580	3960	12278	1.54	2.44	151.22	2.430	254	254

Table 18-3d

That gives us a new average acceleration of 2.44 knots per second, and hence a third estimate of speed after one second is 152.44 knots. If we do one more calculation using that speed, we'd find that the new value of average acceleration would still be 2.44 knots per second.

time	TAS	thrust op eng	spindown factor	thrust inop eng	total thrust	μW	$(C_D\text{-}\mu C_L)qS$	accel kt/sec	avg accel	av TAS	ΔV	ΔS	ΣS
0	150.00	28944	1	28944	57887	3960	11890	3.34					
1	152.44	28836	0.233	6744	35580	3960	12280	1.54	2.44	151.22	2.440	255	255

Table 18-3e

In other words, by repeating this calculation – a process called *iteration*, we have arrived at an accurate value of speed after the first second of time following the engine failure. Then by repeating this iterative process, we can calculate the speed after two seconds, after three seconds, and so on, up to six seconds, and from that we can find the distance traveled during this time period of six seconds following the engine failure.

time	TAS	thrust op eng	spindown factor	thrust inop eng	total thrust	μW	$(C_D\text{-}\mu C_L)qS$	accel kt/sec	avg accel	av TAS	ΔV	ΔS	ΣS
0	150.00	28944	1	28944	57887	3960	11890	3.34					
1	152.44	28836	0.233	6744	35580	3960	12280	1.54	2.44	151.22	2.440	255	255
2	153.80	28776	0.086	2489	31265	3960	12500	1.18	1.36	153.12	1.360	259	515
3	154.91	28727	0.036	1042	29769	3960	12681	1.04	1.11	154.36	1.110	261	775
4	155.92	28683	0.016	463	29146	3960	12847	0.98	1.01	155.42	1.010	262	1037
5	156.88	28640	0.008	232	28872	3960	13005	0.95	0.96	156.4	0.960	263	1300
6	157.81	28599	0	0	28599	3960	13160	0.91	0.93	157.35	0.930	266	1566

Table 18-3f

engine-out acceleration after spindown

Following the engine failure and spindown illustrated above, it may be necessary to continue to accelerate to V_R, depending on the value of V_1.

We have said that V_R is 160 knots. That speed occurs well after engine failure and spindown, so we'll need to continue the step integration on up to 160 knots. The technique is exactly the same.

	time	TAS knots	thrust op eng pounds	spindown factor	total thrust pounds	acceleration ft/sec/sec	accel knots/sec	seg S	ΣS
eng fail	0.0	150.0	28942	1.000	57884	5.6	3.3		0
V1	1.0	152.5	28880	0.233	35609	2.6	1.5	265	265
	2.0	153.8	28847	0.086	31327	2.0	1.2	248	513
	3.0	154.9	28819	0.036	29856	1.8	1.0	262	775
	4.0	155.9	28793	0.016	29246	1.7	1.0	262	1037
	5.0	156.9	28769	0.006	28942	1.6	1.0	264	1301
	6.0	157.9	28746	0.000	28746	1.6	0.9	266	1567
	7.0	158.8	28722	0.000	28722	1.5	0.9	267	1834
	8.0	159.7	28700	0.000	28700	1.5	0.9	269	2103
VR	8.5	160.0	28692	0.000	28692	1.5	0.9	108	2238

Table 18-4

Calculating the Flare Distances

What we call the "flare distance" is defined as the distance from the point at which the airplane is rotated to its takeoff attitude to the point at which it reaches a height of 35 feet above the takeoff surface. Keep in mind here that we're discussing dry runway performance in this chapter; the flare distance under some other conditions is measured to 15 feet instead of 35 feet. That will be discussed in the chapter entitled "Takeoff On Non-Dry Runways".

For the all-engine takeoff distance, we need to compute the distance traveled from the point at which the rotation is initiated until the airplane reaches a height of 35 feet above the takeoff surface, with all engines operating. For the engine-inoperative accelerate-go distance, we need to calculate the flare distance from V_R to V_2 at 35 feet with an engine failed.

The flare parameters are determined experimentally over a wide range of thrust-to-weight ratios during the flight testing of a new airplane. The testing establishes both the flare time from V_R to 35 feet and also the speed at 35 feet, whether for all engines operating (V_{35}) or with an engine failure (V_2).

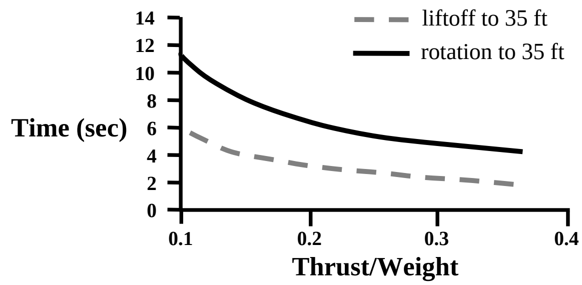


Figure 18-7

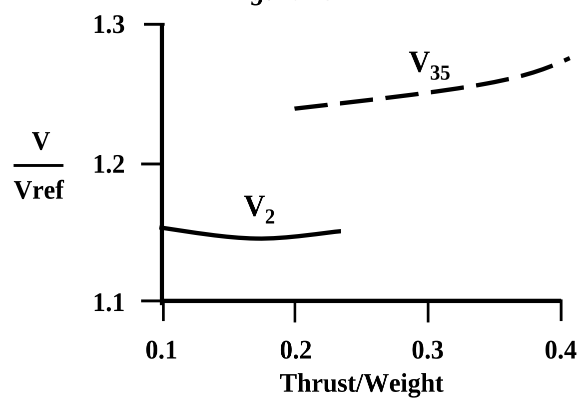


Figure 18-8

You'll notice that the V_2 speed, which by definition is the speed at 35 feet after rotating at V_R <u>with an engine inoperative</u>, is shown for only the range of thrust to weight ratios that would be expected for engine-inoperative conditions. The V_{35} speed is shown for thrust to weight ratios that would be expected for an all-engine takeoff. You'll see also that the speeds V_2 and V_{35} are plotted in terms of a ratio of the speed to an arbitrary reference speed V_{ref} for the same flap setting. Doing so allows us to produce one such plot that is valid for all flap settings. In this plot, the reference speed V_{ref} should not be confused with the "landing reference speed" which is also called V_{ref}.

Since distance is equal to time multiplied by speed, the flare distance is found by multiplying the flare <u>time</u> by the average <u>speed</u> between the rotation speed and the speed at 35 feet. Thus, for the all-engine case:

$$S_{flare} = \left(\frac{V_R + V_{35}}{2}\right) \times \Delta t_{R\text{-}35} \qquad \text{(eq. 5)}$$

where S_{flare} is the distance from V_R to 35 feet
V_R is the rotation speed
V_{35} is the speed at 35 feet
$\Delta t_{R\text{-}35}$ is the flare time from rotation to 35 feet

For the engine-inoperative accelerate-go case, the flare distance would be:

$$S_{flare} = \left(\frac{V_R + V_2}{2}\right) \times \Delta t_{R\text{-}35} \qquad \text{(eq. 6)}$$

Let's see the flare distances for the two example cases we've calculated above.
In the all-engine takeoff case, we said that V_R was 160 knots. Given the flight test flare data shown above, for our *T/W* of 0.2375 and a V_{ref} of 144 knots, we could find that V_{35} is 180 knots and that the flare time is 5.7 seconds. Thus, for our example calculation of the all-engine takeoff distance, the flare segment would be:

$$\left(\frac{160 + 180}{2}\right) \times 1.6878 \times 5.7 = 1635 \textit{ feet}$$

For our engine-inoperative accelerate-go case, using the flight test data with our T/W of 0.1196 we would find that the flare time will be nine seconds, and the V_2 will be 165 knots, so the engine-inoperative flare distance for our example conditions will be:

$$\left(\frac{160 + 165}{2}\right) \times 1.6878 \times 9.0 = 2468 \textit{ feet}$$

Calculating the Deceleration Distances

Now we've come to the most difficult segment to compute – the deceleration segment of the all-engine accelerate-stop distance. It's the most complex segment because many things occur during this segment. Recall the sequence of events in an all-engine accelerate-stop:

- the airplane accelerates with all engines operating from brake release until it reaches the V_1 speed;
- as a conservatism, the sequence includes a two-second period of constant velocity at the V_1 speed (the Amendment 25-92 definition, discussed above);
- the brakes are applied two seconds after reaching V_1, beginning the deceleration segment;
- a short interval of time after brake application, the thrust levers are retarded to the idle position;
- after another short interval of time, the speedbrakes are deployed. The airplane is now fully configured for stopping;
- the airplane decelerates to a stop.

The short intervals between brake application and thrust lever retardation, and between thrust lever retardation and speedbrake deployment, are established by flight test.

Complicating the calculation is the fact that the engine thrust is changing as the airplane decelerates, the drag and lift forces are decreasing with the decreasing velocity, and the speedbrakes change the drag and lift forces abruptly when they're deployed. Because the engines have been retarded to the idle position, they continue to produce a small but not insignificant amount of thrust during the deceleration.

wheel brakes and the braking coefficient μ_B

At this point in our discussion, we need to spend some time discussing the airplane braking force because this is one of the primary parameters affecting the deceleration distance.

Braking is the result of the interaction between the airplane's tires and the runway surface. In order for a tire to create a braking force, the tire must be made to roll more slowly than the free-rolling state. This is called *tire slip*. Slip is the sliding movement of the tire tread relative to the runway surface.

Before brake application, the wheels are almost freely rolling. As you'll recall from our discussion above about the "rolling friction" during acceleration before the brakes are applied, the wheel isn't really rolling completely freely – there's a slight amount of friction required to overcome the internal friction within the wheel and the energy expended in flexing the tires.

Now imagine that we apply the brake, very lightly at first. As we do so, some small amount of torque is generated by the brakes. It is counteracted by a friction force between the tire and the ground. The friction force is the product of the download on the wheel and the available coefficient of friction μ between the tire and the runway surface.

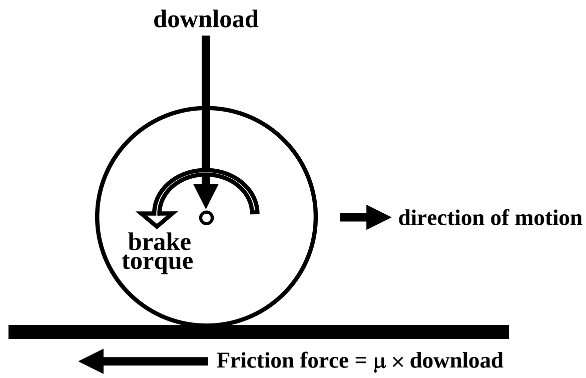


Figure 18-9

That friction force, multiplied by the tire's radius, produces a torque equal to, and in the opposite direction to, the torque produced by the brake. In order to produce this friction force, there must necessarily be a small amount of slippage of the tire's tread over the pavement.

As we increase the brake pressure, increasing the brake torque, the tire-to-ground friction force necessarily increases to produce an increase in the counteracting torque.

But an interesting thing happens as we continue to increase the brake pressure: at first, the tire-to-ground friction will increase, but at some pressure we'll see that the braking force will actually begin to decrease. If we continue to increase the brake pressure beyond that point, ultimately the wheel will "lock up" – stop rotating. A non-rotating wheel will still generate some friction force as the tread skids along the surface, but it's less than the maximum that's possible.

slip ratio

Engineers who work on braking systems – either automobiles or airplanes – define a parameter called *slip ratio*. This is a measure of the wheel speed of the vehicle relative to the speed of the vehicle itself, and it is directly related to the effectiveness of the vehicle's brakes.

Let's simplify the discussion a bit. Let's loosely define the *wheel speed* of a vehicle as being equal to the wheel's rotation rate multiplied by the radius of the tire on that wheel. This assumes that the tire is completely circular, which isn't completely accurate, but it's good enough for our discussion here. So you could think of the wheel speed as being the linear velocity of a point on the tread of the vehicle's tire as it rotates.

By that definition, if a vehicle's brakes are locked up and the wheel is not rotating, the wheel speed is zero. The tire is skidding along the pavement at the same speed as the vehicle: the relative speed between the tire's tread and the pavement is equal to the vehicle speed.

On the other hand, when a wheel is free-rolling along the pavement, the wheel speed is the same as the vehicle speed: there is absolutely no slippage between the wheel and the pavement. That is, the relative speed between the tire's tread and the pavement is zero.

Slip ratio is defined as:

$$slip\ ratio\ = \frac{(vehicle\ speed - wheel\ speed)}{vehicle\ speed}$$

From this, you see that a free-wheeling tire will have a slip ratio of zero and the tire of a locked wheel will have a slip ratio of one.

The amount of braking force available from the wheel brakes and tires depends on slip ratio.

Light application of a wheel's brakes will induce some small amount of slip, for a small slip ratio. Heavier braking will further retard the wheel's rotation rate, increasing the slip ratio.

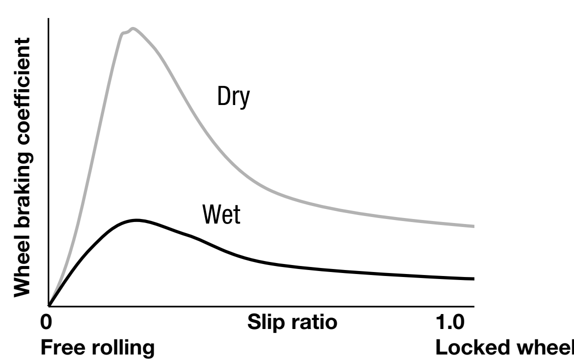


Figure 18-10

The illustration shows that the maximum wheel braking effectiveness occurs around a slip ratio of 0.1 to 0.15, for either a dry runway or a wet runway. Notice that the brake effectiveness will be substantially less on a wet runway than it will on a dry runway; this will be discussed in detail in a later chapter entitled "Takeoff on Contaminated Runways".

anti-skid systems

All Boeing commercial jet airplanes are equipped with *anti-skid* systems. The anti-skid system is an adjunct to the airplane's wheel braking system. Its function is to modulate as necessary the hydraulic pressure metered to the brakes, to enable them to produce the optimum slip ratio and also to prevent locked wheels which could result in airplane skidding.[1]

It does this by sensing the rates of rotation of each of the wheels. As long as the wheel rotation rates appear normal, the anti-skid system takes no action. However, whenever it senses a rotation rate that it considers improper for the given conditions, it will modulate the brake hydraulic pressure in such a way as to return the wheel rotation rate to the expected values.

anti-skid limited and torque limited

To illustrate these two terms, let's think again of an airplane brake, wheel and tire as they interact with the ground.

Suppose, for the moment, that the brakes are rather weak – that is, they're not capable of developing large amounts of braking torque. And let's suppose further that we have very good friction capability between the tire and the runway surface. In such a case, we would be able to apply the maximum brake torque that's available, <u>without exceeding the amount of friction force available to counteract it</u>. This would be called a *torque limited* condition – the torque capability, being less than the friction capability, is the determining factor in the stopping performance.

Suppose now, on the other hand, that the brakes are extremely effective, capable of a large amount of torque. Let's say also that the friction capability isn't as good in this example. Now it would be possible, by exerting the maximum available brake torque, to lock up the wheel – cause it to stop rotating – because the torque would be greater than the friction's capability to counteract it.

That's where the anti-skid system comes in. By modulating the brake pressure, the anti-skid system <u>prevents</u> wheel lockup and brings the slip ratio back from a ratio of one – a locked and skidding tire – to the optimum slip ratio. That's why this is called an "*anti-skid limited* condition; the friction/anti-skid are less than the torque capability and hence is the determining factor in the stopping performance.

One of our Boeing performance instructors once conducted a demonstration of braking force that illustrates what we're talking about. Using a bicycle, first he asked one of his students, a young lady of relatively low weight, to accelerate to some speed and then apply maximum effort braking to bring the bicycle to a full stop.

When the student applied the brakes, she was able to stop the rotation of the bicycle's wheels, and the bicycle slid to a stop by virtue of the friction between its tires and the ground. That is, the torque she could apply with the brakes was greater than the friction force available at her light weight. Had the bicycle been equipped with an anti-skid system, wheel lockup would have been

1. Those of you who drive vehicles having an ABS – anti-lock braking system – installed in the vehicle's brake system have essentially the same protection as an airplane with an anti-skid system.

prevented and the stopping distance would have been optimized. So this was an anti-skid limited case – the friction capability was less than the brake torque capability.

The instructor, a man of more substantial weight than the student, then mounted the bicycle, and he too accelerated and then applied the brakes. In this case, due to his weight the friction force was much greater. The instructor was unable to stop the wheels' rotation. The stopping force, in this case, was limited by the bicycle's brake torque capability – a torque limited condition.

determining braking forces

To determine the braking forces an airplane is capable of producing, a number of accelerate-stop test runs are conducted:

- the airplane is accelerated to a predetermined test stopping speed;
- the pilot stops the airplane by applying brakes, retarding the thrust levers to idle, and deploying the speedbrakes;
- the distance to decelerate from the point of brake application to a full stop is measured.

These tests are conducted with the airplane center of gravity at its most forward location, which results in a greater percentage of the airplane's weight on the (unbraked) nose landing gear and less weight on the main landing gear; this is the conservative case, since less weight on the main landing gear results in less retarding force from the brakes, resulting in longer stopping distances. The tests are also conducted over a range of weights and with a range of brakes-on speeds.

Knowing the test conditions of wind, runway slope, engine idle thrust, and other parameters including the airplane's lift and drag coefficients in the ground attitude, we can determine what value of airplane braking force F_B that was required to stop the airplane in the distance demonstrated in the test.

From these tests, we're able to plot the computed braking force in two different ways: first, as a function of average weight on the landing gear in the test stop, and second as a function of the airplane's kinetic energy at the time the brakes are applied in the test stop.

Let's look at an example of the first of these two:

The average weight on the landing gear is defined as $(W - \bar{L})$ where $\bar{L}$ is the average airplane lift force over the stopping distance in the test, from brake application to the full stop.

At lower values of $W - \bar{L}$ the brake force increases with the average weight on the wheels. At some specific value of average weight, however, the brake force becomes constant. That is the point at which the brake force ceases to be anti-skid limited and becomes instead torque-limited.

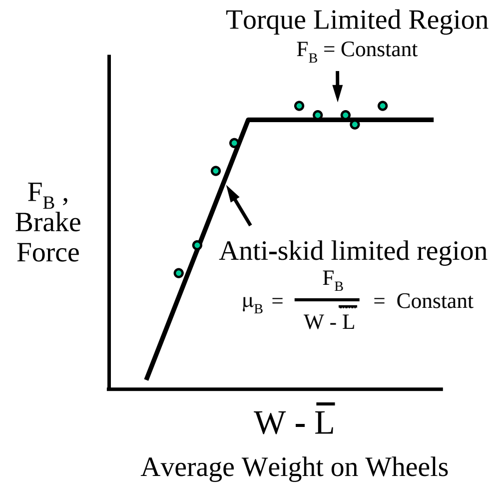


Figure 18-11

In either region, the airplane braking coefficient μ_B is defined as:

$$\mu_B = \frac{F_B}{W - \bar{L}} \qquad \text{(eq. 7)}$$

As shown in the illustration, at lighter weights the brake force F_B is a linear function of $(W - \bar{L})$. That is, the value of μ_B is constant. In the torque-limited region, the value of F_B is a constant, and hence the value of μ_B to be used in a particular stopping distance calculation will depend on the value of $W - \bar{L}$

The illustration to the right shows the second way that we graph the airplane braking force F_B. Here you see it plotted as a function of the "initial braking energy".

The initial braking energy is defined here as:

$$\textit{initial braking energy} = WV_{B_g}^2$$

where V_{B_g} is the "brakes-on" ground speed

From the equation, you'll realize that this isn't truly kinetic energy, since it uses weight rather than mass, but for our purposes it's a perfectly acceptable indicator of the kinetic energy at the times the brakes were applied.

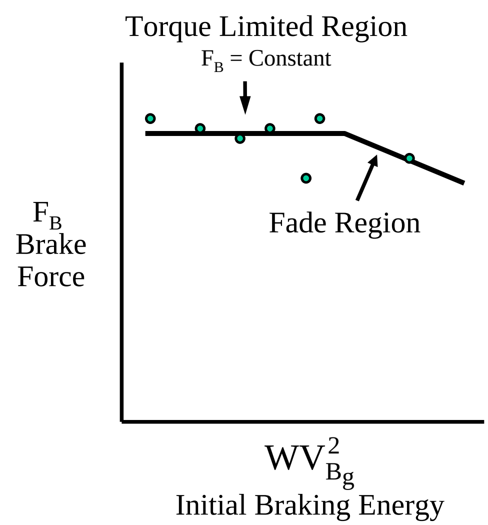


Figure 18-12

At the lower values of initial braking energy, the brake force is a constant at its torque limited value – the same value of brake force as you saw in the first graph above. However, in the illustration, you see that there's a region at high initial braking energy values marked "fade region". In this context, "fade" refers to *brake fade*. Fade is a characteristic of steel brakes operated at high kinetic energy levels. During a high-energy stop, the heat builds up internally in the brake. Eventually, the rotors and stators can't produce as much friction because the brake materials literally begin to melt. Carbon brakes are not prone to this phenomenon because they have much higher melting temperature than steel brakes. The higher the initial braking energy, the greater the degree of brake fade.

Since we now have the braking force plotted as a function of two different variables, for any given takeoff, knowing both the average weight on the wheels and the initial braking energy, we will look at both charts and will take the lower value of F_B of the two. We'll then convert it to a value of μ_B by dividing by $W - \bar{L}$.

Let's look now at the process for calculating the stop.

forces on the airplane during a stop

The forces acting on the airplane during a stop are slightly different from the forces on the airplane during acceleration.

During the acceleration segment, we had engine thrust acting to produce acceleration. During deceleration, however, we have only that portion of thrust that remains after the throttles have been retarded, which is a function of time. If the runway has a downhill slope, a small component of the airplane's weight will work against the deceleration.

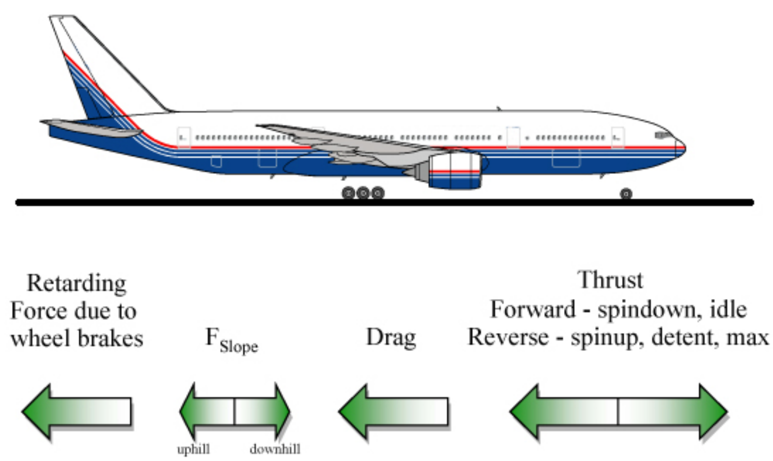


Figure 18-13

Working to produce the deceleration will be the aerodynamic drag of the airplane, the component of the airplane's weight along the runway if the runway has an uphill slope, and the retarding force due to the application of the wheel brakes. Notice also that when the engines are in reverse thrust, they will also provide a retarding force. However, as you'll see later, for dry runway deceleration calculations we take no credit for reverse thrust.

Summarizing the forces on the airplane:

$$\Sigma forces = T - D - \mu_B(W - L) - W\sin\phi$$

where μ_B is the "airplane braking coefficient"

As we did previously, we'll substitute $\frac{W}{g}a$ for Σforces and rearrange:

$$a = \frac{g}{W}[T - D - \mu_B(W - L) - W\sin\phi] \qquad \text{(eq. 8)}$$

The term $\mu_B(W - L)$ represents the retarding force that is generated by the wheel brakes. The airplane braking coefficient μ_B is multiplied by the weight on the landing gear, which equal to the weight of the airplane minus any lift being generated by the airplane.

calculating the stopping distance

Calculation of the stopping distance is a step-integration process similar to the process for calculating the acceleration distance, except that the step integration increment is based on time, rather than velocity as was the case in the acceleration calculation. This is necessary because the engine thrust is known as a function of time after engine failure or retardation of the thrust levers to the idle position.
Step integration as a function of time causes an exact calculation to become iterative, just as you saw above when we calculated the acceleration distance from V_{EF} to rotation. If the calculation is computerized, however, the additional calculation effort is trivial.

Using the current rules governing the accelerate-stop sequence, a table of step integration calculation of the stopping acceleration is shown in the following illustration. Notice that time is measured from the point of brake application. You'll see that the sequence of events calls for retardation of the throttles to idle at 0.3 seconds after brake application, and spoiler deployment another 0.4 seconds later. The values of 0.3 and 0.4 are derived from the flight tests of this airplane model. The values of C_D and C_L are 0.0631 and 0.340 respectively for the speedbrakes retracted condition and 0.1427 and -0.25 respectively for the speedbrakes deployed condition. The value of μ_B is 0.38. These are all representative values for a Boeing airplane.

	Time seconds	Spindown factor	GS knots	Σthrust pounds	Drag pounds	W-L pounds	FB pounds	accel ft/sec/sec	accel knots/sec	S feet	ΣS feet
V1	0.0	1.000	152.5	57603	9668	187908	71405	0	0		0
Brakes on	2.0	1.000	152.5	57603	9668	187908	71405	-3.1	-1.9	515	515
Throttle chop	2.3	1.000	151.9	57603	9597	188291	71550	-3.2	-1.9	77	592
	2.7	0.990	151.0	57027	9502	188799	71744	-3.2	-1.9	102	694
speedbrakes up	2.7	0.990	151.0	57027	21489	277647	105506	-9.4	-5.6	0	694
	3.3	0.640	147.7	36866	20550	276002	104881	-11.9	-7	151	845
	4.3	0.300	140.6	17281	18637	272650	103607	-14.1	-8.3	243	1088
	5.3	0.220	132.3	12673	16491	268891	102178	-14.2	-8.4	230	1318
	6.3	0.180	123.8	10369	14457	265327	100824	-14.1	-8.3	216	1534
	8.3	0.140	107.2	8064	10825	258964	98406	-13.6	-8	390	1924
	10.3	0.112	91.1	6452	7819	253699	96406	-13.1	-7.8	335	2259
	12.3	0.092	75.5	5299	5378	249422	94780	-12.7	-7.5	281	2540
	16.3	0.064	45.4	3687	1941	243400	92492	-12.2	-7.2	408	2948
	22.7	0.060	0.0	3456	0	240001	91200	-11.8	-7	241	3189

Table 18-5

The time begins at V_1. Following V_1 is the distance computed for two seconds at V_1. Next is the brake application two seconds after V_1, then throttle chop at 2.3 seconds and speedbrake deployment at 2.7 seconds after V_1. You see that the stopping distance from V_1 is 3189 feet.

Adding the Distances Together

Now we have all the segments of the three example cases we've been working on. It's time to add them together.

the all-engine takeoff distance

In this case, all we need is the all-engine distance to accelerate to a rotation speed of 160 knots, plus the flare distance.

The total distance is 7073 feet from brake release to 35 feet. But remember that this must be multiplied by a factor of 1.15, so the certified all-engine takeoff distance would be 8133 feet.

	GS - knots	TAS - knots	accel ft/sec/sec	s - feet	Σs - feet
brake release	0	0	9.0		0
	20	20	8.7	64	64
	40	40	8.4	199	264
	60	60	8.0	346	610
	80	80	7.6	510	1120
	100	100	7.1	697	1817
	120	120	6.6	917	2734
	140	140	6.0	1183	3917
	150	150	5.6	713	4630
rotation	160	160	5.3	808	5438
35 feet	180	180		1635	7073

Table 18-6

the engine-out accelerate-go distance

Now that we know the all-engine acceleration distance to V_{EF}, and the engine-out acceleration distance from V_{EF} to V_R, and the engine-out flare distance from V_R to V_2, we have all the pieces we need.

Adding the flare distance to the table for the all-engine acceleration distance to V_R gives us the table to the right.

You see that the certified engine-inoperative accelerate-go distance would be 9309 feet.

	TAS - knots	time seconds	accel ft/sec/sec	S feet	ΣS feet
brake release	0.0		9.0		0
	20.0		8.7	64	64
	40.0		8.4	199	264
	60.0		8.0	346	610
	80.0		7.6	510	1120
	100.0		7.1	697	1817
	120.0		6.6	917	2734
	140.0		6.0	1183	3917
engine failure	150.0	0.0	5.6	713	4630
V1	152.5	1.0	2.6	265	4895
	153.8	2.0	2.0	248	5143
	154.9	3.0	1.8	262	5405
	155.9	4.0	1.7	262	5667
	156.9	5.0	1.6	264	5931
	157.9	6.0	1.6	266	6197
	158.8	7.0	1.5	267	6464
	159.7	8.0	1.5	269	6733
VR	160.0	8.5	1.5	108	6841
35 feet	180.0	9.0		2468	9309

Table 18-7

the all-engine accelerate-stop distance

We know the distance to accelerate to V_1 with all engines operating.

We can easily calculate that two seconds of distance at V_1 would be equal to 152.5×2×1.6878 = 515 feet.

We have also calculated the distance to decelerate from V_1 to a full stop with the thrust levers retarded to idle thrust.

Adding them together produces the table to the right.

You see that the certified all-engine accelerate-stop distance would be 8015 feet.

	TAS - knots	time seconds	accel ft/sec/sec	S feet	ΣS feet
brake release	0		9.0		0
	20		8.7	64	64
	40		8.4	199	264
	60		8.0	346	610
	80		7.6	510	1120
	100		7.1	697	1817
	120		6.6	917	2734
	140		6.0	1183	3917
	150		5.6	713	4630
V1	152.5	0.0	0	196	4826
Brakes on	152.5	2.0	-3.1	515	5341
Throttle chop	151.9	2.3	-3.2	77	5418
	151.0	2.7	-3.2	102	5520
speedbrakes up	151.0	2.7	-9.4	0	5520
	147.7	3.3	-11.9	151	5671
	140.6	4.3	-14.1	243	5914
	132.3	5.3	-14.2	230	6144
	123.8	6.3	-14.1	216	6360
	107.2	8.3	-13.6	390	6750
	91.1	10.3	-13.1	335	7085
	75.5	12.3	-12.7	281	7366
	45.4	16.3	-12.2	408	7774
stop	0.0	22.7	-11.8	241	8015

Table 18-8

Effects of Flap setting and Thrust on Takeoff Distances

From the preceding paragraphs, you can see some factors that will influence the distances we calculate using the methods we've demonstrated. Some are obvious, some perhaps less so. Two of these are the flap setting and the thrust rating. Let's look at each of these two in turn. Then in following sections, we'll examine closely the effects of wind, runway slope, weight and V_1 on the distances.

We'll demonstrate their effect on the ground acceleration distance; from this you'll be able to understand how they will also affect the other distance segments.

flap setting

You know from the chapter entitled "Aerodynamic Devices" that flaps enhance a wing's lifting capability at low speeds. Accompanying the increase of the wing's lift coefficient, however, is an increase of the drag coefficient.

So how is it that a larger flap setting results in a shorter takeoff distance? Since both C_L and C_D increase, there is not much net effect on the term (C_D-μC_L). On one Boeing airplane, for example, at flap setting 5 the value of (C_D-μC_L) is 0.075 and for flaps 15 it's 0.084.

The principal reason why a larger flap setting results in shorter takeoff distances is simply that the larger flap settings, by providing more lift, allow lower takeoff speeds V_R and V_2. Thus the net effect of selecting a larger flap setting will be a shorter distance.

thrust

This has a direct effect on acceleration. Any reduction of thrust will result in a reduction of acceleration, with a resulting increase in acceleration distances.

Why reduce thrust? Why not simply use full thrust for takeoff under all conditions? In the chapter entitled "Reduced Thrust For Takeoff and Climb" later in this book we'll discuss this subject. Summarizing briefly: using less than the allowable thrust results in lower internal temperatures and pressures in the engine as well as lower rotor RPMs. Over time, this will have the desirable effects of improving engine reliability and reducing engine maintenance costs.

Using the same data as previously, here is the step integration assuming a 10% reduction of thrust:

You see in this example that a 10% thrust reduction has caused the acceleration distance to increase from 4630 feet to 5268 feet, a 14% increase in the distance.

V - GS	V - TAS	ACCEL - ft/s/s	ACCEL - kt/s	S-Step-ft	Sum S
0	0	8.0	4.8		0
20	20	7.8	4.6	72	72
40	40	7.5	4.4	223	295
60	60	7.2	4.2	389	684
80	80	6.7	4.0	574	1258
100	100	6.3	3.7	788	2046
120	120	5.8	3.4	1042	3088
140	140	5.2	3.1	1356	4444
150	150	4.9	2.9	824	5268

Table 18-9

Effect of Runway Slope on Takeoff Distances

The AFM limits the allowable runway slope to 2% uphill or downhill. There are some exceptions to this limitation, but they are granted only in the form of special Flight Manual appendices that may be purchased by an airline that needs to operate from a runway having a greater slope.

The exact reasons for the 2% limitation are lost in the sands of time, having been with us since at least the days of the 707. One consideration may be the fact that visual cues are important to a pilot in judging his approach path when making a non-instrument approach to a runway. A sloped runway can cause the pilot, using his normal visual cues, to be either high on the approach to a downhill runway or low on the approach to an uphill runway.

Let's take the case of the acceleration distance we looked at above. We had a 240,000-pound airplane, and if it were on a 2% uphill slope runway, the retarding effect of the weight would be 240,000 × 0.02, or 4800 pounds. The equation F = ma tells us that 4800 pounds of additional retarding force results in a 0.64 feet per second per second decrease in acceleration. The acceleration distance of 4630 feet on a level runway becomes 5109 feet on a runway having a 2% uphill slope.

A downhill-sloped runway will enhance the acceleration, producing a shorter acceleration distance. Conversely, an uphill slope will yield shorter deceleration distances, where a downhill slope will yield longer deceleration distances.

Effect of Wind On Takeoff Distances

Again, let's illustrate the effect of wind on takeoff distances by seeing how it affects the ground acceleration distance; from this you'll be able to understand its effect on the other distance segments.

Looking at Equation 3, what's affected by the wind? Nothing is affected. The thrust is a function of airspeed. The dynamic pressure q is a function of airspeed. The two weight terms of μW and ϕW aren't a function of speed at all, nor is the term (C_D-μC_L).

So how is it that a headwind is helpful? By giving us, so to speak, a head start on the takeoff acceleration. Say we have a 20 knot headwind, and we need to calculate the distance to accelerate to an airspeed of 150 knots with all engines operating. When the airplane is positioned, unmoving, at the end of the runway ready to begin the takeoff roll, it already has a twenty-knot airspeed. Instead of needing to accelerate over a total speed increase of 150 knots as we did when there was no wind, a 20 knot headwind means that now we need to accelerate over a total speed increase of only 130 knots.

Accounting for the 20 knot headwind, the two tables that we saw above for zero wind become instead:

V - GS	V - TAS	DYNAMIC - q	F - THRUST	F - SLOPE	µW	(CD-µCL)qS	ACCEL - ft/s/s	ACCEL - kt/s
0	20	1.35	34653	0	3960	211	8.7	5.2
20	40	5.42	33775	0	3960	845	8.4	5.0
40	60	12.19	32896	0	3960	1902	8.0	4.8
60	80	21.67	32017	0	3960	3382	7.6	4.5
80	100	33.86	31139	0	3960	5284	7.1	4.2
100	120	48.75	30260	0	3960	7609	6.6	3.9
120	140	66.36	29381	0	3960	10357	6.0	3.5
130	150	76.18	28942	0	3960	11889	5.6	3.3

Table 18-10a

The acceleration distances change as shown here:

You can see that the 20 knot headwind reduced the acceleration distance from 4630 feet down to 3546 feet.

GS - knots	TAS - knots	accel - ft/sec/sec	S - feet	ΣS - feet
0	20	8.7		0
20	40	8.4	66	66
40	60	8.0	208	274
60	80	7.6	364	639
80	100	7.1	542	1181
100	120	6.6	750	1931
120	140	6.0	1001	2932
130	150	5.6	614	3546

Table 18-10b

Factored Winds Used in Distance Calculations

The illustration of the effect of wind on the ground acceleration distance is correct, however it's not exactly the way wind effects are computed in the AFM and AFM-DPI.

In the example immediately above, we used a twenty-knot wind as an example. We added the twenty knots to the groundspeed to arrive at the airspeed. This had the effect of giving us a twenty-knot "head start" on the acceleration distance.

However, the AFM and AFM-DPI will not do the same as we did above. Yes, they will account for the effect of wind in the manner we have demonstrated – but they will incorporate three corrections into the value of reported wind velocity that they will actually use in their calculations.

reported wind

Whenever we want to calculate an allowable takeoff weight, we'll need to know – among other things – the wind. The value of wind to use in the calculation can come from any one of a number of sources: the current meteorological information for the airport such as an ATIS report, directly from the control tower or ground control radio frequencies, from a statistical study of winds for that airport, or you may simply assume one or more values.

In any event, you are going to use a *reported* wind. In this context, "reported" means that it is a wind strength and direction that actually were, or would have been, reported by a source such as the control tower or airport meteorology service. That wind is measured at some height above the ground. This reported wind value must be corrected in three ways before using it to calculate the distances:

- if the wind is at any angle to the direction of the takeoff runway, it's necessary to calculate and use only that component of the wind that is parallel to the takeoff runway direction;
- the velocity of the wind that is reported at some height such as the height of the control tower is not the same as the velocity of the wind that will be experienced by the wing of the airplane during its takeoff. It is necessary to apply a height correction to the reported wind to correct it to the value of wind at the height of the airplane's wing above the runway;
- an operational correction factor must be applied to the reported wind, as specified in FAR Section 25.105(d).

Let's discuss each of these in turn.

wind component for takeoff

This is simple trigonometry.

For performance purposes, we use only the component of the wind that is parallel to the direction of the runway. We can safely ignore the second-order effects of the crosswind component on the takeoff performance, because the crosswind component has negligible effect on the airplane's acceleration.

Remember that runway numbers are the magnetic direction of the runway (that is, the compass heading of the runway), divided by ten and rounded to the nearest integer. Thus, a runway having an orientation of between 025° and 035° magnetic would be referred to as runway 03. The same runway, used in the opposite direction, would be runway 21.

Recall also that the direction of reported winds are given as the direction from which the wind is coming, in degrees magnetic.

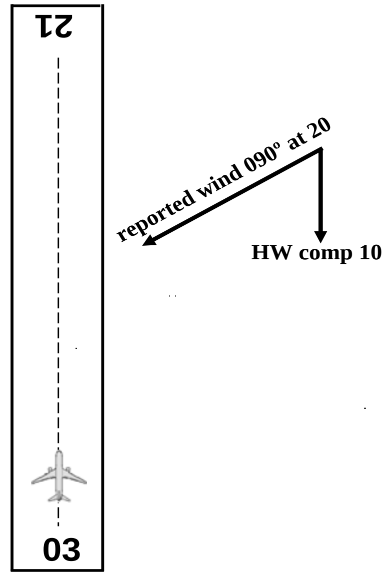


Figure 18-14

Finding the headwind component is done by simply multiplying the reported wind velocity by the cosine of the angle between the runway direction and the wind direction.

In the example shown, if the takeoff runway is 03 and the reported wind is 090 degrees at 20 knots, the angle between the runway and the wind would be 60 degrees, the cosine would be 0.5, and thus the takeoff wind component would be a 10 knot headwind.

While there is no limit to the amount of headwind component allowed for a takeoff, there is a tailwind limit of either ten or fifteen knots, depending on the airplane, the airline and the certifying agency. This tailwind limit is specified in the Airplane Flight Manual.

For normal operations, there is no limit on the crosswind component; instead, the AFM will specify a *maximum demonstrated crosswind*, which is usually on the order of approximately 30 knots. Some operators choose to use the maximum demonstrated crosswind as their maximum allowed crosswind component. Others choose to select their own limit based on their operational experience; this will require operational approval. For autoland operations, and for landings with certain components of the airplane inoperative, crosswind limits are applied.

height correction

In an earlier chapter entitled "Flow Near a Surface", we discussed boundary layers and showed that the velocity of air flow immediately adjacent to a surface is different from the velocity at some distance from that surface. The same is true when considering the wind velocity for takeoff. Wind is nothing more than an air flow over the surface of the earth. As such, the wind velocity is zero right at the runway surface and increases as height above the runway surface increases.

Takeoff winds are reported by some person or agency such as the control tower, ATIS or the like. They read the wind velocity and direction from indicators at their location, and those indicators are displaying the velocity and direction sensed by an anemometer[1]. That anemometer is installed at some known height above the runway surface.

You'll recall from the chapter entitled "Flow Near a Surface" that airflow velocities decrease when measuring closer to a surface – the effect called a boundary layer. This is also true of wind velocities close to the ground, and so it is that the wind velocity closer to the ground will be less than the velocity sensed by the anemometer.

For calculating takeoff performance, the value of wind that is truly relevant is the wind velocity at the height of the airplane's wing. That height is taken as the height above the runway at the wing's Mean Aerodynamic Chord (MAC). Unless the wing MAC height is the same as the anemometer height, however, the wind velocity experienced by the wing during takeoff will be different from the anemometer wind – the "reported" wind.

Tests have shown that an acceptably accurate mathematical relationship between the wind at the anemometer height and the wind at the wing height is:

$$\frac{\textit{wind velocity at wing MAC height}}{\textit{wind velocity at anemometer height}} = \left(\frac{\textit{wing MAC height}}{\textit{anemometer height}}\right)^{\frac{1}{7}} \qquad \text{(eq. 9)}$$

The problem is that different airports may have different anemometer heights. In order to eliminate the need to use different wind height corrections for different airports, <u>today's regulatory agencies permit the assumption of a standard value of reporting height of ten meters.</u>[2]

1. An anemometer, if you're not familiar with that term, may be defined as any instrument designed to measure the velocity of wind.
2. In older FAA documents, the standard reporting height was 50 feet.

In the graph to the right, we show the value of wind at height h above the runway, based on equation 1 above, for an assumed wind of ten knots at the standard reporting height of ten meters. You see that if the wing MAC height were ten feet, then the reported wind of ten knots would correspond to a wind of 8.4 knots at the wing MAC.

What if the wind were 10 knots at a reporting height of 50 feet? In that case, using the same equation, the wind velocity at an MAC height of ten feet would be 8.0 knots – a difference of only 0.4 knots. Hence you can see that a reasonable variation in anemometer height will have a negligible effect on the wind velocity at the MAC; for that reason, the assumption of a standard reporting height of ten meters is considered to be acceptably accurate.

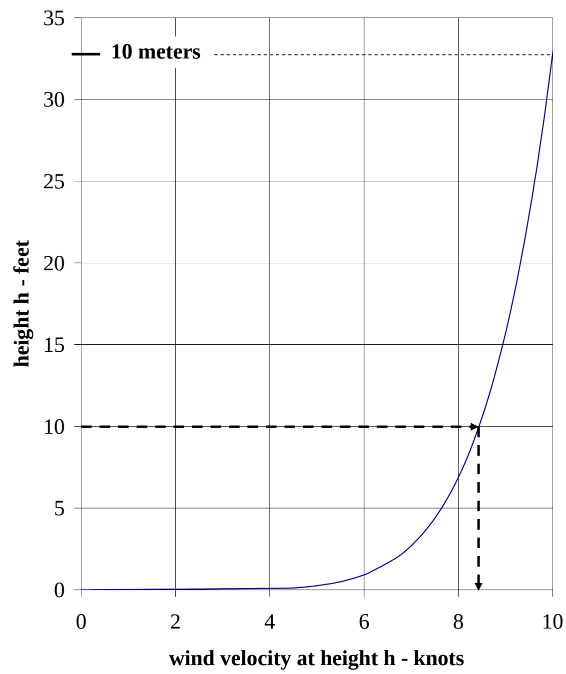


Figure 18-15

operational correction factor

Winds are rarely constant. Wind velocities and directions both tend to vary somewhat over any given period of time. To account for this variation, FAR25.105(d) states:

> *The takeoff data must include, within the established operational limits of the airplane, the following operational correction factors:*
> *(1) Not more than 50 percent of nominal wind components along the takeoff path opposite to the direction of takeoff [i.e. headwinds], and not less than 150 percent of nominal wind components along the takeoff path in the direction of takeoff [i.e. tailwinds].*

Thus, given a ten-knot headwind component at the wing MAC, the takeoff weight calculation may use only a five-knot wind. On the other hand, given a five-knot tailwind component at the MAC, the calculation must be based on a 7.5-knot tailwind.

summarizing the wind correction factors

All three of the corrections described above must be applied to the reported takeoff wind before performing the takeoff weight calculation. If we go back to our example of a reported wind of 20 knots at 090° when runway 03 is being used for takeoff by an airplane having a wing MAC height of ten feet:

- the wind component along the runway will be ten knots headwind, at the standard height of ten meters;
- the wind component at an MAC height of ten feet will be 8.4 knots headwind;

- the 50% rule will allow us to use only 4.2 knots headwind component in our calculations.

HOWEVER: the AFM charts and all takeoff software applications apply the height correction and the 50%/150% correction for you automatically. In our example here, you would need to correct only for the wind direction, hence you would go the AFM charts or the take-off software with a ten-knot headwind.

Effect of Weight and V_1 On Takeoff Distances

In the following chapter entitled “Field Length Limit Takeoff Weight”, we’re going to define what field length limit weight is, and you’re going to see how to determine it. First you’ll see how to find the limit weight for the usual all-engine takeoff. We’ll then talk about the weight limit for the two event/engine-out takeoff cases: the engine-inoperative accelerate-go and the all-engine accelerate-stop. For the event/engine-out takeoff cases, you’re going to learn about a powerful tool for understanding the subject of determining the maximum weight and its associated V_1 for the given runway parameters.

But before we can proceed to that discussion, we need to lay some groundwork for it by talking now about the effects of weight and V_1 on the takeoff distances.

In the following section, we will be using distances calculated by AFM-DPI for a 747-400 on a sea level standard day with no wind, a level runway, and flaps 10.

effect of weight on the all-engine takeoff distance

Obviously, the all-engine takeoff distance isn’t affected by V_1 since there is no event and there is no engine failure, simply acceleration to V_R, rotation, liftoff, and initial climb to 35 feet.

You can see that this relationship, for a given pressure altitude, temperature, wind and runway slope, is a simple thing, almost linear.

Now that you’ve seen the effect of weight on the all-engine takeoff distance, let’s see how weight and V1 will affect the all-engine accelerate-stop distance and the engine-inoperative accelerate-go distance.

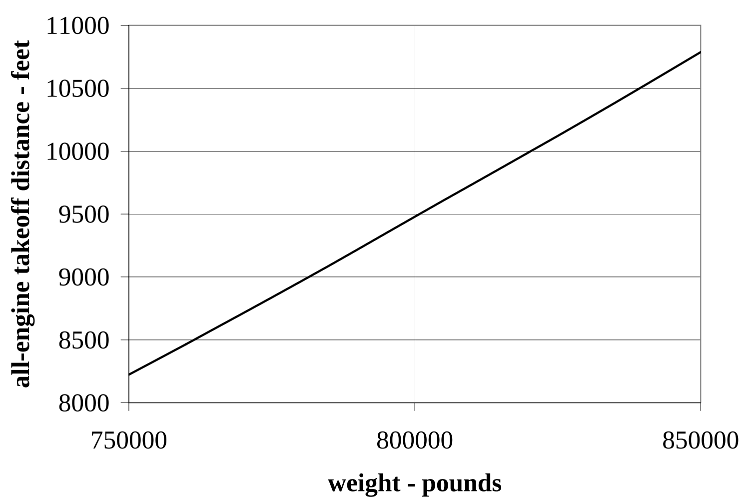


Figure 18-16

effect of weight and V_1 on the all-engine accelerate-stop distance

When looking at the following charts of distance versus V_1, it's important to remember that there's nothing magic about V_1. For the purposes of calculating distances, you can set V_1 arbitrarily to any value(s) of speed you wish, within the allowable range of V_{1MCG} to V_R or V_{MBE}, as discussed in the chapter entitled "Speeds".

Having decided on the V_1 value, and remembering that V_{EF} occurs one second before V_1, by definition, we can use the methods shown previously to calculate the distances.

Recall that the all-engine accelerate-stop distance is that distance required to accelerate the airplane, with all engines operating, and at speed V_1 initiate the RTO procedure. The calculated distance also, you'll remember, includes an arbitrary distance conservatism equal to two seconds at the speed V_1.

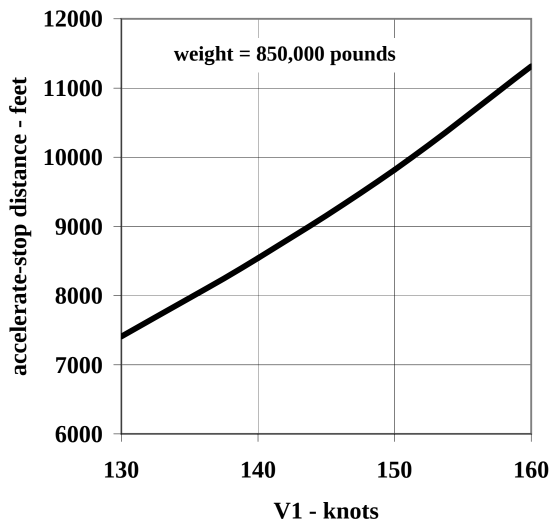


Figure 18-17

As you would expect, increasing the V_1 will increase the distance to accelerate to V1 and also the distance required to stop from that speed; hence, the accelerate-stop distance increases with increasing V_1 as shown in the graph to the right.

In the chart here, we've shown just one weight. How will changing the weight affect the accelerate-stop distance?

That's pretty clear: less weight means less mass and that means more rapid acceleration and also more rapid deceleration after the brakes are applied.

In the chart to the right, we're showing the accelerate-stop distances for 750,000 pounds and 850,000 pounds. You see that for our example conditions the distances increase by an average of about a thousand feet over this range of weight.

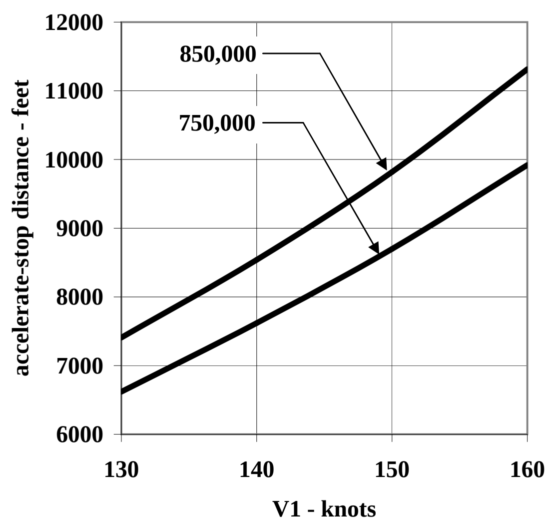


Figure 18-18

effect of weight and V_1 on the engine-out accelerate-go distance

This is a little less obvious than the effect on the accelerate-stop distance, yet when you recall the structure of the accelerate-go distance, it's hopefully not hard to understand.

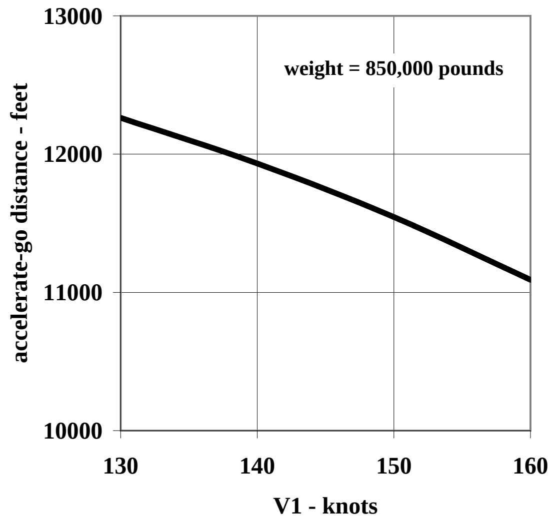


Figure 18-19

The accelerate-go distance calls for all-engine acceleration only up to the point at which the engine fails. Following that, the remainder of the acceleration to V_R is conducted with an engine inoperative, and hence the acceleration is less.

When we calculate distances, we can assume the engine failure speed V_{EF} to be any value we wish. And remember that V_1 occurs only a few knots – one second – after V_{EF}. So let's say that V_1 is equal to V_R – indeed, this can be the case under some conditions. Setting V_1 equal to V_R means that all but one second of the acceleration to the rotation speed is conducted with all engines operating, and that ensures a relatively short acceleration distance.

On the other hand, if we set a ridiculously low V_{EF} of, say, 50 knots, what would happen to the acceleration distance to V_R? Think about it: you'd have all engines operating only up to 50 knots, and the remainder of the acceleration would have to be conducted with an engine inoperative. It stands to reason, doesn't it, that the acceleration distance to V_R will be greater than the case with $V_1 = V_R$? That's exactly what you see in the chart of accelerate-go distance versus V_1 speed. You see that for our example conditions a thirty-knot increase of V_1 actually reduced the accelerate-go distance by more than one thousand feet.

What's the weight effect on the accelerate-go distance? The graph to the right shows that the accelerate-go distance will decrease with decreasing weight. This is logical, since less weight means less mass and that means better acceleration.

In the case of the accelerate-go distance, notice that decreasing the weight from 850,000 pounds to 750,000 pounds will reduce the distances by more than 3000 feet. This is a much greater sensitivity to weight change than was the case of the accelerate-stop distance seen above.

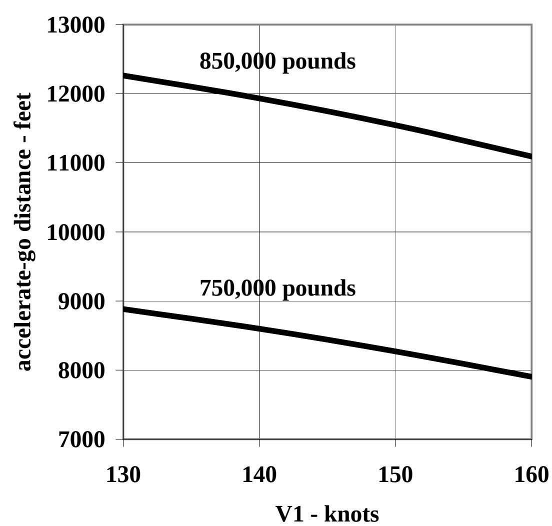


Figure 18-20

Balanced and Unbalanced Takeoffs

You may encounter the terms *balanced V_1* and *balanced field* length when determining field length limit weight and the associated V_1 for various runway characteristics. To illustrate the meaning of "balanced" in this context, here's a new kind of graph for you. This is sometimes referred to as an *X-plot*.

The X-plot graph shown to the right combines the accelerate-go and accelerate-stop lines into one plot of distance versus V_1. The lines are plotted for the same weight, in this case 800,000 pounds. Where the two lines cross you can read the "balanced V_1" – the V_1 that will produce equal accelerate-go and accelerate-stop distances – of 153 knots. The balanced field length – that distance that is the same for both accelerate-go and accelerate-stop – is about 9700 feet.

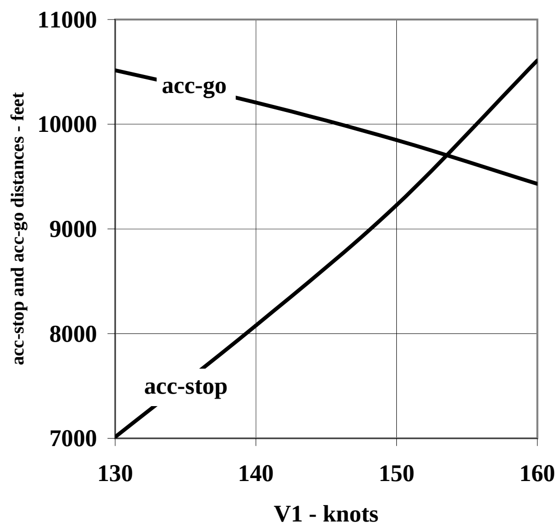


Figure 18-21

Thus, *if you had a 9700-foot runway*, under these example conditions you could perform either the accelerate-go or accelerate-stop scenarios at a weight of 800,000 pounds just within the available runway length. But you could do it with only a single unique value of V_1: 153 knots. If you tried to reject a takeoff after 153 knots, your RTO would go off the end of the runway. If you tried to continue a takeoff following an engine failure more than one second before 153 knots, you'd be unable to reach the required height of 35 feet before passing the end of the runway.

We will discuss this concept of balanced and unbalanced V_1 values in more detail in the following chapter, and we'll be showing you some examples of unbalanced takeoff conditions.

Chapter 19: Field Length Limit Takeoff Weight

Introduction

One of the most complex subjects in calculating the performance of a commercial jet airplane is that of takeoff. So many factors are involved: runway length, runway condition, clearway, stopway, slope, wind, altitude, temperature, flap setting, engine thrust setting, and airplane system malfunctions such as anti-skid system inoperative or brakes deactivated.

In the preceding chapter you saw how to compute the three takeoff distances: the all-engine takeoff distance, the all-engine accelerate-stop distance, and the engine-out accelerate-go distance. It's not difficult to compute the takeoff distance for the all-engine takeoff case. The computation is relatively trivial. As you've seen, though, calculating the all-engine accelerate-stop distance and the accelerate-go distance with engine failure is a bit more complex. Still, given today's digital computing capabilities, producing distance data for all three cases takes merely fractions of a second.

Much of the work for a performance engineer, though, is the reverse process: given a runway length and the other relevant variables, he's asked to determine the maximum allowable takeoff weight for that runway under those conditions. Doing so for the usual condition of all engines operating is relatively simple. It's a much more complex task, however, to calculate the allowable weight when considering the takeoff with an engine failure and the accelerate-stop condition with all engines operating.

In this chapter we're going to tie these all together and show their interrelationships with weight and V_1. You're going to learn about the *web chart*, which is the main AFM tool for manually calculating field length limit weight for earlier Boeing airplanes. Even now in the days of high-speed computers and AFM-DPI, the web chart is still a very powerful tool for visualizing and understanding the relationships between the takeoff distances, weight, and V_1. All performance engineers should strive for a thorough understanding of it regardless of whether or not they will ever use one in their work.

Terminology

In writing this chapter, given the complexity of the subject, we need to be very careful in the words we use. Every effort will be made to keep our discussion simple, clear and easy to understand. From this point onward in this chapter, then, let's agree on some terminology:

- We will refer to the *all-engine takeoff*, meaning the takeoff consisting simply of all engine acceleration, rotation and initial climb to 35 feet.
- We will refer to the *event/engine-out* takeoff, meaning the takeoff that considers the engine-out accelerate-go distance and the lesser of the event-caused or engine-out accelerate-stop distances.

- We will talk about *field length limit weight* meaning the weight of an airplane at which the takeoff distances required will just equal the distances available. The field length limit weight does not include any consideration of other factors that might limit the allowable takeoff weight such as climb or obstacle clearance.

There are really two different field length limit weights:

- the weight for an all-engine takeoff;
- the weight that considers both (1) a one engine inoperative accelerate-go case, and (2) either an engine-out accelerate-stop case OR an event-caused all-engine accelerate-stop case, whichever is more conservative. As previously discussed, the event-caused all-engine accelerate-stop case is usually more conservative.

For any given set of takeoff conditions, these two weights will be different, and we are required to use the smaller of the two.

The Basics

First, we need to discuss some fundamentals that relate to the takeoff distances.

takeoff distance and takeoff run

The takeoff distance and the takeoff run are two different distances, and the difference between them is important as you'll see in a later paragraph when we discuss *clearway.*

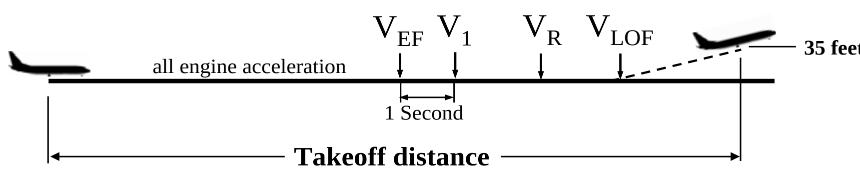


Figure 19-1

In the illustration to the right, you see the definition of takeoff distance for the engine-out accelerate-go case: it's the distance from the point at which the airplane begins its takeoff roll to the point at which the lowest point on the airplane reaches a height of 35 feet above the takeoff surface after liftoff.

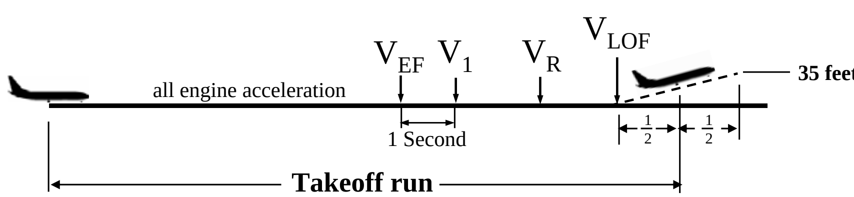


Figure 19-2

As shown to the right, the takeoff run for the engine-out accelerate-go case is the distance from the beginning of the takeoff roll to the point at which the airplane is one-half of the distance from the point of liftoff to the point at which it's 35 feet above the takeoff surface.

For the case of the all-engine takeoff, the takeoff distance is defined as the distance from brake release to 35 feet plus an additional 15% as shown in the illustration.

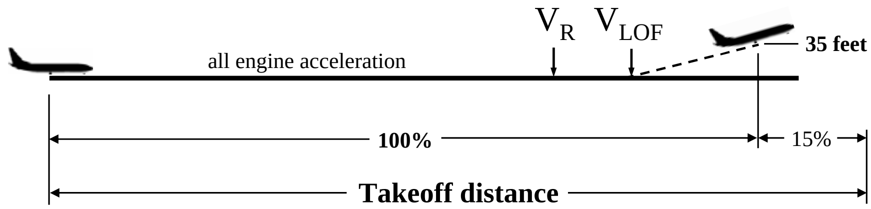


Figure 19-3

For the all-engine takeoff, the takeoff run is equal to the computed distance from the beginning of the takeoff roll to the point at which the airplane is one-half of the distance from liftoff to 35 feet, plus an additional 15% as shown in the illustration.

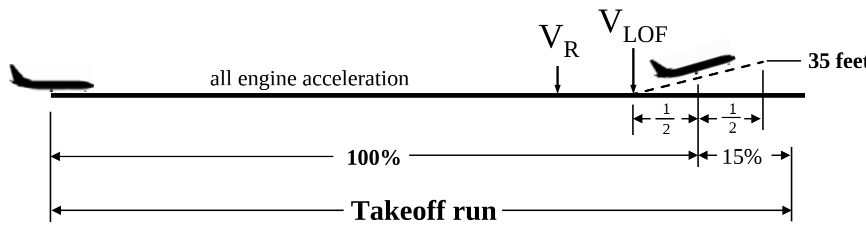


Figure 19-4

stopway

The term *stopway* refers to an area beyond the end of the takeoff runway that may be used for decelerating the airplane in the event of a rejected takeoff. It's sometimes referred to as an "overrun area".

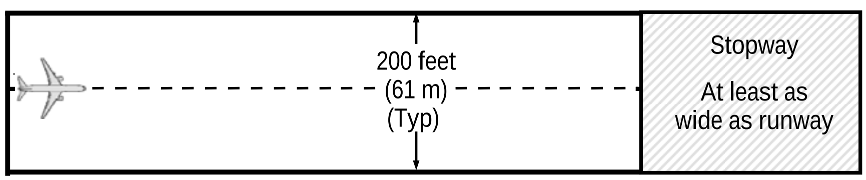


Figure 19-5

The total distance available for the accelerate-stop maneuver is the sum of the runway length plus the stopway length. If the stopway length is to be included in the total accelerate-stop distance available, however, it must meet some legally-defined criteria:

- It must be at least as wide as the runway;
- It must have its center on the same line that is the extended centerline of the runway;
- It must be designated for use to decelerate an airplane if that airplane conducts a rejected takeoff;
- It must have the capacity to hold the airplane's weight without causing any structural damage during a rejected takeoff.

You'll see later in this chapter how stopway will affect the allowable field length limit weight and V_1 speed.

clearway

The term *clearway* refers to an area beyond the end of the runway that may be used as part of the takeoff distance. The amount of clearway which may be used is, however, strictly limited.

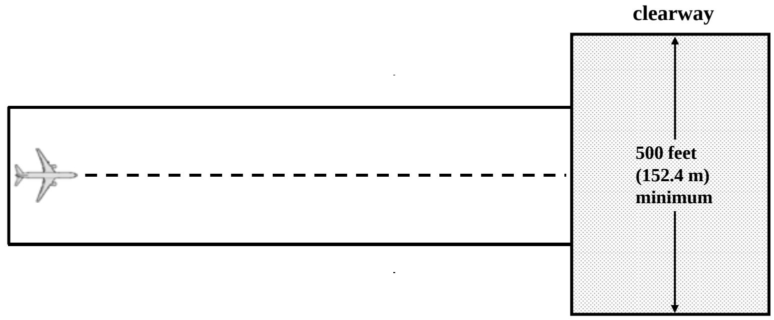


Figure 19-6

The total distance available for the accelerate-go maneuver is the sum of the runway length plus the clearway length. However, in order to be used as part of the takeoff distance, the clearway must also meet some legally-defined criteria:

- It must be at least 500 feet (152.4 meters) wide;
- It must have its center on the same line that is the extended centerline of the runway;
- It must be under the control of the airport authority;
- It must be designated for use as part of the takeoff distance;
- It must contain no object or terrain that is above a 1.25% plane (threshold lights may protrude above the plane if their height above the end of the runway is 26 inches or less and if they are located to each side of the runway.)

maximum usable clearway

As we said above, there's a limitation on the use of clearway. It's phrased this way:

The takeoff run may not exceed the length of the runway.

You'll recall that the definition of the takeoff run says that it's the distance from the beginning of the takeoff roll to the point which is halfway between the liftoff point and the point at which the airplane reaches 35 feet above the takeoff surface (plus an additional 15% in the case of the all-engine takeoff).

Saying that the takeoff run may not exceed the length of the runway, then, is simply another way of saying that the amount of clearway that may be used as part of the takeoff distance may not exceed one half of the distance from liftoff to 35 feet (the "flare distance").

You may also think of it this way: the maximum allowable clearway is equal to the takeoff **distance** minus the takeoff **run**.

For the engine-out case:

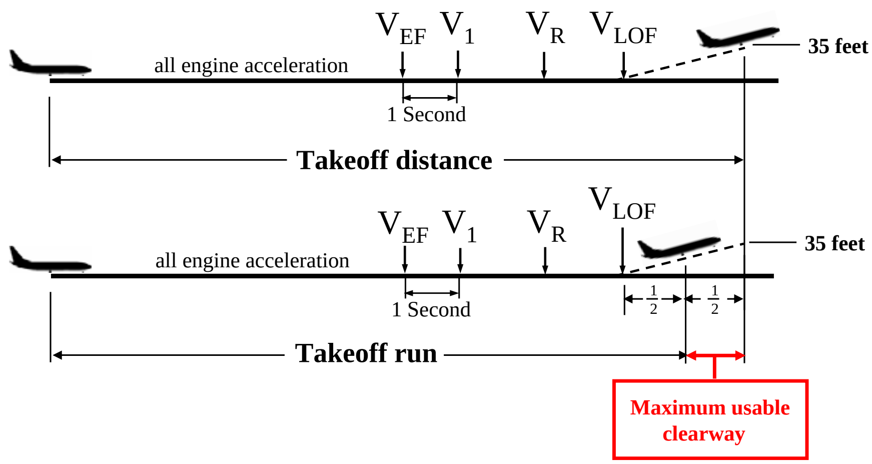


Figure 19-7

For the case of the take-off with all engines operating:

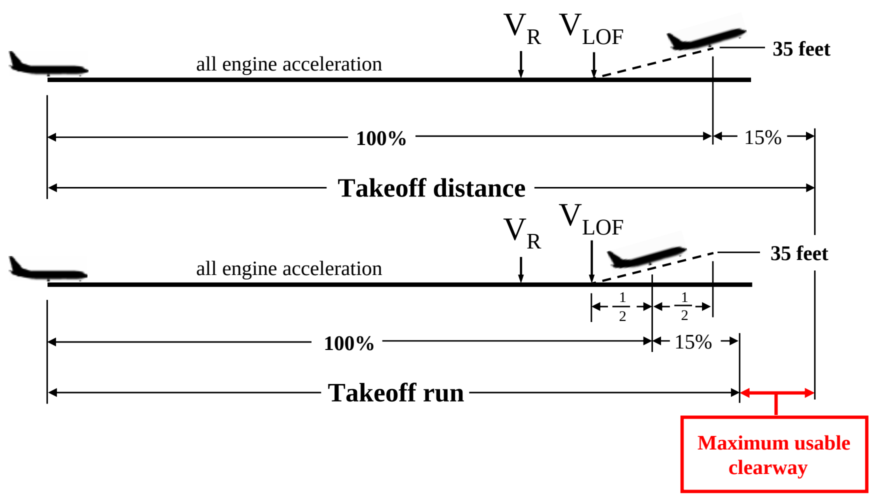


Figure 19-8

The actual values of the maximum usable clearway depend on the thrust-to-weight ratio and the dynamic characteristics of the airplane. To give you a rough idea of the magnitude: for a 737 on short runways, the maximum allowable clearway is around 500 feet; on long runways, it's around 900 feet. For a 747 on a short runway it's around 600 feet, for a long runway it's around 750 feet. More exact values may be found in the FPPM and also in Section 4 of the AFM.

Here are two things about clearway and stopway to discuss briefly. Think about this: is it possible for a runway to have a stopway at its end that is not usable also as a clearway? One might think that since a stopway is a hard-surfaced area which obviously contains no obstacles, it should also be usable for clearway.

The problem, though, is that the clearway must contain no obstructions above a 1.25% plane within a band 250 feet either side of the extended runway centerline; stopway needs to be only as wide as the runway. It's possible, therefore, that a runway might have some obstruction just to the side of the stopway area, but within 250 feet of the runway centerline. In such a case, the stopway could not be used for clearway.

Conversely, it's possible to have clearway without having stopway. This would be the case when, for example, the clearway lies over water at the end of the runway, or when the terrain drops down at the end of the runway, like a cliff.

lineup allowances

In the preceding chapter entitled "Calculating Takeoff Distances", you saw how we can accurately compute the all-engine takeoff distance and the engine-out accelerate-go and accelerate-stop distances.

These calculated distances, though, are sometimes called "point distances" because they do not consider the length or geometry of the airplane. They are, in other words, the distances that would be traveled <u>by a single point</u> on an airplane during a takeoff or a rejected takeoff.

For example: suppose that we could begin a takeoff with the airplane's main landing gear right at the exact beginning of the runway. For simplicity, let's forget for the moment about clearway and stopway, making it easier to visualize.

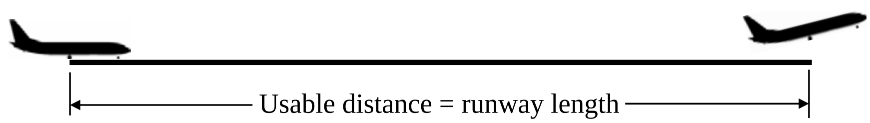


Figure 19-9

Since the takeoff distance is defined as the distance from brake release to the point at which the lowest point of the airplane (usually the main landing gear) reaches a height of 35 feet above the takeoff surface, then in this case <u>the usable takeoff distance is the same as the runway length</u>.

Many runways and the taxiways accessing them aren't arranged in such a way that the airplane can begin its takeoff roll with the main gear at the beginning of the runway. In such cases, the situation is different.

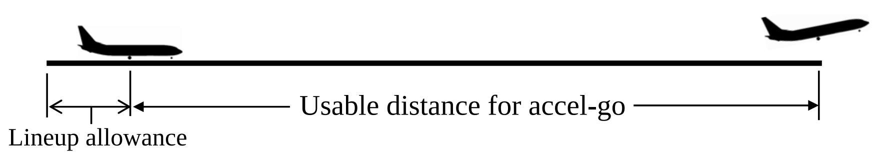


Figure 19-10

In the accelerate-go case shown to the right, you see that the distance that's usable for the accelerate-go case is less than the runway length, by the amount of distance from the beginning of the runway to the location of the main landing gear at the point at which the airplane begins the takeoff roll. This is the lineup allowance for the accelerate-go case.

The lineup allowance for the accelerate-stop case is different:

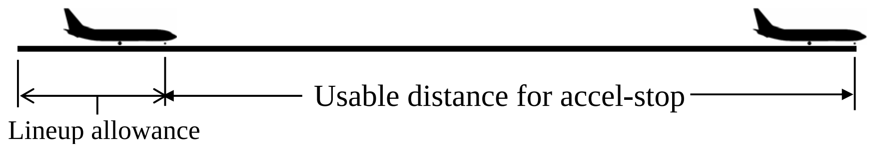


Figure 19-11

In the accelerate-stop case, we assume that the airplane has reached a complete stop when the nose landing gear is exactly at the end of the runway, so you can say that our “single point” is the nose landing gear. The lineup allowance for the accelerate-stop case, then, is the distance from the beginning of the runway to the location of the nose landing gear at the point at which the airplane begins the takeoff roll.

These lineup allowances are published in the Flight Planning and Performance Manual and another document called the “Takeoff Safety Training Aid”. The allowances are provided for a 90° turn onto the runway, and for a backtrack followed by a 180° turn. <u>It should be noted that these are the minimum possible line-up allowances for these maneuvers. Actual line-up allowances will depend on flight crew technique but will not be less than these published minimum values</u>.

By subtracting the lineup allowances from the available accelerate-go and accelerate-stop distances, we have accounted for the difference between the “point” distances calculated by the takeoff software and the actual distances used by a real airplane.

Failure to subtract the lineup allowance from the available runway length will produce a field length limit weight which is slightly unconservative because the airplane’s geometry has not been allowed for. JAR-OPS regulations now <u>require</u> the operator to apply the appropriate lineup allowance where the design of the runway and taxiway don’t allow lineup right at the runway end.

TORA, TODA and ASDA

You’ll frequently see these three terms used as part of a description of a runway, its stopway, and its clearway. Specifically:

TORA means <u>T</u>ake<u>O</u>ff <u>R</u>un <u>A</u>vailable. From the discussion above, you’ll understand then that it means the runway length available, since the takeoff run is not allowed to exceed the length of the runway.

TODA is the <u>T</u>ake<u>O</u>ff <u>D</u>istance <u>A</u>vailable; this is the length of the runway plus any clearway that has been designated as available for use as part of the takeoff distance, regardless of whether or not we can use all of it for takeoff, minus the accelerate-go lineup allowance discussed above.

ASDA is the <u>A</u>ccelerate-<u>S</u>top <u>D</u>istance <u>A</u>vailable; this is the length of the runway plus any stopway that has been designated as available for use as part of the accelerate-stop distance, minus the accelerate-stop lineup allowance discussed above.

The All-Engine Field Length Limit Weight

We reminded you at the beginning of this chapter that the all-engine field length limit weight calculation is a relatively simple matter. Since the all-engine case considers only an accelerate-go with all engines operating, the performance engineer needs only to calculate the takeoff distances required for the given conditions of altitude, temperature and so on, over a range of runway lengths. Then, for a given runway length, the task of finding the field length limit weight is simple.

For the older Boeing airplanes, those having the printed Airplane Flight Manual, as opposed to the AFM-DPI software, finding the all-engine field length limit weight requires two steps:

- First, beginning with the runway length available, a chart is used to make adjustments to that length for the effects of clearway, runway slope, wind, and – if applicable – the use of engine anti-ice protection for takeoff. After applying these adjustments, the user has now a "corrected takeoff distance".
- This "corrected takeoff distance" is simply the length of a runway having no wind, no slope, no clearway, and no anti-icing use that would yield the same all-engine field length limit weight as the actual runway.
- Next, the user enters a second chart with the corrected takeoff distance, pressure altitude and temperature, and reads on a weight scale the all-engine field length limit weight.

As you can see, this is a relatively simple matter.

For older non-AFM-DPI airplanes for which the user has the Boeing takeoff software such as BPS or STAS and the necessary database, entering the takeoff parameters into the software yields the most limiting of all of the takeoff weight limits; it does not show the different limits separately.

For the later Boeing airplanes, those having the AFM-DPI software instead of the AFM charts, the user simply enters all of the relevant parameters into the program, instructs it to compute the takeoff weight limits, and gets from it a list of all of the takeoff weight limitations for the given conditions, including the more critical of the all-engine and engine-out/event field length limit weights.

The Event/Engine-Out Field Length Limit Weight

This takeoff is more complex in that it considers both the all-engine event-caused accelerate-stop distance (or the engine-out accelerate-stop distance if that's more conservative) and the engine-out accelerate-go distance. It's further complicated by the need for a speed criterion during the takeoff on which the pilots will base their decision to continue or reject a takeoff should some event occur. That speed, of course, is the speed referred to as V_1.

The engine-out accelerate-go distance and the event/engine-out accelerate-stop distance, for any given set of conditions, are functions of not only the weight but also of the V_1 speed. The weight and V_1 must satisfy both the available accelerate-go distance and the available accelerate-stop distance. This makes the event/engine-out case considerably more complex than the all-engine case.

distance, weight, V_1 and the web chart

Intricate relationships exist between the weight, the distances, and V_1. The relationships aren't always intuitively obvious.

You've already seen how to compute the accelerate-go and accelerate-stop distances for given conditions of weight and engine failure speed. Now we want to develop the relationships between the weight, the V_1 speed, and the distances. Bear in mind, please, that the following discussion of the web chart deals only with the event/engine-out takeoff. A field length limit weight derived from the web chart must always be compared to the all-engine field length limit weight.

the distances

For use in the following discussion we've computed the accelerate-go and accelerate-stop distances for four different V_1 speeds at each of five different weights. We assumed a sea level standard day with no wind or slope, just to simply life a bit. Here's a table of the distances we computed using AFM-DPI for a 747 at flaps 10:

weight	V_1	acc-go dist	acc-stop dist
850,000	130	12263	7408
850,000	140	11932	8540
850,000	150	11545	9818
850,000	160	11091	11315
825,000	130	11366	7210
825,000	140	11046	8309
825,000	150	10674	9518
825,000	160	10238	10960
800,000	130	10515	7013
800,000	140	10207	8078
800,000	150	9849	9228
800,000	160	9431	10609
775,000	130	9674	6816
775,000	140	9378	7848
775,000	150	9034	8962
775,000	160	8638	10263
750,000	130	8883	6619
750,000	140	8599	7619
750,000	150	8270	8698
750,000	160	7904	9919

Table 19-1

Look first at just the top four lines, showing the distances for the four different values of V_1 at a weight of 850,000 pounds.

You notice that the accelerate-go distance is decreasing with increasing V_1. That's exactly what we should expect. Increasing the V_1 speed has the effect, at constant weight, of moving it closer to the rotation speed; this means that more of the takeoff acceleration will be made with all engines operating, and less of it with an engine inoperative. The result? The accelerate-go distance will decrease as V1 increases.

You notice also that the accelerate-stop distance is increasing with increasing V_1. Again, this is what we expect; raising the V_1 means longer acceleration distances since the airplane is accelerating to a higher speed; it also means longer deceleration distances since the speed and hence the kinetic energy at the brakes-on point is higher and the brakes will need more distance to bring the airplane to a stop. Thus the accelerate-stop distance will increase as V_1 increases.

Now compare the distances at the same V_1 but at two different weights. For example, look at a V1 of 130 knots at 850,000 pounds and at 825,000 pounds. What's happening? Both distances are greater at the higher weight. That's no surprise: more weight means more mass and that means that both acceleration and deceleration distances will increase.

a line of weight for different V_1 values

Let's take the data shown above and put it into a graphical form. First of all, let's simply show one line of accelerate-stop and accelerate-go distances for one weight with the four different V_1 values:

Here we have graphed the line of 850,000 pounds weight, with V_1 values from 130 to 160 knots. It shows us what we already know: for constant weight, as V_1 increases, the accelerate-stop distance increases but the accelerate-go distance decreases. You can see that the accelerate-stop distance is more affected by the V1 increase than is the accelerate-go distance.

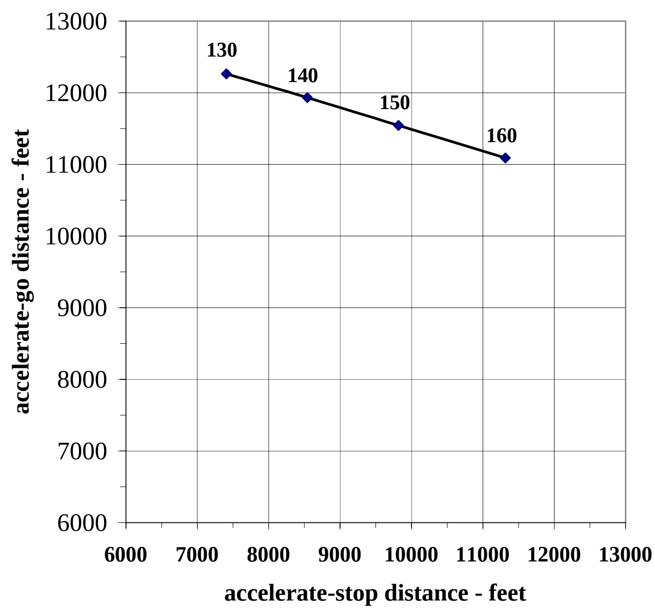


Figure 19-12

a family of weight lines

Here we have graphed lines for each of the five different weights, again for the same four values of V_1 of 130, 140, 150 and 160 knots. We now have a family of weight lines, each showing accelerate-stop and accelerate-go distances for a constant weight over the same range of V_1.

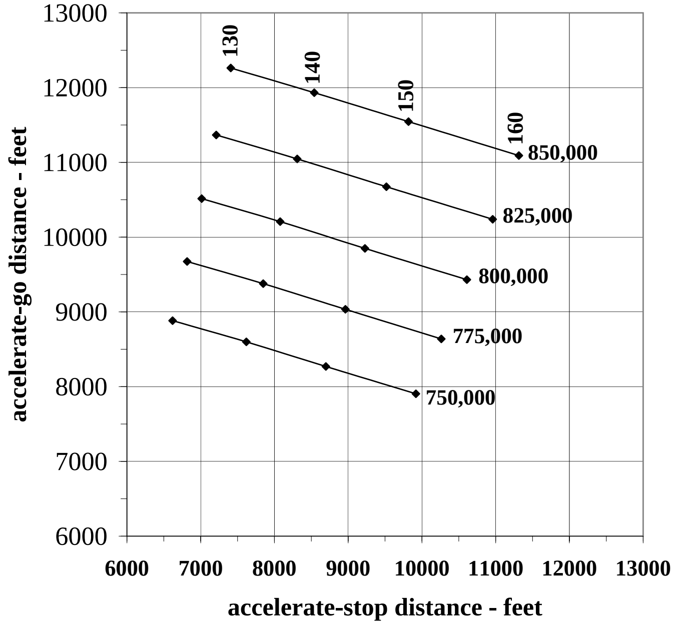


Figure 19-13

a line of V_1 for different weights

Let's change the emphasis for a minute, and instead of making lines of weight, we'll make a line of constant V_1 over a range of weights from 750,000 pounds up to 850,000 pounds.

Here, we see that for a given V_1 of 130 knots, increasing the weight increases both the accelerate-stop and accelerate-go distances, as we would expect. That's happening simply because as the weight increases, the increasing <u>mass</u> of the airplane results in longer acceleration and deceleration distances.

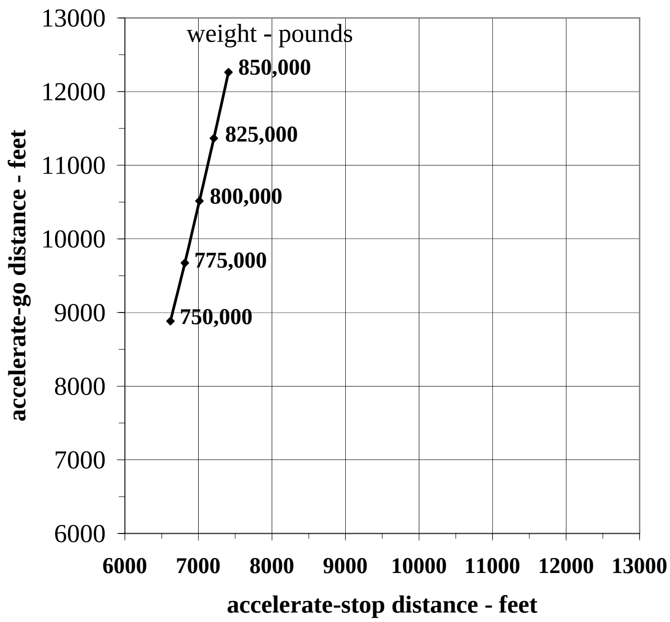


Figure 19-14

You can see clearly in this illustration the fact that the accelerate-go distance is much more affected by the weight than is the accelerate-stop distance. A 25,000-pound increase of weight increases the accelerate-go distance by about 850 feet, but the same weight change increases the accelerate-stop distance by only about 200 feet.

a family of V_1 lines

As we did above, now we'll graph a family of lines of V_1 for a range of weights.

Now we have lines of constant V1 over a range of weight.

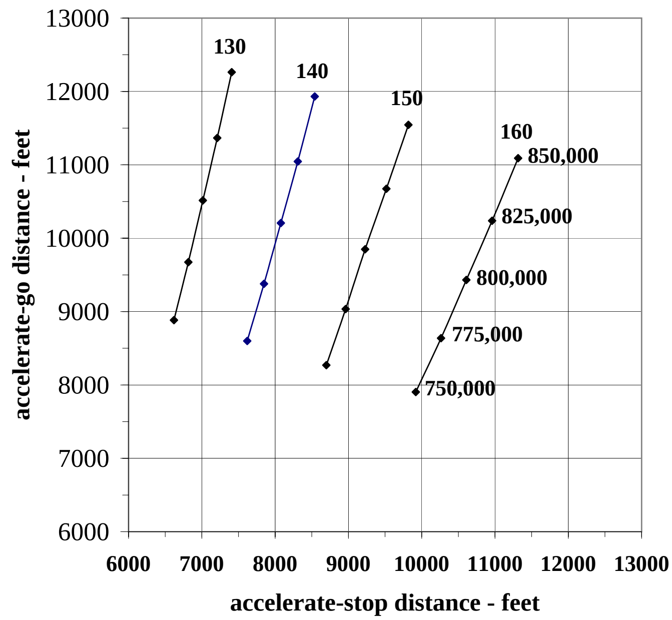


Figure 19-15

tying it all together – the web chart

Let's show the family of weight lines and the family of V_1 lines on the same chart. This at last is the "web chart" that we've been talking about.

To keep it visually simpler, we have graphed data for only a limited range of weights and V_1 speeds, but obviously it could be done over wider ranges of weight and V_1 just as well.

What does it do for us? It allows us to see at a glance the relationships between weight, V_1, and the accelerate-go and accelerate-stop distances. For any given combination of weight and V_1, we can see immediately the corresponding accelerate-stop and accelerate-go distances.

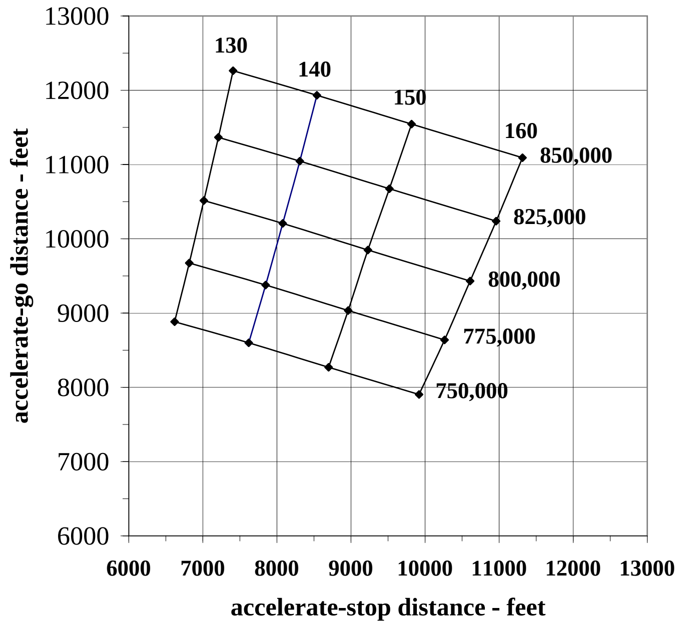


Figure 19-16

But what's more important, it allows us to see immediately, for any given runway length, exactly what the allowable field length limit weight and the corresponding V_1 would be. Let us remind you once again, however, that this weight and V_1 are only valid for the event/engine-out takeoff case.

using the web chart, example 1

Here's an example of a very simple use of the web chart. We're supposing that we have a runway that is 10,000 feet long with no clearway and no stopway. For simplicity we'll ignore lineup allowances in these examples. The accelerate-stop distance available is therefore 10,000 feet, and the accelerate-go distance available is also 10,000 feet.

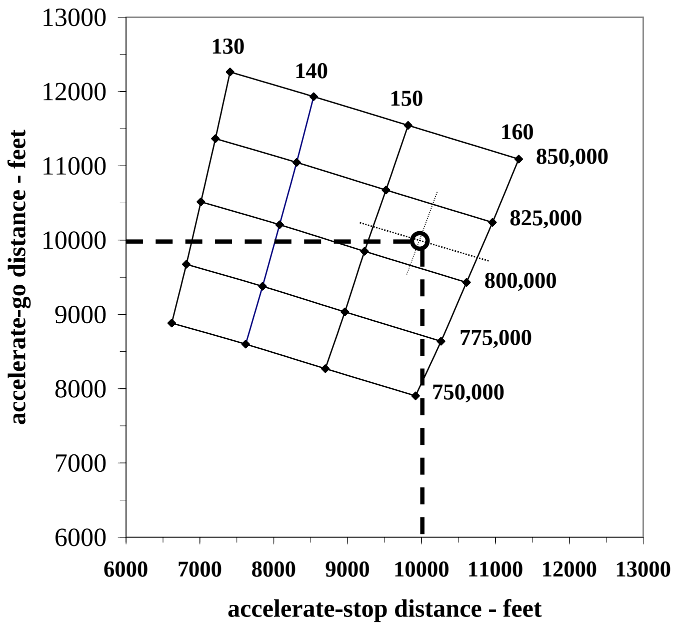


Figure 19-17

When we draw lines into the web chart with those lengths, the two lines intersect at a point.

By interpolation between the nearest lines, we can see that for a 10,000 foot runway with no clearway and no stopway, the event/engine-out field length limit weight would be 810,000 pounds, with a V_1 of 154 knots.

Please understand that this V_1 of 154 knots is the ONLY acceptable value of V_1 at this takeoff weight. If you use a V_1 greater than 154 knots, for example 160 knots, your accelerate-stop distance required would increase to approximately 10,750 feet and thus you would exceed the available accelerate-stop distance of 10,000 feet. If you use a lower V_1, for example 150 knots, your accelerate-go distance required would increase to approximately 10,200 feet and thus you'll exceed the available accelerate-go distance of 10,000 feet.

using the web chart, example 2

Here's a slightly different example. We still have a runway length of 10,000 feet with no clearway, but we have added 500 feet of stopway. The accelerate-stop distance available has increased from 10,000 feet to 10,500 feet. Should that yield a takeoff weight increase?

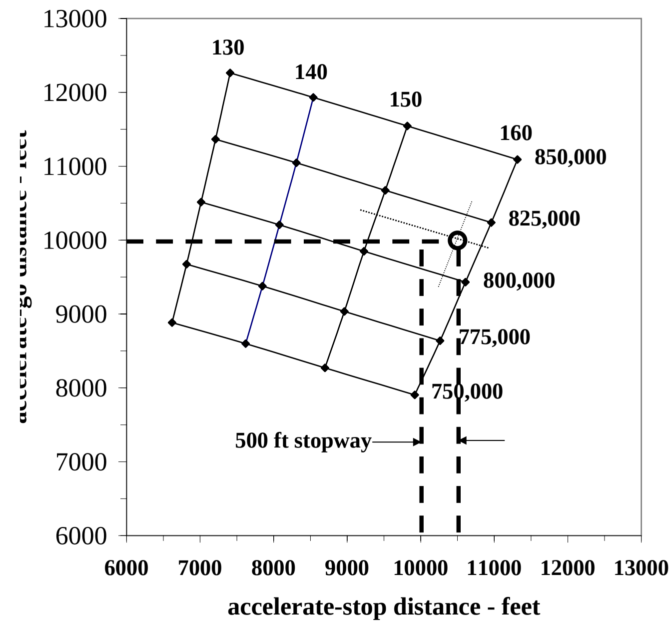


Figure 19-18

In the chart on the right, you see that now the field length limit weight has indeed increased, from 810,000 pounds to 815,000 pounds. So how do we accomplish that weight increase without violating the available 10,000 feet of accelerate-go distance? <u>By increasing the V_1 from 154 knots to 157 knots</u>. You can see that immediately on the web chart. The airplane has maintained the same accelerate-go distance but it's obtained a 5000-pound weight increase, by increasing the V_1 by three knots. The combination of a 5000-pound weight increase and a three-knot V_1 increase will just exactly consume the added 500 feet of stopway.

using the web chart, example 3

Here's another example of how a knowledge of the web chart can yield substantial benefits.

In this case, we have the same 10,000-foot runway with no stopway, but we've added 500 feet of <u>clearway</u>. We'll assume that all of the clearway is usable for takeoff. The accelerate-go distance available has increased from 10,000 feet to 10,500 feet. Will that clearway give a benefit, as the stopway did?

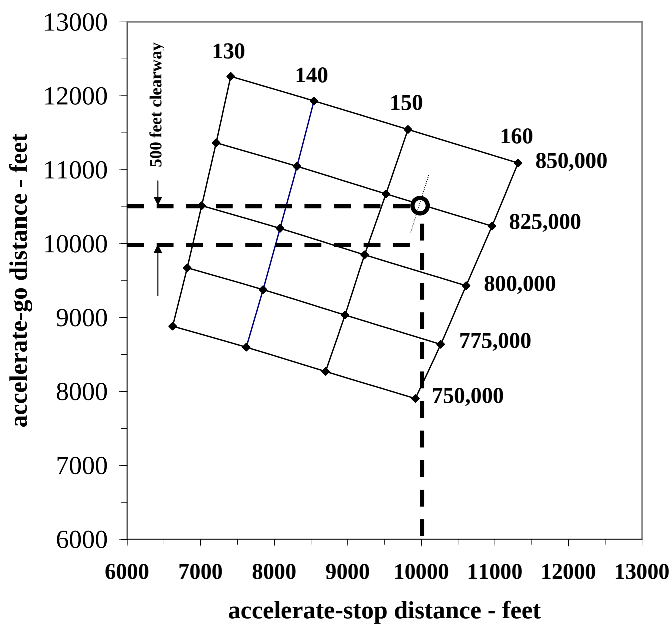


Figure 19-19

The web chart shows the answer – yes. In fact, we can get approximately 15,000 pounds more takeoff weight from that clearway, <u>but we must reduce the V_1 by one knot, from 154 to 153 knots</u>. This decrease makes it possible to keep the same 10,000 foot accelerate-stop distance despite the substantial weight increase.

range of V_1

In each of the examples above, we have used the web chart to find the field length limit weight and the corresponding V_1 speed that, taken together, meet the available engine inoperative accelerate-go and all engine accelerate-stop distances. What the web chart makes clear is that <u>if an airplane is taking off at the event/engine-out field length limit weight, there is only one single unique value of V_1 that is acceptable. If any other V_1 is used, one or the other of the available distances will be exceeded. This is not only a violation of the regulations, it is also potentially hazardous</u>.

But how about the case in which the takeoff weight will be <u>less</u> than the event/engine-out field length limit weight?

It makes sense that the airplane could reject the takeoff at a speed somewhat above the V_1 that corresponds to the event/engine-out field length limit weight. Since the airplane is lighter than the weight that needs all of the available distance, the acceleration and deceleration distances both will be less than they would at the limit weight.

Similarly, it would be possible for the airplane to <u>continue</u> a takeoff even if an engine fails more than one second prior to V_1, because the lighter weight yields a higher acceleration.
Let's illustrate this using the web chart.

Here is the same web chart, using a 10,000-foot runway having no clearway and no stopway. Once again we see that the field length limit weight would be 810,000 pounds and the corresponding V_1 would be 154 knots.

But in this case, let's say that the actual takeoff weight will be only 790,000 pounds. Perhaps it's a shorter flight and the plane doesn't have a heavy fuel load, perhaps the weight is limited by some other consideration such as obstacles, or by the all-engine field length limit weight. There can be many reasons why a takeoff is made at less than the event/engine-out field length limit weight.

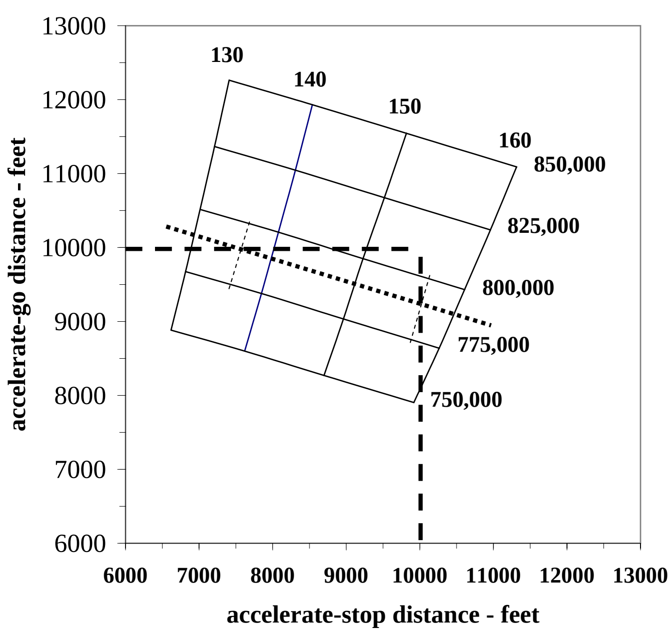


Figure 19-20

We've drawn a weight line on this chart at a weight of 790,000 pounds.

What does the web chart illustrate?

First, it shows that at this weight a V_1 as slow as 136 knots would yield acceptable accelerate-go distances. If using a V_1 slower than 136 knots, however, the accelerate-go distance would exceed the 10,000-foot runway length.

Second, it shows that at this weight, a V_1 as fast as 157 knots would yield acceptable accelerate-stop distances. A V_1 faster than 157 knots, however, would cause the accelerate-stop distance to exceed the 10,000-foot runway length.

In other words, on a 10,000-foot runway at a weight of 790,000 pounds, there is actually a range of acceptable V_1 speeds, all the way from 136 to 157 knots. Any V_1 within that range will yield accelerate-go and accelerate-stop performance within the available distances.

Obviously, no matter how wide the V_1 range may be for a given set of conditions, the constraints on V_1 still apply: V_1 may not exceed V_R or V_{MBE}, whichever is less, and it may not be less than V_{1MCG} (or V_{MCG} for the earlier airplanes).

how the V_1 range concept is useful

“All right,” you say, “this V_1 range idea is intellectually stimulating, but what practical value does it have?” That’s a good question; in fact there are a number of ways it can be useful.

Example: some years ago, a large airline had experienced a number of rejected takeoff over-run incidents. Disturbed by this, they wanted to take action to reduce the likelihood of over-runs in the future.

By studying the web chart data for their airplanes, they were able to establish a very simple policy which they called “reduced V1”. It was simply this: for each 1000 pounds that an airplane’s scheduled takeoff weight was below the field length limit value, for that takeoff the pilot would reduce the scheduled takeoff V_1 speed by one knot. By reducing V_1 they gave themselves additional stopping distance margin. Even a few knots of V_1 reduction offers a substantial decrease in the accelerate-stop distance required. This simple method resolved their over-run problem.

Here’s another example: suppose that your takeoff weight is limited by an obstacle. By allowing the V_1 to be increased within the V_1 range, the accelerate-go distance is reduced and the distance from the end of the takeoff to the obstacle is increased. This will allow an increase to the obstacle-limited weight.

And another example: if a takeoff weight is limited by brake energy, as is sometimes the case particularly at high-altitude high-temperature airports, the V_1 range allows the V_1 to be reduced, increasing the brake energy-limited weight.

Calculating Field Length Limit Weight and V_1

You now have all of the knowledge that you need for calculating, for any given conditions, the field length limit weight.

using AFM-DPI

If you're calculating takeoff performance for an airplane having AFM-DPI, your job is relatively simple.

Recall that AFM-DPI is the *Airplane Flight Manual-Digital Performance Information* software application. It takes the place of all of the Flight Manual Section 4 takeoff performance charts. AFM-DPI uses *first principles* calculations to generate takeoff, enroute climb and landing data. A first principles calculation of takeoff weights means that the software is rigorously calculating takeoff performance from fundamental data such as thrust, lift and drag, and so on. A first-principles method is the most accurate method available for computing takeoff performance.

AFM-DPI allows the user to make a *single-point calculation* of the field length limit weights for a single combination of parameters. Alternatively, it will let you compute the weights for any desired range of parameters, such as a range of temperatures; it will even let you find the limit weights for <u>two</u> independent ranges of variables, such as temperature and wind.

If you provide AFM-DPI with the runway parameters, it can give you the more critical of the all-engine and event/engine-out field length limit weights; if, on the other hand, you give it a weight or a range of weights, it can give you the corresponding all-engine and event/engine-out takeoff distances.

AFM-DPI also has a plotting capability allowing the user to graph one or more dependent variables as a function of the independent variable, for example the accelerate-stop and accelerate-go distances as a function of V_1.

It's beyond the scope of this document to instruct the reader on the use of AFM-DPI. A comprehensive User's Guide is available.

AFM-DPI is most effectively used as a tool for parametric studies of takeoff performance. It is <u>not</u> designed to produce finished tables of takeoff analysis data such as are used by most airlines. To do that, other software applications (for example, STAS) and their databases are available from Boeing. For day-to-day production of standard takeoff analysis tabulations, these are the appropriate tools, not AFM-DPI.

It's beyond the scope of this document to provide instruction on the use these other software tools. Comprehensive User Guides are available from Boeing; additionally, Boeing offers classes on the use of these tools.

using other takeoff software

Non-AFM-DPI airplanes are those that have detailed performance charts in Section 4 of the AFM. Computerized takeoff analysis for these airplanes is made possible by the availability of computer databases consisting of tabulations of the AFM takeoff charts.

The software tools mentioned above are designed to work with these databases also. As with the AFM-DPI airplanes, they allow the user to specify input parameters such as runway data, airport pressure altitude and so on, and obtain a takeoff analysis output in a number of user-selectable different output formats.

The AFM computer databases and other software applications can be obtained through Boeing.

using paper AFMs

It's beyond the scope of this document to instruct the readers on the solution of takeoff performance problems using the charts in Section 4 of the AFM for a non-AFM-DPI airplane. Those AFMs include an "example appendix" illustrating the methods for solving various types of takeoff problems utilizing the charts.

Obviously, given the large number of variables that affect takeoff performance, the AFM charts are numerous and very detailed. Some charts are relatively simple to use – for example, the charts for climb-limited weight – others are quite complex.

One example of the latter are the web charts used for determining field length limit weight and V_1. In principle they are the same as the web charts we've been showing you in this chapter, and they appear similar, but on closer examination you'd see that they're somewhat different.

Since you know that thrust, lift and drag depend on flap setting, altitude and temperature, you may have realized that the web charts we've shown above can be valid only for one specific flap-altitude-temperature combination. No other combination of flap setting, altitude and temperature would have the identical accelerate-go and accelerate-stop distances. Distances are also affected by wind and runway slope. Accelerate-stop distances will be affected by event/engine-out brake conditions such as a wheel brake deactivated, which is permitted for revenue service provided its effect is accounted for. The axes of the AFM web charts are "corrected" distances, which are the actual distances available corrected for wind, slope, and brake system condition. The lines of the web chart are lines of V_1/V_R rather than V_1, and "corrected runway length" instead of weight

These differences are necessary in order to "generalize" the web charts to make them valid over a range of conditions. The end result of using the AFM charts is the same as you saw above – you will have determined the field length limit weight and V_1 – but the exact method is slightly different in order to make it possible for one chart to cover a range of conditions. For the 747-400, for example, the AFM uses five different web charts to cover the entire environmental envelope. Other AFMs may use a different number of web charts.

Chapter 20: Takeoff on Non-Dry Runways

Introduction

Yes, we agree that the title of this chapter sounds a bit strange. What, after all, is a "non-dry" runway?

Originally, we were planning to call this chapter "Takeoff on Contaminated Runways", but then we realized that the word "contaminated" – although commonly used in this context – is imprecise and means different things to different people. We decided to use the term "Non-Dry" for this chapter.

In the chapter entitled "Calculating Takeoff Distances" we dealt with the takeoff performance of an airplane on a dry runway. In this chapter, we'll be discussing takeoff on runways that are either wet, slippery, or covered with slush, standing water or snow. You'll see that there are many similarities in how we'll calculate the takeoff distances, but there are some significant differences also. While the calculation of performance on a wet runway is relatively simple, you'll see that the task of doing the same thing for a runway that's covered with slush, for example, is far more complex.

Different runway conditions will have different effects on the acceleration and deceleration characteristics of an airplane. Wet and slippery runways will affect the airplane's deceleration capability without affecting its acceleration. Standing water, slush and loose "compactible" snow will affect an airplane's acceleration capability as well as its deceleration. In either case, the end result of these effects is the necessity for a change to the field length-limited takeoff weight and the corresponding V_1 in order to maintain the required operating margins of safety.

On non-dry runways for which there exist obstacles that limit the takeoff weight, the obstacle-limited weight will also be affected because of changes to the takeoff distances and the resulting change in the obstacle distance from the end of the takeoff.

In this chapter, we're first going to discuss the effects of each of these types of non-dry runway conditions on the takeoff distances. Then, after that's all finished, we'll look at the way the takeoff weight and V_1 will be affected by different contaminants. Finally, we'll look at the way that non-dry runway data is made available to operators of Boeing airplanes.

Terminology

The terminology relating to non-dry runways used by the regulatory agencies and elsewhere can be confusing and may be somewhat different from that used by Boeing. It will be instructive to discuss this, to avoid any misunderstanding later as we continue our discussion of the subject.

To begin: both the FAA (Advisory Circular 25-13) and the JAA (JAR-OPS 1 Subpart F) view runways as being either dry or wet or contaminated.

dry runways

The term *dry* should be obvious to all; there's no ambiguity about that.

wet runways

The FAA has no rigorous definition of wet runway, saying only that it is "a runway that is neither dry nor contaminated". Given their definition of a "contaminated" runway, quoted below, one can assume that <u>a wet runway for FAA purposes is one having slush or standing water of a depth equal to or less than one eighth of an inch, or 3.2 millimeters</u>.

JAR-OPS 1 is a little more detailed:

> *A runway is considered wet when the runway surface is covered with water, or equivalent...*[3mm or less in thickness]*...or when there is sufficient moisture on the runway surface to cause it to appear reflective, but without significant areas of standing water.*

Can we really measure water/slush depths of three millimeters? That's arguable; obviously, this definition is a bit arbitrary but the intent is clear: to attempt to separate those depths that will <u>not</u> affect acceleration from those that <u>will</u>.

contaminated runways

FAA Advisory Circular 25-13 defines a contaminated runway as follows:

> *A contaminated runway is a runway where more than 25 percent of the required field length, within the width being used, is covered by standing water or slush more than 0.125 inch (3.2 mm) deep, or that has an accumulation of snow or ice. However, in certain other situations it may be appropriate to consider the runway contaminated. For example, if the section of the runway surface that is covered with standing water or slush is located where rotation and liftoff will occur, or during the high speed part of the takeoff roll, the retardation effect will be far more significant than if it were encountered early in the takeoff while at low speed. In this situation, the runway might better be considered "contaminated" rather than "wet".*

JAR-OPS 1.480 is only slightly different:

> *(2) Contaminated runway. A runway is considered to be contaminated when more than 25% of the runway surface area (whether in isolated areas or not) within the required length and width being used is covered by the following:*
>
> > *i) Surface water more than 3 mm (0·125 in) deep, or by slush, or loose snow, equivalent to more than 3 mm (0·125 in) of water;*

(ii) Snow which has been compressed into a solid mass which resists further compression and will hold together or break into lumps if picked up (compacted snow); or

(iii) Ice, including wet ice.

Some of these contaminants – slush, for example – will affect both the airplane's ability to accelerate <u>and</u> its ability to decelerate. Some other contaminants such as ice will affect the airplane's deceleration but <u>not</u> its acceleration. For that reason, runway contaminants are divided into two different categories: *solid contaminants* and *loose contaminants*.

- <u>Solid contaminants:</u> these contaminants affect deceleration but have no effect on acceleration. This category includes ice and compact snow.
- <u>Loose contaminants:</u> these contaminants contribute a component of <u>drag,</u> retarding an airplane's motion and thus affecting <u>both</u> acceleration and deceleration. This includes slush or standing water more than 0.125 inches deep, and loose snow.

A contaminated runway is thus a runway having some portion of its surface (the regulatory guideline being 25 percent or more) covered with either a solid contaminant or a loose contaminant.

Background Information

wet runways

Prior to the release of FAR Part 25 Amendment 25-92 in 1998, airplane manufacturers were not required to certify wet runway performance data in the FAA-Approved Airplane Flight Manual. For over 30 years, however, Boeing has provided ***advisory*** wet runway takeoff data for those airplanes.

Until Amendment 25-92 was enacted, there was no formalized regulatory method for establishing the airplane braking coefficient μ_B for wet runways. To meet British CAA certification requirements then in effect, Boeing did do wet runway testing on the 707, 727, 737-100/-200, and the 747-100.

The data resulting from these tests was used for the AFM calculations for those specific airplanes and was the basis for using one half of the dry braking coefficient for certification of the 757, 767 and 747-200 to UKCAA standards.

This testing, together with the results from some other tests, led to the Boeing recommendation that an airplane braking coefficient of 0.2 should be used for wet runway <u>advisory</u> data.

<u>It must be clearly understood that the value of 0.2 is nothing more than an accepted value for a parameter that in fact has a considerable range of values</u>. The construction characteristics of different runways vary widely. As a result, wet braking capability can also vary greatly depending on the specific surface. The value of 0.2, approximately half that of dry braking, was selected as it satisfactorily represents the airplane's performance on the runways that were tested. If operating

into an airport having runways with worse braking characteristics, it's incumbent on the operator to make provision for that lower level of stopping performance by using a lower braking coefficient.

Amendment 25-92 now mandates for the first time that wet runway takeoff performance must be included in the certified data for the airplane. Accordingly, the AFM-DPIs for the airplanes certified under Amendment 25-92 incorporate certified wet runway takeoff data. The amendment specifies the method for calculating the wet runway airplane braking coefficient as a function of tire pressure and anti-skid system efficiency.

As discussed above, wet runway data has been included in the AFMs for UKCAA, JAA and EASA operators for many years.

wet skid-resistant runways

Some runways in the world are specially treated in a manner intended to improve the takeoff airplane braking coefficient when wet as compared to runways having no such treatment.

A *skid-resistant runway* is one that has been specially treated to provide better braking effectiveness when wet. The treatment typically takes one of two forms:

- The runway may be *grooved*. Grooving means cutting channels into the runway surface. Those grooves are square or rectangular in cross section. The FAA standard groove configuration calls for grooves that are 1/4 inch (6 mm) in depth, 1/4 inch (6 mm) in width, and 1 1/2 inches (38 mm) in spacing. These grooves facilitate the movement of surface water away from an airplane's tires, improving friction.
- The runways may be surfaced with a *porous friction course*, or PFC. A "course" is a layer. Like grooved runways, the porosity of this top layer also facilitates the movement of surface water away from an airplane's tires, improving friction.

Grooved runways or runways having a porous friction course both offer improved wet runway stopping performance but don't achieve the same capability as a dry runway. The amount of improvement is dependent on runway material (PFC) and groove spacing, as well as runway maintenance.

As it does for the smooth wet runways, Amendment 25-92 specifies the method for calculating the skid-resistant wet runway braking coefficient of friction as a function of tire pressure and anti-skid system efficiency.

It's worth pointing out here that the Airplane Flight Manual for Amendment 25-92 airplanes provides a restriction on taking the performance advantage when operating from a runway listed as skid-resistant:

> *Takeoff performance on wet skid resistant surfaces, such as grooved runways or porous friction course runways, is accessed in AFM-DPI by selecting WET SK-R. This data may be used only if the runway is constructed and maintained to meet the Friction Level Classification for Runway Pavement Surface defined in*

AC 150/5320-12C dated 3/18/97 or its equivalent. Operational approval of the wet skid resistant data must be obtained from the appropriate regulatory authority.

The information necessary to determine whether the runway is constructed and maintained to the standards listed in this FAA Advisory Circular is not readily available in databases. If it's desirable to take advantage of this performance improvement, the airline should contact the individual airports.

contaminated runways

At the time of this writing, December 2008, there are no FAA regulations concerning operation on contaminated runways.

FAA Advisory Circular (AC) 91-6A states:

The operations manual of the air carrier and commercial operator...should include specific instructions for the flightcrew on each type of turbojet aircraft showing the gross weight reduction, V_1 speed adjustments, and/or additional runway length required for the conditions described. These instructions should outline details of the methods to be used in determining runway conditions at departure time.

The AC also provides some interesting background:

a. Early in the operation of turbojet aircraft, it was determined that adjustment factors should be applied to the takeoff data in order to maintain the aircraft performance requirements as specified in the SR-422 series of the Civil Air Regulations and the Federal Aviation Regulations when water, slush, and/or snow are on the runway. The first test, using a Boeing 707 airplane, with a slush depth of 6/10 inch on the runway, showed that retardation of acceleration on takeoff was of such consequence that an offload from the maximum gross weight should be made for a critical field length.

b. In August 1961, further slush tests were conducted at the National Aviation Facilities Experimental Center (NAFEC) by the Federal Aviation Agency/National Aeronautics and Space Administration using the agency's Convair 880/22M type transport. The test was designed to obtain data regarding the retardation effects of slush and the effects of aquaplaning on the aircraft's takeoff performance, as well as aircraft control problems and damage encountered when operating in a runway slush environment.

c. The tests at NAFEC were conducted on a slush covered section of a 10,000-foot runway at depths of 0 to 2.0 inches and at velocities of 80 to 160 knots. The retardation forces measured from the deceleration data were considerably greater than those predicted from earlier wheel and tire drag tests and theoretical studies which neglected the factors of slush spray impingement and aquaplaning. Impingement of slush against the aircraft and landing wheels contributed significantly to slush *drag forces. At velocities above 120 knots, aquaplaning occurred and as a result, drag forces were reduced.*

Advisory data showing the performance effects of runway contamination has been produced and made available to operators of all Boeing commercial jet airplanes.

For the 707 and 727 airplanes, the data was published only for standing water or slush. For the 737 and subsequent airplanes, data has been provided for slippery runways as well.

January 2009 note: The FAA has begun a rule-making process addressing takeoff from contaminated runways. It is expected that comprehensive regulatory material governing operation on non-dry runways will be issued within a few years. The proposed rule making will add contaminated runway accountability for takeoff to FAR Part 25, as has already been done for wet runways. This will apply to new airplane certifications. The operating rules (for example, FAR Part 121) however will require operators of every airplane to account for both wet and contaminated runways.

Significant Differences From Dry Runway Certification

There are several elements of non-dry runway takeoff distance calculations that are significantly different from the method you've already seen for dry runways. Those two are the screen height and the use of reverse thrust.

screen height

The "screen height" is the height of the airplane's lowest point (usually the main landing gear) above the takeoff surface that defines the point at which the takeoff distance is considered to end. The screen height for dry runways is 35 feet. For contaminated runways the screen height used for calculating the takeoff distances with engine failure has historically been taken as 15 feet. FAR Part 25 Amendment 25-92 defines 15 feet as the engine-out screen height for wet or wet skid resistant runway takeoffs for airplanes certified under that Amendment.

The reduced screen height has the effect of reducing the accelerate-go distance significantly when compared to the distance to 35 feet.

credit for use of reverse thrust

dry runways

Dry runway takeoff data does not (with very few exceptions, such as the 737-300/-400/-500 certified to CAA rules) include credit for thrust reverse in the accelerate-stop calculations.

non-dry runways

Historically, Boeing has taken credit for reverse thrust when producing data for non-dry runways. This has been accepted by the FAA. The UKCAA rules, and now the JAA and European certification rules, also allow credit for reverse thrust under these conditions.

For airplanes certified under Amendment 25-92 rules: (737NG, 747-400F, 747-8, 757-300, 767-400, 777-200LR, 777-300ER, 787)

- The wet and wet skid-resistant runway certified all-engine takeoff RTO distances take credit for all engines in reverse thrust;
- The wet and wet skid-resistant runway certified engine-out RTO distances take credit for two symmetric engines in reverse thrust for the 747, and one engine in reverse thrust for the two-engine airplanes;
- The contaminated runway (non-certified, advisory only) takeoff distances consider both the all-engine and engine-out cases. All-engine RTO distances take credit for all engines in reverse; RTO distances for the engine-out case take credit for two symmetric engines in reverse thrust for the 747, and one engine in reverse thrust for the two-engine airplanes;

For airplanes certified under Amendment 25-42 rules: (777-200 and 777-300, except the ER and LR models)

- Wet and wet skid-resistant runways are not certified, as they are for Amendment 25-92 airplanes. The advisory wet runway data considers both the all-engine and engine-out RTO cases. The all-engine RTO distances take credit for all engines in reverse thrust; engine-out RTO distances take credit for two symmetric engines in reverse thrust for the 747, and one engine in reverse thrust for the two-engine airplanes;
- The contaminated runway (non-certified, advisory only) rejected takeoff distances consider both the all-engine and engine-out cases. All-engine RTO distances take credit for all engines in reverse; RTO distances for the engine-out case take credit for two symmetric engines in reverse thrust for the 747, and one engine in reverse thrust for the two-engine airplanes.

For all other Boeing models:

- There were no provisions for certification of wet or wet skid-resistant runway takeoff data;
- For all non-dry runway conditions, all of the advisory RTO distance data is based on the engine-out condition. Credit for reverse thrust is based on single-engine reverse thrust on a two-engine airplane, and symmetric two-engine reverse thrust on the 747.

Takeoff Distances on Solid Contaminants

The takeoff distances on runways having solid contaminants are affected differently from those distances on runways having loose contaminants. First, let's look at the distances on runways covered with solid contaminants.

Let's discuss the all-engine accelerate-go distance on a runway having solid contaminants, then the engine-out accelerate-go distance under the same conditions. Following that, we'll look at the two factors that make the accelerate-stop distances on runways having solid contaminants different from the distances on a dry runway: reverse thrust and the reduced airplane braking coefficients.

all-engine takeoff distance

Although the screen height is reduced from 35 to 15 feet on contaminated runways for the engine-out case, it doesn't change for the all-engine takeoff case.

Since solid contaminants don't have an adverse effect on acceleration, and since the screen height is unchanged for the all-engine case, the all-engine takeoff distance is unaffected by solid contaminant conditions.

Calculating this distance is done in exactly the same manner as for the dry runway, and the distances will be the same as they are for dry runways.

engine-out accelerate-go distance

This is only slightly different from the dry runway condition.

The distance for the acceleration from zero speed to rotation speed is identical to that for the dry runway case. The distance from rotation to the screen height – the end of the accelerate-go distance – is where the difference lies.

We discussed above the reduction of the screen height for the engine-out accelerate-go case from 35 to 15 feet. The flare distance for this case is calculated in the same manner as for the dry runway case, except that a different graph of time versus thrust-weight ratio is used. This relationship is established through flight testing.

Thus the calculation of this distance is done in the same manner as for a dry runway, with the exception of a different flare distance.

For the same V_1, the engine-out accelerate-go distance, on runways having solid contaminants, will be less than the corresponding distance on dry runways, due to the reduced screen height.

accelerate-stop distances: reverse thrust credit

Reverse thrust is of greater value on a wet runway than it is on a dry runway. This is simply due to the fact that it's a larger percentage of the total airplane retarding force available on a wet runway, because of the reduction of the braking coefficient when the runway is wet.

Reverse thrust isn't a constant amount of force: it's time and speed dependent. In the chapter entitled "Calculating Takeoff Distances" you saw the "engine spindown" graph. There's a plot similar to that one that shows the transition from forward thrust to reverse thrust as a function of time.

In the figure to the right we show an example of what such a typical time history could look like, and we've added the plot of spindown to forward idle for comparison.

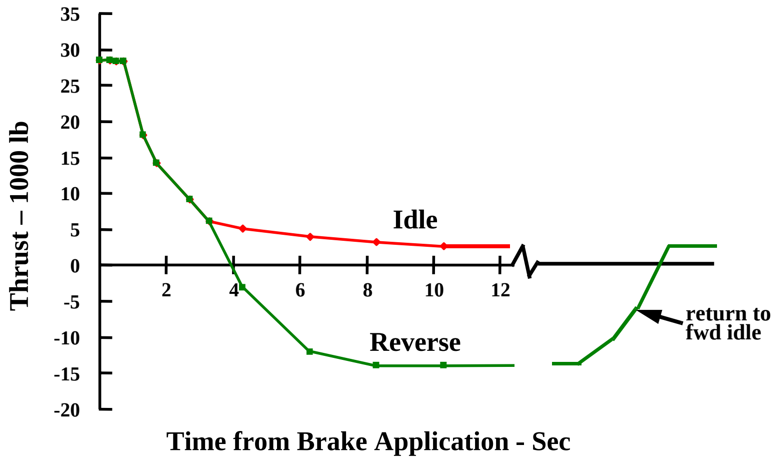


Figure 20-1

The speed effect on thrust that we mentioned above arises from the fact that the standard procedure for use of reverse thrust says "...field length permitting, initiate movement of the reverse thrust levers to reach the reverse idle detent by taxi speed". Thus there is a decrease in reverse thrust at the lower speeds during the stop maneuver. This is accounted for in the calculations.

As you can see, since the reverse thrust is a function of time – and of speed – the step integration calculation of the stopping distance will be more difficult than it is for the dry runway case.

accelerate-stop distances: airplane braking coefficient

In general: a wet runway has less friction available for stopping an airplane in an emergency. How much the runway friction is reduced by moisture on the surface of the runway is a function of the material and techniques of runway construction. Runway construction materials and techniques result in a runway surface microtexture and macrotexture. FAA Advisory Circular 150-5320-12 describes microtexture and macrotexture:

> *2-2. SURFACE TEXTURE AND DRAINAGE. In discussing the effects of pavement texture on friction and hydroplaning, two terms commonly used to describe the pavement surface are microtexture and macrotexture. Microtexture refers to the fine scale roughness contributed by small individual aggregate particles on pavement surfaces which are not readily discernible to the eye but are apparent to the touch, i.e. the feel of fine sandpaper. Macrotexture refers to visible roughness of the pavement surface as a whole. Microtexture provides frictional properties for aircraft operating at low speeds and macrotexture provides frictional properties for aircraft operating at high speeds. Together they provide adequate frictional properties for aircraft throughout their landing/takeoff speed range. The primary function of macrotexture is to provide paths for water to escape from beneath the aircraft tires. This drainage property becomes more important as the aircraft speed increases, tire tread depth decreases, and water depth increases. All three of these factors contribute to hydroplaning. Good microtexture provides a degree of "sharpness" necessary for the tire to break through the residual water film that remains after the bulk water has run off. Both properties are essential in providing skid-resistant pavement surfaces.*

Textural appearances, however, can be deceiving. A rough looking surface could provide adequate drainage channels for the water to escape, but the fine aggregate in the pavement may consist of rounded or uncrushed mineral grains that are subject to polishing by traffic, thereby causing the pavement surface to become slippery when wet. Likewise, a less rough looking surface, that may even have a shiny appearance when wet, will not necessarily be slippery if it has good microtextural properties.

With the enactment of Amendment 25-92 in 1998, a detailed method for determining wet runway braking coefficients is now in place.

Amendment 25-92 provides equations for the wet runway "maximum braking coefficient (tire-to-ground)" as a function of tire pressure and airplane ground speed. The equations are based on work by the Engineering Sciences Data Unit of London, as published in their paper number 71026, and are:

tire pressure (psi)	maximum braking coefficient (tire-to-ground)
50	$-0.0350\left(\frac{V}{100}\right)^3 + 0.306\left(\frac{V}{100}\right)^2 - 0.851\left(\frac{V}{100}\right) + 0.833$
100	$-0.0437\left(\frac{V}{100}\right)^3 + 0.320\left(\frac{V}{100}\right)^2 - 0.805\left(\frac{V}{100}\right) + 0.804$
200	$-0.0331\left(\frac{V}{100}\right)^3 + 0.252\left(\frac{V}{100}\right)^2 - 0.658\left(\frac{V}{100}\right) + 0.692$
300	$-0.0401\left(\frac{V}{100}\right)^3 + 0.263\left(\frac{V}{100}\right)^2 - 0.611\left(\frac{V}{100}\right) + 0.614$

where V is the true ground speed in knots

Linear interpolation is allowed for tire pressures other than those listed.

As an example, for a tire pressure of 200 psi the maximum friction coefficient defined by the equation will be as shown in the figure to the right.

For its determination of the airplane maximum braking coefficient on wet runways Boeing uses a tire pressure which is approximately at the top of the range of tire pressures for that airplane.

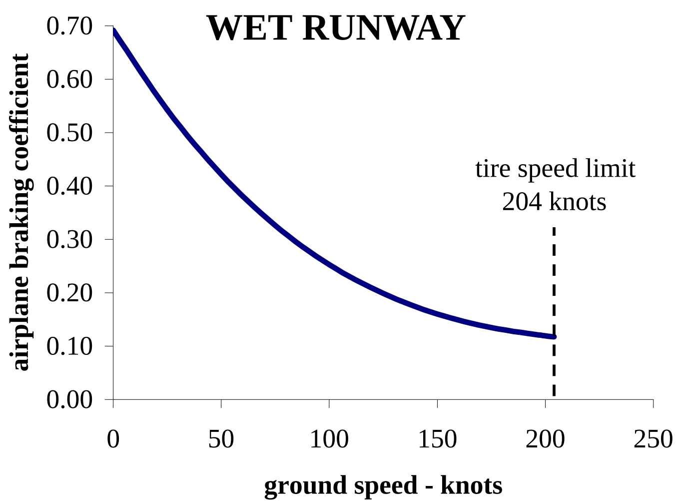


Figure 20-2

This "maximum" coefficient of friction, however, is not the one that will be used for calculating the accelerate-stop distances, because the amendment goes on to say:

> *The maximum tire-to-ground wet runway braking coefficient of friction must be adjusted to take into account the efficiency of the anti-skid system on a wet runway. Anti-skid system operation must be demonstrated by flight testing on a smooth wet runway, and its efficiency must be determined...*

In lieu of flight-testing the anti-skid system, Section 25.109 allows manufacturers to use adjustments provided in the Section – which they call *efficiency values* – to adjust the maximum braking coefficients. To date, Boeing has elected to establish the anti-skid adjustments by flight testing.

Based on the equations and the results of our flight testing, then, Boeing arrives at a definition of the wet runway airplane braking coefficient for the given airplane, as shown in this figure.

Because the airplane braking coefficient is a function of airplane ground speed, the calculation of the stopping distance on wet runways does not use a single constant value of μ_B as we do for dry runways. The step integration of stopping distance will use a changing value of μ_B as the speed decreases.

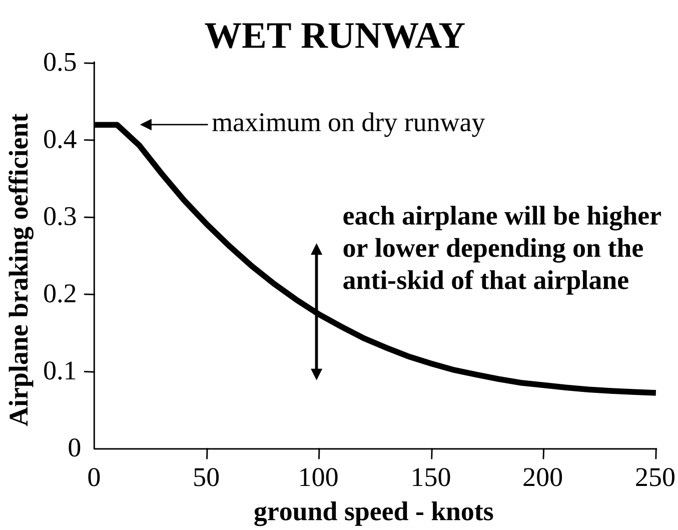


Figure 20-3

wet runways, prior to 25-92

Recall the discussion on airplane braking coefficient in the chapter entitled "Calculating Takeoff Distances". You learned there that the airplane braking coefficient μ_B on dry runways is determined during the flight test program of a new airplane by a series of airplane accelerate-stop tests. The brake force is plotted as a function of two different parameters: the average weight on the wheels during the stop, and the initial braking energy. Each of these plots yields a brake force and, from that, a braking coefficient of friction. The airplane braking coefficient of friction to be used is then the smaller of the two values determined from the brake force in the anti-skid limited region and the brake force determined for the torque limited region.

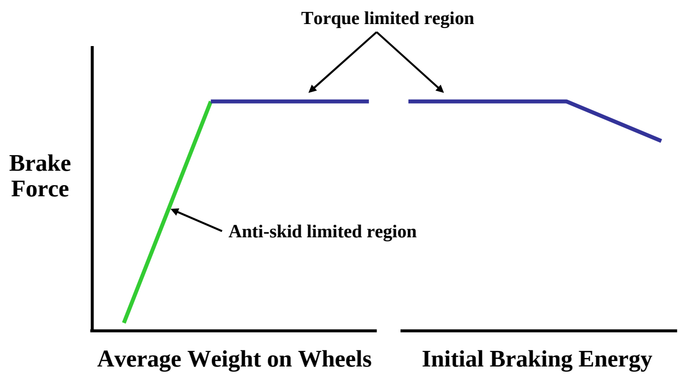


Figure 20-4

In the anti-skid limited region, the brake force is a variable depending on the average weight on the wheels, but the airplane braking coefficient μ_B is constant.

In the case of a wet runway, as the figure to the right illustrates, the braking coefficient will almost always be anti-skid limited. This is due to the fact that while the torque limited brake force is unaffected by slippery runways, the anti-skid limited brake force is adversely affected. Thus, for any reasonable amount of average weight on the wheels, the brake force coming from tire-to-ground friction is substantially less than the force that can be developed from the brake torque.

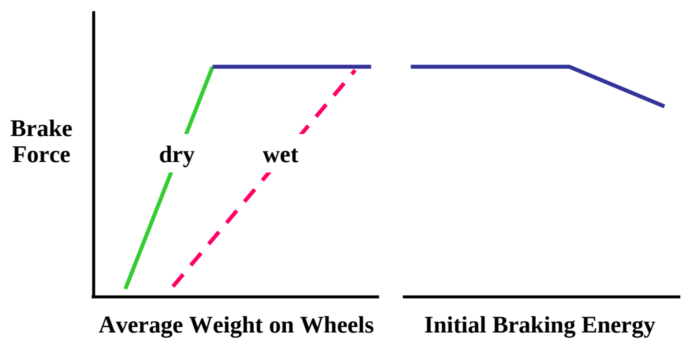


Figure 20-5

In calculating the accelerate-stop distances on wet runways prior to Amendment 25-92, the airplane braking coefficient used for the calculation is a constant throughout the stop, as it was for the dry runway condition.

wet skid-resistant runways

The braking coefficient for the wet skid-resistant runway is, like the wet runway case, a varying value of μ_B as a function of groundspeed. Testing has shown that wet skid-resistant runways offer typically from 75 to 90 percent of dry runway braking.

As is the case with smooth wet runways, FAR Section 25.109 provides an equation for calculating the maximum braking coefficient as a function of tire pressure using equations similar in form to those for smooth wet runways. This maximum braking coefficient must then be adjusted to account for the anti-skid system efficiency. Adjustment factors are provided; alternatively, the manufacturer may establish its own adjustment factors through flight testing.

The airplane braking coefficients on wet skid-resistant runways are utilized in the distance calculations in the same way as they for runways that are not skid-resistant.

The FPPM and PEM do not contain any data for wet skid-resistant runways. Both the FAA and EASA versions of AFM-DPI do contain that data for airplanes certified to the Amendment 25-92. Thus, an operator is able to produce certified takeoff data for wet skid-resistant runways for those airplanes.

solid contaminants other than wet or wet skid-resistant

There is no universally accepted relationship between runway description, reported braking action, and airplane performance. The airplane's actual performance may well be different for the same description of the runway surface or the pilot-reported braking action. Boeing has chosen, based on experience, a relationship of reported braking action to airplane braking coefficient. This relationship has been used to create the published data.

As previously discussed, Boeing publishes data labeled "good" based on an airplane braking coefficient of 0.2. For airplanes prior to Amendment 25-92, this was also the value of braking coefficient recommended for wet runways.

Boeing considers "medium" or "fair" braking conditions to be equivalent to a braking coefficient of 0.1. This is what would be expected under conditions of dry snow at an outside air temperature colder than 15 °C, or ice or slush. Published Boeing data for medium braking is based on a coefficient of 0.1.

Boeing feels a braking coefficient of 0.05 is an acceptable representative value for "poor" braking conditions. This would be advisable, for example, for conditions of melting ice. Published Boeing data for poor braking is based on a coefficient of 0.05.

With the future implementation of new regulations, now under development as mentioned earlier, the preceding paragraphs will become obsolete. The new rules will provide more explicit guidelines for calculating performance on contaminated runways. This document will be updated to reflect these changes as necessary.

accelerate-stop distances on solid contaminants

The basic process for calculating accelerate-stop distances, whether all-engine or engine-out, is essentially the same as for dry runways: step integration, just as you saw in the chapter entitled "Calculating Takeoff Distances".

Calculating the accelerate-stop distances on runways having solid contaminants, however, is clearly a bit more complex than for dry runways:

- On wet and wet skid-resistant runways, the coefficient of friction is a function of the airplane's velocity;

- Reverse thrust is a function of time and velocity.

Takeoff Distances on Loose Contaminants

Up to this point, we have considered those runway conditions that have no effect on an airplane's acceleration during takeoff. Now it's time to look at the conditions called "loose contaminants": standing water, slush, wet snow, or dry (loose) snow. These are considerably more complex, and they will affect the airplane's deceleration capability as well as its acceleration.

Note: *contaminant drag* is the term traditionally given to the additional drag imposed on an airplane due to the presence of a contaminant on the runway. For simplicity in this discussion, whether we're talking about slush or standing water or wet snow, we'll refer to the increment of drag they cause as contaminant drag.

physics of contaminant drag

The physics of takeoff on a runway having loose contaminants are similar to those on a dry runway, with one notable exception: the addition of the drag on the airplane resulting from the material which is covering the runway, be it standing water, slush, or wet snow.

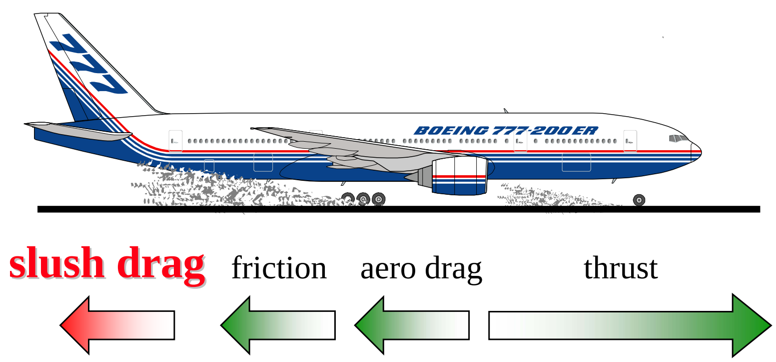


Figure 20-6

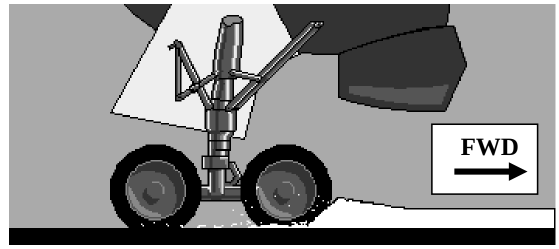
(Sharp-eyed readers will have noticed that in the diagram we've left out one more force: the component of weight due to runway slope. We've done that only to simplify the drawing slightly – it's still a real force that must be accounted for when appropriate. For now, we'll say that we have a level runway.)

Contaminant drag actually has two elements: *displacement* drag and *impingement* drag.

As illustrated in the figure to the right, displacement drag results from the energy required for the landing gear tires to *displace* the contaminant – that is, to move it out of their way as the airplane rolls along the runway.

Impingement drag results from the airplane kinetic energy lost due to the impact of contaminant on parts of the body. The passage of the wheels through the contaminant causes a very powerful spray to be thrown up; due to its density and the velocity at which it strikes the airplane, it creates considerable impact force on the airplane. Since this impact force is in an aftward direction, it subtracts from the airplane's kinetic energy.

Displacement drag



Impingement drag

Figure 20-7

The contaminant impact can actually cause physical damage to an airplane. As a result of this, and because of the increasingly adverse effect of loose contaminants on takeoff performance as depth increases, <u>the FAA and JAA both state specifically that takeoff is prohibited on runways having more than 1/2 inch (FAA) or 12.7 millimeters (JAA) of loose contaminant.</u>

There's one exception to the statement above: the latest EASA regulations on non-dry runways permit up to 15 millimeters of depth instead of the earlier 12.7 millimeters, which corresponds to the FAA's maximum depth of one-half inch. At the time of this writing (March 2009) however, Boeing takeoff software still limits the maximum depth to 12.7 millimeters.

hydroplaning

Hydroplaning (also sometimes referred to as "aquaplaning") is a dynamic condition encountered by an airplane's tires when operating on runways covered with loose contaminant.

At <u>low</u> speeds on a runway having loose contaminant there is adequate time for the contaminant to move away from an airplane's tires as it accelerates down the runway for takeoff. The tires remain in solid contact with the runway surface. The presence of the contaminant does result in an increase of the airplane's drag, as discussed above, but there are no other adverse effects.

However, as an airplane accelerates in loose contaminant, the tires cause an increase of pressure in the contaminant in the area immediately ahead of them. When that pressure becomes sufficiently great, it forces a wedge of fluid underneath the tires' leading edges, thus lifting the tires out of contact with the runway surface resulting in a loss of traction.

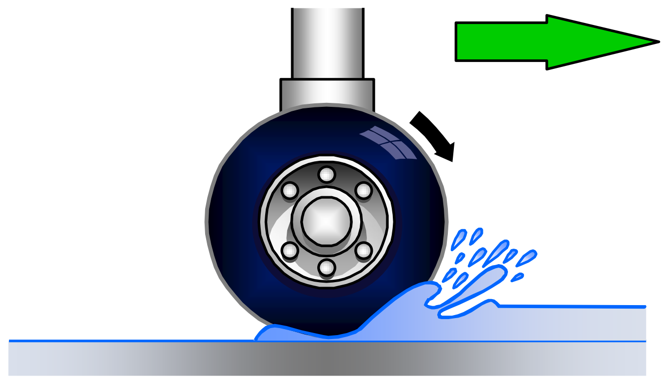


Figure 20-8

The speed at which hydroplaning commences during an acceleration is known as the "hydroplaning speed" V_{HP}. It's a function of tire pressure.

The accepted equation for the hydroplaning speed is:

$$V_{HP} = 8.63\sqrt{\frac{tire\ pressure}{\sigma}} \qquad \text{(eq. 1)}$$

where V_{HP} is the hydroplaning speed in knots
σ is the contaminant specific gravity
tire pressure is expressed in pounds per square inch (psi)

Traditionally, Boeing has used values of 0.85 and 1.00 for the contaminant specific gravity of slush and standing water respectively.

It's worth noting here that EASA presently uses a slightly different version of equation 1:

$$V_{HP} = 9.0\sqrt{tire\ pressure}$$

the takeoff distance calculation process

The acceleration of the airplane from the start of the takeoff roll to liftoff is divided into three different speed ranges:

- <u>From brake release to the hydroplaning speed</u>. In this speed regime, the distance calculation follows the same method as before, but includes the additional drag of the contaminant as a function of speed;
- <u>From hydroplaning speed to rotation speed</u>. In this speed range, the tires are out of contact with the runway surface; the contaminant drag initially increases slowly, then as the hydroplaning effect increases the contaminant drag begins to decrease. As the speed increases further, the contaminant drag decreases more rapidly.

- From rotation speed to liftoff. When the airplane is rotated, the nose landing gear tires are instantaneously lifted out of the contaminant, causing an abrupt discontinuity in the contaminant drag. Beyond V_R, as the speed increases toward the liftoff speed, the contaminant drag decreases more and more rapidly.

The calculation process used by Boeing for determining the distance is considered proprietary and may not be discussed here.

In Figure 20-10, to the right, for one Boeing model taking off in half an inch of water with all engines operating you can see how the slush drag varies with velocity at ground speeds approaching and then exceeding the hydroplaning and rotation speeds.

Up to the hydroplaning speed V_{HP} the slush drag increases with the square of the velocity. At V_{HP} you see that the hyroplaning is beginning to affect the slush drag as the tires lose contact with the takeoff surface.

At the rotation speed V_R there is an instantaneous discontinuity in the contaminant drag due to the lifting of the nose landing gear from the runway, out of contact with the contaminant.

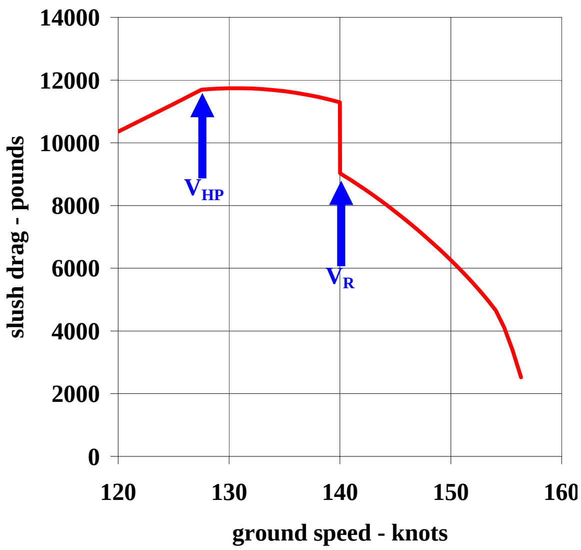


Figure 20-10

Following rotation, as the main landing gear tires lift out of the contaminant, the drag decreases rapidly and becomes equal to zero at the liftoff speed.

For the engine-out accelerate-go distance, the effect of the contaminant will look much the same as you see for all engines operating, except of course the acceleration will be greatly reduced following the engine failure.

For the accelerate-stop distances, the slush has two different effects, and the effect of the slush depth is different in these two cases. For acceleration, increasing contaminant depth has an adverse effect, since the contaminant drag is a function of the area of the tire exposed to the slush – which depends directly on the depth. On the other hand, increasing contaminant depth actually improves the deceleration because of the increasing drag.

effects of slush on acceleration and deceleration

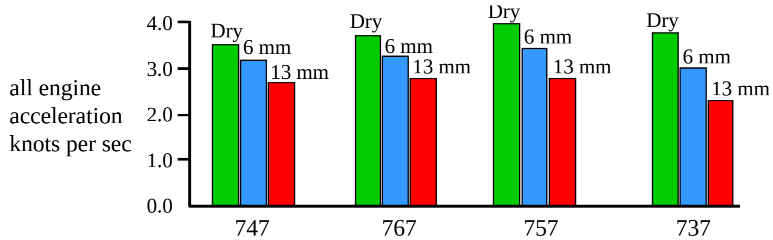


Figure 20-11a

All-engine acceleration: in six millimeters (1/4 inch) of slush, you can see that there's a 10 to 20 percent reduction in the all-engine acceleration. A slush depth of 13 millimeters (1/2 inch), the maximum, causes a 20 to 40 percent reduction in the all-engine acceleration. That's reasonable. How about the acceleration capability following an engine failure?

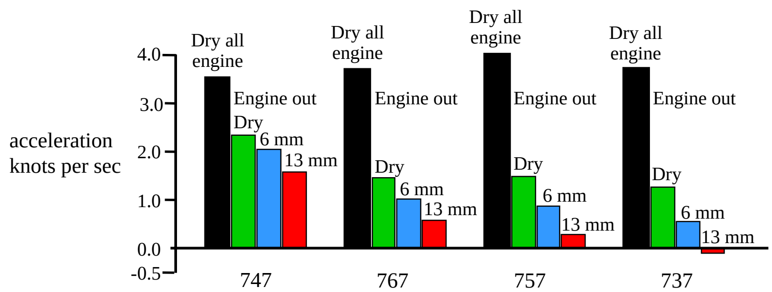


Figure 20-11b

Engine-out acceleration: to the right we show the effect of runways covered with slush or standing water on engine-out acceleration. You see that in six millimeters of slush, there's a 15 to 50 percent reduction in the engine-out acceleration capability.

In 13 millimeters of slush, there's a 30 to 110 percent reduction in the engine-out acceleration. More than 100 percent loss of acceleration? Yes.

Because of this fact, if constrained to a balanced takeoff, the 737 could not take off in 12.7 millimeters of slush or standing water. (There is, however, an operational method for getting around this condition, given a sufficiently long runway: by setting V_1 equal to V_R. That will unbalance the takeoff distances, making the accelerate-stop distance longer but enabling the airplane to conduct the accelerate-go.)

You can see that when computing the engine-out takeoff distances on runways having loose contaminants, it's necessary to ensure that the airplane has adequate acceleration to continue the takeoff following the engine failure. This is done by checking the engine-out acceleration capability at speeds below the rotation speed. If the acceleration doesn't meet a specified minimum threshold then mitigating action must be taken. On Boeing airplanes the minimum acceleration value used and the method for mitigation has changed over the years. It should also be pointed out that this issue is more common on smaller airplanes.

In general on older airplanes such as the 737-100 through -500, the 757-200, and the 767-200/-300 the minimum engine-out acceleration considered was zero feet per second per second. When the engine-out acceleration fell below the zero threshold, the typical mitigation strategy was to set V_1 equal to V_R, in essence not allowing an engine failure to be considered until flying speed had been reached. The field length in this case would be based on an accelerate-stop distance from a V_1 equal to V_R, resulting in very long takeoff distances. This mitigation method had the advantage of ensuring that such takeoffs could still be conducted, providing adequate runway length was available.

In general, on the newer airplanes like the 777, 737NG, 757-300 and 767-400 the minimum engine-out acceleration considered is 0.5 feet per second per second. When the engine-out acceleration falls below this check value, the computation is discontinued. The maximum takeoff weight allowed is that weight for which the engine-out acceleration exceeds the threshold.

<ins>Engine-out deceleration:</ins>
You saw previously that the airplane braking coefficient is greatly reduced in slush.

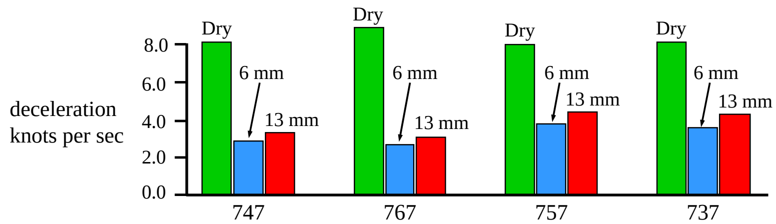


Figure 20-11c

However, the slush <ins>does</ins> add drag to the landing gear, at least partially offsetting the loss of braking coefficient. You can see too that a greater contaminant depth is <ins>better</ins> than a lesser depth because of the greater slush drag.

calculating the loose contaminant takeoff distances

Armed with the above information on the effects of loose contaminants, it’s now possible to calculate the takeoff distances.

While the calculation isn’t trivial, it follows the same basic step-integration method as for the dry case. The dependence of the thrust on time following the throttle chop and the dependence of the liftoff relief factor f_{lof} on time following rotation necessitate a complex time-based step integration to arrive at the distance. The underlying methods, however, are the same as those you’ve seen previously and will not be demonstrated here.

Summary: Takeoff Distances on Loose and Solid Contaminants

In the chart to the right we show the accelerate-go and accelerate-stop distances for a variety of contaminants, all at the same weight (645,882 lb) and V_1 (160.6 knots) – the field length limit weight and V_1 for an 11,000 foot dry runway.

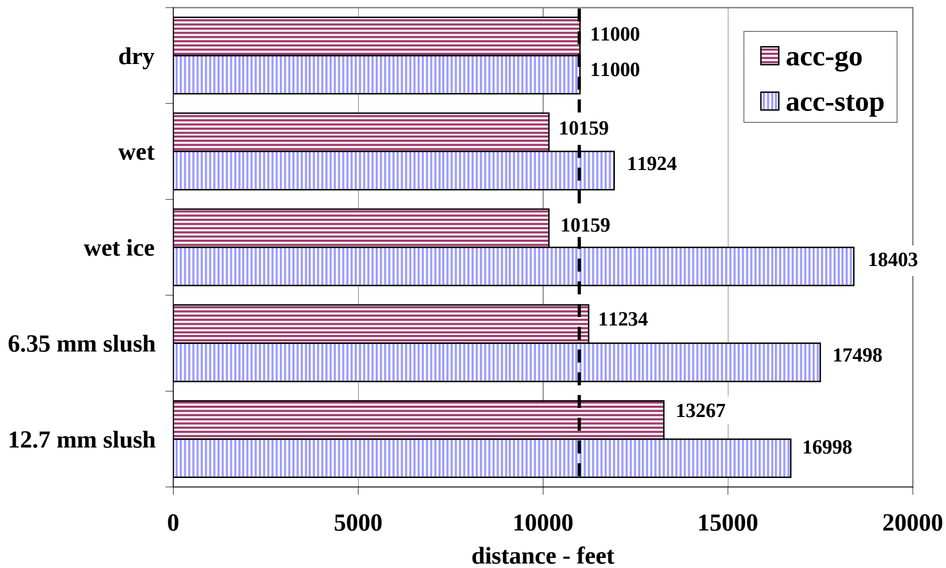


Figure 20-12

Notice that the wet runway accelerate-go distance is less than for the dry runway. This is due to the decrease of the screen height and the fact that there is no loss of acceleration capability on a wet runway. The accelerate-stop distance, however, is somewhat increased because of the reduction of the airplane braking coefficient – offset somewhat, however, by the credit for reverse thrust.

For the runway contaminated with wet ice, the accelerate-go distance is the same as the wet runway case; notice, though, that the accelerate-stop distance is greatly increased because of the substantial loss of braking capability.

For the runway with 6.35 millimeters (1/4 inch) of slush, you see that the screen height reduction almost completely offsets the effect of the added slush drag, so there's only a slight increase in the accelerate-go distance. The accelerate-stop distance, however, increases by almost 6500 feet.

Contrast that with the distances for 12.7 millimeters (1/2 inch) of slush: here, the accelerate-go distance is further increased, as we would expect because of the increased slush drag; the accelerate-stop distance is <u>decreased</u> however because of the additional slush drag's effect on the deceleration capability.

Effects of Contaminants on Weight and V_1 Speed

Now that you're familiar with the effects of contaminants on the accelerate-go and accelerate-stop distances, it will be easier to understand their effects on the field length limit weight and V_1. Let's start with wet smooth runways, then look at the effects of the other contaminants.

wet runways

Clearly, the reduction of the airplane braking coefficient will tend to increase accelerate-stop distances. In the engine-out accelerate-stop case, that is partially offset by the use of reverse thrust. The engine-out accelerate-go distances can be expected to decrease due to the reduction of the screen height from 35 to 15 feet, especially since there is no additional drag on the landing gear on a wet runway.

To illustrate the effect of wet runway operation on the field length-limited weight and V_1, we ran some wet runway data using typical takeoff conditions. For simplicity, we ran only two weights and two V_1 values, using AFM-DPI to compute the accelerate-stop and accelerate-go distances for the given values. The results are as follows:

weight (lb)	V_1	acc-stop dist DRY	acc-go dist DRY	acc-stop dist WET	acc-go dist WET
640,000	150	9375	12179	10274	11358
640,000	165	11586	9939	12545	9119
650,000	150	9509	12838	10369	11979
650,000	165	11772	10515	12671	9664

Table 1: Table 20-1

The data above defines, in effect, one cell of a web chart, bounded by weight lines of 640,000 and 650,000 pounds and V_1 lines of 150 and 165 knots, for both dry and wet runways. (If you've forgotten the web chart – shame on you! – refer back to the chapter entitled "Field Length Limit Takeoff Weight".)

If we look just at the web chart cell for the dry runway conditions, it's as shown to the right.

Each corner of the cell corresponds to one of the four weight-V_1 points in the table above.

We've also added a point on the graph corresponding to an 11,000 feet runway with no clearway and no stopway. By interpolation between the weight and V_1 lines, you can see that the field length-limited weight for this condition would be about 646,000 pounds with a V_1 of 161 knots.

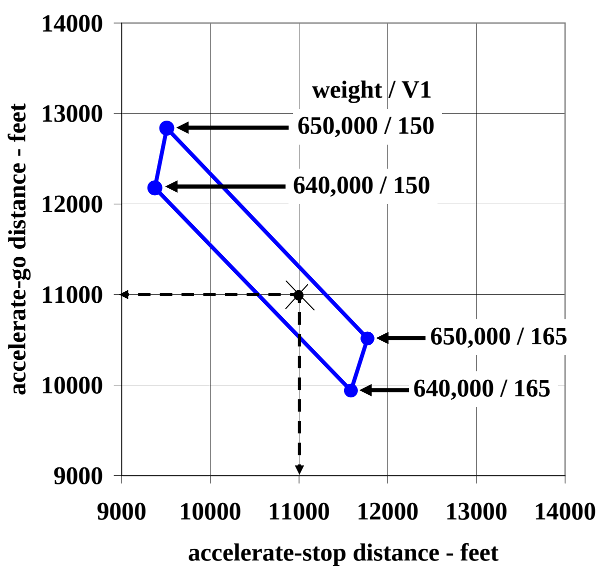


Figure 20-13

Now look at the same chart with the wet runway data added. You see that the wet runway cell is below and to the right of the dry runway cell. That agrees with what we would expect: somewhat shorter accelerate-go distances because of the screen height decrease, but longer accelerate-stop distances.

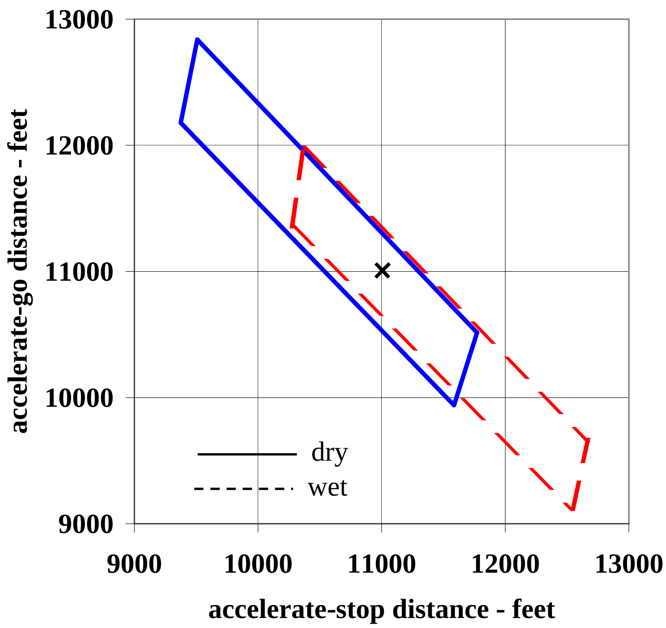


Figure 20-14

Look where the 11,000 foot runway point lies within the wet cell: at a weight of 645,000 pounds and a V_1 of 155 knots.

Thus you see that, for these conditions, the field length-limited weight for these conditions will decrease by about 1000 pounds with a V_1 decrease of nine knots when the runway is wet, to ensure that the required distances don't exceed the available distances.

other contaminants

In the wet runway example above, we needed a weight reduction of about a thousand pounds and a V_1 reduction of about nine knots to "re-balance" the takeoff distances, so that the new weight and the new V_1 on a wet runway will yield the same accelerate-stop and accelerate-go distances as the dry runway weight and V_1 will require on a dry runway.

In the table to the right, we show the weight and V_1 for a balanced 11,000 foot takeoff distance, for different runway conditions.

runway condition	weight (pounds)	V1 (knots)
dry	645,882	160.6
wet	644,625	154.6
wet ice	580,294	117.7
6.35 mm slush	567,404	130.1
12.7 mm slush	543,564	135.8

Table 2: Table 20-2

For the wet runway, we're showing in the table the exact values of weight and V_1 for the balanced condition. What we saw in the web chart above were rounded off for convenience.

Notice the very large reduction of V_1 on the runway covered with wet ice.

Notice also that the weight on a runway having 12.7 mm slush is less than the weight for the runway having 6.35 mm of slush, but the V_1 is greater.

A more general way of showing the effects of the different contaminants can be seen in the graph to the right.

The upper portion of the plot consists of lines of balanced takeoff distance versus weight for a dry runway and for contaminated runways.

The lower portion of the plot consists of lines of weight versus V_1 for dry and contaminated runways.

You'll see that the plot agrees with the table just above it for an 11,000 foot runway.

You'll be seeing this graph again a little later.

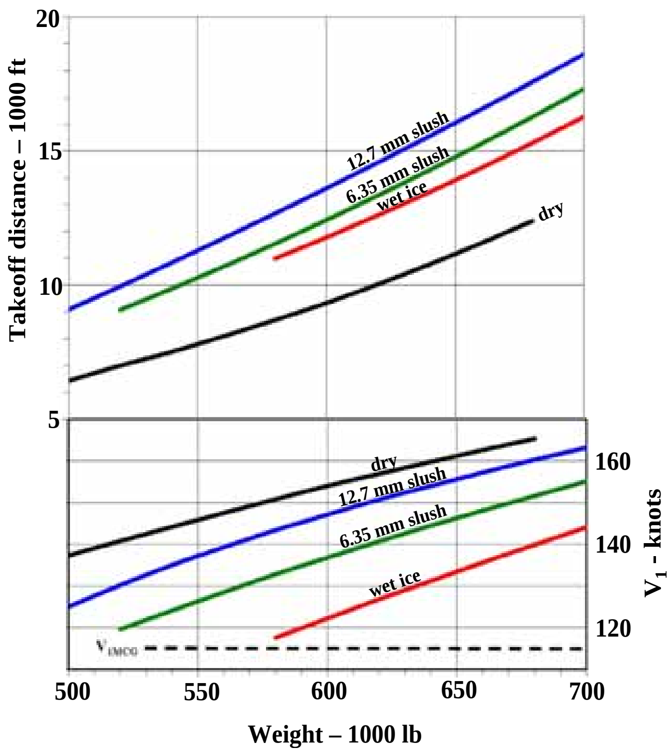


Figure 20-15

Presentation of Contaminated Runway Data

availability of software

Software enabling the user to compute takeoff weight and V_1 values for different runway contamination conditions is available for almost all 737 and later Boeing models. The software is able to compute and print a tabulation of the allowable takeoff weight and speeds for a given runway under any runway conditions desired, for a range of temperatures and winds.

The SCAP specification includes the protocols for calculation of data for non-dry runways. Boeing's SCAP-compliant software application called STAS, or other equivalent applications, is able to compute data for non-dry conditions.

equal-distance weight and V_1 decrements

The effects of runway contamination can be presented in the form of decrements.

Contaminated runway data appears in the FPPMs and/or PEMs for most airplanes. In these documents, the effects of the contaminants are shown as weight and V_1 decrements that are applied to the dry runway values.

The weight and V_1 decrements are derived from an *equal distance* concept. That is, after correctly applying the weight and V_1 decrements to the dry runway values, takeoff at the corrected weight and V_1 will yield the same takeoff distances as would be required on the dry runway.

The graph at the right is a repetition of the graph shown a few paragraphs previously. Here we have added an example of how the equal distance concept works:

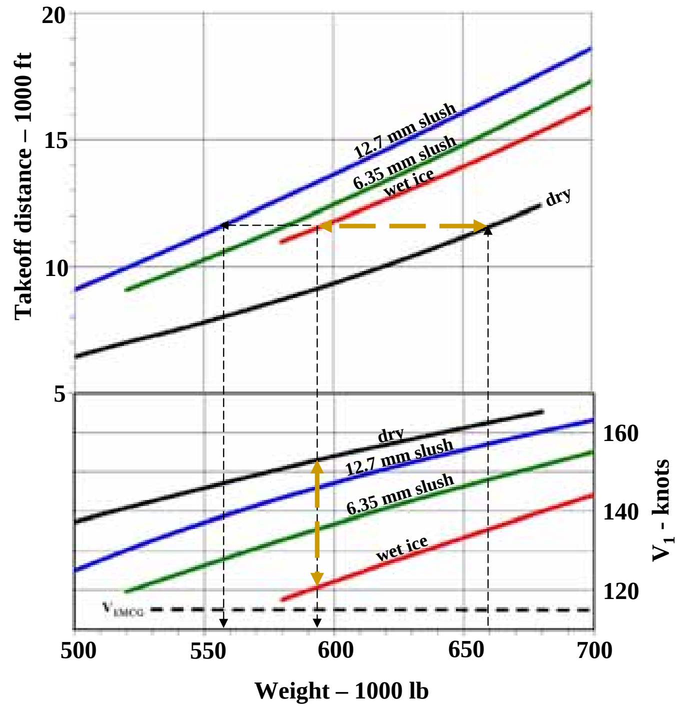


Figure 20-16

Say for example that we have a dry runway field length limit weight of 660,000 pounds. Enter the graph at the bottom with that value. Proceed upward to the dry runway takeoff distance line. From there proceed to the left to the distance line for the particular contaminant you're studying. Let's say, for example, that's wet ice.

You see from the heavy arrow that the weight decrement from dry to wet ice, *at the same takeoff distance,* will be about 68,000 pounds (from 660,000 down to 592,000). Now proceeding downward to the lines of V_1, you see that the reduction of V_1 will be about 32 knots (from 153 down to 121).

Using the same method, you will see that for a runway covered with 12.7 millimeters (1/2 inch) of slush, the weight decrement would be about 103,000 pounds with a V_1 decrement of about eight knots.

tabulated weight and V_1 decrements

While it's beyond the scope of this document to explain in detail the use of specific FCOM, FPPM and PEM performance data, it's worth talking at least generally about the presentation of the tabulated data for contaminated runway conditions in those documents.

As we mentioned before, Boeing has for many years provided information as a function of airplane braking coefficient and/or reported braking action. Good braking data is based on an airplane braking coefficient of 0.2, fair or medium braking data is based on a coefficient of 0.1, and poor braking data is based on a coefficient of 0.05. For the earlier airplanes not certified under Amendment 25-92, Boeing recommends using a braking coefficient of 0.2 for wet runways.

The data is provided in the form of weight and V_1 reductions; the weight reduction is applied to the dry runway field length-limit weight, and the V_1 reduction is applied to the dry runway V_1 *corresponding to the new decremented weight*. The tables which are shown as examples below are for the same 777-200 as the weight and V_1 graphs shown above, so you can see how they agree with each other.

The weight decrement tables are as shown below for slush-covered runways:

Slush/Standing Water Takeoff
Maximum Reverse Thrust
Weight Adjustment (1000 LB)

DRY FIELD/OBSTACLE LIMIT WEIGHT (1000 LB)	SLUSH/STANDING WATER DEPTH								
	0.12 INCHES (3 mm)			0.25 INCHES (6 mm)			0.50 INCHES (13 mm)		
	PRESSURE ALTITUDE (FT)			PRESSURE ALTITUDE (FT)			PRESSURE ALTITUDE (FT)		
	S.L.	4000	8000	S.L.	4000	8000	S.L.	4000	8000
700	-71.7	-83.3	-94.9	-84.2	-95.8	-107.4	-111.5	-123.1	-134.7
660	-68.4	-80.0	-91.6	-79.3	-90.9	-102.5	-102.9	-114.5	-126.1
620	-64.4	-76.0	-87.6	-73.8	-85.4	-97.0	-94.0	-105.6	-117.2
580	-59.7	-71.3	-82.9	-67.7	-79.3	-90.9	-84.7	-96.3	-107.9
540	-54.3	-65.9	-77.5	-61.0	-72.6	-84.2	-75.1	-86.7	-98.3
500	-48.3	-59.9	-71.5	-53.7	-65.3	-76.9	-65.1	-76.7	-88.3
460	-41.6	-53.2	-64.8	-45.8	-57.4	-69.0	-54.8	-66.4	-78.0
420	-34.3	-45.9	-57.5	-37.4	-49.0	-60.6	-44.1	-55.7	-67.3
380	-26.3	-37.9	-49.5	-28.3	-39.9	-51.5	-33.1	-44.7	-56.3
340	-17.7	-29.3	-40.9	-18.7	-30.3	-41.9	-21.8	-33.4	-45.0
300	-8.9	-20.5	-32.1	-9.0	-20.6	-32.2	-10.4	-22.0	-33.6

Table 20-3

Recall the example we used above of a dry runway having a field length limit weight of 660,000 pounds. We saw from the graph that *for the same takeoff distance* on a runway covered with 1/2 inch of slush the new weight would need to be approximately 103,000 pounds less than the dry weight. Look now in the table above for 0.50 inches of slush at sea level, and you'll see the weight decrement of 102,900 pounds. That agrees very well.

V1 Adjustment (KIAS)

WEIGHT (1000 LB)	SLUSH/STANDING WATER DEPTH								
	0.12 INCHES (3 mm)			0.25 INCHES (6 mm)			0.50 INCHES (13 mm)		
	PRESSURE ALTITUDE (FT)			PRESSURE ALTITUDE (FT)			PRESSURE ALTITUDE (FT)		
	S.L.	4000	8000	S.L.	4000	8000	S.L.	4000	8000
660	-19	-15	-11	-14	-10	-6	-4	0	0
620	-21	-17	-13	-16	-12	-8	-5	-1	0
580	-22	-18	-14	-18	-14	-10	-7	-3	0
540	-24	-20	-16	-20	-16	-12	-10	-6	-2
500	-26	-22	-18	-22	-18	-14	-14	-10	-6
460	-27	-23	-19	-24	-20	-16	-17	-13	-9
420	-28	-24	-20	-25	-21	-17	-20	-16	-12
380	-28	-24	-20	-26	-22	-18	-21	-17	-13
340	-27	-23	-19	-25	-21	-17	-21	-17	-13
300	-26	-22	-18	-24	-20	-16	-21	-17	-13

Table 20-4

Continuing the example, the new field length limit weight on the contaminated runway would be just slightly less than 560,000 pounds; for that weight, with 0.50 inches of slush at sea level, you see that in the table just above that the V_1 reduction would be about eight or nine knots. Again, this agrees with the eight knot decrement we got from the graph of weight and V_1.

The table just below shows the performance on slippery runways for three different reported braking action conditions. Above, we looked at a case of an icy runway and saw that for a dry runway field length limit weight of 660,000 pounds the weight decrement would be about 68,000 pounds. You can see in the table below that for “poor” braking conditions, which is the braking action we would suggest for use with a wet icy runway, a braking coefficient of 0.05, the weight penalty is 68,300 pounds on a sea level runway. That agrees with what we saw in the graph above.

Slippery Runway Takeoff
Maximum Reverse Thrust
Weight Adjustment (1000 LB)

DRY FIELD/OBSTACLE LIMIT WEIGHT (1000 LB)	REPORTED BRAKING ACTION								
	GOOD			MEDIUM			POOR		
	PRESSURE ALTITUDE (FT)			PRESSURE ALTITUDE (FT)			PRESSURE ALTITUDE (FT)		
	S.L.	4000	8000	S.L.	4000	8000	S.L.	4000	8000
700	0.0	-3.6	-7.2	-31.7	-35.3	-38.9	-68.2	-71.8	-75.4
660	0.0	-3.6	-7.2	-35.2	-38.8	-42.4	-68.3	-71.9	-75.5
620	0.0	-3.6	-7.2	-37.3	-40.9	-44.5	-67.3	-70.9	-74.5
580	-3.2	-6.8	-10.4	-38.2	-41.8	-45.4	-65.1	-68.7	-72.3
540	-5.6	-9.2	-12.8	-37.8	-41.4	-45.0	-61.8	-65.4	-69.0
500	-7.0	-10.6	-14.2	-36.1	-39.7	-43.3	-57.3	-60.9	-64.5
460	-7.1	-10.7	-14.3	-33.1	-36.7	-40.3	-51.7	-55.3	-58.9
420	-6.2	-9.8	-13.4	-28.8	-32.4	-36.0	-44.9	-48.5	-52.1
380	-4.1	-7.7	-11.3	-23.3	-26.9	-30.5	-36.9	-40.5	-44.1
340	-0.9	-4.5	-8.1	-16.5	-20.1	-23.7	-27.8	-31.4	-35.0

Table 20-5

two-engine versus four-engine airplanes

For two-engine airplanes, the result of applying these adjustments is a decrease of both the weight and the V_1. For four-engine airplanes, the situation is a bit different:

- On a wet runway (braking coefficient 0.2), the usual result is a V_1 reduction with the weight reduction shown as zero;
- on a runway having a braking coefficient of 0.1 or 0.05, the usual result is a reduction to both field length-limit weight and the V_1.

Look at the table immediately below. This table is for a 747.

Slippery Runway Takeoff
2 Engine Reverse Thrust
Weight Adjustment (1000 KG)

FIELD/OBSTACLE LIMIT WEIGHT (1000 KG)	REPORTED BRAKING ACTION								
	GOOD			MEDIUM			POOR		
	PRESSURE ALTITUDE (FT)			PRESSURE ALTITUDE (FT)			PRESSURE ALTITUDE (FT)		
	S.L.	4000	8000	S.L.	4000	8000	S.L.	4000	8000
400	0	0	0	-5	-5	-5	-17	-17	-17
380	0	0	0	-6	-6	-6	-17	-17	-17
360	0	0	0	-6	-6	-6	-15	-15	-15
340	0	0	0	-6	-6	-6	-14	-14	-14
320	0	0	0	-5	-5	-5	-12	-12	-12
300	0	0	0	-5	-5	-5	-11	-11	-11
280	0	0	0	-4	-4	-4	-10	-10	-10
260	0	0	0	-3	-3	-3	-9	-9	-9
240	0	0	0	-3	-3	-3	-7	-7	-7

Table 20-6

Why the difference? Simply because a four-engine airplane’s all-engine takeoff distance is usually greater than the engine-out takeoff distance because of the 1.15 factor that must be applied.

Therefore the field length-limit weight is typically limited by the all-engine takeoff requirement, and even on a wet runway that will still be the case.

For lower braking coefficients, however, because of the increase in the accelerate-stop distance, the all-engine takeoff distance is less than the engine-out distance, hence the engine-out case becomes limiting and the weight must be adjusted for the effect of the loss of braking capability.

minimum control speed considerations

There's one additional factor which can affect the allowable weight and V_1 on a contaminated runway. Although it's seldom a consideration on wet runways, it can be a problem when the runways are covered with ice, wet ice, or compact snow.

You'll understand that the lower the airplane braking coefficient, the greater must be the weight and V_1 reductions necessary to hold the same takeoff distances. What if the V_1 decrement, when applied to the dry runway V_1 for the decremented weight, causes the wet V_1 to be less than the minimum control speed V_{MCG}? That's forbidden by the regulations: we know that the minimum allowable value of V_1 is equal to V_{MCG} for the older airplanes or V_{1MCG} for the later airplanes, as appropriate. (To refresh your memory, you can review that topic under the heading of "minimum V_1" in the chapter entitled "Speeds").

To provide operators with data that will allow them to avoid this situation, Boeing provides one additional tabulation relating to slippery runways in the FPPM and the PEM. That is a chart showing the maximum weight with V_1 set equal to the minimum V_1 value at which the takeoff distance will be equal to the runway length available.

If you look at one of these tables, such as the one shown to the right for a 737, you'll see that the table presents the value of field length limit weight for the case of $V_1 = V_{1MCG}$. A limit weight is not provided at runway lengths such that the wet V_1 will be greater than the minimum value.

You see that for wet conditions the minimum V_1 becomes limiting only on relatively short runways on which the dry runway weight and V_1 are necessarily low. The wet runway V_1 reductions are typically quite small, on the order of five to ten knots, so unless the dry runway V_1 is low, the corrected V_1 will usually remain above the minimum.

V1(MCG) Limit Weight (1000 LB)

ADJUSTED FIELD LENGTH (FT)	GOOD		
	PRESSURE ALTITUDE (FT)		
	S.L.	4000	8000
3800	83.3		
4200	109.5	75.3	
4600	134.7	101.9	67.3
5000	160.0	127.2	94.0
5400		152.4	119.6
5800			144.8
6200			
6600			
7000			
7400			
7800			
8200			
8600			
9000			
9400			
9800			
10200			
10600			

Table 20-7

By using the slippery runway data as discussed above, the user arrives at two different field length limit weights: one for a V_1 equal to the dry V_1 minus a decrement, and the other for V_1 set equal to the minimum value. The smaller of those two weights is the limiting value.

Chapter 21: Climb Angle and Rate of Climb

Introduction

When an airplane is in flight and the available thrust is just equal to the airplane's drag, the airplane is capable of constant-velocity level flight – but it can't accelerate or climb. On the other hand, when there is a surplus of thrust -- that is, there's more thrust available than the drag at that condition -- then the airplane could accelerate, or could climb, or could do a combination of the two. Acceleration and climb can be traded for each other: by accepting less acceleration, the climb angle may be increased, or by accepting a smaller climb angle, the airplane can accelerate more rapidly. Surplus thrust is also involved in turning flight in climb. That will be discussed later in this chapter.

The minimum allowable angles for an airplane's climb capability near the ground during takeoff and approach are specified by regulatory agency requirements. Climb performance at altitude can also be important. Bear in mind that climb angles are not always positive: during driftdown following an engine failure, for example, climb angles may be substantially less than zero. We therefore need to be able to compute an airplane's climb performance under a variety of conditions.

Climb Physics

Let's examine the physics relating to an airplane in climb.First we'll discuss some terminology, then we'll develop the equations that will allow us to calculate an airplane's climb performance.

angle of climb

When climbing, an airplane follow a path whose angle to the horizontal we call γ.

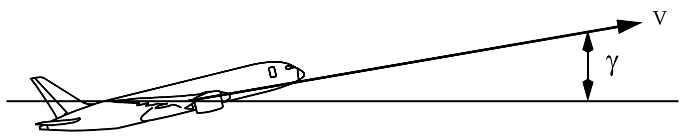


Figure 21-1

In order to generate the necessary lift, the airplane's wing will be at some angle of attack to the path of flight, as discussed in the chapter entitled "Lift and Drag".

The longitudinal axis of the airplane in climb will therefore be at an angle relative to the flight path. This angle is called α_B, the <u>body</u> angle of attack. It depends on the speed, airplane weight, and air density, in accordance with the equation for lift coefficient, and the relationship between angle of attack and C_L.

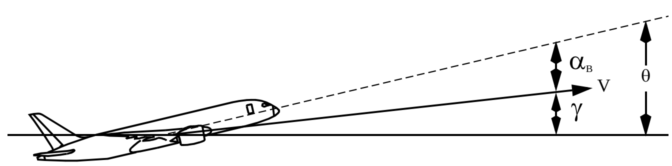


Figure 21-2

The angle of the body's longitudinal axis relative to the horizontal is thus the sum of γ and α_B and it's called θ. This is the angle that will be seen by a pilot on his flight director.

climb gradient

In much of our performance work, it's standard practice to describe an airplane's climb path angle as a *gradient*, expressed in percent. "Gradient" in this context can be described as the amount of altitude gained over a given amount of horizontal distance traveled. Gradient then is simply <u>the tangent of the climb path angle γ</u>. To express a gradient in <u>percent</u>, it's only necessary to multiply the tangent of the climb path angle by 100. Thus:

$$\textit{gradient in percent} = 100 \times \tan\gamma$$

For example, one of the regulatory requirements for engine inoperative climb after takeoff specifies a minimum engine-inoperative climb gradient of 3%. That's equivalent to a climb path angle of 1.72 degrees.

rate of climb

An airplane's *rate of climb* is the product of its true velocity *V* and the sine of its climb path angle γ. The rate of climb, or "R/C" is its vertical velocity, which can be also be expressed as $\frac{dh}{dt}$, where *h* is the airplane's height above the ground. Thus, we can say that

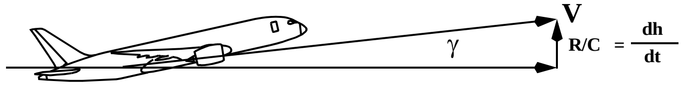


Figure 21-3

$$\textit{rate of climb R/C} = \frac{dh}{dt} = V\sin\gamma \qquad \text{(eq. 1)}$$

Just a moment ago, we mentioned a climb gradient of three percent, equivalent to a climb path angle of 1.72 degrees, as one of the regulatory minimum. That doesn't sound like much, but at a speed of 150 knots TAS, for example, that would be a rate of climb of 456 feet per minute.

forces acting on the airplane in flight

There are a number of forces acting on an airplane in flight. The obvious ones are its weight, its lift, its drag and its thrust. However, there are two other forces that must be considered.

<u>*acceleration during climb*</u>

In commercial airplane operation, the climb after takeoff is conducted at constant airspeed. That airspeed is the plane's indicated or calibrated airspeed, seen by the pilot or used by the autopilot. As the airplane climbs, the <u>true</u> airspeed is gradually increasing – in other words, the airplane is <u>accelerating</u>. That acceleration is at a slow rate – for example, on a standard day if the true airspeed at sea level is 150 knots, it will have increased by only four knots as the airplane passes 1000 feet. But the airplane <u>is</u> accelerating, and to cause acceleration requires force in accordance with Newton's equation $F = ma$.

It's the force of engine thrust that produces climb. If some (small) amount of engine force is needed to produce acceleration, then there is (slightly) less thrust available to produce climb. The thrust available for climb is therefore equal to the thrust produced by the engines minus the thrust used for acceleration.

We can express the thrust lost to acceleration as $\frac{W}{g}\frac{dV}{dt}$ which you'll recognize as simply $F=ma$ expressed in terms of the airplane's weight W, the acceleration of gravity g and the airplane's rate of change of true airspeed $\frac{dV}{dt}$.

climb angle change during climb

It's also possible that the climb path angle might be changing as the airplane climbs. That requires another force in the direction perpendicular to the flight path. Just as it required force (in this case, thrust) to produce acceleration along the flight path, it also requires a force (in this case, *an amount of lift)* to produce any rate of change to the climb path's vertical component.

An airplane following a flight path that exhibits a changing gradient is flying in an arc. That arc has a radius, and the airplane following the arc exhibits an angular rate about the arc's center. The force required to produce this changing gradient is a centripetal force, following the equation:

$$F = mr\omega^2$$

The airplane's velocity V along the arc can be shown as:

$$V = r\omega$$

where r is the radius of the arc
ω is the angular rate

and we can also say that:

$$\omega = \frac{d\gamma}{dt}$$

and thus:

$$F = \frac{W}{g}\frac{d\gamma}{dt} \times V$$

Hence, a force in accordance with this equation is required to create a rate of change of gradient.

summations of forces

The forces can be resolved into components along the flight path and perpendicular to it, as shown in the illustration.

You see the four principal forces, as well as the smaller forces resulting from any longitudinal or vertical acceleration.

For steady-state climb, the net force perpendicular to the flight path must be zero, and the net force parallel to the flight path must also be zero.

Those two relationships can be expressed mathematically as:

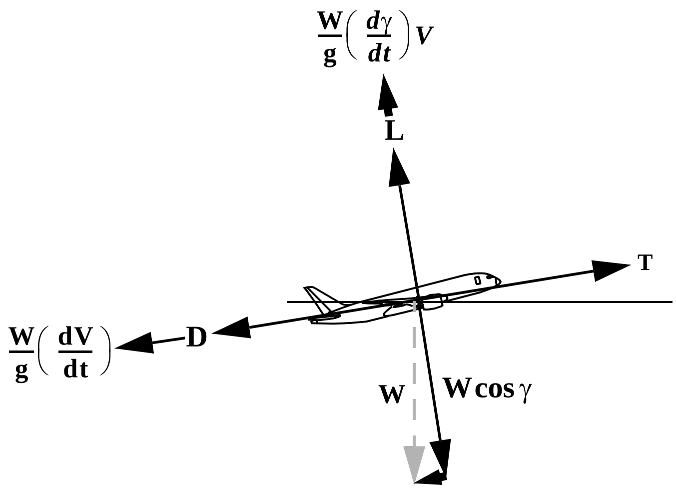


Figure 21-4

$$T - D - W\sin\gamma - \frac{W}{g}\frac{dV}{dt} = 0 \text{ along the flight path, and} \quad \text{(eq. 2)}$$

$$L + \frac{W}{g}\frac{d\gamma}{dt}V - W\cos\gamma = 0 \text{ perpendicular to the flight path.} \quad \text{(eq. 3)}$$

calculation of the climb angle γ

If we rearrange equation 2:

$$T - D = W\sin\gamma + \frac{W}{g}\frac{dV}{dt} \quad \text{and thus:} \quad \frac{T-D}{W} = \sin\gamma + \frac{1}{g}\frac{dV}{dt} \quad \text{(eq. 4)}$$

Substitute $\frac{dV}{dh}\frac{dh}{dt}$ for $\frac{dV}{dt}$, and remember that $\frac{dh}{dt} = V\sin\gamma$ from equation 1 above. Thus:

$$\frac{T-D}{W} = \sin\gamma + \frac{V}{g}\frac{dV}{dh}\sin\gamma = \sin\gamma\left(1 + \frac{V}{g}\frac{dV}{dh}\right) \quad \text{(eq. 5)}$$

This gives the equation for the climb path angle γ:

$$\gamma = \sin^{-1}\left[\frac{\frac{T-D}{W}}{\left(1 + \frac{V}{g}\frac{dV}{dh}\right)}\right] \quad \text{(eq. 6)}$$

The equation for the gradient would then be:

$$gradient\ \% \ =\ 100\ tan\gamma\ =\ 100\ tan\left\{ sin^{-1}\left[\frac{\dfrac{T-D}{W}}{\left(1+\dfrac{V}{g}\dfrac{dV}{dh}\right)}\right]\right\} \qquad \text{(eq. 7)}$$

Thus far, we've looked at the summary of forces parallel to the flight path. Let's look briefly at the forces <u>perpendicular</u> to the flight path, shown as equation 3 above. Rearranging the terms of the equation:

$$W cos\gamma - L\ =\ \frac{W}{g}\frac{d\gamma}{dt}V \qquad \text{(eq. 8)}$$

If we were dealing with substantial rates of change of gradient with time, the term $\frac{W}{g}\frac{d\gamma}{dt}V$ might be significant. However, for reasonably steady-state climb, this term is insignificant and thus <u>it is ignored in most commercial jet airplane performance work</u>. That leaves us with:

$$W cos\gamma - L\ =\ 0 \qquad \text{or} \qquad L\ =\ W cos\gamma$$

There's another simplification that is used in some climb angle calculations. Much of our work deals with very small climb angles – as you'll see in the next chapter, the regulatory minimum allowable climb angles specified for takeoff are less than two degrees. For climb angles as small as these, it's sufficiently accurate to say that $cos\gamma = 1$. That is, lift is equal to weight.

Using that simplification, let's go back to equation 6 and modify it a bit:

$$\gamma\ =\ sin^{-1}\left[\frac{\dfrac{T}{W}-\dfrac{D}{L}}{\left(1+\dfrac{V}{g}\dfrac{dV}{dh}\right)}\right] \qquad \text{(eq. 9)}$$

The term D / L is equivalent to C_D / C_L which gives this form of the climb angle equation:

$$\gamma\ =\ sin^{-1}\left[\frac{\dfrac{T}{W}-\dfrac{C_D}{C_L}}{\left(1+\dfrac{V}{g}\dfrac{dV}{dh}\right)}\right] \qquad \text{(eq. 10)}$$

From equation 10, we can calculate the climb gradient:

$$gradient\ \% = 100 \times tan\left\{ sin^{-1}\left[\frac{\dfrac{T}{W} - \dfrac{C_D}{C_L}}{\left(1 + \dfrac{V}{g}\dfrac{dV}{dh}\right)} \right] \right\} \quad \text{(eq. 11)}$$

If we're dealing with small climb angles, as is particularly the case when we're calculating climb with an engine inoperative, it's also sufficiently accurate to say that the tangent of an angle is equal to the sine of the angle, and thus that:

$$gradient\ (percent) = 100 \times \left[\frac{\dfrac{T}{W} - \dfrac{C_D}{C_L}}{\left(1 + \dfrac{V}{g}\dfrac{dV}{dh}\right)} \right] \quad \text{(eq. 12)}$$

At this point we need to emphasize that the two simplifying assumptions we have made: first, that lift equals weight, and second that the tangent of the climb angle is equal to the sine of the angle, <u>are only acceptable for relatively small angles of climb</u>. For more precise calculations, and especially when calculating climb angles with all engines operating, equations 6 and 7 should be used instead of equations 10 and 12.

acceleration factor

Look at the denominator of equation 12.

What is this term $\left(1 + \dfrac{V}{g}\dfrac{dV}{dh}\right)$? It's usually referred to as the *acceleration factor.* You'll recall from the earlier discussion that the term $\dfrac{W}{g}\dfrac{dV}{dt}$ represents the force needed to cause the airplane to accelerate as it climbs; the acceleration factor comes from this term. The acceleration factor is a correction for the fact that the airplane isn't climbing at constant true airspeed. It's quite small for typical takeoff conditions, having a value of 1.03 or so; for higher altitudes and speeds, however, the acceleration correction can be much greater.

equations for acceleration factor

The equations for the acceleration factor may be summarized as follows:

<u>For standard day below the tropopause:</u>

constant Mach number: $1 - 0.133184M^2$ (eq. 13)

constant calibrated airspeed: $1 + 0.7\,M^2\,(\phi - 0.190263)$ (eq. 14) constant equivalent airspeed: $1 + 0.566816\,M^2$ (eq. 15)

For non-standard day below the tropopause:

constant Mach number: $-\,0.133184\;M^2\left(\dfrac{T_{STD}}{T}\right)$ (eq. 16)

constant calibrated airspeed: $1 + 0.7M^{\,2}\left[\phi - 0.190263\left(\dfrac{T_{STD}}{T}\right)\right]$ (eq. 17)

constant equivalent airspeed: $1 + 0.7M^{\,2}\left[1 - 0.190263\left(\dfrac{T_{STD}}{T}\right)\right]$ (eq. 18)

For standard or non-standard day above the tropopause:

constant Mach number: 1 (eq. 19)

constant calibrated airspeed: $1 + 0.7\,M^2\,\phi$ (eq. 20)

constant equivalent airspeed: $1 + 0.7\,M^2$ (eq. 21)

In the above equations, the term ϕ is defined as:

$$\phi = \frac{(1 + 0.2M^{\,2})^{3.5} - 1}{0.7M^2(1 + 0.2M^{\,2})^{2.5}} \qquad \text{(eq. 22)}$$

In typical performance engineering work, when needing to compute a climb gradient it's most probable that the climb speed schedule used will be one of constant calibrated airspeed, since that's how the airplane is flown at lower altitudes for which the majority of gradient calculations are performed. The chart on the right shows the acceleration factor as a function of airspeed and altitude, for constant calibrated airspeed.

For convenience, a more detailed copy of the chart is included in the appendix of this book.

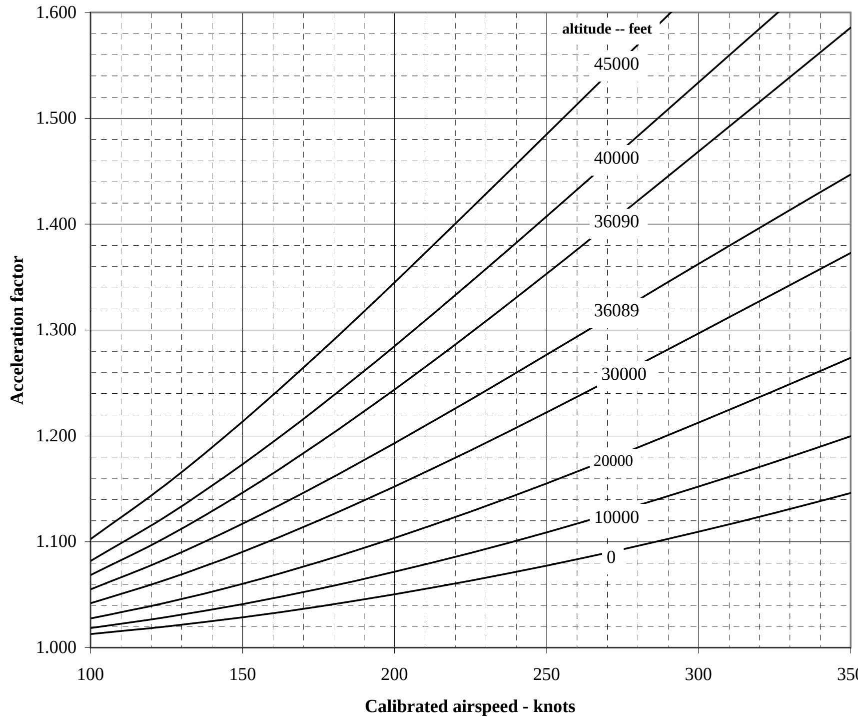


Figure 21-5

You'll observe from the chart that for typical takeoff speeds and altitudes, the acceleration factor will be very near to unity -- typically below 1.05. While small, the acceleration factor shouldn't be ignored since doing so will lead to unconservative results.

maximizing the climb gradient

Certain performance engineer tasks may call for maximizing the climb gradient available at a given weight, or maximizing the allowable weight for a specified climb gradient.

By reference back to equation 10, you can see that there are a number of ways to increase an airplane's climb gradient toward its maximum achievable value. Here's the heart of the gradient equation:

$$\textit{gradient is a direct function of } \left(\frac{T}{W} - \frac{C_D}{C_L} \right)$$

If thrust and weight are constant values, then maximizing the gradient would be simple: just minimize the value of C_D/C_L.

As you could see in the discussion on drag polars in an earlier chapter, finding the minimum value of C_D/C_L and the speed at which it occurs is simply a matter of drawing a line from the origin of the drag polar (the point at which both C_L and C_D are zero) tangent to the drag polar line. The C_D and C_L values at that point will produce the minimum available value of C_D/C_L. The C_L value at that point can be used to determine the *speed* at which the minimum C_D/C_L will occur.

For sailplanes, this technique will work just fine, since the term T/W is zero; that is, all gradients are simply $gradient = -100 \tan^{-1}\left(\frac{C_D}{C_L}\right)$. That's logical: a sailplane, having no thrust, is always flying at a negative gradient. Sailplane pilots frequently talk about flying at the speed for "max L over D".[1] "Max L over D" is the same thing as saying "the minimum ratio of C_D over C_L". Flying the sailplane at that speed, then, ensures that its glide is at the best possible angle and the sailplane can maximize its distance flown for the altitude lost.

Since Boeing airplanes DO have engines, however, it's necessary to account for thrust. Older engines such as the low bypass ratio JT8s and JT3s exhibited thrust that was relatively unaffected by speed. Newer engines, however, display considerable thrust change with speed, and for these airplanes the maximum climb gradient may not occur at the speed for best drag ratio. It will be necessary to compute the gradient for a range of speeds and find the maximum gradient and the speed for maximum gradient from the calculations.

Climb Angle With One or More Engines Inoperative

When one or more engines are inoperative, climb gradient capability will be reduced due not only to the loss of a significant portion of the thrust but also because of an increase of drag due to two factors.

One component of this drag increase is that caused by the inoperative engine. The other component is due to the deflections of the airplane's flight controls that are required to maintain directional control in the condition of asymmetric thrust.

windmilling and spillage drag

An inoperative engine causes an increment of drag simply because the engine is being caused to rotate by the flow of air entering the inlet rather than by the energy coming from the combustion of fuel. It is acting as a windmill acts, extracting energy from the air flowing past it to cause its rotation – hence the name "windmilling" drag, designated as D_{WM}. The coefficient of this drag force is designated as $C_{D_{WM}}$

Not all of the air entering the inlet of an inoperative engine will be able to pass through the engine, so some amount of the air will spill out of the inlet causing some more drag. This is usually called "spillage" drag.

1. They also talk about flying at "V min sink", More about that later when we discuss rate of climb.

The engine's contribution to the drag increase experienced in engine inoperative flight is the sum of the windmilling drag increase and the spillage drag

A graph of this drag increase for a typical Boeing airplane is shown here.

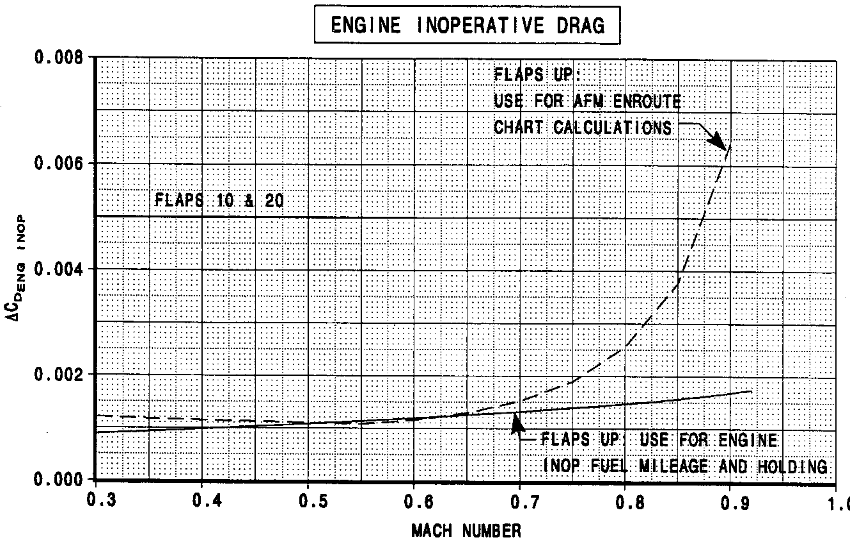


Figure 21-6

control drag

Pilots are trained in the proper technique for flying with an engine inoperative: maintain wings level with control wheel, trim the ailerons for zero wheel deflection, hold rudder as needed to maintain constant heading. When flying in this manner, the airplane will be slightly "cross-controlled", in a slight skid. The deflection of the ailerons and rudder add a significant increment of drag. This is referred to as *control drag*.

yaw moment

The airplane will yaw toward the failed engine due to a yawing moment generated by the asymmetric thrust condition.

The yawing moment would be given by

$$yaw\ moment = (T + D_{WM}) \times l_e \qquad \text{(eq. 23)}$$

where l_e is the engine moment arm as shown in the illustration to the right.

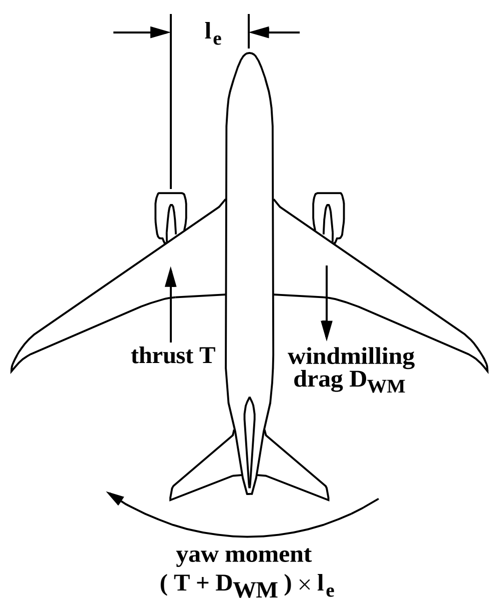


Figure 21-7

yaw moment coefficient

The yawing moment coefficient C_N then is calculated by

$$C_N = \frac{(T + D_{WM}) \times l_e}{Sqb} \qquad \text{(eq. 24)}$$

where T is the thrust of the operative engine
S is the reference wing area

q is the dynamic pressure
b is the wingspan

The wingspan b is used in the denominator to keep the coefficient dimensionless.

control drag coefficient

As stated above, control drag is simply that increase of airplane drag which results from the deflection of the flight controls needed to keep the airplane in a wings-level, constant-heading condition.

Control drag can be reduced to a coefficient form which is generalized as a function of the airplane's yawing moment coefficient C_N. This is called the "control drag coefficient" or "yaw drag coefficient", denoted as ΔC_{D_Ψ}

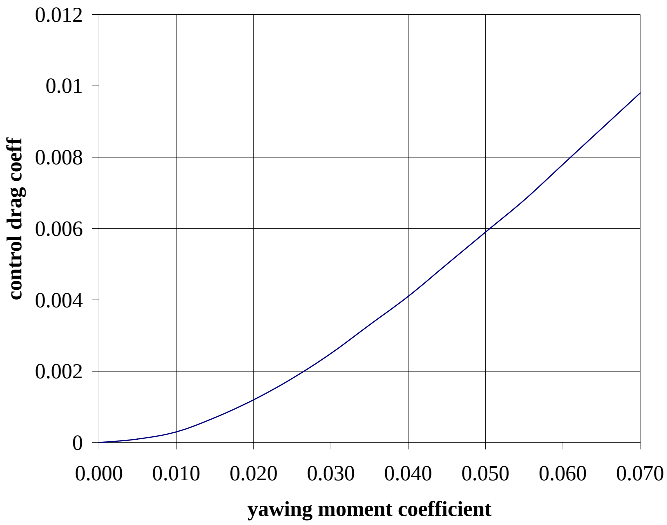


Figure 21-8

climb angle equation for the engine-out condition

Let's look back at equation 6, and see how to modify it to account for the extra drag in an engine inoperative condition. Equation 6 says:

$$\gamma = \sin^{-1}\left[\frac{\frac{T-D}{W}}{\left(1+\frac{V}{g}\frac{dV}{dh}\right)}\right]$$

Let's insert into the equation the effect of the two elements of drag increase:

$$\gamma = \sin^{-1}\left[\frac{\frac{T-D-\Delta D_{WM}-\Delta D_{control}}{W}}{\left(1+\frac{V}{g}\frac{dV}{dh}\right)}\right] \qquad \text{(eq. 25)}$$

For small angles of climb, this equation can be expressed as:

$$\gamma = \sin^{-1}\left[\frac{\dfrac{T-D}{W} - \dfrac{(\Delta C_{D_{WM}} + \Delta C_{D_\Psi})}{C_L}}{\left(1 + \dfrac{V}{g}\dfrac{dV}{dh}\right)}\right] \quad \text{(eq. 26)}$$

or:

$$\gamma = \sin^{-1}\left[\frac{\dfrac{T}{W} - \dfrac{(C_D + \Delta C_{D_{WM}} + \Delta C_{D_\Psi})}{C_L}}{\left(1 + \dfrac{V}{g}\dfrac{dV}{dh}\right)}\right] \quad \text{(eq. 27)}$$

Rate of Climb

Rate of climb can be found from the climb angle simply by knowing the flight path angle and the climb true airspeed. Remember equation 1 which specifies that R/C = $V_{true} \sin \gamma$. Therefore:

$$\textit{rate of climb R/C} = V_{true}\left[\frac{\dfrac{T-D}{W}}{\left(1 + \dfrac{V}{g}\dfrac{dV}{dh}\right)}\right] \quad \text{(eq. 28)}$$

For the engine-inoperative case, that becomes

$$\textit{engine-out rate of climb} = V_{true}\left[\frac{\dfrac{(T - D - \Delta D_{WM} - \Delta D_{control})}{W}}{\left(1 + \dfrac{V}{g}\dfrac{dV}{dh}\right)}\right] \quad \text{(eq. 29)}$$

Remember always that when airspeeds are provided in knots, they must be multiplied by 1.6878 to convert them to feet per second before evaluating the equation, so that the result of the calculation will be a rate of climb in feet per second. Airspeeds in knots may be multiplied by 101.27 before evaluating the equation in order to obtain the rate of climb in feet per minute.

Maximum Gradient and Maximum Rate of Climb

We discussed above, briefly, how one might find the speed for the maximum possible climb gradient, and you have seen how to calculate the gradient for any given conditions.

If we graph the climb gradient as a function of speed for given conditions of weight, altitude and temperature, it would look like this:

Observe that the speed for the maximum gradient for these conditions is approximately 145 knots. One might think that this speed would also produce the maximum rate of climb, but such is not the case.

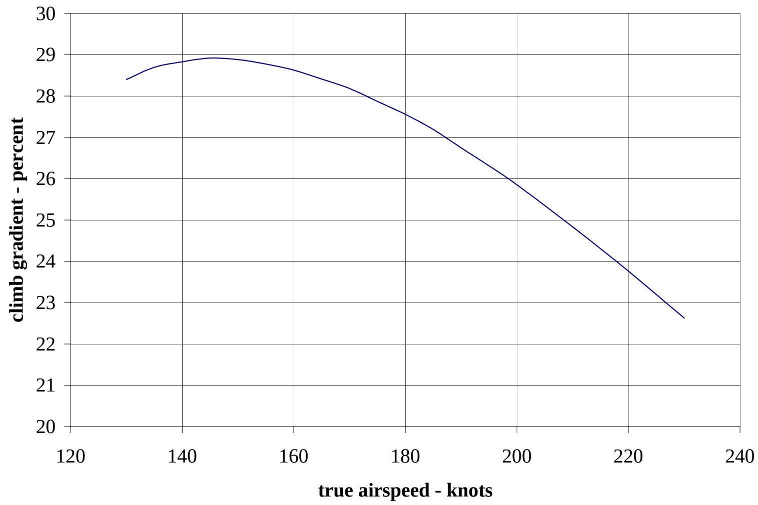


Figure 21-9

Here is the graph of rate of climb for the same conditions as the gradient chart above:

In this chart, you see that the greatest rate of climb occurs at approximately 220 knots, some 75 knots faster than the speed for best gradient. Why?

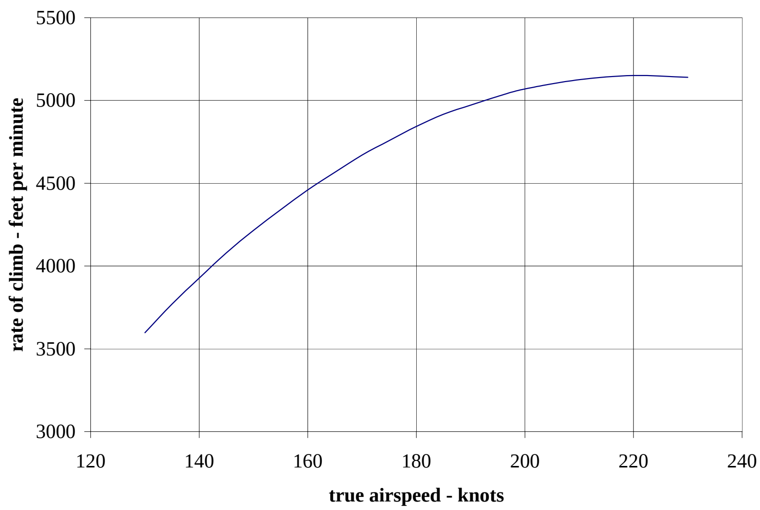


Figure 21-10

This occurs because as speed increases above the speed for best gradient, the rate of *decrease* of the term $\frac{T}{W} - \frac{C_D}{C_L}$ is less than the rate of *increase* of velocity: thus the product of the two increases. This will continue until the two rates become equal as speed increases.

When the sailplane pilot we mentioned before talks about "flying at max L over D" he's trying to maximize the distance he can fly for the amount of altitude lost. On the other hand, when he speaks of "flying at V min sink" he means that he's flying at the speed for the minimum rate of descent ("sink rate"), which is the sailplane's equivalent to the speed for best rate of climb. In this case, the sailplane pilot is trying to maximize his time in the air instead of trying to maximize the distance he can fly. The pilot must know both speeds about his sailplane and must use the appropriate one.

Climb Gradient in a Turn

Suppose that an airplane is climbing while performing a turn. What, if anything, happen to its climb gradient and rate of climb?

Look at an illustration of an airplane in turning flight:

In the banked turn, the lift must be greater than the weight, in order to support the weight of the airplane. If the lift is greater, the drag will be greater also. *This increase in drag will cause a loss of gradient.*

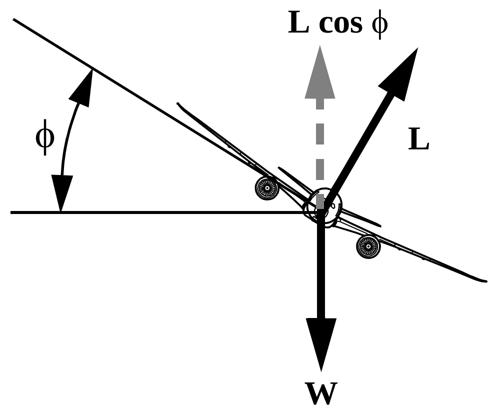


Figure 21-11

In the vertical direction, we see that:

$W = L \cos \phi$

where ϕ is the angle of bank

and thus the amount of lift needed in the bank will be:

$$L = \frac{W}{\cos\phi} \qquad \text{(eq. 30)}$$

Looking back at figure 3 at the beginning of this chapter, from which we derived the equation for the climb angle, what has changed? As far as the climb angle equation is concerned, the only thing that has changed is the drag acting on the airplane, resulting from the increased lift.

The change in the climb angle, then, would be:

$$\left[\frac{\dfrac{T - D_{unbanked}}{W}}{1 + \dfrac{V}{g}\dfrac{dV}{dh}} - \frac{\dfrac{T - D_{banked}}{W}}{1 + \dfrac{V}{g}\dfrac{dV}{dh}}\right] = \frac{\dfrac{(D_{banked} - D_{unbanked})}{W}}{1 + \dfrac{V}{g}\dfrac{dV}{dh}} \qquad \text{(eq. 31)}$$

This would reduce to (for smaller angles of climb):

$$\Delta\gamma = \frac{\dfrac{\Delta C_D}{C_{L\ unbanked}}}{1 + \dfrac{V}{g}\dfrac{dV}{dh}} \qquad \text{(eq. 32)}$$

where $\Delta\gamma$ is the reduction of gradient
ΔC_D is the drag coefficient difference between banked and unbanked conditions

Looking at a typical flaps-down drag polar, the ΔC_D can be found as you see in this illustration:

Since every airplane has a different drag polar the climb angle decrements will differ from one model to another. What we're showing here is just for illustration purposes.

For convenience, charts of gradient decrement are published in the Boeing Performance Engineer's Manuals.

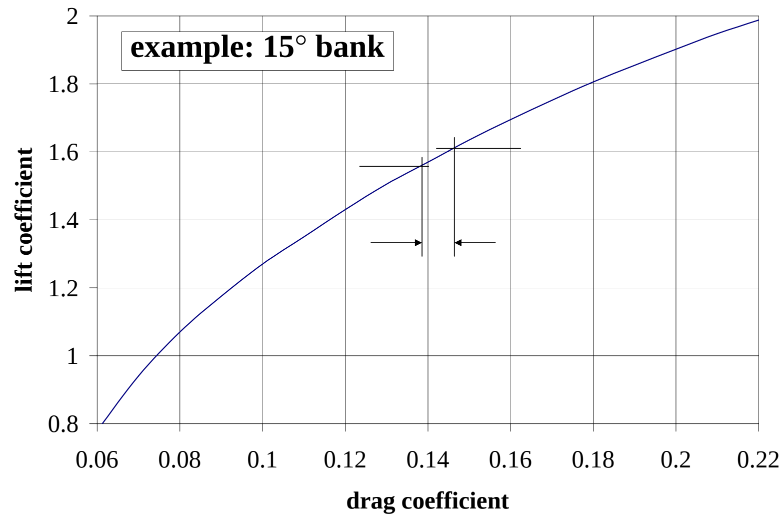


Figure 21-12

Gradient - Acceleration Trades

We established at the beginning of this chapter that any time an airplane has thrust available that is greater than the drag in the given conditions, the airplane can either climb, or accelerate, or some combination of the two.

It's sometimes convenient to be able to calculate these trade-offs between gradient and acceleration. Look again at the equation for the summation of the forces along the flight path:

$$T - D - W\sin\gamma - \frac{W}{g}\frac{dV}{dt} = 0 \qquad \text{(eq. 2 repeated)}$$

Dividing by W and rearranging the terms gives:

$$\sin\gamma = \left(\frac{T-D}{W}\right) - \frac{1}{g}\frac{dV}{dt} \qquad \text{(eq. 33)}$$

You'll recognize the term $\frac{T-D}{W}$ as being equal to the sine of the climb angle available for a given amount of thrust, drag and weight, when using all of the surplus thrust for climb. You'll also recognize the term $\frac{dV}{dt}$ as being acceleration; we'll call it a. So we could re-write equation 32 in terms of climb angle available, climb angle, and an acceleration term:

$$\sin(\text{climb angle}) = \sin(\text{available climb angle}) - \frac{a}{g} \qquad \text{(eq. 34)}$$

Equation 33, then, just shows that if *some* of the surplus thrust available will be used for climb and some of it for acceleration, then the sum of the acceleration term a/g and the sine of the gradient γ will be equal to the sine of the total climb angle available.

For smaller angles of climb, it's acceptable to say that:

$$sin(climb\ angle) = tan(climb\ angle) = \frac{gradient}{100}$$

Thus, for reasonably small angles of climb,

$$\frac{gradient}{100} = \frac{gradient\ available}{100} - \frac{a}{g}$$

In the first term, *gradient* is sometimes referred to as the "residual gradient" -- that is, the gradient that remains when some portion (but not all) of the surplus thrust is used to produce acceleration a. This relationship is true for any airplane, it is not a function of the performance capability of any given airplane model. Thus:

$$\frac{residual\ gradient}{100} = \frac{available\ gradient}{100} - \frac{a}{g} \qquad \text{(eq. 35)}$$

When doing calculations of this nature, caution must be used to ensure that units are consistent. Since the value of g which is customarily used, 32.174, is in feet per second per second, then the acceleration must also be expressed in feet per second per second.

Another way of looking at this equation is to say:

$$available\ gradient = residual\ gradient + 100 \times \frac{a}{g} \qquad \text{(eq. 36)}$$

That makes sense: for a given set of conditions, there's only one value of available gradient, but there can be any combination of residual gradient and acceleration; more acceleration means less climb angle, more climb angle means less acceleration.

Using this information, it's possible to produce a chart showing the trade-offs between gradient and acceleration.

Here's a chart of available gradient versus residual gradient, for a number of values of acceleration.

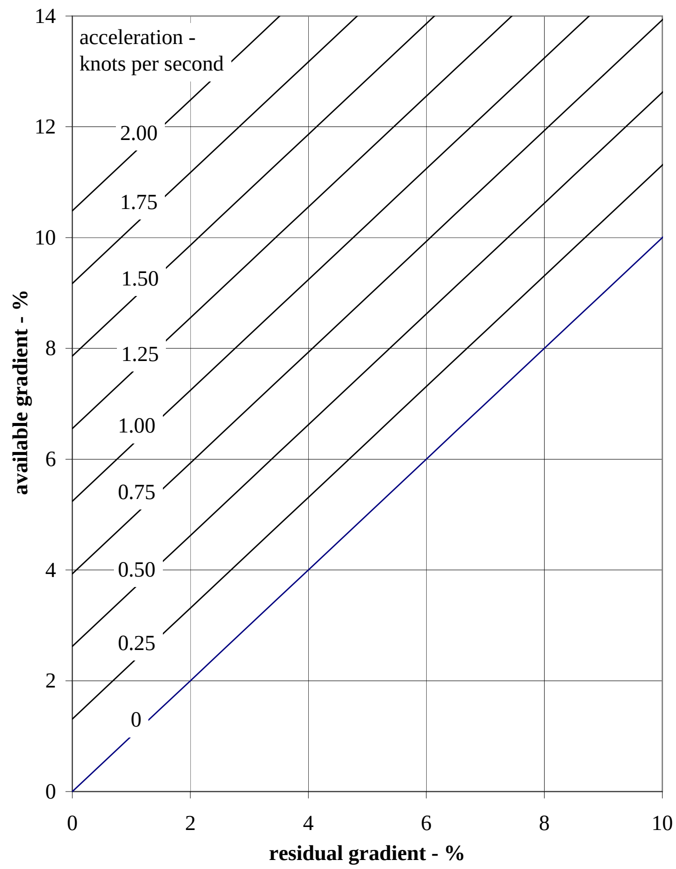


Figure 21-13

Look at the line for zero acceleration rate. You see that everywhere along this line, the residual gradient is equal to the available gradient. That's logical: we're not using any surplus thrust for acceleration, so all of it is going to producing climb. The residual gradient will be equal to the available gradient.

But for all other lines of acceleration, you see that the residual gradient will be equal to the available gradient minus $100 \times \frac{a}{g}$ in accordance with equation 36. Try it, you'll see.

For example: suppose you're given a gradient available of ten percent, and a desired acceleration rate of one knot per second. What's the residual gradient?

One knot per second is equal to 1.6878 feet per second per second. Thus:

$$\frac{residual\ gradient}{100} = \frac{10}{100} - \frac{1 \times 1.6878}{32.17405}$$

and the residual gradient will be 4.75%.

Chapter 22: Climb Limit Takeoff Weight

Introduction

In an earlier chapter we talked about the field length limit takeoff weight. You saw that this is one of the constraints placed on an airplane's allowable takeoff weight, and that it's a direct function of the runway, clearway, and stopway lengths. It thus ensures that the weight will result in takeoff distances that are within the available runway parameters, with an acceptable margin of safety.

In the next chapter, we'll be discussing the obstacle limit takeoff weight. This is another of the constraints on an airplane's allowable takeoff weight, and this constraint depends directly on the location and height of any obstacles that must be considered. Here too, as in the case of the field length limit, the weight is dictated by the physical characteristics of the takeoff environment.

In this chapter, we will be discussing the climb limit takeoff weight. It is important that we make very clear the distinction between the climb limit takeoff weight and the obstacle limit takeoff weight. The two are totally unrelated, except that they're both concerned with an airplane's ability to climb. The takeoff climb limit weight is not dictated by any physical characteristics of the takeoff environment. Rather, you might find it helpful to think of it as the mandatory minimum level of the airplane's *surplus energy* condition during takeoff.

Surplus energy could be defined as energy in excess of that needed for steady state level unaccelerated motion. Surplus energy can be used for either climb or for acceleration (or some combination of the two – refer to the chapter entitled "Calculating Climb Gradient and Rate of Climb for additional discussion of this.) It is convenient for the regulations to mandate a minimum energy level in terms of an airplane's climb gradient capability, because an airplane's angle of climb is a direct function of the surplus energy available, but this gradient requirement has nothing to do with obstacles.

The Takeoff Vertical Profile

The path of an airplane during and after takeoff can be seen in several different ways: as a horizontal profile and as a vertical profile.

A horizontal profile would be the flight path as seen by an observer looking down on an airplane's movement from above. The horizontal movements are east/west/north/south or left/right. The vertical profile is the flight path as seen by an observer who looks at the airplane's movement from the side. The vertical movements then are level paths – such as cruise – or up and down following some climb or descent angle.

The horizontal profile has no relevance to this chapter. It does have great relevance to the subject of obstacle clearance, which we'll discuss in the next chapter. Here, we're going to be talking about the airplane's vertical profile after the airplane has passed the end of the takeoff distance.

Takeoff Climb Segments

This seems like a good time to introduce the concept of the *takeoff climb segments*. We'll be discussing them in much greater detail in the next chapter when we get into the topic of obstacle clearance, but it will be helpful to give you a basic idea of the segments here in this chapter before we talk about the regulatory requirements on climb performance following takeoff.

It is standard industry practice to divide an airplane's vertical profile after takeoff into "segments". These are distinctly separate pieces of the profile, each characterized by a different configuration or thrust settings. <u>They are all based on the assumption that one engine has failed during the takeoff</u>. Here they are:

first segment

The first segment begins at the point at which the retraction of the landing gear is initiated following liftoff. It continues to the point at which the gear is totally retracted and the gear doors (except for the 737, which has none except for the nose gear) are closed.

The initiation of landing gear retraction is assumed to begin three seconds after liftoff. First, the landing gear doors will open (except for the 737), then the gear will retract into the wheel wells, and finally the gear doors will close, leaving the landing gear in a "clean" configuration. Because of the changing configuration during this segment, the drag is not constant. The worst drag condition will occur when the gear doors are open with the gear still fully extended (or, for the 737, when the gear is fully extended).

The speed is not a constant throughout the segment; it begins at a speed slightly above the liftoff speed. The airplane is assumed to accelerate during the segment until it achieves V_2 speed, after which the speed remains constant at V_2. The thrust on the operative engine(s) remains constant at the takeoff thrust rating throughout the segment, except for second-order speed effects.

As a result of the changing airplane drag and the changing speed, the gradient is not constant during the first segment.

second segment

Second segment begins at the point at which the landing gear is fully retracted – in other words, at the end of the first segment. It ends at the leveloff height, but not less than 400 feet above the takeoff elevation.

The flaps remain at the takeoff setting throughout the segment; the thrust setting on the operative engine(s) remains constant at the takeoff setting, but because of altitude and temperature effects, the thrust will be decreasing as the airplane climbs.

The second segment is conducted at V_2 speed, but remember that V_2 is an indicated/calibrated speed, not a true airspeed. The <u>true</u> airspeed gradually increases slightly as the airplane climbs and the air density decreases.

Because of the changing thrust and true airspeed, the climb gradient isn't constant during the second segment – rather, it gradually decreases. It will be at its greatest value at the beginning of second segment.

third segment

This segment is conducted at constant altitude. The altitude of third segment may legally be as low as 400 feet, although operators are allowed to select a higher altitude and many airlines do so. Should obstacles exist in the takeoff flight path, the third segment may be conducted at higher altitudes as needed for obstacle clearance. This will be discussed at length in the next chapter.

The third segment is an acceleration segment in level flight; it is during this segment that the airplane will retract its flaps from the takeoff setting to the zero-flaps "clean" configuration. The airplane's speed increases since all of the surplus thrust is now available for acceleration, allowing the speed to increase from V_2 up to the appropriate flaps-up speed. The third segment ends when it achieves the clean configuration and is at the flaps-up climb speed. Flap retraction is accomplished following a published schedule of speeds for each step of the retraction; this speed schedule is selected to preserve adequate margins of stall at all points during the flap retraction process.

The thrust setting on the operative engine(s) is held constant throughout the third segment at the takeoff setting. At the end of third segment, when the airplane is in the clean configuration at its flaps-up climb speed, the thrust will be reduced to the Maximum Continuous Thrust (MCT) setting. MCT is used instead of Maximum Climb Thrust because this is an engine inoperative condition.

In the course of a normal takeoff with all engines operating, the flaps will be retracted during climbout rather than in level flight since adequate thrust exists to allow both climb and acceleration at the same time. The thrust will be reduced to the Maximum Climb Thrust (MClT) after flap retraction is completed.

final segment

This segment is conducted at Maximum Continuous Thrust and at the "final climb speed" which is the flaps-up climb speed appropriate to that airplane at that weight.

Final segment begins at the end of third segment. It ends at the conclusion of the takeoff profile, which is usually considered to occur at 1500 feet above the takeoff airport. Should obstacles exist necessitating a continuation of final climb to a higher altitude, then the final segment ends after all obstacles are cleared.

The thrust setting is constant throughout final segment, but because of decreasing air density the thrust is not constant. The final climb segment gradient will be at its greatest value at the beginning of the segment and will gradually decrease after that point.

Regulatory Requirements

The regulations governing commercial air transportation are very specific about the climb capability of an airplane after takeoff.

FAA regulations

Note: In some places we have inserted ellipses (...) to indicate that we have omitted text that has no relevance to this chapter's discussions. Also, we have added emphasis to some words to assist in the following discussions.

Defining the minimum allowable climb angles during an airplane's climb profile after liftoff, FAR Section 25.121 reads as follows:

> ***(a) Takeoff; landing gear extended****. In the critical takeoff configuration existing along the flight path (between the points at which the airplane reaches V_{LOF} and at which the landing gear is fully retracted)...without ground effect, the steady gradient of climb must be positive for two-engine airplanes, and not less than 0.3 percent for three-engine airplanes or 0.5 percent for four-engine airplanes, at V_{LOF} and with --*
> *(1) The critical engine inoperative and the remaining engines at the power or thrust available when retraction of the landing gear is begun...and*
> *(2) The weight equal to the weight existing when retraction of the landing gear is begun...*
>
> ***(b) Takeoff; landing gear retracted****. In the takeoff configuration existing at the point of the flight at which the landing gear is fully retracted...without ground effect:*
> *(1) The steady gradient of climb may not be less than 2.4 percent for two-engine airplanes, 2.7 percent for three-engine airplanes, and 3.0 percent for four-engine airplanes at V2 with:*
> *(i) the critical engine inoperative, the remaining engines at the takeoff power...and*
> *(ii) The weight equal to the weight existing when the airplane's landing gear is fully retracted...*
>
> ***(c) Final takeoff****. In the en route configuration at the end of the takeoff path...:*
> *(1) The steady gradient of climb may not be less than 1.2 percent for two-engine airplanes, 1.5 percent for three-engine airplanes, and 1.7 percent for four-engine airplanes, at V_{FTO} with---*
> *(i) The critical engine inoperative and the remaining engines at the available maximum continuous power or thrust; and (ii) the weight equal to the weight existing at the end of the takeoff path...*

In item (a), you can see that this requirement occurs at a point falling within the first segment as discussed above. The minimum gradient specified, depending on the number of engines, will be computed at the liftoff speed and "in the critical takeoff configuration existing along the flight path..." meaning simply at the worst drag condition.

In item (b), this requirement occurs at the beginning of the second segment – the gear up point.

In item (c), this requirement occurs at the end of the final segment, 1500 feet above the takeoff elevation.

EASA regulations

The European Aviation Safety Agency, in its Certification Specifications for Large Aeroplanes CS-25, Book 1 Subpart B, contains precisely the same climb requirements as the FAR25 requirements quoted above. CS 25.121 corresponds to FAR Part 25 Section 25.121.quoted above. Thus the requirements to both FAA and EASA/JAA operators are identical.

wind corrections to required gradients

We said earlier that the gradient requirements quoted above are unrelated to the requirements for obstacle clearance; instead, they're a means of mandating a minimum acceptable level of surplus energy during a takeoff. Because of this fact, the gradients specified are zero-wind gradients – they are not corrected for the wind expected during the takeoff. The airplane's surplus energy state is not a function of the wind encountered during the takeoff.

Obviously, however, when we're concerned with obstacle clearance we must consider the wind since the wind affects the airplane's path over the ground and hence its ability to clear any obstacles. We'll be discussing this in the next chapter.

Calculating Climb Limit Weights

As you've seen above, what we term a *climb limit weight* is nothing more than the weight, under the specified conditions, that will yield a climb gradient exactly equal to the required minimum value. The computed second segment gross gradient, for a four engine airplane for example, at the second segment climb limit weight will be exactly three percent at V_2 at the gear-up point.

From the regulations quoted above, you now know that there are three criteria for climb limit weights: the first segment, second segment and final segment requirements. Each of these yields a different value of weight since they are computed for different condition. The smallest of those three climb limit weights, then, will be the takeoff climb limit weight.

Repeating here the equation for small climb gradients from the last chapter, and allowing for a failed (windmilling) engine since the required gradients all assume an engine failure:

$$gradient\ (percent) = 100 \times \left[\frac{\dfrac{T}{W} - \dfrac{(C_D + \Delta C_{D_{WM}} + \Delta C_{D_\psi})}{C_L}}{\left(1 + \dfrac{V}{g}\dfrac{dV}{dh}\right)} \right] \qquad \text{(eq. 1)}$$

factors affecting climb capability

From equation 1 you can readily see the factors that will affect the airplane's climb capability. The principal factors will be the thrust, the weight, and the ratio of drag to lift. The yaw drag and the windmilling drag of the inoperative engine don't vary much, and the acceleration factor $\left(1 + \frac{V}{g}\frac{dV}{dh}\right)$ at typical takeoff conditions is very small and varies only slightly.

What affects the thrust? The main factors are pressure altitude and temperature. The engine air-bleed configuration has an effect, although it's relatively small. Takeoff thrust reduction by derate or the assumed temperature method will also affect the thrust, but we'll be discussing that subject in a later chapter.

What affects the drag-to-lift ratio? The primary influence is the flap setting. Here's an example for one of the Boeing twinjets at typical second segment climb speeds and weights:

flap position	$\frac{C_D}{C_L}$
1	0.075
5	0.083
15	0.089
20	0.101

Table 22-1

While that variation in drag-to-lift ratio seems quite small, in terms of gradient capability it's very significant – and in terms of climb limit weight, as you'll see, it's even more significant. Other things remaining the same (thrust, weight, windmilling drag, yaw drag and acceleration factor) you can see that the effect of the flap setting causes a substantial difference in the gradient between flaps 1 and flaps 20. Later in this chapter you'll see how significant this effect is on the climb limit takeoff weight at the different flap settings.

Climb speed will also influence the drag-to-lift ratio but it's a second-order effect.

iterative solution for the climb limit weight

There are two ways in which the climb limit weight for a given set of conditions may be determined. The first of these is to perform a rigorous *iterative*[1] calculation that will determine exactly the climb limit weight for any given conditions.

For a rigorous solution, equation 1 can be rearranged to solve for *W:*

1. Iterative: characterized by or involving repetition. You'll see what we mean in just a minute.

$$W = \frac{T}{\frac{gradient\ \%}{100}\left(1 + \frac{V}{g}\frac{dV}{dh}\right) + \left(\frac{C_D + \Delta C_{D_{WM}} + \Delta C_{D_\psi}}{C_L}\right)} \text{ ‘} \qquad \text{(eq. 2)}$$

Where $\left(1 + \frac{V}{g}\frac{dV}{dh}\right)$ is the “acceleration factor”

T is the thrust of the operating engine(s)

$(C_D + \Delta C_{D_{WM}} + \Delta C_{D_\psi})$ is the total airplane drag coefficient including the yaw drag ΔC_{D_ψ} and the windmilling drag $\Delta C_{D_{WM}}$

But there’s a problem here, isn’t there? We know that the lift and drag coefficients depend on speed, as does the thrust. Let’s say that we’re calculating a second segment climb limit weight, so the speed is V_2, and V_2 is a function of weight. So we can’t solve for weight without knowing the weight. What to do?

This is something that spreadsheet software sometimes refers to as a “circular calculation”. We can’t find an exact solution by using the equation just once. Instead, it’s going to be necessary to iterate: assume a weight, for that weight find the speed, thrust, acceleration correction and force coefficients, and use them to calculate a weight. For the first iteration – the first time you use equation 2 – the weight you find from the equation probably won’t be the same as the weight you assumed...unless you’re amazingly lucky, or an amazingly good guesser.

So repeat the calculation using equation 2 with the new weight. Now the value of W you find will be closer to the weight found by your first iteration. Simply repeat this process until the weight you calculate by your application of equation 2 agrees with the weight you assumed when finding the variables needed to use the equation.

solving for climb limit weight by interpolation

The other method for determining the climb limit weight for a given set of conditions is simply to assume a series of weights and solve for the gradient using equation 1. Then, having thus established the relationship of weight to gradient, find the weight corresponding to the desired gradient. Overall, this is probably easier than solving by iteration. Programming the solution to equation 1 is a relatively simple matter using available thrust data, the airplane drag polar, the windmilling drag and yaw drag data, and the known speed for that flight segment as a function of flap setting and atmospheric conditions.

Shown to the right is a sample graph of second segment climb gross gradient for a range of weights on a sea level standard day.

This is a two engine airplane, so the second segment gross climb gradient requirement is 2.4 percent.

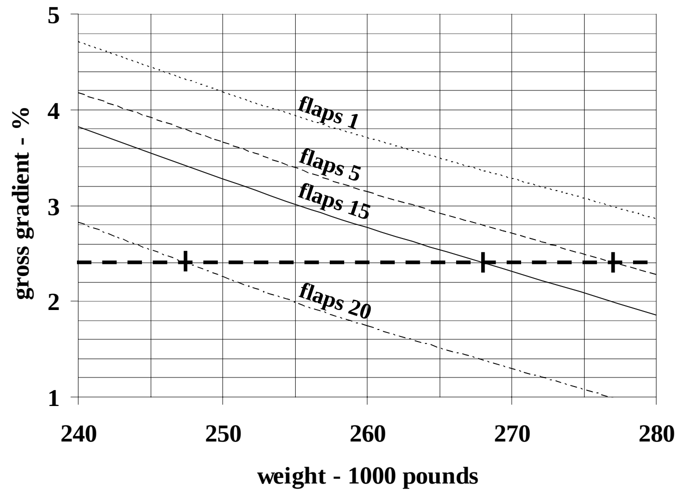


Figure 22-1

At flaps 5, for example, you can see that the second segment climb limit weight would then be 277,000 pounds, and would be 268,000 pounds at flaps 15 and 247,500 pounds at flaps 20. This clearly illustrates the large effect of flap setting on climb limit weights, despite what seems like a small change in the drag-to-lift ratio.

Maximum Takeoff Weight – Climb Limits

Up to this point, we have been talking about the first, second and final segment climb limit weights. Each of these three weight limits is different.

However, when talking about the allowable takeoff weight for a given set of conditions, we have to take the smallest of those three weight limits, in order to ensure that the allowed takeoff weight will meet or exceed all climb gradient requirements.

In many but not all instances, the second segment climb limit weight is the smallest of the three. Under some conditions, the final segment may be the most limiting.

In either case, the Airplane Flight Manual provides a chart that graphically shows the allowable takeoff weight as limited by the climb requirements. This chart has historically had several different names. The earlier AFMs usually called the chart “Maximum Takeoff Weight (Climb Limits), and the later AFMs call it “Takeoff Climb Limits”.

Regardless of the title of the chart, it presents the most limiting weight. In some instances the weight may be limited by one requirement at some conditions and by another requirement at some other conditions. For some of the 707 airplanes, for example, the chart entitled “Maximum Takeoff Weight (Climb Limits)” displays the following comment near the top of the page: “Note: limits based on second segment climb to 37 °C, final segment climb above 37 °C.”

Boeing takeoff analysis software will always compute and display the most limiting of the climb weights.

Presentation of Climb Limit Weight Data

Once the climb limit weights have been determined, they may be presented in a variety of ways.

the Airplane Flight Manual

The AFM presents the climb limit weight data in a graphical format such as you see below.

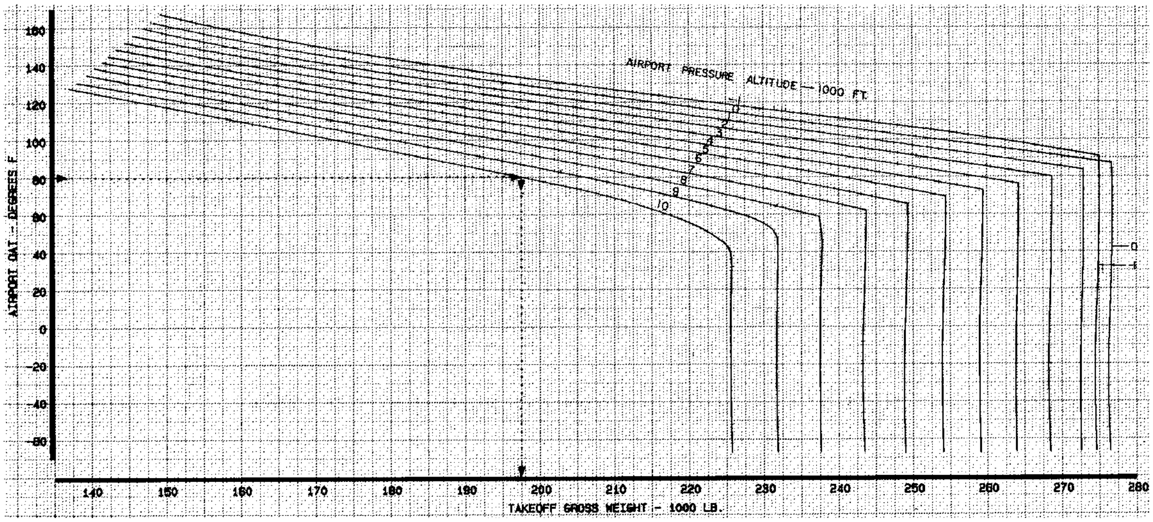


Figure 22-2

Knowing the flap setting, the takeoff pressure altitude and temperature, and the airbleed configuration the determination of the climb limit weight is straightforward. The chart format may vary somewhat between different airplane models; in this example chart, the takeoff temperature is shown along the vertical axis, the takeoff pressure altitude is shown as a series of lines on the graph, and the climb limit weight is read along the horizontal axis.

A graphical presentation such as this makes it easy to see the effects of temperature and altitude on the thrust and thus on the climb limit weight. You can see that the weights are constant at the lower temperatures where the engine thrust is flat-rated. Above the flat-rated temperatures, you can see that the thrust and therefore the climb limit weight will decrease with increasing temperature.

the Flight Planning and Performance Manual

The FPPM contains airplane performance data for use by customer personnel for flight planning purposes, including the determination of the takeoff weight limits. The charts and tables in the FPPM are presented in a simplified format, making them easier to use than the AFM charts.

A typical FPPM climb limit weight chart is shown to the right.

In this chart, note the sliding scale on the left allowing the user to find the climb limit weight for all of the available flap settings in a single chart.

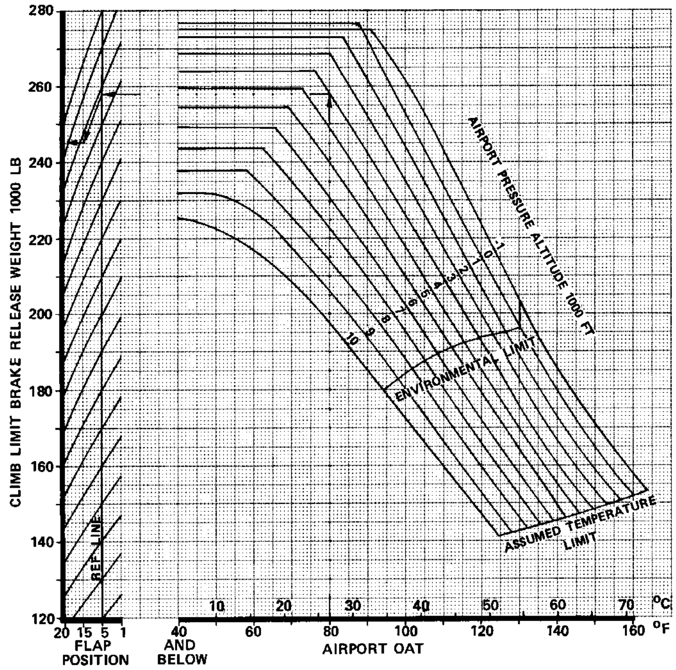


Figure 22-3

the AFM-DPI

For airplanes provided with the AFM-DPI software in lieu of a paper Airplane Flight Manual, the AFM-DPI may be used for a single-point calculation of all takeoff weight limitations, including the climb limit weight, for user-specified takeoff conditions. Alternatively, the AFM-DPI may be used to calculate the limit weights for a range of altitudes and/or temperatures; the resulting data can be automatically plotted by AFM-DPI or it may be plotted by the user in any other desired format.

the Boeing software

For all Boeing airplanes, both those having paper AFMs and those having AFM-DPI, software is available that allows the user to prepare *takeoff analyses*. These are pages of allowable takeoff weight data for specific runways for a range of temperatures at a given airport pressure altitude.

The software allows the user to choose from among a number of user-selectable output formats. A typical output format is shown below:

```
ELEVATION    20 FT                                                  HJGY

*** FLAPS 20 ***   AIR COND OFF    ANTI-ICE OFF        JIGGYVILLE
                                                       COOLCOUNTRY
747-400      80C2B1F                       10% DERATE   DATED 25-JAN-1999
*A* INDICATES OAT OUTSIDE ENVIRONMENTAL ENVELOPE
   MAX BRAKE RELEASE WT-LB, LIMIT CODE AND TAKEOFF SPEEDS FOR ZERO WIND
 TAKEOFF    OAT     CLIMB     ** RWY    13R      **     ** RWY    31L      **
   N1      DEG F    LIMIT     WEIGHT    V1  VR  V2    WEIGHT    V1  VR  V2

     .0     140A    664500    644700*  142 148 156    648100F  143 148 156
     .0     130A    690300    662000F  143 150 158    664900F  143 150 158
  102.0     120     746600    701800F  144 154 163    705200F  145 154 163
  102.6     110     784100    725600F  146 156 166    729300F  147 156 166
  103.2     100     818000    748800F  147 158 168    752100F  147 159 169
```

Figure 22-4

You can see that in this output format the outside air temperature (OAT) is shown in the second column from the left, and the corresponding takeoff climb limit weight is shown in the third column.

Chapter 23: Obstacle Limit Takeoff Weight

Introduction

In the preceding chapter we talked about the climb limit takeoff weight. We defined it as the weight that would yield a climb gradient equal to the mandatory minimum value, allowing for the failure of an engine before reaching V_R speed during the takeoff. We emphasized that the climb limit weight is not intended to consider obstacles in the takeoff flight path.

Now it's time to consider obstacle clearance. Here too, we're required by the regulations to allow for an engine failure. That engine failure might occur at the critical engine failure speed V_{EF}, or it might occur at a later point. We'll be showing you the relevant regulatory requirements a bit later in this chapter.

Each airport is different when considering obstacle clearance, and each must be carefully checked and analyzed for the effects of its obstacles if any exist. Obstacles can range from very close-in to very distant, and there are a number of techniques for dealing with obstacles having different characteristics.

First of all, it may be possible to avoid the obstacles by following a departure procedure that will satisfy the regulatory requirements for clearing obstacles laterally. If that's not possible, then you'll have to calculate the weights that will allow the airplane to clear the obstacles vertically by the required amounts.

It may be that the obstacles along the flight path aren't limiting – that is, the maximum weight allowing obstacle clearance is greater than some other weight limitation such as the field length limit weight or the climb limit weight. Still, it's the performance engineer's job to determine if this is the case. If obstacles do limit the takeoff weight, then the engineer must decide on the optimum takeoff procedure.

The best procedure might be to level off for flap retraction at a lower altitude (often a good procedure for more distant obstacles); alternatively, it might be better to level off for flap retraction later at a higher altitude (a good procedure for closer obstacles).

Other options exist for optimizing the obstacle clearance procedure, such as extending the flaps-down climb beyond the usual altitude for flap retraction (extended second segment climb), or using takeoff thrust beyond the usual five minute limit after beginning the takeoff (ten minutes takeoff thrust) or using takeoff speeds that are increased above their normal values (improved climb technique) or simply increasing V_1 and unbalancing the takeoff distances. We'll discuss each of these in this chapter.

Clearly, this is a complex subject, but it must be considered. True, engine failure is extremely uncommon on today's airplanes, but we must assume that a failure could happen on any takeoff.

We’re going to deal first with straight-out departures as they’re less complex than turning departures. After we’ve acquainted you with the regulations and the methods of calculating obstacle clearance for straight-out departures, we’ll go on to talk about turning departures.

Definitions

There is some terminology used in this chapter with which you may not be familiar, so let’s take a few minutes here to discuss it.

gross flight path

An airplane’s climb gradient, as we would compute it from the basic data including the drag polar and the thrust data, is called the *gross gradient*, and the flight path so computed is called the *gross takeoff flight path*.

net flight path

The *net takeoff flight path* is defined in FAR Section 25.115 and the corresponding EASA regulations as follows:

> *(a) The takeoff flight path shall be considered to begin 35 feet above the takeoff surface at the end of the takeoff distance...*
> *(b) The net takeoff flight path data must be determined so that they represent the actual takeoff flight paths...reduced at each point by a gradient of climb equal to--*
> *(1) 0.8 percent for two-engine airplanes;*
> *(2) 0.9 percent for three-engine airplanes; and*
> *(3) 1.0 percent for four-engine airplanes...*

The critical wording here is “...The net flight path data must be determined so that they represent the actual flight paths reduced at each point by a gradient of climb...” By “<u>actual</u> flight paths” the regulation means the <u>gross</u> flight paths as defined above. <u>Net performance is thus the airplane’s computed (gross) performance decreased by a specified margin of conservatism</u>.

After applying the required decrement to the gross data we have the *net gradient* and the *net takeoff flight path*.

Government regulations specify that the <u>net</u> takeoff flight path must have a vertical clearance of 35 feet above all obstacles lying within a defined area. The intent of defining obstacle clearance in terms of <u>net</u> performance is to account for the fact that an airplane’s climb gradient might be slightly less than we computed. That difference could be due to operational variations that can reasonably be expected in gross weight, thrust, airplane drag, pilot technique, wind effects, and so on.

The gross and net flight paths will be discussed in detail later in this chapter.

brake release point

The point at which the pilot "releases the brakes" – that is, where the airplane begins its takeoff roll – is referred to as the *brake release point*. The end of the runway at which the airplane begins its takeoff roll is called the *brake release end of the runway.*

departure or liftoff end of the runway

The end of the runway over which the airplane passes after liftoff is referred to as the *departure end of the runway* or, in some places, as the *liftoff end of the runway.* Both terms are in common usage.

reference zero

FAR Section 25.115 quoted above says "The takeoff flight path shall be considered to begin 35 feet above the takeoff surface at the end of the takeoff distance..." The point at which the airplane achieves a height of 35 feet above the takeoff surface – which you'll remember is defined as the end point of the takeoff – is called *reference zero*. In other words, "reference zero" is the end point of the takeoff distance and the beginning of the takeoff climb profile.

In obstacle clearance calculations, the location of reference zero along the runway centerline is of critical importance. Since reference zero is the beginning point of the takeoff flight path's vertical profile, the airplane's height depends on its distance from reference zero, not from its distance from the end of the runway.

To determine whether or not an airplane will clear the obstacles after liftoff, it's therefore necessary to know the obstacle distances from reference zero.

It's essential to remember that reference zero can be located before the departure end of the runway, at the departure end of the runway, or beyond the departure end of the runway. The first of these cases occurs whenever the takeoff weight is less than the field length limit takeoff weight. The second case occurs when the takeoff weight is exactly equal to the field length limit takeoff weight when that weight is based on a takeoff distance that doesn't include clearway. The third case occurs when the airplane is at its field length limit takeoff weight when that limit weight is based on a takeoff distance that does include some clearway.

The preceding paragraphs are important because the distances to obstacles in the takeoff path are customarily specified in terms of distance from either the brake release end of the runway or the departure end of the runway. The engineer doing a hand-calculation of obstacle clearance must determine every obstacle's distance from reference zero. The takeoff analysis software must also make that determination.

Remember too that the distance from brake release to reference zero – that is, the takeoff distance – depends on the takeoff weight, among other things. If the takeoff weight changes, the obstacle distances from reference zero will change. Changing the takeoff conditions by selecting an unbalanced V_1 can change the obstacle distances from reference zero. These effects must be accounted for when performing an obstacle analysis.

magnetic and true directions

True directions are those referenced to lines of longitude or latitude, as if oriented to the true north pole of the earth. A true direction of zero degrees would be directly to the north along a line of longitude; 90 degrees would be directly to the east along a line of latitude, and so on.

Magnetic directions are those referenced to the magnetic north pole of the earth. Since the magnetic north pole is located at some considerable distance from the true north pole, there can be considerable difference between true and magnetic directions. Complicating this subject further, the difference between magnetic and true direction, called *magnetic variation* or, sometimes, *magnetic declination*, depends on your location on the earth. The variation also changes over time, since the magnetic north pole moves, however slowly.

In Seattle, for example, the magnetic variation is approximately 20 degrees east (+20°), meaning that a compass needle is pointing 20 degrees to the east of the true north pole. In the northeastern states of America, the variation is on the order of 20 degrees west (-20°). Thus, magnetic direction plus or minus the variation equals the true direction.

runway number

Since we frequently deal with runway numbers (for example, Boeing Field runway 13 right) and may want to relate it to the reported direction of the wind, it's important to know that *runway number* is the magnetic direction in which the runway is pointing, rounded to the nearest ten and then divided by ten. Thus Boeing Field runway 13R has a magnetic direction of between 125 and 135 degrees. The "R" simply means that there are two runways having the same number, 13L and 13R, and this is the right-hand one of the two as you look in the runway direction.

Seattle-Tacoma International Airport airport in Seattle has three parallel runways. Taking off or landing to the south, you would use runway 16 left (16L), runway 16 center (16C), or runway 16 right (16R). Taking off or landing to the north, you would use 34L, 34C, or 34R.

Runway numbering uses magnetic heading simply because the compasses in a cockpit are magnetic devices (with the exception of some navigation data displays in the more recent airplanes that have computer-generated true direction information). The convention of numbering runways by their magnetic direction is logical – it's simply to have the runway number agree with what the pilot sees on his (magnetic) compass, hopefully reducing any possibility of confusion as to which is the correct runway to use. If true direction were used for runway numbers, then for example when lined up for takeoff on Boeing Field runway 13R the magnetic compass would read approximately 110 degrees.

What if your runway is pointing at the magnetic north pole, a direction of 000 degrees. Will it be called runway 00? No. On a compass, 360 degrees is the same as 000 degrees and it's customary to call a north-pointing runway by the number 36 instead of 00.

reported wind

A reported wind such as 270/10, "two seven zero at ten" means that the wind is coming from a magnetic direction of 270°, west, with a velocity of ten knots. Thus if you're on runway three six, that's a left crosswind of ten knots with no headwind or tailwind component; if you're on runway two seven, it's a ten knot headwind, with no crosswind component.

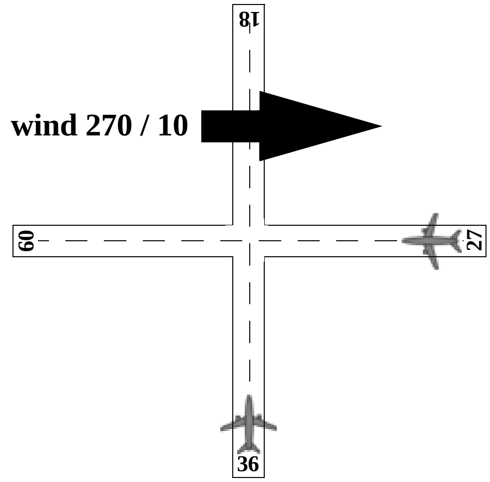


Figure 23-1

Winds on the ground are reported in magnetic direction simply to facilitate calculation of a wind's direction relative to the direction of takeoff or landing, since runway designations are also in magnetic units.

Although takeoff winds are reported in the magnetic direction they're from, winds aloft are reported in terms of the true direction they're from. That facilitates calculation of their direction relative to a route of flight plotted on a navigation chart since they use grid lines of latitude and longitude, which are true directions.

heading and heading angle

The compass direction (either true or magnetic) in which the longitudinal axis of an airplane is pointing is called its *heading* or *heading angle*. Thus, for example, if the airplane's axis is pointed directly to the east, its heading angle is 90 degrees.

track and track angle

The compass direction (either true or magnetic) of an airplane's path over the ground is referred to as its *track* or *track angle*. Suppose, for example, that the pilot of an airplane is flying it such that it's following a road that points directly to the west. The airplane's track angle is then 270 degrees.

drift and drift angle

When there is no wind, an airplane's heading angle is the same as its track angle. In such a case, if the airplane is flying over a road that points directly to the south, its heading angle and its track angle will both be 180 degrees.

When there is a wind, however, if the direction of that wind is at some angle to the airplane's heading, it will cause the airplane's track over the ground to be different from its heading. The airplane will be "drifting" sideways. The difference between the heading angle and the track angle is called the *drift angle*.

The drift angle is found from the vector sum of the airplane's velocity vector and the wind's velocity vector. For example, an airplane heading directly north (heading angle 000 degrees) at a true airspeed of 150 knots (55 smoots per microfortnight) with a crosswind reported as 270 at ten knots will have a drift angle of four degrees to the right. The track angle then would be 004 degrees.

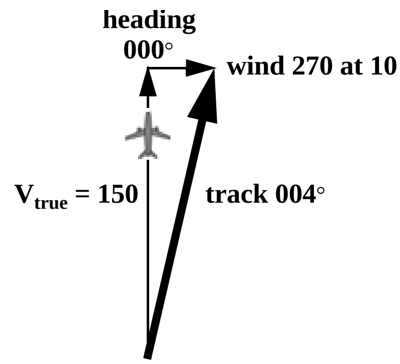


Figure 23-2

If the pilot wanted to maintain a <u>track</u> angle of 0 degrees, he would need to correct for the drift by flying with an airplane <u>heading</u> of 356 degrees.

We'll be demonstrating how to calculate a flight path corrected for wind later in this chapter.

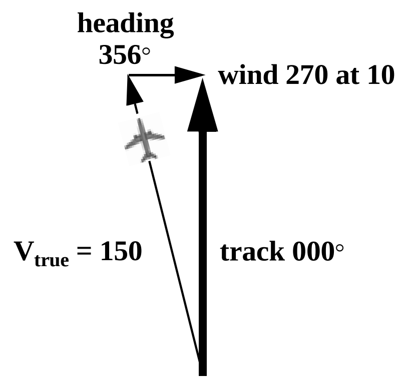


Figure 23-3

splay

The term *splay* can be used in several ways. When used as a verb, one definition is "to spread or flare". We'll be using the word as a noun, describing one characteristic of the area called the *Obstacle Accountability Area (OAA)* – an area within which all obstacles must be considered for their possible effect on the allowable takeoff weight. A little later in this chapter we'll be discussing the regulatory requirements for obstacle clearance, and you'll see how the FAA regulations describe the OAA, but to state it briefly: one characteristic of the OAA is that it gradually widens as distance beyond the end of the takeoff distance increases. This widening shape is called splay, and the rate of the widening can be described as a ratio, such as "sixteen to one", numerically written as 16:1. A splay of 16:1 means simply that the line defining the edge of the OAA becomes one foot (or meter) farther from the centerline of the OAA for each 16 feet (or meters) of distance increase from the end of the takeoff distance available.

half-width

The term *half-width* in the context of obstacle clearance refers to the distance from the centerline of an obstacle accountability area to the edge of the area, measured perpendicular to the centerline.

You'll sometimes see splay dimensions described as half-widths. Thus for a splay of 16:1 for example you can say that the *half-width* of the OAA (that is, the distance from the centerline of the OAA to each side) is $0.0625 \times D$, where D is the distance from the end of the takeoff.

Before we begin to look at the body of regulatory material governing obstacle clearance, let's first find out where to obtain obstacle data.

Sources of Obstacle Data

You'll be seeing much of FAA Advisory Circular 120-91 later in this chapter, as it's the definitive source of guidance on obstacle clearance requirements for those following FAA methods. It's interesting, as a beginning to our discussion, to see what they say about the sources of obstacle data:

> ***5. IMPLEMENTATION....****The FAA expects operators to use the best available data for airport obstacle analysis and to continually review and use improved data as it becomes available...*
>
> ***6. SOURCES OF OBSTACLE DATA.*** *Operators are expected to use the best and most accurate available obstacle data for a particular airport at the time of analysis. Data sources do not require specific FAA approval. Operators should be aware that an airport Obstruction Chart (OC), Type A chart, or any other single source may not include all the pertinent information necessary for doing a takeoff analysis.*

The following is a brief guide to some of the available sources of obstacle information.

Operators should be aware that there is frequent disagreement between different sources of obstacle data for the same runway. It's the operator's responsibility to verify the accuracy of obstacle data you use for the takeoff analyses.

ICAO

The International Civil Aviation Organization (ICAO) is headquartered in Montreal, Canada. It is an agency of the United Nations; its purpose is to develop and codify the principles and techniques of international air navigation and to promote the planning and development of international air transport to enhance safe and orderly growth.

airport characteristics data bank

In the past, ICAO published an airport information database called the Airport Characteristics Data Bank (ACDB) but this has been out of publication since approximately 2005 and existing copies should be discarded as the information may be out of date.

Aeronautical Information Publication (AIP)

One of ICAO's many functions is to establish standards for the publication of aeronautical information for use by airlines and other aviation organizations. One such document defined by ICAO is the Aeronautical Information Publication (AIP). This document is usually published by the civil aviation authority of a country, or by another agency on their behalf. The purpose of the AIP is to provide aeronautical information relevant to air navigation. It is designed to be a manual providing details of regulations, procedures and other information pertinent to flying aircraft in the particular country to which it relates.

One section of most AIPs is the AD (aerodromes) section. It contains details and charts of the airports of that country, including obstacle data. Some of the most useful charts for our work are the Aerodrome Obstacle type A charts that present, in detail, the obstacle heights and locations relative to the runway.

To obtain the Aerodrome Obstacle charts, you can contact the responsible aviation agency of the country in which the airport is located. In some cases they may be available from the airport authority itself.

Experience has shown that AIPs in some parts of the world may be unreliable sources of obstacle data, as they are not always kept up to date with the latest information. AIP users should do their best to ensure that the AIP data they use is up to date.

IATA

The International Air Transport Association (IATA) is an international industry trade group of airlines headquartered in Montreal, Quebec, Canada. Among other services, IATA publishes and maintains a database of airport information known as the Airport and Obstacle Database (AODB), which is available to operators with a monthly or yearly subscription fee. This database provides data in digital and graphical formats for nearly 3000 airports.

FAA in the Unites States

Some obstacle data is available from the United States Federal Aviation Administration. The FAA as a source however is inconsistent in its ability to provide information.

commercial sources

There are a number of commercial sources of airport data, including Jeppesen-Sandersen, Lido Aircraft Performance Engineering, SITA, and others. Their services are provided for a fee.

Regulatory Requirements

FAA

The heart of the regulatory requirements for obstacle clearance is contained in FAR Section 121.189. We have omitted some text where shown by ellipses (...) that is not relevant to current airplanes.

> *...(d) No person operating a turbine engine powered transport category airplane may take off that airplane at a weight greater than that listed in the Airplane Flight Manual--*
> *...(2) In the case of an airplane certificated after September 30, 1958...that allows a net takeoff flight path that clears all the obstacles either by a height of at least 35*

feet vertically, or by at least 200 feet horizontally within the airport boundaries and by at least 300 feet horizontally after passing the boundaries.

So, in summary, either (a) the airplane's horizontal flight path must clear all obstacles laterally by a specified margin of distance, or (b) the airplane's net flight path must clear all obstacles vertically by at least 35 feet.

vertical clearance

Clearing an obstacle by 35 feet doesn't sound like much of a safety margin, but you'll remember that the net flight path is defined as the actual computed flight path (the gross path) minus a substantial performance decrement as a safety margin

We'll discuss the gross and net flight paths in much more detail later in this chapter, but just for a quick example: if a two-engine airplane is 10,000 feet past reference zero, the difference between net and gross heights will be 0.8 percent of 10,000 feet, or 80 feet. Thus an airplane 10,000 feet past reference zero could count on clearing an obstacle by 115 feet, rather than the 35 feet mandated by the FARs for the net path.

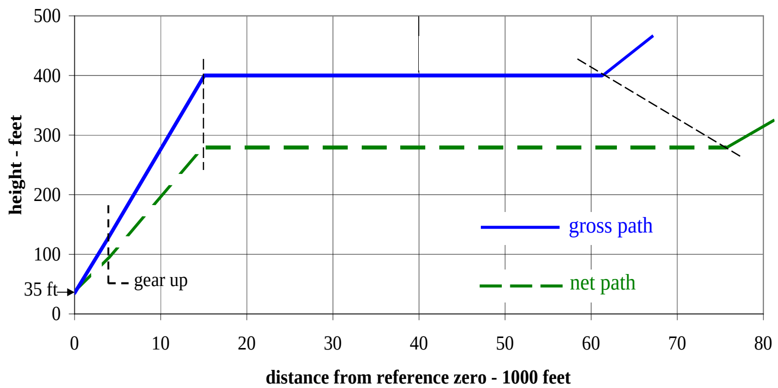


Figure 23-4

horizontal clearance

You saw in FAR 121.189 above that an airplane's horizontal clearance of an obstacle must be at least 200 feet within the airport boundaries, or at least 300 feet after passing the boundaries. FAA Advisory circular 120-91 provides additional guidance on this subject. Quoting from the AC:

4. BACKGROUND. Sections 121.177, 121.189, 135.367, 135.379, and 135.398 specify required takeoff and performance operating limitations. These limitations include determination of the takeoff flightpath that meets specified obstacle clearance requirements (both vertical and horizontal) in the event of an engine failure. Sections 121.189, 135.379, and 135.398 specify AFM compliance, and part 25 provides requirements for establishing the AFM performance data. While the AFM provides detailed instructions for determining the vertical clearance, it offers little guidance on the lateral clearance requirements. This AC provides information for determining safe clearance from obstacles for the actual flightpath, and for considering factors that may cause a divergence of the actual flightpath from the intended flightpath. This AC also provides guidance and acceptable lateral criteria to assist an operator in developing takeoff procedures and allowable weights for operational use.

The AC goes on to say:

...This AC will focus on two methods that may be used to identify and ensure clearance of critical obstacles: the Area Analysis Method and the Flight Track Analysis Method. The two methods may be used in conjunction with each other on successive portions of the analysis...

***a**. The Area Analysis Method defines an obstacle accountability area (OAA) within which all obstacles must be cleared vertically. The OAA is centered on the intended flight track and is acceptable for use without accounting for factors that may affect the actual flight track relative to the intended track, such as wind and available course guidance.*

***b.** The Flight Track Analysis Method is an alternative means of defining an OAA based on the navigational capabilities of the aircraft. This methodology requires the operator to evaluate the effect of wind and available course guidance on the actual ground track. While this method is more complicated, it can result in an area smaller than the OAA produced by the Area Analysis Methods...*

An Obstacle Accountability Area, or OAA, is simply an area centered on the intended takeoff departure flight path from a runway. Any object within the OAA that could be an obstacle to the takeoff flight path <u>must</u> be analyzed for its possible effect on the takeoff weight. Objects or obstacles lying <u>outside</u> the OAA need not be considered in any takeoff analysis for that runway.

The Flight Track Analysis Method of defining an OAA is complex; it allows for the use of "ground-based course guidance" such as localizer, VOR, ADF or DME capability. It also allows for the use of "airplane performance-based area navigation capabilities" such as Global Positioning System (GPS), Area Navigation System (RNAV), Initial Reference System (IRS) or required navigation performance (RNP) capabilities. If you're faced with the need to develop an obstacle clearance analysis that would fall into the Flight Track Analysis Method please consult the AC for details, as further discussion of this method is outside the scope of this chapter.

The Advisory Circular's description of the Area Analysis Method is relatively straightforward. It details two types of departures: (a) straight-out departures or those when the intended track or airplane heading is within 15 degrees of the extended runway centerline heading, and (b) departures involving turns of the intended track or when the airplane heading is more than 15 degrees from the extended runway centerline heading. The second of these two will be discussed later in this chapter.

The description of the OAA for a straight-out departure reads as follows:

11. AREA ANALYSIS METHOD

***a.** During straight-out departures or when the intended track or airplane heading is within 15 degrees of the extended runway centerline heading, the following criteria apply:*

> ***(1)*** *The width of the OAA is 0.0625D feet on each side of the intended track (where D is the distance along the intended flightpath from the end of the runway in feet), except when limited by the following minimum and maximum widths.*
>
> ***(2)*** *The minimum width of the OAA is 200 feet on each side of the intended track within the airport boundaries, and 300 feet on each side of the intended track outside the airport boundaries.*
>
> ***(3)*** *The maximum width of the OAA is 2,000 feet on each side of the intended track.*

The AC goes on to provide guidance on several other points of interest:

> ***...11.c.(3)*** *The distance to an obstacle within the OAA should be measured along the intended track to a point abeam[1] the obstacle.*
>
> ***(4)*** *When an operator uses the Area Analysis Method, the operator does not need to separately account for crosswind, instrument error, or flight technical error within the OAA...*
>
> *...**(6)*** *One or more turns of less than 15 degrees each, with an algebraic sum of not more than a 15 degree change in heading or track, may be analyzed as a straight-out departure.*
>
> ***(7)*** *No accountability is needed for the radius of the turn or gradient loss in the turn for a turn with a 15 degree or less change in heading or track.*

Note in particular paragraph (4) above. We'll be talking more about that a little later.

1. The term *abeam* in this context means that when the airplane is at the point along the intended track at which the obstacle is "abeam", then a line drawn from the airplane's position to that obstacle will be at a 90° angle relative to the airplane's track.

We can illustrate the above definition of the OAA for a straight-out departure graphically:

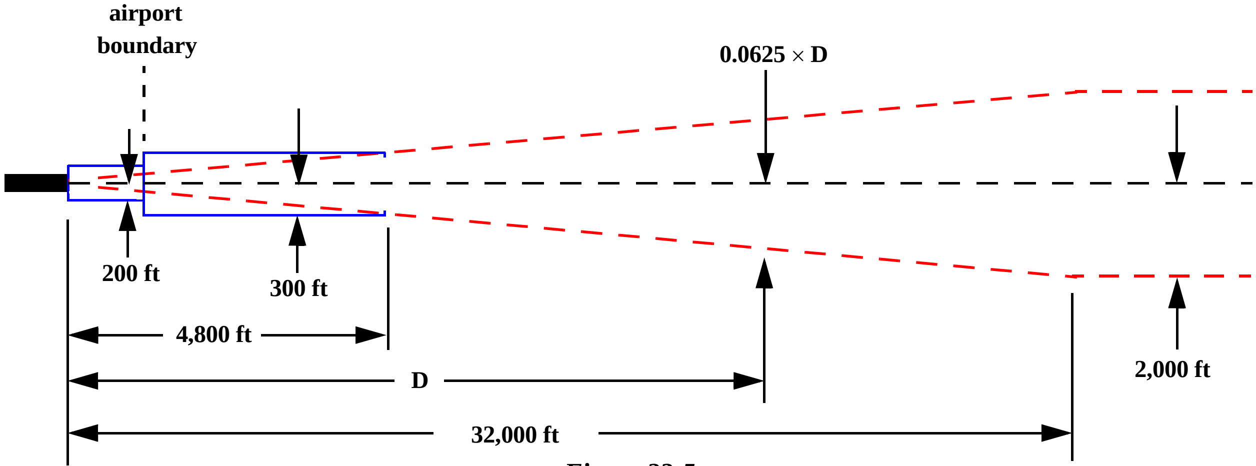


Figure 23-5

You see that the OAA is centered on the extended runway centerline. The width of the OAA can be expressed as the distance from the runway end multiplied by 0.125 – that is, a half-width equal to the distance from the runway end multiplied by 0.0625.

The OAA has, however, a minimum half-width of 200 feet where it lies within the airport boundaries and 300 feet after passing the airport boundary. That 300 foot minimum half-width continues until passing a point 4,800 feet from the end of the runway.

When passing a point 32,000 feet from the runway end, the OAA reaches its greatest half-width of 2,000 feet.

JAR-OPS

JAR-OPS 1.495 contains the following wording:

> *(a) An operator shall ensure that the net takeoff flight path clears all obstacles by a vertical distance of at least 35 feet or by a horizontal distance of at least 90 m plus 0.125 × D where D is the horizontal distance the aeroplane has travelled from the end of the take-off distance available or the end of the take-off distance if a turn is scheduled before the end of the take-off distance. For aeroplanes with a wingspan of less than 60 m* [this includes all Boeing airplanes except the 747-400, 747-8, and 777] *a horizontal distance of half the airplane wingspan plus 60m, plus 0.125 × D may be used...*
>
> *...(d) When showing compliance with subparagraph (a) above for those cases where the intended flight path does not require track changes of more than 15°, an operator need not consider those obstacles that have a lateral distance greater than:*

(1) 300 m, if the pilot is able to maintain the required navigational accuracy through the obstacle accountability area (see AMC OPS 1.495(d)(1) & (e)(1); or

(2) 600 m, for flights under all other conditions.

Notice that the JAR-OPS regulation does not refer to an obstacle accountability area, as the FAA regulation does – but in effect, by specifying a lateral clearance requirement, it is creating an OAA. We can use the term “obstacle accountability area” or “OAA” to refer to either the FAA or JAA requirements.

To the right, we show graphically the JAR-OPS obstacle accountability area. Notice in particular the splay: it is at 0.125 times the distance from the end of the takeoff (an 8:1 ratio), whereas the FAA splay is 0.0625 times the distance, a 16:1 ratio.

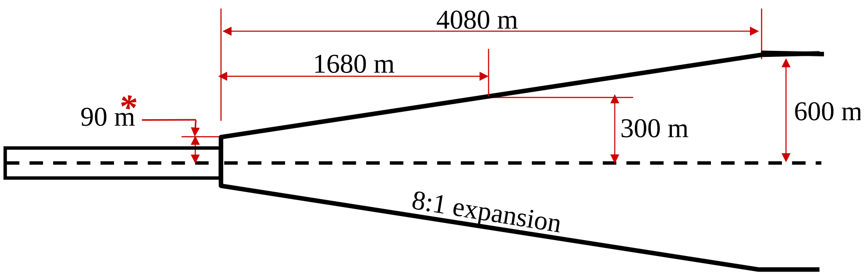


Figure 23-6

Notice that the lateral clearance requirement reaches a maximum of 300 meters if the pilot is able to maintain a specified navigational accuracy, otherwise it reaches a maximum of 600 meters during a straight-out departure. AMC OPS 1.495(d)(1) & (e)(1) mentioned above is the “Acceptable Means of Compliance” that provides guidance on what is meant by “specified navigational accuracy”. Refer to that AMC for details.

Any obstacle that can’t be cleared by the distances specified above must then be cleared vertically by 35 feet in the net flight path.

ICAO

ICAO Annex 6 defines takeoff obstacle clearance[1] in a manner that is identical to that of the JAR-OPS regulation, with one exception: the maximum half-widths. The JAR-OPS says that, for a straight-out departure, an operator need not consider those obstacles that have a lateral distance greater than 300 meters, if the pilot is able to maintain the required navigational accuracy through the obstacle accountability area, or 600 meters for flights under all other conditions.

Annex 6 puts it a little differently:

3.1.1 Where the intended track does not include any change of heading greater than 15 degrees,

1. Annex 6, Attachment C, Example 3, Paragraph 3.1

a) for operations conducted in VMC by day, or

b) for operations conducted with navigation aids such that the pilot can maintain the aeroplane on the intended track with the same precision as for operations specified in 3.1.1 a),

obstacles at a distance greater than 300 m on either side of the intended track need not be cleared.

3.1.2 Where the intended track does not include any change of heading greater than 15 degrees for operations conducted in IMC, or in VMC by night, except as provided in 3.1.1 b)...obstacles at a distance greater than 600 m on either side of the intended track need not be cleared.

Departure Procedures

Engine reliability these days is amazingly high, so the incidence of engine failure during takeoff or initial climb is extremely low. Still, an engine may fail at any point during a flight so it's necessary to consider departures for both the all engine case and the engine failure case.

all engines operating

For many runways around the world, pilots are required to follow specific departure procedures following takeoff. These are usually referred to as "SIDs" (Standard Instrument Departures), or "DPs" (Departure Procedures) or simply "departures". The most common reason for standard departures is simply to facilitate smooth and speedy flow of departing and arriving traffic at the airport. There are, however, a number of other reasons for these specific departure procedures: one is for terrain or obstacle avoidance or clearance, another reason is for avoidance of another airport's traffic area or some prohibited area.

These departure procedures are based on the U. S. Standard for Terminal Instrument Procedures (TERP) or an equivalent such as ICAO Procedures for Air Navigation Services – Aircraft Operations (PANS-OPS). These standards are for normal departures with all engines operating.

TERP and PAN-OPS procedures specify still-air climb path angles in terms of feet per nautical mile. For any given angle of climb specified in feet per nautical mile, it's a simple calculation to find the corresponding rate of climb, knowing the climb true airspeed:

$$\text{rate of climb (feet per minute)} = \frac{\text{true airspeed (knots)}}{60} \times \text{rate of climb (feet per NM)}$$

These departure procedures require at least an angle of climb of 200 feet per nautical mile (NM). Pilots should not confuse TERP angles of climb with <u>rates</u> of climb. An angle of climb of 200 feet per nautical mile, for example, is a geometric climb <u>gradient</u> of 3.3%, equal to path <u>angle</u> of 1.9 degrees; this corresponds to a rate of climb of 500 feet per minute at a true airspeed of (for example) 150 knots. If a greater angle of climb than 200 feet per nautical mile is necessary for a specific procedure, that angle of climb will be specified for the departure.

A TERP requirement is to be treated "as a plane which must not be penetrated from above until reaching the stated height, rather than as a gradient which must be exceeded at all points in the path."

As an example, the ELMAA7 departure from Seattle-Tacoma International Airport (KSEA) runway 16L requires the pilots to maintain a minimum angle of climb of 560 feet height gain per nautical mile until passing through 3,000 feet. Pilots accepting this departure procedure as part of their takeoff clearance are responsible for ensuring that their airplane is capable of delivering the specified minimum angle of climb.

It is essential that the following statement be understood clearly: these procedures are based on normal all engine operation. Advisory Circular 120-91 discusses this:

> ***7. TERPS CRITERIA VERSUS ONE-ENGINE-INOPERATIVE REQUIREMENTS.***
>
> *(a)...Thus, one-engine-inoperative obstacle clearance requirements and the all-engines-operating TERPS requirements are independent, and one-engine-inoperative procedures do not need to meet TERPS requirements. Further, compliance with TERPS all-engines-operating climb gradient requirements does not necessarily assure that one-engine-inoperative obstacle clearance requirements are met. TERPS typically use specified all-engines-operating climb gradients to an altitude, rather than certified one-engine-inoperative performance. TERPS typically assume a climb gradient of 200 feet per nautical mile (NM) unless a greater gradient is specified. For the purposes of analyzing performance on procedures developed under TERPS or PANS-OPS, it is understood that any gradient requirement, specified or unspecified, will be treated as a plane which must not be penetrated from above until reading the stated height, rather than as a gradient which must be exceeded at all points in the path. Operators must comply with* [FAR] *requirements for the development of takeoff performance data and procedures. There are differences between TERPs and one-engine-inoperative criteria, including the lateral and vertical obstacle clearance requirements. An engine failure during takeoff is a non-normal condition, and therefore takes precedence over noise abatement, air traffic, SIDs, DPs, and other normal operating conditions.*

departures considering engine failure

As you see, all of the above discussion about all engine departures immediately becomes irrelevant if an engine fails, and we must consider that possibility. Every airline has the responsibility to examine every airport into which it operates to examine obstacle clearance weights and procedures assuming the failure of an engine at any point during the takeoff.

We emphasize "at any point during the takeoff" because the takeoff obstacle environment at some airports is such that the critical situation occurs <u>not</u> when an engine fails at V_1 but instead when it fails during climbout <u>while the pilots are following the all engine departure procedure</u>. It's thus necessary for the airline to consider both possibilities – the usual case of engine failure at V_1, as well as failure at a later time – when analyzing the airport. AC 120-91 again says it best:

> ***(1)*** *The most common procedure to maximize takeoff weight when significant obstacles are present along the normal departure route is to use a special one-engine-inoperative departure routing in the event of an engine failure on takeoff. If there is a separate one-engine-inoperative departure route, then the obstacles along this track are used to determine the maximum allowable takeoff weight for that runway.*
>
> ***(2)*** *Consideration should be given to the possibility of an engine failure occurring after passing the point at which the one-engine-inoperative track diverges from the normal departure track. Judicious selection of this point would simplify the procedure and minimize the difficulty of this analysis. This is generally achieved by keeping the two tracks identical as far as is practical.*
>
> ***(3)*** *In some cases, two or more special one-engine-inoperative tracks may be required to accommodate all the potential engine failure scenarios.*
>
> ***(4)*** *Analysis of an engine failure after takeoff may require the use of performance data in addition to that provided in the AFM.*

Paragraph (4) in that quotation refers to the fact that the AFM does <u>not</u> include any charts of climb gradient for the all engines operating condition. Later in the AC it says:

> *Unless otherwise authorized, AFM data must be used for one-engine-inoperative takeoff analysis. It is recognized that many AFMs generally contain only the one-engine-inoperative performance for loss of an engine at V_1 on takeoff. All-engines-operating performance must also be considered to determine the airplane's flightpath in the event of an engine failure at a point on the flightpath after V_1. The best available all-engines-operating data should be used consistent with best engineering practices. Operators may find appropriate acceptable data in various sources, such as: community noise documents, performance engineer's handbooks, flight characteristic manuals, and manufacturers' computer programs.*

It should be clearly understood that the distinction between gross and net climb performance relates only to the requirements of FAR 121.189. The requirement of AC120-91 for consideration of engine failure <u>after</u> the takeoff does not mandate a gross-net margin of conservatism.

In a previous chapter, you have seen how to calculate the all engines operating gradients; it's a relatively simple calculation involving the drag polar and thrust tables from the Performance Engineer's Manual published by Boeing. A step integration procedure will allow you to calculate the all engine flight path up to the point at which you assume engine failure.

Boeing Climbout Program

A software application called the Boeing Climbout Program (BCOP) is a Windows-based Graphical User Interface (GUI) application. The BCOP application will analyze the performance of SIDs, STARS, go-around and engine-out procedures. For a unique airframe/engine combination and user specified aircraft configuration, BCOP uses specific airport characteristics and user specified vertical and lateral profiles to produce three dimensional flight path information. A subset of the 70 BCOP output parameters available includes latitude, longitude, altitude, speed, climb gradient, rate of climb, time, fuel, ground track distance, and aircraft heading. BCOP has the added advantage of allowing engine failure to be specified at any point in the departure, facilitating analyses such as we're discussing here for engine failure after takeoff.

While BCOP can't be used for obstacle limit weight calculation, by producing vertical profiles it greatly facilitates that work. Inquiries about BCOP should be directed to the Boeing Commercial Airplanes Flight Operations Engineering Group.

Accounting For Winds

Any wind encountered during the takeoff flight path will affect the airplane's vertical and/or horizontal profile.

headwinds and tailwinds

When there exists a wind, or a component of wind, along the airplane's takeoff flight path, that wind will have two effects on the airplane's takeoff performance.

First, when there's a headwind or a tailwind, an airplane's ground speed will be different from its airspeed. That difference will be equal to the velocity of the wind. Thus an airplane climbing at a true airspeed of 150 knots in a ten knot headwind component will have a ground speed of 140 knots.

How about an airplane's <u>rate of climb</u> in a headwind or a tailwind? That <u>isn't</u> affected by wind. An airplane's <u>vertical</u> speed will be the same, for a given set of conditions, regardless of wind. It's a function only of thrust required and thrust available.

Since an airplane's horizontal speed <u>is</u> affected by wind but its vertical speed <u>isn't</u>, the angle of the climb – the angle whose tangent is equal to the airplane's vertical speed divided by its horizontal speed – <u>is</u> a function of wind. A headwind, by reducing the airplane's horizontal speed without affecting its vertical speed, results in a greater climb angle over the ground; a tailwind produces a smaller climb angle over the ground.

By how much does a headwind or tailwind change an airplane's climb gradient? It's not a big change, but it shouldn't be neglected: for example, if climbing at a V_2 of 150 knots true airspeed, a zero wind gradient of 2.4 percent would become a 2.57 percent gradient in a ten knot headwind.

But headwinds and tailwinds have a second effect: they will change the takeoff distance from brake release to reference zero – and that changes the distance from reference zero to any obstacles.

A headwind thus has two favorable effects: it increases the climb gradient over the ground, and it increases the obstacle distances from reference zero. These two effects can combine to produce substantial takeoff weight increase. A tailwind, on the other hand, with its two adverse effects, will have result in a substantial weight penalty.

required headwind and tailwind conservatism requirement

Clearly, any headwind or tailwind component of the wind existing during a takeoff must be accounted for, whether using either the area analysis method or the flight track analysis method, as it will affect the airplane's vertical path and hence its height above any obstacles.

That's true, but in case you didn't already know this, we need to tell you that there's a special requirement in FAR Part 25 on this subject:

> *25.105 (d) The takeoff data must include, within the established operational limits of the airplane, the following operational correction factors:*
>
> *(1) Not more than 50 percent of nominal wind components along the takeoff path opposite to the direction of takeoff, and not less than 150 percent of nominal wind components along the takeoff path in the direction of takeoff...*

The FAA is thereby introducing a conservatism in how headwind and tailwind components of the reported takeoff wind may be accounted for: recognizing that the actual wind angle and velocity at the time of takeoff may be somewhat different from the reported wind angle and velocity, an operator may not take credit for more than one-half of a headwind component, and must use at least one and one-half times any reported tailwind component.

When using flights from a Boeing Airplane Flight Manual, this wind conservatism is already included. Say, for example, that the reported takeoff wind will be ten knots headwind; reading the charts using the data shown for ten knots headwind will actually produce an answer for five knots headwind. Reading the charts using the data shown for ten knots tailwind will actually produce an answer for fifteen knots tailwind. Therefore, the user should not apply the 50% or 150% corrections to the reported wind before entering the charts; to do so would cause a conservatism of double the required amount.

crosswinds

A wind or component of wind that is perpendicular to an airplane's horizontal path will result in a drift angle, as discussed above.

You'll remember that the AC says:

When an operator uses the Area Analysis Method, the operator does not need to separately account for crosswind, instrument error, or flight technical error within the OAA.

It's sometimes advisable, however, to account for crosswinds when they will alter the horizontal path in such a way as to create a potential obstacle hazard. To determine whether or not this will be the case for a given runway, it may be necessary to plot the horizontal flight path considering possible crosswinds.

plotting the horizontal flight path with wind

For a straight-out departure, plotting the horizontal profile in a wind is a simple matter.

Think of the airplane as flying in a big block of air. If that block of air is stationary, that's a zero wind condition, and the airplane will fly straight out the extended runway centerline.

Now imagine that the block of air is moving as the airplane is flying through it. The direction that the block is moving and the speed at which it's moving are the wind direction and the wind velocity respectively. The path of the airplane through the block of air is unchanged, but because the block of air is moving at the same time that the airplane is flying through it, the airplane's path over the ground will be different from its path through the block of air. Thus its path over the ground is altered by the distance and direction that the block of air has moved.

If the wind has any crosswind component, the flight path – that is, the horizontal profile – will exhibit some drift.

For the purposes of this discussion, let's call the extended runway centerline the Y direction, and the sideways drift is the X direction, it's simple to plot the path of the airplane in the wind. Take, for example, the case of an airplane climbing at a speed of 150 knots, with a ten-knot wind blowing at an angle of 135 degrees to the direction of flight – that would be a wind having a seven knot headwind component and a seven knot left crosswind component. In one second, the airplane will fly $150 \times 1.6878 = 253.2$ feet. The block of air in which the airplane is flying will move $10 \times 1.6878 \times \sin(135°) = 11.9$ feet to the right and 11.9 feet in the direction opposite to the direction of flight.

Calculate the airplane's location at ten second intervals. Thus, for example, after ten seconds in no wind the airplane has flown 2531 feet; in that same ten seconds, the block of air has moved 119 feet to the right and 119 feet back toward reference zero, so the X coordinate with wind is 119 feet and the Y coordinate is 2412 feet.

	x		y	
time - sec	no wind	with wind	no wind	with wind
0	0	0	0	0
10	0	119	2532	2412
20	0	239	5063	4825
30	0	358	7595	7237
40	0	477	10127	9649
50	0	597	12659	12062

Table 23-1

Graphing the table of data: the solid line represents the path of the airplane in no wind. Each mark represents the distance flown in ten seconds. The broken line represents the path of flight of the airplane in a ten knot wind at an angle of 135° to the direction of the flight path.

For comparison, we're showing also the flight path with a 5 knot right crosswind.

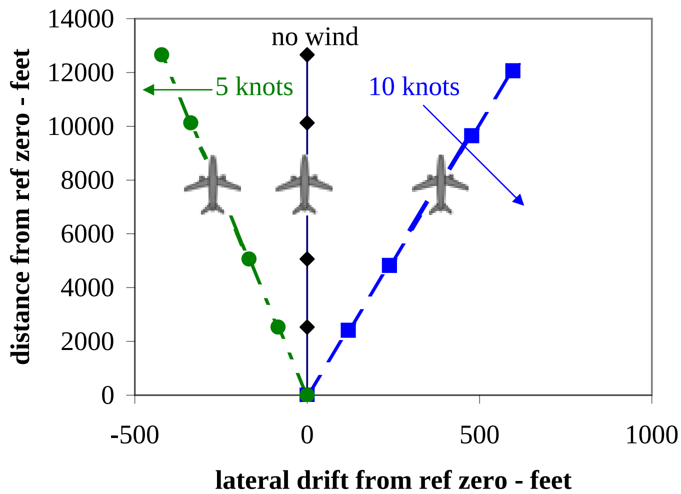


Figure 23-7

at this point, a note of caution

We pointed out to you above that the FAA Advisory Circular on obstacle clearance does not require the person doing an obstacle analysis to account for crosswinds.

When you think about it and play with some numbers as we've done just above, you'll recognize immediately that if an airplane is maintaining runway heading after takeoff, <u>even a modest crosswind will cause the airplane to drift outside of the obstacle accountability area</u>. For example, a ten knot crosswind at a climb speed of 150 knots is a 15:1 ratio, which lies <u>outside</u> of the 16:1 ratio OAA side lines specified by the FAA Advisory Circular. A twenty knot crosswind, which isn't uncommon, will cause an airplane's flight path to lie outside the JAR-OPS 8:1 ratio.

Clearly, if your takeoff area has critical obstacles, particularly if they lie outside of but close to the obstacle accountability area, you must take precautions to ensure that the airplane can't accidentally fly into the obstacle. You might specify crosswind limitations, for example, or a path that will give greater clearance to the obstacles. The best option would be to define a ground track for departure that utilizes existing navigation aids such as the Flight Management Computer (FMC) together with Global Positioning System (GPS) inputs. However the operator chooses to do so, <u>it is the responsibility of the operator to ensure that its obstacle analyses provide acceptable safety margins under any takeoff conditions that may reasonably be expected</u>.

plotting the vertical flight path with wind

We showed you above that winds with any headwind or tailwind component will alter an airplane's vertical profile, just as winds with any crosswind component will alter the airplane's horizontal profile. While crosswinds produce a <u>lateral</u> drift, headwinds or tailwinds produce a drift along the direction of the flight, thus affecting the airplane's height at any given distance from reference zero.

In the table to the right, we calculate the horizontal and vertical distances in ten second intervals. We are assuming a true airspeed of 150 knots, a headwind component of 20 knots, and a gradient of 2.4 percent. The height gain per second isn't affected by the wind, but the horizontal distance traveled is.

time seconds	horiz dist 0 wind	horiz dist 20 kt HW	height - feet
0	0	0	0
10	2531	2193	61
20	5062	4387	121
30	7593	6580	182
40	10124	8774	243
50	12655	10967	304

Table 23-2

The zero wind horizontal distance comes from:

$$distance = airspeed\ (knots) \times time\ (sec) \times 1.6878 \times \cos(atan(0.024))$$

The last term in the equation is simply a correction for the fact that the true airspeed is along the flight path, not over the ground; it converts it to a true groundspeed, knowing that the climb angle is a 2.4 percent gradient. (In this example, the correction is less than 0.05 knots, hardly worth bothering.)

Graphing the data in the table above yields the plot shown to the right, comparing the vertical profile with no wind to that with a 20 knot headwind.

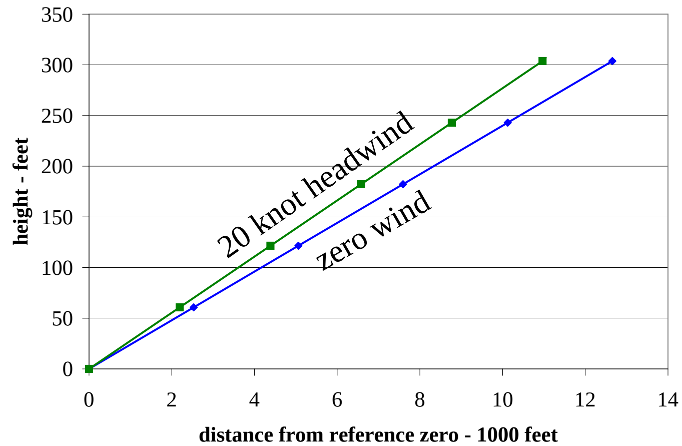


Figure 23-8

Cold Weather Effects on Obstacle Analysis

At this point, before we start looking in detail at an airplane's vertical takeoff profiles, it's appropriate to remind you of the effect of cold temperatures on altimetry, as discussed in the chapter entitled "Measurement of Altitude".

Weather colder than ISA causes an airplane's altimeters, which display pressure height, to show altitudes that are greater than the airplane's true altitude – a potentially dangerous condition. Thus, when designing and publishing obstacle clearance procedures for runways that may experience cold-weather conditions the possible effects of altimeter error must be considered.

The obstacle heights that we use to solve obstacle clearance problems are actual measured heights. The obstacle clearance charts in the Airplane Flight Manual, and the takeoff analysis software, all use true heights when performing the analysis of an obstacle limited takeoff. But any height data published to the flight crews relating to an obstacle clearance takeoff procedure can be in error if altimetry errors are not considered, because the pilots have no knowledge in the cockpit of their true height – they know only what they see on their altimeters, which is pressure height.

It's therefore essential, when conducting cold-weather takeoffs at obstacle limited airports, that the pilots must be provided with enough data that they may know the pressure heights that are important to them, such as obstacle avoidance leveloff heights. Thus:

- either the pilots should be provided with obstacle avoidance data that is written in units of pressure height, OR
- the pilots should be provided with obstacle avoidance data written in units of true height, AND provided with the necessary altimeter correction data such as that shown in the chapter entitled "Measurement of Altitude".

Performance engineers solving obstacle clearance problems using the AFM or Boeing takeoff analysis software applications should be aware of that source's treatment of cold-weather altimetry effects:

- The Airplane Flight Manual charts solve obstacle problems in terms of geometric (i.e. actual) height. The AFM provides an additional chart that allows the user to convert geometric height to pressure (i.e. altimeter) height or to create a table of altimeter error as a function of temperature;
- AFM-DPI, and the Boeing Performance Software application (BPS), generate outputs expressed only in units of geometric height. For pilot use, persons using AFM-DPI or BPS for obstacle clearance solutions must convert those solutions to altimeter heights or provide altimeter error data to the pilots;
- Persons using SCAP-compliant software applications[1] such as Boeing's STAS for any Boeing model can select to generate either geometric height data or pressure (altimeter) height data.

In the last few years we have seen the introduction of various types of performance computers into the cockpit. The first of these was the Boeing Laptop Tool, now called the Onboard Performance Tool (OPT). "Electronic flight bag" applications and the like are increasingly attractive as the size and capability of flight-capable computers improve.

The OPT is able to output obstacle clearance data in terms of either true or pressure heights. Clearly, if it is to be used by pilots in real-time obstacle clearance solutions in the cockpit, it should be set to display its information in pressure heights.

---

1. SCAP is the *Standard Computerized Airplane Performance Interface Specification - Takeoff* published by the International Air Transport Association (IATA). It establishes a set of protocols for computerized takeoff weight analysis software that may be adopted by manufacturers and airlines to facilitate consistency between the data and software provided by different manufacturers or vendors, making it more understandable and usable.

A single SCAP-compliant software package such as Boeing's Standard Takeoff Analysis Software (STAS) can be used to analyze the data for any manufacturer's airplane, provided that the manufacturer's takeoff analysis methods are also SCAP-compliant

The SCAP specification also enables users to write computer code for printing takeoff analysis outputs in their own preferred formats if desired.

The Vertical Profile of the Takeoff Flight Path

Shown below are the net and gross takeoff flight path, in this case for a 777, for a given set of takeoff conditions. Obviously, the paths will be different if the weight or any of the other conditions change.

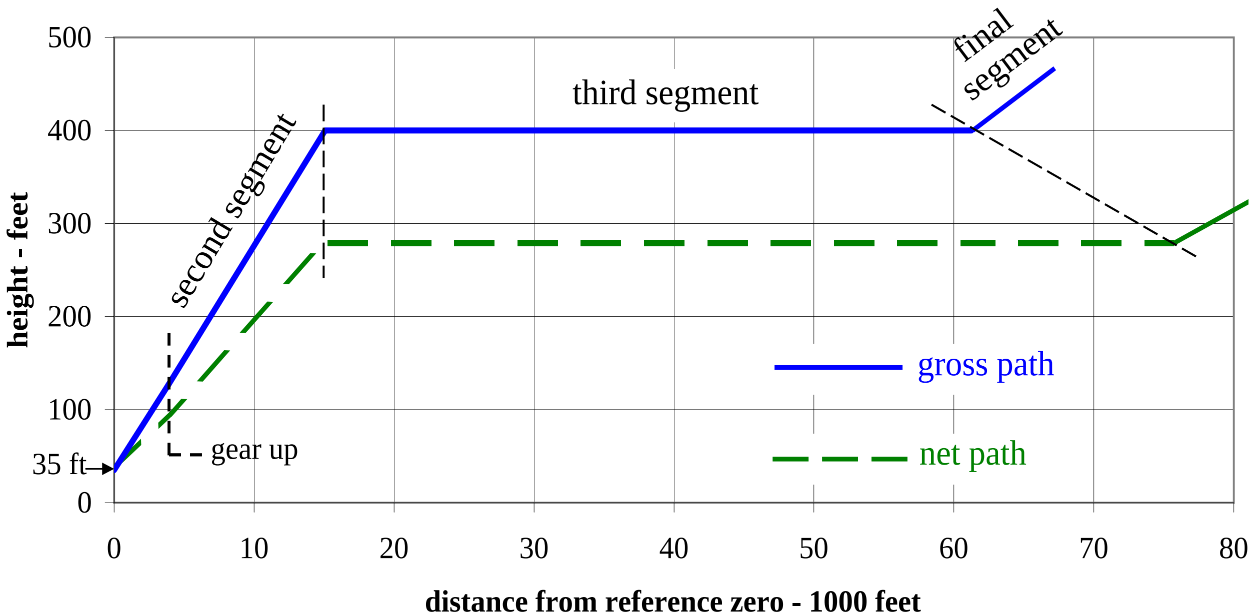


Figure 23-9

Before we look at the illustration in greater detail, allow us to emphasize a point that's not always clearly understood. The illustration above, as well as illustrations from other sources, might possibly mislead a reader into believing that segments of climb such as the second segment above are straight lines – that is, constant gradient. That isn't true for several reasons: the thrust is decreasing throughout the climb, and the lift and drag are also affected by changes in the true airspeed with increasing height. As a result, the climb gradient is always decreasing as the airplane's height increases. Over a short height gain of only 400 feet as shown above, the change is very slight. Later in the chapter, though, you'll see some illustrations which show how much a climb profile is affected by altitude.

(As a quick example to illustrate the magnitude of the effect: looking at a typical 747 AFM, we can find that if the second segment gradient were five percent at 400 feet, it would have decreased to only four percent by the time it reached a height of 2000 feet. Your calculation methods must account for this deterioration of climb capability with increasing height. Failure to do so will lead to unconservative results.)

gross and net path

In the illustration above you see the two profiles marked "gross path" and "net path". The upper line is the gross path, shown as a solid line, and the lower line is the net path, shown as a broken line. An airplane flying along the gross path will be at an altitude known as the *gross altitude* and an airplane flying along the net path will be at an altitude known as the *net altitude*.

Notice that we begin both takeoff flight paths at same point at the end of the takeoff – that is, at the point where the airplane reaches a height of 35 feet above the takeoff surface. From the earlier chapter entitled “Climb Limit Takeoff Weight”, you’ll recall that from 35 feet to the point marked “gear up” is the first segment; from the gear up point to the beginning of the level flight segments is the second segment; the level flight segments are the third segment, and after the end of the level flight will be the final segment.

gross and net height

The difference between net and gross heights can be calculated. Up to the beginning of third segment, it’s very simple. Think about it: if the difference in gradient beginning at 35 feet is (in this example of a two engine airplane) 0.8%, then the difference between gross and net height, for the same distance traveled, is simply:

$$net\ height\ =\ gross\ height - 0.008 \times D$$

where D is the distance from reference zero.

Look, for example, at the beginning of third segment in the illustration above. The third segments begin at a distance of 15073 feet from reference zero, and the gross height at level-off is 400 feet. You could calculate then that the net height at the beginning of third segment would be

$$400 - 0.008 \times 15073\ =\ 279.4\ feet$$

You’ll observe that this agrees with the net level-off height shown in the illustration.

This example illustrates the conservatism introduced by the requirement to use net performance for obstacle clearance. In the example, the difference between gross and net height is 120.6 feet. The difference between gross and net will of course be less when closer to reference zero, greater when farther from reference zero.

net and gross third segment distances

You see in the illustration that the net path third segment distance is greater than the gross path third segment distance. That’s logical, isn’t it? You know from the preceding chapter that surplus thrust can be used for climb or for acceleration. When an airplane levels off in order to accelerate (in other words, when it begins the third segment) then the amount of acceleration available depends on the gradient that was available. A higher climb gradient capability yields faster acceleration in level flight.

Since the net climb gradient is less than the gross gradient by a specified margin, then it follows that the acceleration in the net path third segment will be less than the acceleration in the gross path third segment.

During third segment, the object of leveling the airplane’s path is to accelerate from V_2 to the final climb speed. That change of speed will be the same in both the gross or net paths. Thus, since the net third segment acceleration is less, the time and distance to accelerate in third segment will be greater in the net path than it is in the gross path.

obstacle height

Remember that "net height" is an artificial thing – it's a conservatism ordered by the regulatory agencies. The airplane is actually considered to follow the gross flight path, but for the purposes of calculating obstacle clearance, we are required to use the net flight path, in order to allow for reasonable operational variation in weight, thrust, and so on.

Now, let's add one more height to this discussion: the obstacle height. Think of it as the maximum height of an obstacle that the airplane can clear with the legally required margin of safety as it proceeds along its departure path. You know that the net flight path must clear obstacles by 35 feet (50 feet under JAR-OPS in a banked turn of 15 degrees or greater, but let's forget about that one for now) and therefore the maximum obstacle height along the path is equal to net height minus 35 feet.

With this discussion in mind, let's re-draw the previous illustration:

What you have now is a practical obstacle clearance chart: it shows you the height of the obstacles it can clear as a function of distance from the end of the takeoff. (Of course, this chart is valid for only one altitude-temperature-flap-weight combination.)

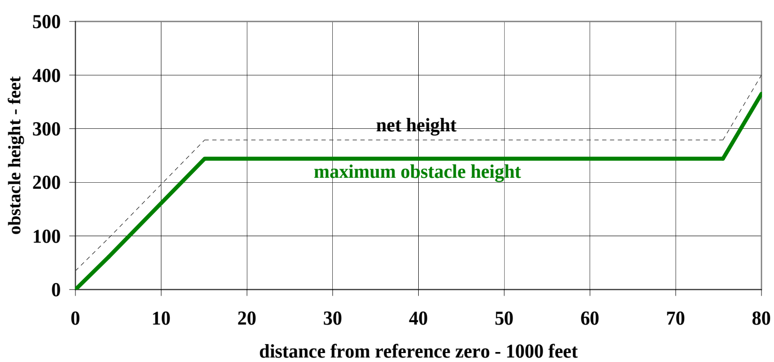


Figure 23-10

A number of profile options are available to the performance engineer when calculating obstacle limit takeoff weights. Let's explore some of those options.

varying the third segment height

In the profile shown above, the third segment height (level-off height) was set at 400 feet. It can, however, be flown at any desired height, within certain restrictions:

- the minimum level-off height is set by the regulations at 400 feet (FAR 25.111(c)(2);
- the maximum level-off height (with one exception that will be discussed later under the heading of "extended second segment climb") is the height at which the airplane may transition to level flight, accelerate and retract flaps, and achieve the final climb speed in the clean configuration within the time limit to which the takeoff thrust setting is certified.

The first of these two restrictions is arbitrary, based on considerations of safety when maneuvering at relatively low altitudes. Let's make this absolutely clear: the 400-foot restriction is a gross height, which is the same thing as what the pilots see on their altimeters. The minimum net height will be less, and the minimum obstacle height will be another 35 feet below the net height.

The second restriction is a performance limitation. You'll recall from the chapter entitled "Thrust Ratings" that the takeoff thrust rating for all Boeing models, with the exception of the 787, is restricted to a maximum of five minutes[1]. It is assumed that the third segment acceleration and flap retraction will be accomplished at the takeoff thrust rating. The maximum third segment height, then, is that height to which the airplane can climb after takeoff and then accomplish its acceleration and flap retraction in exactly five minutes from the time of advancing the thrust levers for takeoff. At the end of the third segment, then, the thrust is reduced to the maximum continuous thrust rating.

The illustration to the right shows the two extremes of third segment height. The maximum third segment height in this example lies at an obstacle height of 1535 feet – obviously, it will occur at a different height for any other weight-altitude-temperature-flap combination.

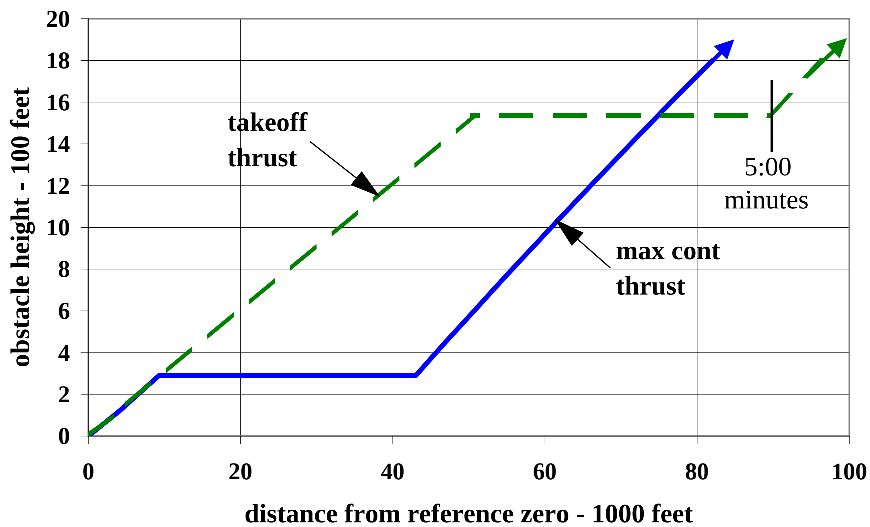


Figure 23-11

The solid line shows the climb profile with the third segment gross height at its minimum of 400 feet. (Be reminded here that the obstacle height is the net height minus 35 feet, and that the level-off gross height of 400 feet in this example is a net height of 326 feet, an obstacle height of 291 feet).

The broken line shows the profile with its maximum level-off height of 1535 feet obstacle height. Note that the end of the third segment at the maximum level-off height coincides with the five minute takeoff thrust limit.

Notice, by the way, that for these example conditions the final segment climb gradient is better than the second segment gradient, even though the thrust is less (since the second segment is flown at takeoff thrust and final segment is flown at maximum continuous thrust). Remember that climb gradient depends not only on the thrust available but also on the drag-to-lift ratio. In this case, the drag-to-lift advantage at flaps up is greater than the loss of thrust at MCT, so the gradient

1. This five minute limit becomes a ten minute limit for operators who have purchased the "Ten-Minute Takeoff Thrust" appendix for their Airplane Flight Manual. For AFM-DPI airplanes, the ten-minute takeoff thrust option is called an "Alternate Performance" More about that later in this chapter.

is better in final climb. As you'll soon see, there are times when you can take advantage of final segment's superior climb capability.

Let's continue the 777 example from above and use it to help us visualize several different types of obstacle problems.

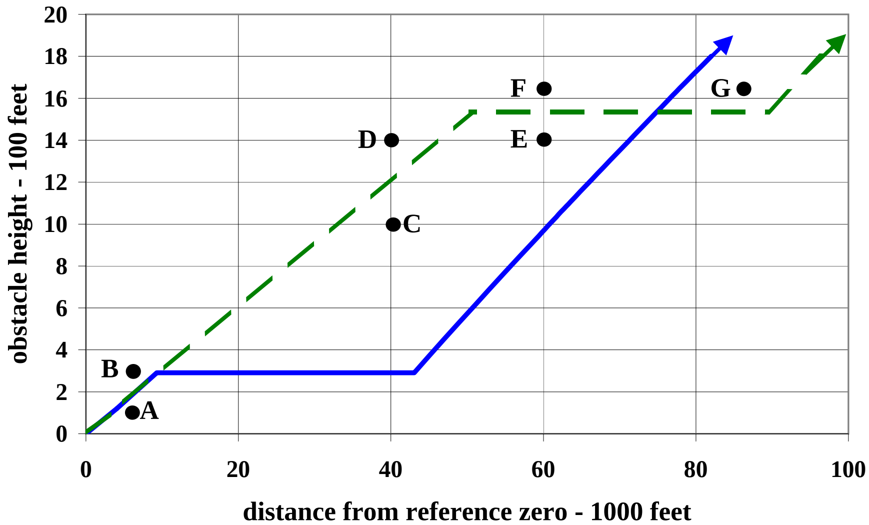


Figure 23-12

Let's say that we have calculated the other weight limits for this airplane on the given runway under the given takeoff conditions. Let's say that the takeoff weight is limited by runway length. Finally, let's say that this illustration shows the profiles for the minimum and maximum third segment heights for the airplane <u>at that field length limit weight</u>. In other words, this illustration shows the takeoff profiles at the maximum possible takeoff weight before obstacles are considered.

<u>second segment obstacles</u>

Now suppose, for example, that there's a close-in obstacle having a height of 100 feet at a distance of 6000 feet from the end of the takeoff. This is shown as point A in the illustration. This is called a *second segment obstacle*. Does this takeoff profile successfully clear obstacle A, or do we need to reduce the takeoff weight? For obstacle A, clearly no weight reduction is necessary since the profile is above the obstacle.

What if the obstacle is at 6000 feet but has a height of 300 feet, shown as point B on the diagram. Here's a problem: the only way to clear this obstacle would be to reduce the takeoff weight. The takeoff weight would then be <u>obstacle</u> limited, rather than <u>field length</u> limited.

How about obstacle C, 1000 feet high at 40,000 feet from reference zero? No problem – obstacle C doesn't necessitate a weight reduction, but it <u>does</u> require delaying the level-off until at least a gross height of 1355 feet (1355 = 1000 obstacle height + 35 feet clearance + 0.008*40000 difference between net and gross heights).

Obstacle D? At that obstacle height, a weight reduction will be necessary.

<u>third segment obstacles</u>

Now let's look at obstacles that lie at a greater distance from reference zero. Look, for example, at points E and F. These are called *third segment obstacles*. Point E is no problem since it lies below the profile. Point F, however, will require a weight reduction – unless we can invoke the use of *extended second segment*, which we'll discuss in just a moment.

final segment obstacles

Look at point G. This is called a *final segment obstacle*. Can the airplane clear this obstacle without needing a weight reduction? The answer is no if you consider it to be a third segment obstacle. But the answer is yes if you consider it to be a final segment obstacle, and base the takeoff departure procedure on leveling off for the third segment at the minimum value. In this instance, early level-off is advantageous because it allows the profile to clear the obstacle in the final segment, whereas to clear it in third segment would require a weight reduction.

The relationship between second segment gradient and final segment gradient is complex, depending on the flap setting, thrust, and other variables. From what you have seen in this example, an obstacle at point G favors early flap retraction and then obstacle clearance in final segment. This is not always the case, but when a distant obstacle is limiting it's worth looking at the possibility of early flap retraction followed by final segment climbout to see whether or not it offers a better obstacle limited takeoff weight.

Because some obstacles are most efficiently cleared using an early level-off procedure, but other obstacles are best cleared using a late level-off procedure, one of the most complex challenges to the performance engineer in calculating obstacle limit weights is the airport having a mix of obstacle types. Optimizing the departure procedure in such a case can be quite complex.

extended second segment climb

Here is another option that can be used by the performance engineer when solving obstacle problems. This option, however, isn't always available. Let's talk about that.

You'll recall that we normally retract flaps and accelerate to the final climb speed in the third segment, at maximum takeoff thrust. Under certain conditions, we may be allowed to level off, retract flaps and accelerate at maximum continuous thrust, instead of takeoff thrust.

FAR Section 25.111 makes a very specific requirement:

> *(c) During the takeoff path determination in accordance with paragraphs (a) and (b) of this section--*
> *...(3) At each point along the takeoff path, starting at the point at which the airplane reaches 400 feet above the takeoff surface, the available gradient of climb may not be less than--*
> *(i) 1.2 percent for two-engine airplanes;*
> *(ii) 1.5 percent for three-engine airplanes; and*
> *[(iii) 1.7 percent for four-engine airplanes.*

Let's emphasize that: at each point along the takeoff path, above 400 feet, the airplane must be capable of a specified minimum gradient. This is not related to the climb limit weights that we discussed in the preceding chapter. This is a different requirement, as it applies at all points along the takeoff path above 400 feet. That includes the portion of the takeoff path during which the flaps are being retracted.

Note especially the words "...available gradient of climb may not be less than..." When the airplane is in third segment, even though the flight path is level the airplane has some available gradient of climb, even though the available gradient is being used for acceleration rather than climb.

During flap retraction, the drag and lift of the wing are changing. At some point during the flap retraction process, the airplane's drag-over-lift will be at its greatest value, meaning that the available gradient will be at its minimum.

Suppose that you could show that even at the worst drag-over-lift condition with one engine inoperative and the remaining engine(s) at maximum continuous thrust you could meet the requirements of 25.111(c)(3)? In that case, it would be permissible for you to do your flap retraction flight segment at maximum continuous thrust, after the five minute limit on takeoff thrust has expired.

This is referred to as "extended second segment climb". It means simply that the airplane will maintain second segment climb until the takeoff thrust rating time limit is reached, and only then will it level off for flap retraction and acceleration with the operative engine(s) set to MCT. But remember: extended second segment climb is not always available as an option. It depends on whether or not you can satisfy the requirement of 25.111(c).

Shown to the right are the climb profiles for the same airplane as before, at the same weight.

The solid line shows the profile with level-off at the maximum value, as you saw in the previous illustration. The five minute takeoff thrust time limit occurs at the end of the third segment.

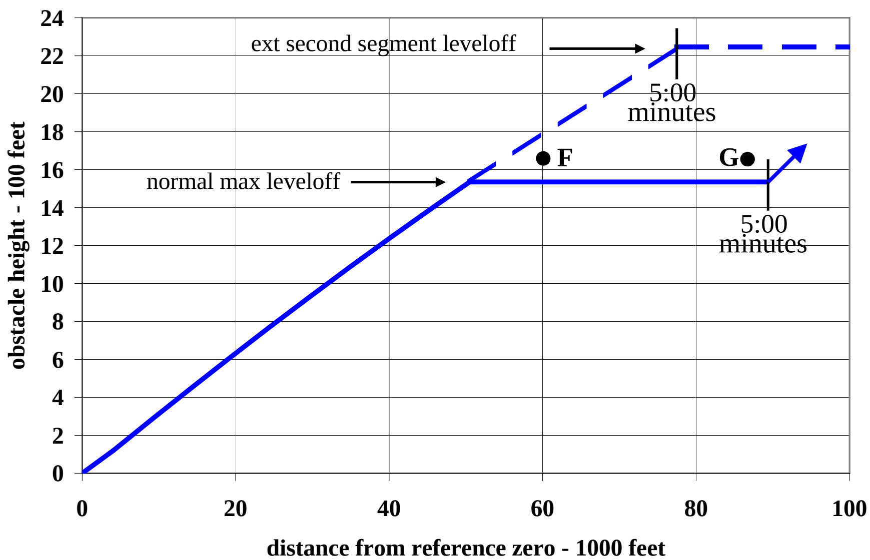


Figure 23-13

With the broken line, we have added the extended second segment, which was available under these conditions. Now, the five minute point occurs at the beginning of the level segment.

You can see that the use of extended second segment climb has, in this example, added another 710 feet to the level-off height. Repeating obstacles at points F and G from the previous illustration you see that extended second segment climb allows us to clear those obstacles without needing any weight reduction.

BE AWARE: the advantage of using extended second segment should be clear from the diagram above for certain types of obstacles such as F and G in the diagram, called “third segment obstacles”. However, you must always keep in mind that extended second segment is not always available – it depends on the airplane weight and other factors.

To illustrate what we mean, let’s give an example: let’s say that you have made an initial takeoff analysis, and it shows that you have some obstacles in the third segment that your net path can’t clear by 35 feet. The initial analysis shows also that, for the given conditions, extended second segment is not possible because you’re too heavy to satisfy 25.111(c).

What to do? Well, you don’t have any choice, do you? You have to reduce the takeoff weight.

As you reduce the takeoff weight from your initial value, two things happen: (a) the maximum leveloff height increases; (b) you improve your gradient capability and approach the weight at which extended segment climb will be permitted.

So you can find a weight at which either (a) the new maximum leveloff altitude clears the obstacles, or (b) at which extended second segment climb becomes possible. The weight at which the first of those two occurs will be the correct solution to your problem.

extended second segment: AFM and AFM-DPI

Just above, we quoted FAR 25.111(c) and showed that it permits us to extend the second segment climb segment beyond the usual maximum leveloff height, provided that compliance with its stated minimum gradient could be demonstrated during flap retraction at maximum continuous thrust. This extension of the second segment gradient might, depending on obstacle heights and distances, offer higher obstacle-limited weights.

In the AFM, a simple check is provided to assist the user in determining whether or not 25.111(c) is satisfied at the given takeoff conditions. For example: on the distant obstacle clearance chart of the 747-400, there is a note stating:

> *Extended second segment and MCT cleanup may be used when final segment gross gradient (1500 feet, zero wind) exceeds percent shown for the appropriate flap position...*

The “percent shown” in the quotation above refers to a little table that shows for a flaps 10 takeoff the final takeoff climb gradient must be equal to or greater than 4.10%, or 4.70% for a flaps 20 takeoff.

This means simply that if, for the given takeoff conditions, the final climb gradient at 1500 feet uncorrected for wind is at least equal to the minimum values given, you will know that the requirement of 25.111(c) is satisfied and extended segment may therefore be used if desired. You only need to refer to the AFM final climb gradient chart to find the gradient for the given conditions, and check that against the requirement of 25.111(c).

If, on the other hand, the calculated final segment gradient for the given takeoff conditions is less than the stated minima, extended second segment is not permitted.

Only in this context does the requirement of 25.111(c) appear in the AFM.

In AFM-DPI the software treats the regulation differently. Since it is conducting a first-principles calculation, it is able to check the calculated flight path against the requirement of 25.111(c) at all points during every takeoff calculation. The user is allowed to select a performance option directing AFM-DPI to use extended second segment if it's available.

ten minute takeoff thrust appendix

In general, Boeing airplanes are certified for five minutes of takeoff thrust. For the 787 airplane, ten minutes of takeoff thrust will be standard. For most other Boeing models, however, an Airplane Flight Manual appendix called the "ten-minute takeoff thrust" appendix is available. For AFM-DPI airplanes, the data is referred to as an Alternate Performance option.

This appendix or Alternate Performance provides the legal authorization to operate the engines to ten minutes of takeoff thrust rather than the usual five minutes.

Section 1 of the appendix, under "General Limitations" states:

> *The time limit on the use of takeoff thrust is increased to 10 minutes provided this use is limited to situations where an engine failure actually occurs and there is an obstacle in the takeoff flight path.*

The appendix or Alternate Performance option is available only as a purchase item, with the exception of the 787. There are relatively few airports having distant obstacles that will benefit from the ten minute appendix, but for those airports the appendix may well be worth the cost since it would allow substantially higher takeoff weights.

The appendix serves simply to extend the second and third segments to greater heights. In the illustration to the right you can compare the five minute limit with the ten minute limit for a typical airplane.

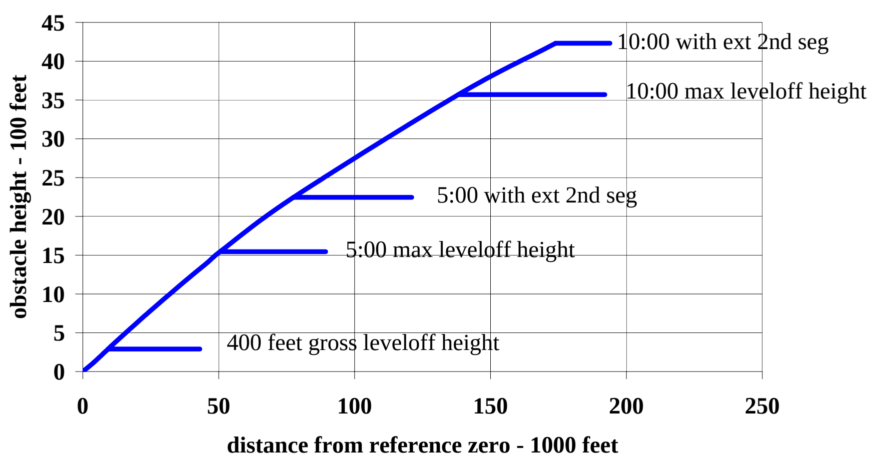


Figure 23-14

comparing 10-minutes takeoff thrust with final segment climb

Clearing a distant obstacle in final climb can be a powerful option, particularly if early level-off is possible. For a given altitude, temperature, weight and flap setting, the final climb gradient is substantially greater than the second segment gradient.

If leveling off at 400 feet, for example, for the conditions we've been using for the examples shown above the gross second segment gradient is 3.6% and the gross final segment gradient is 4.5%, a substantial increase.

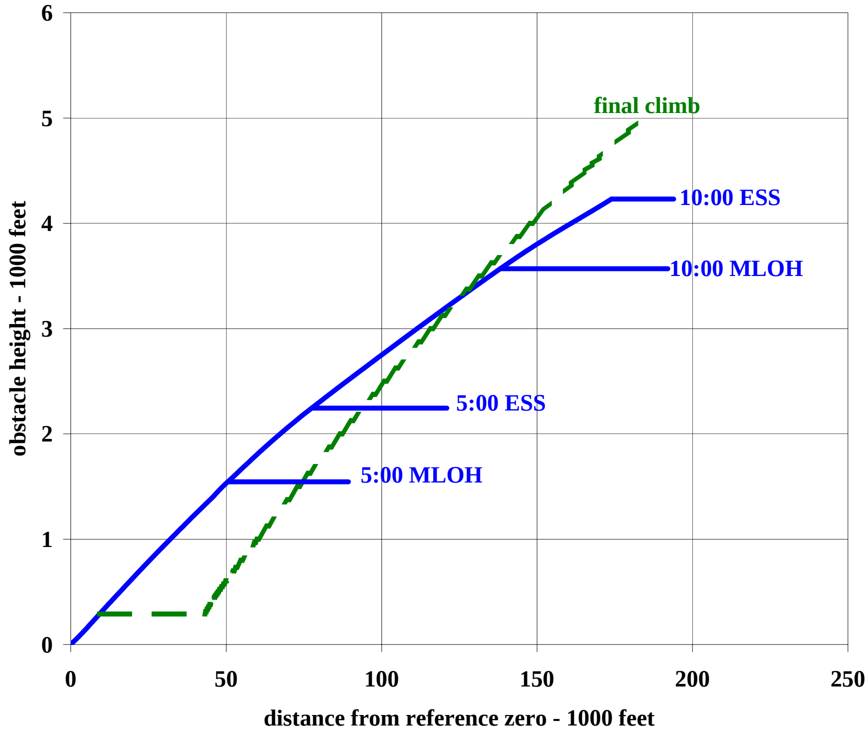


Figure 23-15

The solid line in this illustration represents climb with takeoff thrust up to the five minute maximum level-off height (MLOH) and the five minute extended second segment limit (ESS) altitudes as well as the ten minute MLOH and ESS altitudes.

The broken line represents level-off, acceleration and flap retraction at the minimum allowable gross height of 400 feet followed by climb at maximum continuous thrust.

If airline policy does not permit level-off at an altitude as low as 400 feet, or if closer obstacles make early level-off impossible, then the ten minute takeoff thrust option becomes increasingly attractive. For more distant obstacles, both options should be examined to see which offers greater benefits.

Flap Selection For Obstacle Clearance

Consider what you already know about takeoff distances and climb gradients for different takeoff flap settings.

You know that smaller flap settings require longer takeoff distances than larger flap settings at the same weight, but offer superior climb capability as shown by the broken line in the illustration. You know also that larger flap settings need less takeoff distance than smaller flap settings at the same weight, but have less climb capability as shown by the solid line.

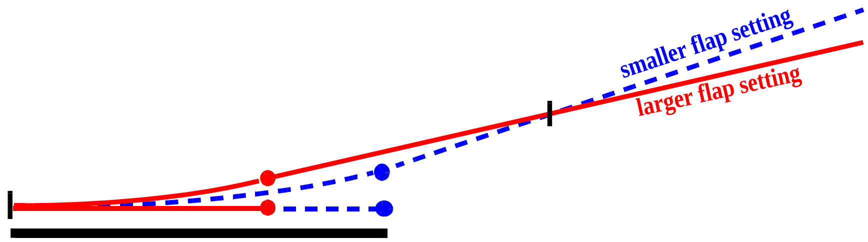


Figure 23-16

What does this suggest? That obstacles farther from the end of the runway may favor a smaller takeoff flap setting, and that obstacles closer to the end of the runway may favor a larger takeoff flap setting.

Thus, whenever you find that your takeoff weight is obstacle limited, you should consider the use of a different flap setting. It may help, it may not.

Two Additional Obstacle Clearance Options

There are two optional techniques available that may – or may not – offer higher obstacle limit takeoff weights. As the pilots are required to follow slightly different takeoff procedures than normal when using these techniques, it may be necessary to educate them about the procedures and their potential benefits, demonstrate to them the simplicity and safety of using them, and seek their endorsement of the new procedures.

Actually, both of these special procedures are very simple, requiring only small changes to the takeoff speeds, but the pilots must be able to determine these new speeds simply and accurately. Making this possible is the job of the performance engineer.

takeoff speeds and takeoff distances

To understand these two techniques, let's first briefly review the subject of takeoff speeds and their effect on the takeoff distances.

In the chapters about takeoff distances, we discussed the effect of the V_1 speed on the accelerate-stop and accelerate-go distances. You saw that increasing V_1 while holding the weight constant has the effect of increasing the accelerate-stop distance but decreasing the accelerate-go distance, whereas decreasing the V_1 would reduce the accelerate-stop distance but increase the accelerate-go distance.

In the same chapter we discussed balanced and unbalanced takeoff distances, learning that clearway and stopway can offer takeoff weight benefits but that the V_1 speed for the takeoff needs to be exactly suited to that combination of distances.

You saw that when the airplane is taking off at its field length limit weight, the V_1 speed associated with that takeoff is the only acceptable value of V_1. You saw that increasing V_1 above its proper value could cause an over-run in the case of a rejected takeoff, and decreasing the V_1 below its proper value could cause the airplane to be lower than the required 35 feet at the end of the runway. Both of these possibilities are potentially hazardous.

Finally, you learned that the takeoff V_R and V_2 speeds are functions of the takeoff weight, the altitude and temperature, and the flap setting.

when limited by obstacles

Now let's say that the airport being analyzed has an obstacle that can not be cleared when taking off at the field length limit weight. The only option then is to decrease the takeoff weight.

That new decreased weight means new reduced takeoff speeds. It also means that the new takeoff accelerate-go and accelerate-stop distances will be less than the distances that are available.

These two facts give us an option – at the new reduced weight we could revise the takeoff speeds if we wanted to, without exceeding the available distances. Fair enough, but why change the speeds?

increasing V_1 for takeoff

Think back to the chapter entitled "Field Length Limit Takeoff Weight", where we introduced you to the web chart. Here's another web chart for you, for a 777 taking off on a 10,000 foot runway having no clearway or stopway.

The computer says that the field length limit weight would be 721,885 pounds with a V_1 of 168.6 knots. You can see that the web chart agrees with those numbers.

But let's say that this runway has an obstacle that is 150 feet high, 5000 feet from the liftoff end of the runway. Then the computer tells me that if I'm using a balanced V_1 the obstacle limit weight would be 712,018

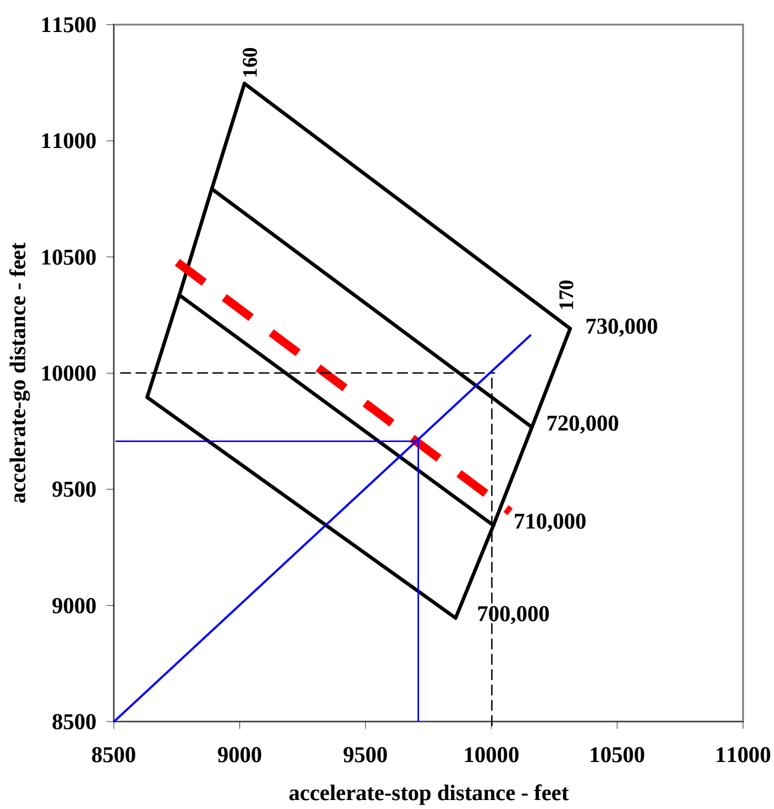


Figure 23-17

with a V_1 of 167.7 knots. On the chart, that weight is shown by the heavy dashed line. The accelerate-go and accelerate-stop distances will both be 9695 feet.

Thus: at a weight of 712,018 pounds with a balanced V_1 of 167.7 knots, the airplane will just exactly clear the obstacle with the required margin of 35 feet below the net flight path.

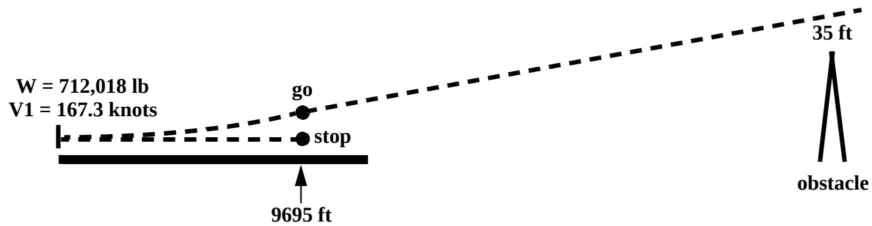


Figure 23-18

The web chart shows that, because the obstacle is limiting the weight to less than the field length limit value, there is now a range of V_1 that's available. The minimum V_1 would be 164.3 knots, and the maximum V_1 would be 169.7 knots.

What would happen if we chose to make the takeoff at the maximum V_1 of 169.7? You know that increasing the V_1 increases the accelerate-stop distance but <u>decreases</u> the accelerate-go distance.

Keeping the weight at 712,018 pounds but using the <u>maximum</u> V_1 of 169.7 knots has the effect shown in the illustration. We now have an unbalanced takeoff. Reference zero has moved almost 240 feet to the left. The accelerate-stop distance is now 10,000 feet, the full length of the runway, since we're using the maximum V_1. Since the weight is unchanged, the <u>climb angle</u> is unchanged and as a result the vertical profile now clears the obstacle by 42 feet.

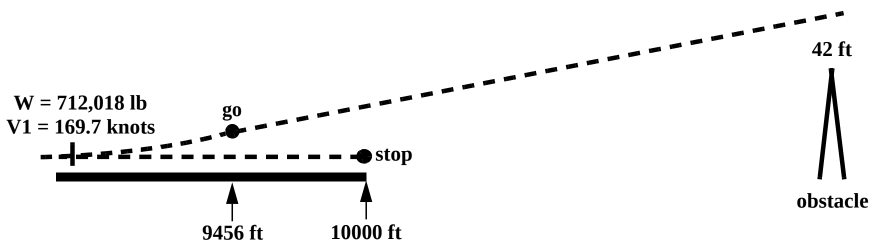


Figure 23-19

Since we're now clearing the obstacle by more than the required 35 feet, we could increase the weight, couldn't we?

Yes. You could increase the weight to 714,914 pounds, with its maximum V_1 of 169.4 knots; this will return your vertical profile to an obstacle clearance of 35 feet.

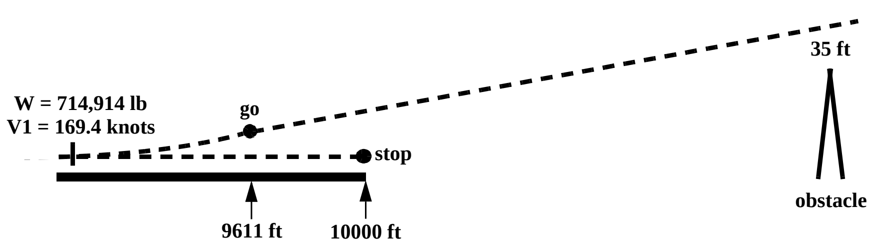


Figure 23-20

Figure 23-21

You have increased your allowable takeoff weight by 2900 pounds without any complex takeoff procedures. (Of course, the pilots will laugh when you tell them to use a V_1 of 169.4 knots, so we'd suggest you round it down to 169 in this example.)

What we've shown you here as an optional technique for improved obstacle clearance weights does mean that the V_1 speeds used for takeoff will be somewhat faster than usual, with the calculated accelerate-stop distance equal to the full runway length. But that doesn't mean that a rejected takeoff begun at a maximum V_1 speed will finish with the nose landing gear exactly at the end of the runway. Remember that there are some conservatisms in the way that the accelerate-stop distance is calculated – principal among which is the fact that no credit is taken for reverse thrust.

improved climb for obstacle clearance

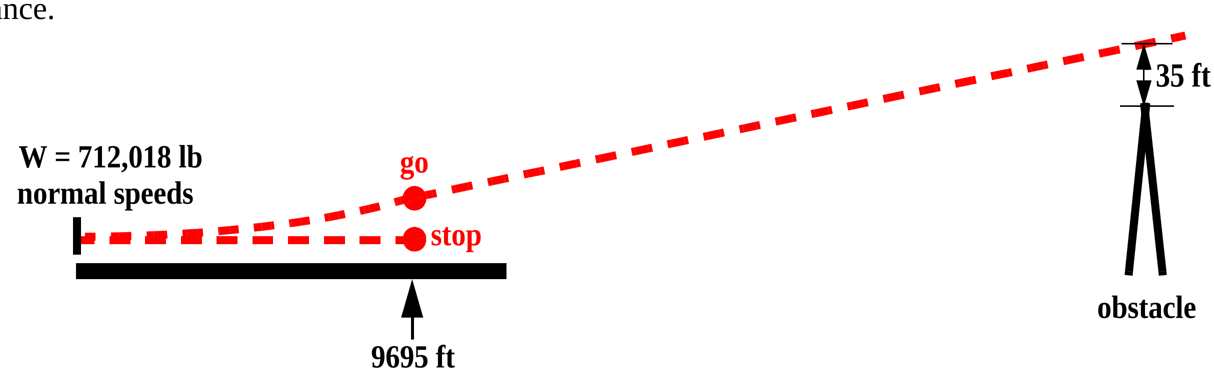
In an upcoming chapter, you'll be learning all about an optional takeoff technique known as *improved climb* that can be used for takeoff. Since we'll be showing you much more about improved climb technique in that chapter, we'll only mention it here briefly in the context of obstacle clearance. This will also be discussed in greater length in that chapter.

For now, all we're going to say is this: if the pilots will increase their takeoff speeds V_R and V_2 above their standard values, a better climb gradient will be achieved, and this fact can be utilized to advantage – sometimes – when calculating obstacle clearance. So ask yourself: when can the pilots increase their takeoff speeds above the normal values?

You know that the takeoff speeds depend on the airplane's weight, not the length of the runway. What if the airplane's takeoff weight is restricted by an obstacle to some value less than the runway limit takeoff weight? Well, then, some portion of the liftoff end of the runway isn't being used, and we could increase the takeoff speeds such that the takeoff distances would increase until they use the entire runway length.

And if the pilots utilize increased takeoff speeds, they'll get a better climb gradient. Will this offer an advantage when limited by obstacles? Sometimes yes, sometimes no.

Let's go back to the illustration we used above when talking about increased V_1 for obstacle clearance.



Now let's increase the takeoff speeds, but not the weight, so as to use the entire runway length:

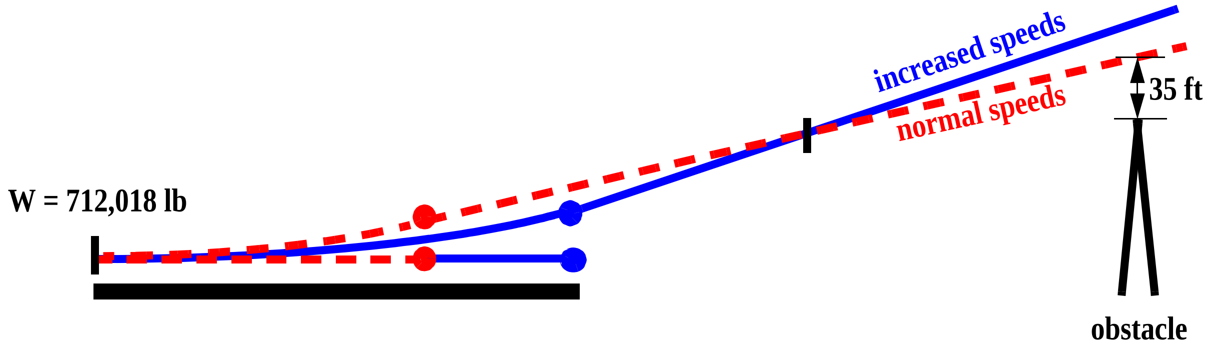


Figure 23-22

What have we achieved by increasing the takeoff speeds without increasing the weight? We've increased the climb gradient, so the vertical profile with increased speeds is now above the profile with normal speeds at the given obstacle distance. We now clear the obstacle by more than the required margin.

Clearly, then, there's some value of weight, greater than the original weight, which together with some speed increase, will just exactly clear the obstacle by the required margin.

But what if the obstacle were closer to the end of the runway? Then, depending on the distance, the vertical profile with increased speeds could be below the profile with normal speeds – and there would be no advantage to increasing the speeds.

Summarizing: the improved climb technique may or may not offer better obstacle limit takeoff weights, depending on the distance to the obstacle.

Turning Flight and Obstacle Clearance

The two topics of turning flight and obstacle clearance are sometimes necessarily taken together for any one of a number of reasons, including:

- A turn following takeoff may allow the takeoff path to avoid obstacles altogether;
- A turn following takeoff may allow the takeoff path to pass over lower obstacles hence allowing better takeoff weights than a straight-out departure;
- A turn may be required after takeoff for reasons other than obstacle clearance – for example, avoidance of a noise-sensitive area – and obstacles may lie within the turning obstacle accountability area.

Everything you have seen in this chapter up to this point has been presented in the context of a straight-out departure. However, all of the techniques and principles you've learned so far are equally applicable to a turning departure. When a turn is designed into a departure procedure, however, there are some additional factors that must be considered.

So let's begin by talking about the physics of turning flight, and then we'll show you how turns will affect your obstacle clearance calculations.

Physics of Turning Flight

NOTE: The following discussion of the physics of turning flight is based on the classical mathematical solutions for turn radius, rate of turn, and bank angle.

Thanks to the increasing ability of airplane flight simulators to model precisely the actual flight behavior of an airplane, Boeing has recently become aware of the fact that while the classical equations are valid for coordinated turns conducted with all engines operating, they do not correctly predict the radius of turn for a given bank angle, or the bank angle required to produce a specific radius of turn, when an engine is inoperative.

For example, we see that when turning toward the failed engine (e.g. turning left when the left engine is inoperative) the resulting turn radius is less than the value predicted by the classical equations. Turning away from the failed engine results in a radius that is greater than that predicted by the classical equations.

The precision of our airplane performance calculations improves with time and with advances in technology.

At the time of this writing, March 2009, we are not yet able to provide guidance in this document on methods for accurate calculation of engine-inoperative turn data. It will be provided when it becomes available.

radius of turn

The radius of the coordinated turn is a function of the airplane's true airspeed and its bank angle.

It can be shown, with the help of a little calculus, that when an object is moving in a circular path, there is an inward (centripetal) acceleration whose magnitude is predicted by:

$$\textit{centripetal acceleration} = \frac{V^2}{r}$$

where V is the true velocity
r is the radius of the turn

Since Newton's equation $f = ma$ applies here, the centripetal force required to produce that centripetal acceleration in a turn will be:

$$\textit{centripetal force} = \frac{mV^2}{r}$$

In the diagram to the right, we show an airplane in a turn with an angle of bank ϕ. If the weight of the airplane is *W*, then the force acting toward the center of the airplane's circular path will be a centripetal force whose magnitude is found from:

$$centripetal\ force = W\tan\phi$$

We can set those two equations for the force equal to each other:

$$\frac{mV^2}{r} = W\tan\phi$$

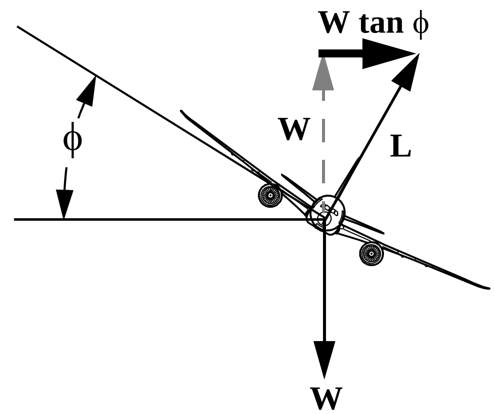


Figure 23-23

So:

$$r = \frac{mV^2}{W\tan\phi} \text{ and since } \frac{W}{m} = g \text{ then:}$$

$$r = \frac{V^2}{g\tan\phi} \qquad \text{(eq. 1)}$$

In this equation *V* must be in units of feet per second and *g* must be in units of feet per second per second if you want to find the radius in feet (meters per second and meters per second per second respectively for a radius in meters). For practical purposes, we can ignore the secondary effects of altitude, latitude and speed on gravitational acceleration and say that $g = 32.174$ feet per second per second. Then, for a value of *V* expressed in knots, which is usual in this sort of problem:

$$r = 0.08854\left(\frac{V^2}{\tan\phi}\right) \qquad \text{(eq. 2)}$$

where 0.08854 is equal to $1.6878^2 \div 32.174$
V is the airplane's true airspeed in knots
r is the radius of the turn in feet.

Be reminded here that the velocity *V* must be true airspeed rather than indicated or calibrated airspeed. To use calibrated airspeed on any day when the density ratio σ is not equal to unity will introduce an error.

At the airspeeds and altitudes that we're usually dealing with in obstacle clearance problems, it's acceptably accurate to say that $V_C = V_e$. That is, the compressibility error is equal to zero and thus calibrated airspeed is equal to equivalent airspeed. You know that the true airspeed can readily be found from:

$$V_{true} = \frac{V_e}{\sqrt{\sigma}} \quad \text{or} \quad V_{true} = V_e\sqrt{\frac{\theta}{\delta}}$$

rate of turn

It is sometimes convenient to know the rate of change of an airplane's heading in a turn.

As an airplane travels a small distance *ds* along a circular arc, it travels through a small angle change $d\theta$. That angle change is given by:

$$d\theta = \frac{ds}{r}$$

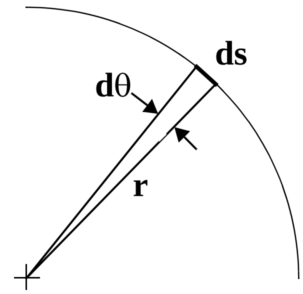


Figure 23-24

The small distance *ds* is equal to the airplane's true velocity multiplied by the small time interval *dt*:

$$ds = V_{true}\,dt$$

and thus:

$$d\theta = \frac{V_{true}dt}{r} \quad \text{so} \quad \frac{d\theta}{dt} = \frac{V_{true}}{r}$$

Since we know that $r = \dfrac{V_{true}^2}{g\tan\phi}$ then:

$$\frac{d\theta}{dt} = \frac{g\tan\phi}{V_{true}} \qquad \text{(eq. 3)}$$

where $\dfrac{d\theta}{dt}$ is the rate of turn in radians per second

g is the gravitational acceleration in feet per second per second

ϕ is the bank angle

V_{true} is the true airspeed in feet per second

For true airspeed in knots and rate of turn in degrees per second:

$$\frac{d\theta}{dt} = 1092.1 \times \frac{\tan\phi}{V_{true}} \qquad \text{(eq. 4)}$$

where $1092.1 = (57.3 \times 32.17405) \div 1.6878$

For example, an airplane flying at 150 knots true airspeed in a 15 degree bank will be turning at a rate of 1.95 degrees per second.

Instrument pilots are familiar with the term *standard rate turn*, particularly in relation to flying a holding pattern A holding pattern consists of two 180° turns connected by straight sectors; this pattern is called usually referred to as a *racetrack pattern* because of its resemblance to the tracks used for horse races. The turn at each end of the racetrack pattern is a standard rate turn, which is defined as a 360° degree turn in three minutes, or three degrees per second.

Should it be necessary to calculate the radius of a standard rate turn, combining equations 2 and 4 from above yields:

$$\textit{standard rate radius of turn} = 32.231 \times V_{true} \qquad \text{(eq. 5)}$$

where V_{true} is in knots

Regulatory Requirements

FAA

In Advisory Circular 120-91 the description of the obstacle analysis area for a turning departure reads as follows:

> ***b***. *During departures involving turns of the intended track or when the airplane heading is more than 15 degrees from the extended runway centerline heading, the following criteria apply:*
>
> ***(1)*** *The initial straight segment, if any, has the same width as a straight-out departure.*
>
> ***(2)*** *The width of the OAA at the beginning of the turning segment is the greater of:*
>
> ***(a)*** *300 feet on each side of the intended track.*
>
> ***(b)*** *The width of the OAA at the end of the initial straight segment, if there is one.*
>
> ***(c)*** *The width of the end of the immediately preceding segment, if there is one, analyzed by the Flight Track Analysis Method.*
>
> ***(3)*** *Thereafter in straight or turning segments, the width of the OAA increases by 0.125D feet on each side of the intended track (where D is the distance along the intended flightpath from the beginning of the first turning segment in feet), except when limited by the following maximum width:*

(4) The maximum width of the OAA is 3,000 feet on each side of the intended track.

This is shown graphically to the right.

Note in particular that in the case of the turning departure, the OAA increases in width at a rate double that for a straight-out departure: 8:1 instead of 16:1.

Notice also that the maximum OAA width for a turning departure is 3,000 feet rather than the 2,000 feet specified in a straight-out departure.

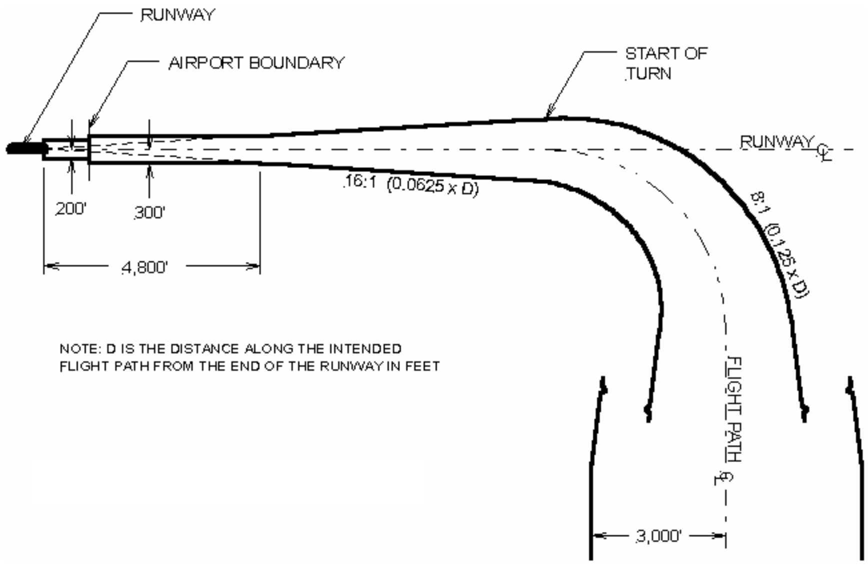


Figure 23-25

JAR-OPS

JAR-OPS 1.495, quoted above where it defines the straight-out departure, goes on to say for turning departures:

...(f) For the purposes of this section, it is assumed that the airplane is not banked before reaching a height of 50 feet, as shown by the...net takeoff net flight path data...in the Airplane Flight Manual, and thereafter that the maximum bank is not more than 15 degrees.

...(c) when showing compliance with subparagraph (a) above:

(1) Track changes shall not be allowed up to the point at which the net takeoff flight path has achieved a height equal to one half the wingspan but not less than 50 feet above the elevation of the end of the takeoff run available. Thereafter, up to a height of 400 ft it is assumed that the aeroplane is banked by no more than 15°. Above 400 ft height bank angles greater than 15° but not more than 25° may be scheduled;

(2) Any part of the net takeoff flight path in which the aeroplane is banked by more than 15° must clear all obstacles within the horizontal distances specified in sub-paragraphs (a), (d) and (e) of this paragraph by a vertical distance of at least 50 ft; and

(3) An operator must use special procedures, subject to the approval of the Authority, to apply increased bank angles of not more than 20° between 200 ft and 400 ft, or not more than 30° above 400 ft (See Appendix 1 to JAR-OPS 1.495(c)(3).

(4) Adequate allowance must be made for the effect of bank angle on operating speeds and flight path including the distance increments resulting from increased operating speeds. (See AMC OPS 1.495(c)(4)).

(e) When showing compliance with sub-paragraph (a) above for those cases where the intended flight path does require track changes of more than 15°, an operator need not consider those obstacles which have a lateral distance greater than:

(1) 600 m, if the pilot is able to maintain the required navigational accuracy through the obstacle accountability area (See AMC OPS 1.495(d)(1) & (e)(1); or

(2) 900 m for flights under all other conditions.

The obstacle accountability area for a departure which includes a turn of more than 15 degrees is shown to the right.

The turn in this example is initiated at a distance of 1000 meters from the end of the takeoff. The bank angle is 15 degrees and the true airspeed is 150 knots – that combination yields a turn radius of 2266 meters.

In this case, the maximum lateral clearance required is 600 meters if the pilot can maintain the required navigational accuracy, otherwise it is 900 meters.

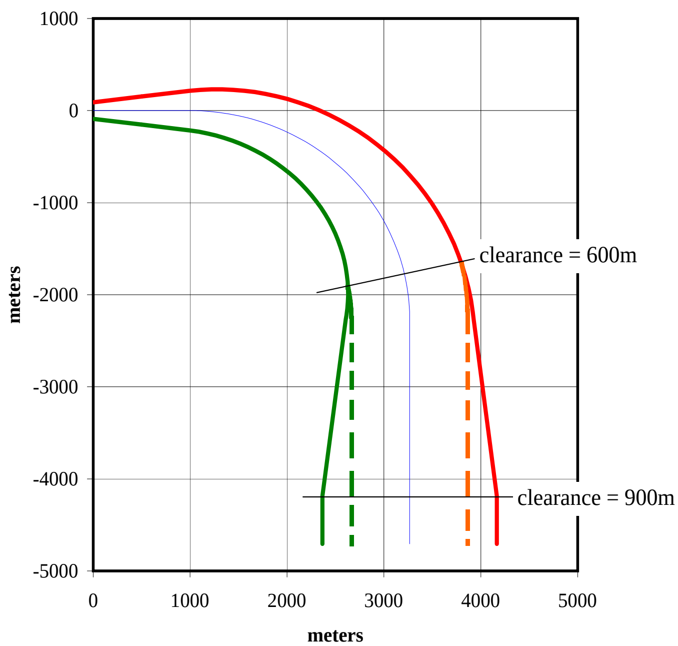


Figure 23-26

Please take note of the JAR-OPS 1.495(c)(2) requirement that the 35 foot obstacle clearance margin increases to 50 feet whenever the airplane is in a banked attitude greater than 15 degrees.

ICAO

Annex 6 obstacle clearance requirements are identical to those of the JAR-OPS regulation, as they are for straight-out departures, with – again – a slight difference in their description of the maximum half-widths. For the turning departure, the maximum half-width is 600 meters for VMC day operations, and 900 meters for IMC or night VMC conditions.

Bank Angle and Turn Height Restrictions

For safety when maneuvering near the ground, regulatory agencies impose limitations on the bank angle and the minimum height above the ground for starting a turn.

FAA

FAR Section 121.189 specifies that a turn shouldn't be started below 50 feet, and that the bank angle shouldn't exceed 15 degrees. Advisory Circular 120-91 broadens that somewhat:

- For a 15 degree bank, the turn shouldn't be started below either 50 feet, or one-half of the wingspan, which ever is greater;
- For a 20 degree bank, the turn may be commenced no lower than 100 feet;
- For a 25 degree bank, the minimum height for starting the turn is 400 feet.
- In some circumstances a bank angle of over 25 degrees may be appropriate but this would require specific evaluation and the approval of the FAA.

JAR-OPS

JAR-OPS 1.495 says that:

- A turn may not be started below either 50 feet or one-half of the span, whichever is greater. Above that height, up to a height of 400 feet, a bank angle of 15 degrees may not be exceeded.
- above 400 feet the bank angle may exceed 15 degrees but may not be greater than 25 degrees.

The same regulation, however, does allow for greater bank angles:

> *1.495(c)(3) An operator must use special procedures, subject to the approval of the Authority, to apply increased bank angles of not more than 20° between 200 ft and 400 ft, or not more than 30° above 400 ft. (See Appendix 1 to JAR-OPS 1.495(c)(3)).*

ICAO

ICAO Annex 6 says simply:

> *...it is assumed that the aeroplane is not banked before the clearance of the net takeoff flight path above obstacles is at least 15.2 m (50 ft) and that the bank thereafter does not exceed 15 degrees.*

Selection of the Bank Angle

In the usual case, the bank angle for the turn is selected to produce the radius of turn that's dictated by the obstacle distances and bearings from the runway. From the discussion above, you know that:

$$r = 0.08854\left(\frac{V^2}{\tan\phi}\right)$$ for a radius in feet with the true airspeed V provided in knots.

Knowing the radius r for the given takeoff situation, this equation can be revised to solve for the bank angle ϕ:

$$\phi = \tan^{-1}\left(\frac{0.08854V^2}{r}\right) \qquad \text{(eq. 6)}$$

Bank Angle Effects

Banking an airplane for a turn will have an effect on the airplane's performance.

gradient decrement in a bank

First of all, you'll recall that an airplane in an angle of bank experiences a small but not negligible loss of climb path angle. You may wish to review this in the chapter entitled "Calculating Climb Angle and Rate of Climb". Depending on how you will be calculating your obstacle limited weights, you may need to account for this effect manually; later software applications can do this for you automatically when the parameters of the turn are provided.

The Performance Engineer's Manual provides gradient decrement data for varying bank angles at the different flap settings as shown in the figure to the right.

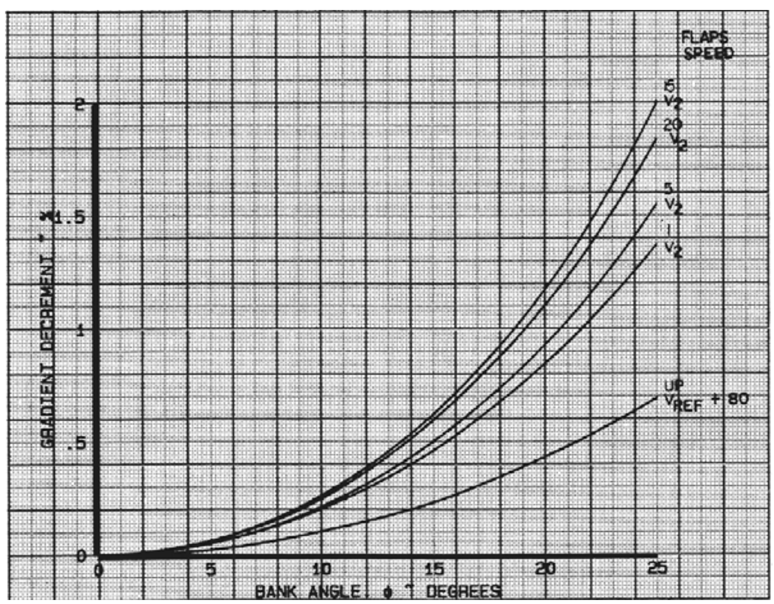


Figure 23-27

<u>This decrement varies from airplane to airplane, it is not the same for all Boeing models.</u>

As an example, notice that the gradient decrement for a climb at V_2 with the flaps at 5 in a 15 degree bank will be 0.5 percent.

(The really sharp-eyed of you probably noticed that the gradient decrement increases at constant bank angle from flaps up to flaps 15, then decreases slightly at flaps 20. This reversal of an apparent trend isn't an error, it's simply due to the shape of the drag polars for flaps 15 and 20.)

This loss of climb performance will affect obstacle clearance and thus must be accounted for. We'll be showing you later one way to account for this loss of climb angle.

reduction of stall margin in a bank

Another equally important consideration is the increase of an airplane's stalling speed in a bank and the resulting decrease in the margin between the airplane's speed and the speeds at which stall warning (stick shaker) and stall would begin.

At the certified V_2 speeds provided by the Airplane Flight Manual, a bank angle of 30 degrees will still be somewhat above the stick shaker speed, avoiding the possibility of actuating the stall warning system when making a turn after takeoff. Why talk about 30 degrees? Because the desire is to have a climb speed adequate to avoid stall warning in a 15 degree bank with an "overshoot" (that is, an unintentional exceedance) of 15 additional degrees. In this way, even if the target 15 degree bank is inadvertently exceeded, there will still be an acceptable margin of speed above stall warning actuation.

Both the FAA and the JAA provide recommended speed increases and gradient decrements for bank angles of 15 degrees and above.

FAA accountability for bank angles

AC 120-91Paragraph 14 is entitled "Analysis of Turns". In part, it says:

> *If bank angles of more than 15 degrees are used, V_2 speeds may have to be increased to provide an equivalent level of stall margin protection and adequate controllability (i.e. V_{MCA} (minimum control speed, air)). Unless otherwise specified in the AFM or other performance or operations manuals from the manufacturer, acceptable adjustments to ensure adequate stall margins and gradient decrements are provided by the following table:*

Bank Angle	Speed	'G' load	Gradient Loss
15°	V_2	1.035	AFM 15° Gradient Loss
20°	V_2 +XX/2	1.064	Double 15° Gradient Loss
25°	V_2 +XX	1.103	Triple 15° Gradient Loss

Table 23-3

> *Where 'XX" = the all-engines-operating speed increment (usually 10 or 15 knots)*

It's appropriate to comment here concerning the table shown above. The use of the gradient losses specified in the table are typically very conservative. A better answer can be obtained by computing specific data for the airplane in question or by using the turning flight path options of the associated Boeing software applications.

JAR-OPS accountability for bank angles

The Acceptable Means of Compliance (AMC) relating to paragraph 1.495(c)(4) above provides additional guidance:

> *1. The Aeroplane Flight Manual generally provides a climb gradient decrement for a 15° bank turn. For bank angles of less than 15°, a proportionate amount should be applied, unless the manufacturer or Aeroplane Flight Manual has provided other data.*
>
> *2. Unless otherwise specified in the Aeroplane Flight Manual or other performance or operating manuals from the manufacturer, acceptable adjustments to assure adequate stall margins and gradient corrections are provided by the following:*

BANK	SPEED	GRADIENT CORRECTION
15°	V2	1×Aeroplane Flight Manual 15° Gradient Loss
20°	V2 + 5 kt	2×Aeroplane Flight Manual 15° Gradient Loss
25°	V2 + 10 kt	3×Aeroplane Flight Manual 15° Gradient Loss

Table 23-4

You see that this table is essentially the same as that provided by the FAA. It is similarly conservative and the comment made following Table 23-3 is equally applicable here.

accounting for stick shaker margin by analysis

You know that whenever an airplane is banked to produce a turn, the wing's lift force necessarily increases in order to maintain a vertical component equal to the airplane's weight. Smaller and smaller radii necessitate higher and higher lift forces and thus higher and higher angles of attack. Ultimately the wing could require an angle of attack at which the stick shaker would actuate.

For any given speed it is a simple matter to calculate the angle of bank at which stick shaker activation would occur.

The Performance Engineer's Manual provides charts of stick shaker speed for a range of weights at all flap settings, as shown to the right.

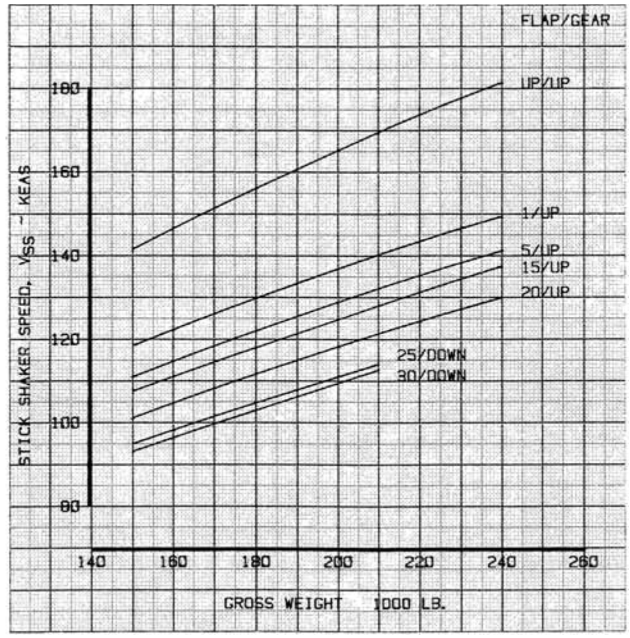


Figure 23-28

Knowing the takeoff weight and flap setting, this chart can be used to find the stick shaker speed V_{ss}. That speed can in turn be used to calculate the lift coefficient at stick shaker using the standard equation for lift coefficient:

$$C_{L_{SS}} = \frac{W}{\frac{1}{2}\rho V_{ss}^2 S}$$

Knowing now the lift coefficient at which stick shaker can be expected, we could find the bank angle at which this lift coefficient will be required by the wing. In a bank, the lift force L is equal to:

$$L = \frac{W}{\cos\phi}$$

where ϕ is the bank angle

We want to find the angle of bank ϕ at the climb speed V_{climb} at which the lift coefficient will be equal to the stick shaker lift coefficient, thus:

$$C_{L_{ss}} = \frac{W}{\frac{1}{2}\rho V_{ss}^2 S} = \frac{\frac{W}{\cos\phi}}{\frac{1}{2}\rho V_{climb}^2 S}$$

From this, we find that:

$$\phi = \cos^{-1}\left(\frac{V_{ss}}{V_{climb}}\right)^2 \qquad \text{(eq. 7)}$$

We should emphasize at this point that the stick shaker speed charts provided in the PEM are based on level flight, and small secondary effects on speed, on the order of a few knots, can occur when considering stick shaker actuation in a turn. If faced with a situation in which the bank angle and stick shaker actuation become a critical factor, we recommend that you contact Boeing for more detailed information.

effect of weight on turn radius

We know that weight has an effect on the takeoff speeds, including V_2. If V_2 varies, then, for a given bank angle, the radius will vary.

Consider, for example, a 737 on a sea level standard day with flaps at position 5 in a 15° bank:

weight (pounds)	V_2 (knots)	turn radius (feet)
140,000	166	9105
130,000	159	8354
120,000	152	7634
110,000	145	6947
100,000	138	6293

Table 23-5

You see that going from 100,000 pounds to 140,000 pounds results in a 45 percent increase in the turn radius.

effects of turn radius variation

One way to specify an obstacle clearance departure procedure is to specify a turn commencing at a particular location or height with a particular bank angle. One complexity introduced by the selection of a specific (constant) bank angle is the variation of the turn radius resulting from variation of the takeoff speed, which in turn results from variation of the takeoff weight.

Since the geometric locations of any obstacles relative to the end of the runway are constant, it follows that the locations of the obstacles relative to the turning flight path will change as weight changes. This can have a number of consequences. Consider the following illustration:

You'll recall that obstacle distance is measured at the point where the flight path is abeam of the obstacle – that is, a line drawn from the airplane's position to the obstacle forms a right angle with the direction of the flight path at that point.

As you can see to the right, the abeam point will be different for different weights. Thus, for the same conditions, the distance to the abeam point from beginning the turn will depend on the weight.

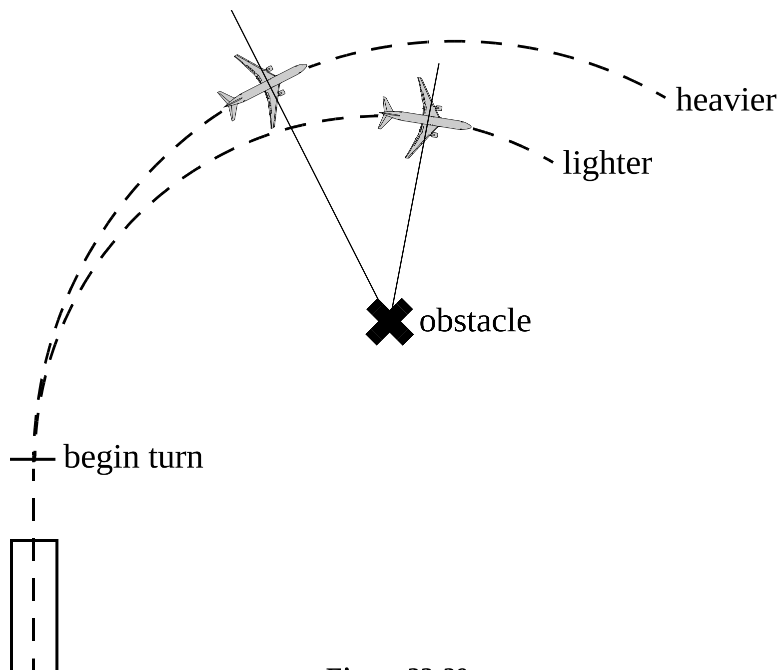


Figure 23-29

It's also possible that the obstacle will lie within the OAA at one weight but not at another. Looking at the illustration, you see that the obstacle might lie within the OAA at the lighter weight but not at the heavier weight.

Turning Flight Paths in a Wind

Earlier in this chapter we discussed the way that an airplane's flight path may be affected by wind during a straight-out departure. The same principles can be applied in calculating a flight path during a turning departure with a wind.

Let's work an example for an airplane climbing at 150 knots true airspeed, in a 15 degree bank. For those conditions, we calculate that the radius of the turn will be 7,435 feet. Although we will be using units of feet throughout the example, you'll understand that the method will work equally well with metric units.

Without considering any wind, this illustrates the parameters of the flight path. Assuming a constant bank angle, the path is circular with a radius *r*, in our example equal to 7435 feet.

We'll continue to call the direction of the runway Y and the direction lateral to that is X.

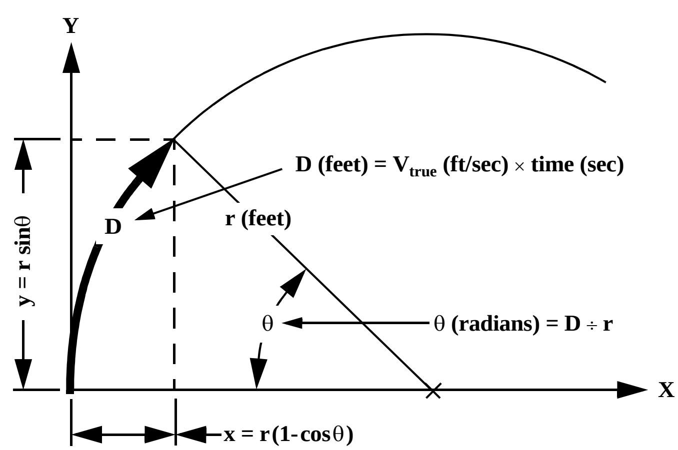


Figure 23-30

The distance *D* flown along the arc is simply equal to the true airspeed multiplied by the time. The subtended angle then is equal to $D \div r$ in radians, or $57.3 \times (D \div r)$ in degrees.

In our example, for each second the airplane will move through an angle of 0.03405 radians, equal to 1.95 degrees of arc.

The distance traveled in the X direction will be equal to $r(1\text{-}\cos\theta)$ and the distance traveled in the Y direction will be $r \sin\theta$.

crosswind effect on a turning departure

Now let's add in a ten knot right crosswind at 90 degrees to the runway direction. That will produce a drift to the left at 17 feet per second. There will be no drift along the direction of the runway centerline.

time - sec	turn angle - deg	X due to turn	Y due to turn	corr X	corr Y
0	0.00	0	0	0	0
1	1.95	4	253	-13	253
2	3.90	17	506	-17	506
3	5.85	39	758	-12	758

Table 23-6

As before, we're calling the lateral drift the X direction and the drift along the runway centerline the Y direction. Thus the "corrected X" will be the X value due to the turn <u>minus</u> the drift of 17 feet per second (since it's a drift to the left). The "corrected Y" will be same as the Y value due to the turn since there's no drift in the Y direction.

Graphing a no-wind turn together with the turn in a ten knot right crosswind, you see how the wind has affected the departure flight path.

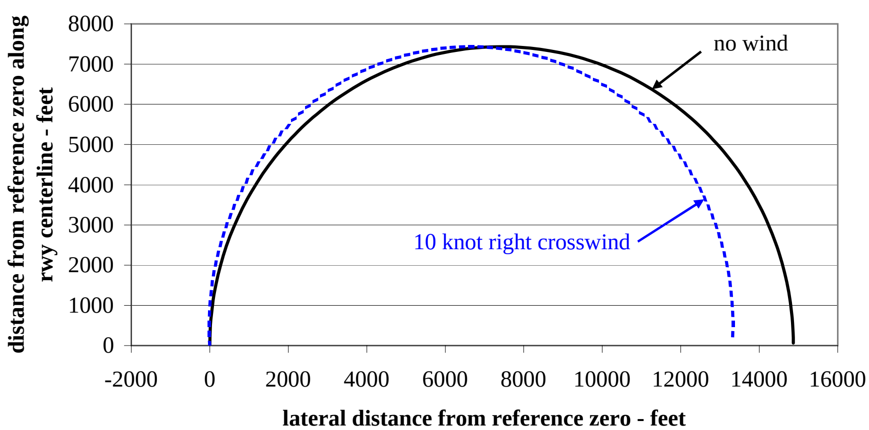


Figure 23-31

headwind effect on a turning departure

If our wind is instead a ten knot headwind with no crosswind component, its effect on the takeoff flight path would be as shown in this illustration. In this case the drift is directly opposite to the direction of flight.

The X value corrected for the wind is the same as the X value due to the turn, since there's no lateral drift.

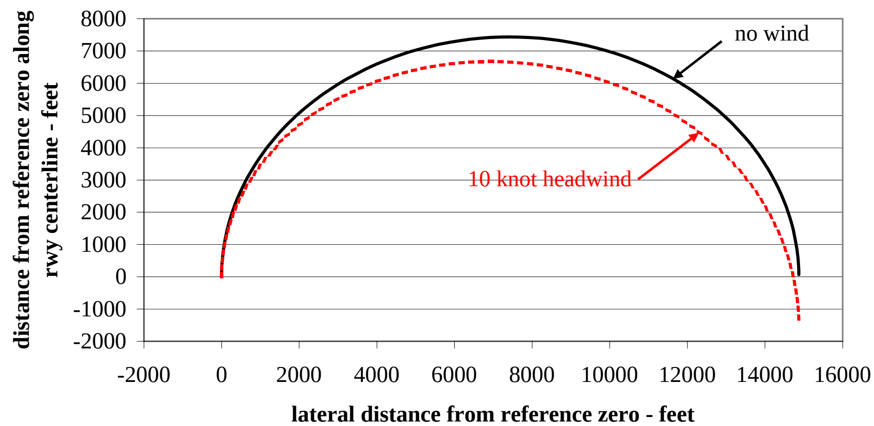


Figure 23-32

wind effect on an OAA

Just as we can calculate a flight path in wind, it's also possible to calculate the shape of an OAA in a wind.

In the illustration to the right, the solid lines show the OAA in no wind, and the broken lines show the effect of a ten knot right crosswind.

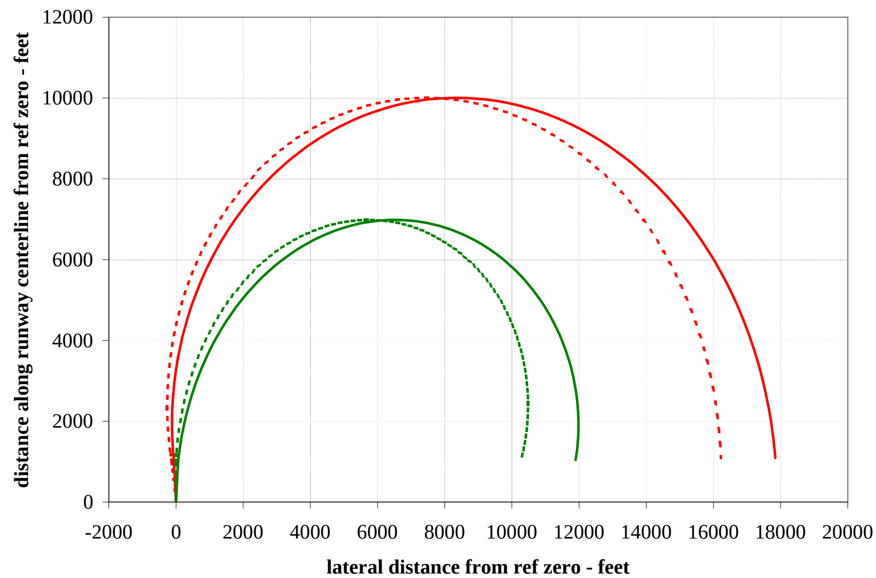


Figure 23-33

effect of wind on bank angle

Here's another complication that will give you an even bigger headache:

Suppose that you have defined an obstacle clearance departure path <u>over the ground</u> that is based on available navigation aids and has a circular arc of a specified radius designed for optimum obstacle clearance. If there's no wind, that's relatively simple: knowing the radius of turn, you can calculate the required bank angle. That in turn defines the gradient decrement that you must include in any vertical profile calculations.

But winds complicate the task of flying a fixed path over the ground.

As every student pilot learns, it's relatively easy to fly a circular pattern in the air: at constant airspeed and throttle, the student just maintains a constant bank angle. And if there's no wind, then the airplane's track over the ground will also be a circle.

But what if there is a wind? Then, as the airplane flies a circular path in the air, the path over the ground will be distorted by the drift as you see to the right.

In a pilot training exercise, an instructor will tell the student to fly a circular path at a constant distance from a fixed point on the ground. This is a maneuver called "turning about a point". To perform this maneuver, the student quickly learns that he must be constantly adjusting the bank angle.

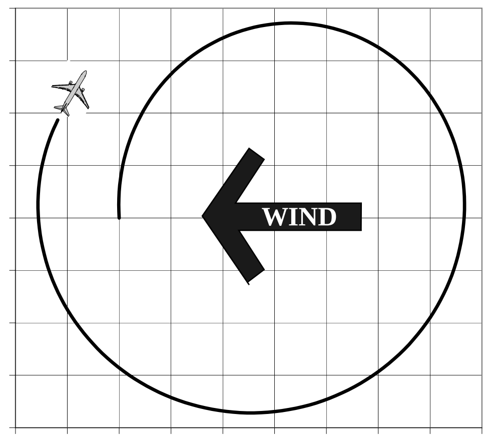


Figure 23-34

At point 1, directly upwind, the bank angle must be at its smallest value. From point 1 through point 2 up to point 3, the bank angle must gradually be increased so that at point 3 the bank angle will be at its greatest value.

From point 3 through point 4 and back to point 1, the bank angle must gradually be decreased.

Since the goal here is to maintain a constant radius *over the ground*, the bank angle can be calculated from:

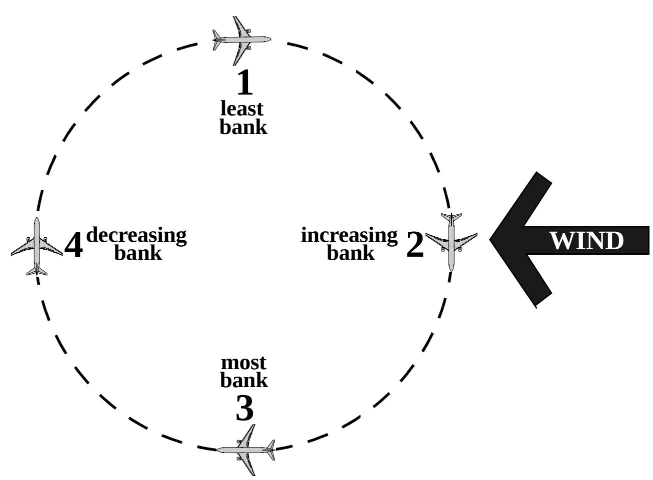


Figure 23-35

$$\phi = tan^{-1}\left(\frac{0.08854 V_g^{\ 2}}{r}\right) \qquad \text{(eq. 7)}$$

where V_g is the true ground speed in knots
r is the desired ground radius in feet

From this you can see that, since we have a situation in which an airplane's bank angle is constantly changing, the climb gradient decrement due to the bank will be varying as it executes a turn of constant ground radius.

Determining the Obstacle Limit Weight

All of the preceding discussion should have prepared you for the task of designing an obstacle clearance departure procedure, and for the manual calculation of an airplane's horizontal and vertical profiles as it follows that procedure. We've tried to make you aware of all of the factors that need to be considered in a detailed analysis.

In today's computer-based engineering environment, however, it's rare that one needs to do the performance calculations manually[1]. With that in mind, let's look at the software applications that are available to assist Boeing operators with obstacle limit takeoff weight calculations.

It's beyond the scope of this document to explain in detail how to use these applications. Boeing offers software training classes at no charge for customer personnel. Please contact the Boeing Flight Operations Engineering Group for more information on this training.

In the following pages we'll discuss the basic capabilities of the earlier software applications and the later ones based on AFM-DPI. We'll briefly discuss some special procedures that are needed when computing obstacle clearance in a turning departure when using the software for the non-AFM-DPI airplanes.

airplanes without AFM-DPI

Airplanes that do not have the digital Flight Manual are supported by earlier software applications that utilize databases made up of tabulations of the relevant AFM charts. Those applications, however, are less sophisticated than the capabilities programmed into AFM-DPI.

While the software for non-AFM-DPI airplanes is capable of obstacle clearance calculations, it is <u>not</u> capable of accounting for turns or for wind variation through a departure procedure. Winds input to the software are only headwind or tailwind components. All obstacles are considered to lie along a straight-out departure path. Further, this earlier software is not capable of solving problems of obstacles in final climb, which you've seen earlier in this chapter can be a powerful option.

When a turning departure is necessary, it's possible to modify the obstacle data to make it suitable for use in the software. This is sometimes referred to as creating an "equivalent straight-out departure". The trick is to <u>raise</u> the obstacle heights by the amount that the vertical profile would be <u>lowered</u> by the gradient decrement resulting from the bank angle in a turn. Properly done, this method yields correct results without the need for the more advanced software.

<u>*STAS and BPS*</u>

The software applications that may be used for non-AFM-DPI airplanes are called Standard Takeoff Analysis Software (STAS) and Boeing Performance Software (BPS).

STAS is a DOS-based program. It is used by first populating several matrices of input information with the airplane, airport and parameters for the analysis such as flap settings, winds and tempera-

1. Thank heavens for that. We've been there and done that, and it's no fun.

tures, and then executing the software. The program outputs are tabulations of allowable takeoff weight for the specified ranges of winds and temperatures, formatted in any one of a number of output formats, selected by the user.

BPS is a Windows–based graphical user interface (GUI). It allows the user to make selections and entries in the same manner as for other similar Windows-based graphical interfaces, making it easier to learn. BPS has the added advantage that it is used not only for takeoff analyses but also for enroute calculations such as climb, cruise and descent; familiarity with the BPS interface thus provides the user with the ability to execute many different types of tasks using the same input system.

creating the equivalent straight-out departure

This process involves, first, the determination of the obstacle locations relative to the turn and straight sections of the departure procedure. From that, it's possible to calculate the effect on airplane height that the bank angle will have as a function of its position along the departure path. Finally, the obstacles within the OAA will be raised by an amount equal to the height loss calculated at the points abeam the obstacles, to create the equivalent straight-out departure obstacle profile.

Let's look at an example.

Let's say that we have decided on a departure consisting of a 180° arc having a constant radius of 7500 feet. This turn will begin when passing 1000 feet from the runway end.

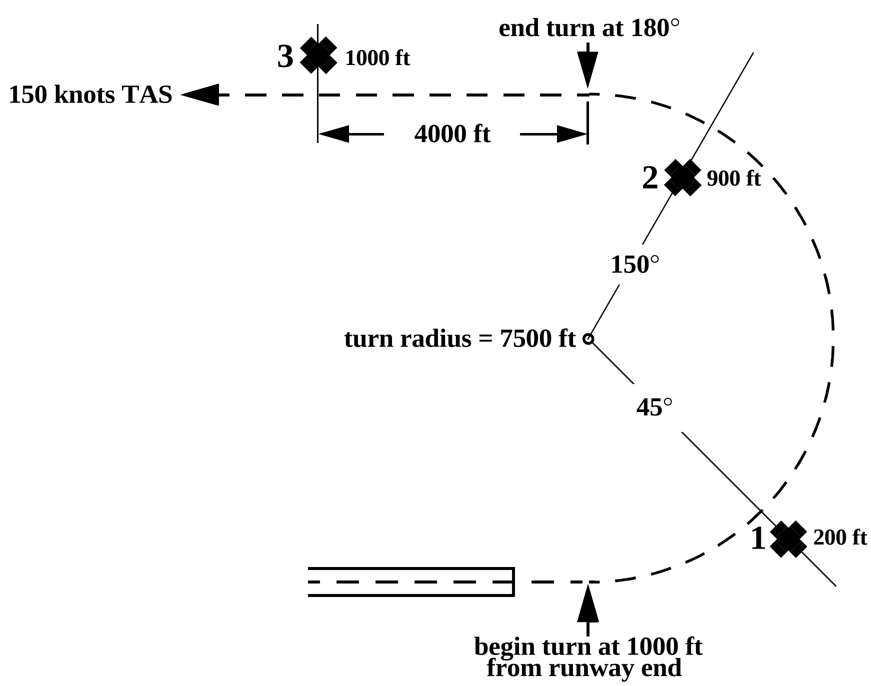


Figure 23-36

We estimate that the climb true airspeed will be approximately 150 knots.

There are three obstacles: number one is 200 feet high, with an abeam point at 45° into the arc. Number two is 900 feet high with an abeam point 150° into the arc. Number three is 1000 feet high and lies 4,000 feet beyond the point at which the arc will end.

The bank angle in the turn, from equation 6, will be:

$$\phi = tan^{-1}\left(\frac{0.08854 \times 150^2}{7500}\right) = 14.875°$$

We'll call it a 15 degree bank.

We need now to find the distances to the abeam points along the departure path. That's made easy by remembering that distance along a circular arc is equal to the subtended angle <u>in radians</u> multiplied by the radius. Remember also that to convert degrees to radians, simply divide by 57.3 or multiply by $\frac{\pi}{180}$.

Thus, the distance from the runway end to obstacle 1 will be $1000 + \left(\frac{45}{57.3} \times 7500\right) = 6890\ feet$.

The distance to obstacle 2 will be $1000 + \left(\frac{150}{57.3} \times 7500\right) = 20{,}634\ feet$

The distance to obstacle 3 will be $1000 + \left(\frac{180}{57.3} \times 7500\right) + 4000 = 28{,}560\ feet$

Knowing the obstacle <u>distances</u>, we need now to calculate their heights corrected for the loss of gradient in the turn. To do that, we first need to know the gradient decrement corresponding to a 15 degree bank.

We saw earlier a sample PEM chart giving gradient decrements in a bank. Remember that the decrement is not the same for all airplane models. For the purposes of this example, though, let's accept the value from that chart, which is 0.5 percent gradient decrement for a 15 degree bank.

To calculate the amount of height lost to the gradient decrement when passing the obstacle abeam points, we just need to calculate the distance traveled in the bank from beginning the turn to each abeam point. Then the height decrement is just the gradient decrement multiplied by the distance.

For obstacle 1: the airplane has been in a bank for 45 degrees when passing the abeam point. That corresponds to a distance of 5890 feet. A gradient decrement of 0.5 percent over a distance of 5890 feet corresponds to a height decrement of 5890 × 0.005 = 29.45 feet; we'll round it to 29.

For obstacle 2: having been in a bank for 150 degrees corresponds to a distance of 19,634 feet with a height decrement of 98 feet.

For obstacle 3: the airplane has made a complete 180 degree turn while banked, a distance of 23,560 feet, a height decrement of 118 feet.

To create now the equivalent straight-out departure, we'll just increment the obstacle heights by the amounts we've calculated; obstacle 1 goes from 200 to 229 feet, obstacle 2 goes from 900 to 998 feet, and obstacle 3 goes from 1000 to 1118 feet.

We see in the plot to the right that obstacle 2 will be the critical obstacle.

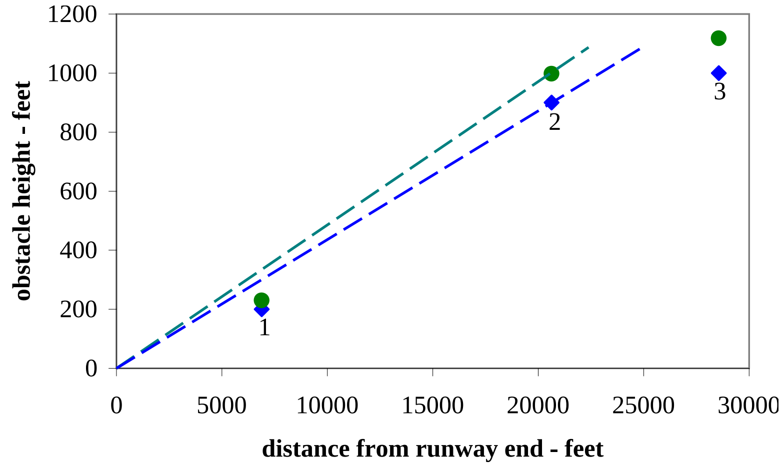


Figure 23-37

airplanes having AFM-DPI

The Airplane Flight Manual - Digital Performance Information (AFM-DPI) is a software application which completely replaces the performance functions of the Airplane Flight Manual with a highly precise software tool.

AFM-DPI is called a "first principles" application, because it calculates the airplane's performance from the basic airplane data: the drag polars, thrust data, and so on. Because it works in this manner, it is much more demanding of computer capability, but in exchange for that it provides calculations that are more precise than those available from its predecessor, the "paper flight manual".

Calculated performance based on the paper AFM is often somewhat conservative when compared to the same performance calculated using a first principles method. That is simply because it isn't possible to create AFM charts that are as accurate under all conditions as first principles calculations, and any loss of accuracy in AFM charts must necessarily be taken in the conservative direction.

For some Boeing airplanes that were originally only provided with paper AFMs, upgrades to AFM-DPIs were later offered as a customer option, with a price tag attached. Despite the price tag of the optional AFM-DPI, however, a number of operators found that the greater accuracy of the AFM-DPI offered substantial takeoff weight increases and hence payload increases, when compared to the same calculations done in the corresponding paper AFM.

The AFM-DPI is also capable of much more complex calculations than one could accomplish using the paper AFM. One noteworthy example of this is the fact that AFM-DPI can handle obstacle clearance problems involving turn profiles. It also can do splay calculations, eliminating obstacles that don't fall within the OAA splay. In AFM-DPI, therefore, it's only necessary to provide the obstacle location data and the parameters of the turn, if any. AFM-DPI can also handle any winds during takeoff, including the capability of calculating a variable gradient decrement in a turn affected by wind, which you'll remember we discussed earlier in this chapter.

The AFM-DPI application is capable of single-point calculations, in which the user provides a set of specific calculation conditions, including a single value of wind, temperature and so on. It is also capable of doing calculations over a range of either one or two parameters.

AFM-DPI does not, however, produce a printed "takeoff analysis" such as we're familiar with – for a given runway, an array of winds and temperatures with the corresponding allowable takeoff weights shown for each combination. To achieve that, the user has the two options: STAS and BPS. Either one is capable of producing an airport analysis for specified ranges of wind and temperature, and both offer a number of standard output formats.

STAS has the added advantage of being open-architecture software, offering the user who is capable of computer programming the ability to create his own formats for output from STAS. Also, STAS is written to the IATA Standard Computerized Airplane Performance (SCAP) specification; thus, STAS is capable of operating with databases and computing engines not only from Boeing but also from other airframe manufacturers, provided of course that their software is fully SCAP-compliant.

For those who are more comfortable working in a Windows environment, BPS can operate with AFM-DPI airplanes, just as it could with non-AFM-DPI airplanes as described above. When used with AFM-DPI airplanes, BPS retains the full functionality of AFM-DPI including obstacle clearance in turns.

Performance Engineer's Tool, PET

New with the 787 airplane, and ultimately usable on all Boeing models, is the Performance Engineer's Tool. PET's capabilities include takeoff analysis, landing analysis, enroute performance, mission studies, flight path analysis, noise calculations, airplane performance monitoring, and integrated data display. As such, for obstacle analyses PET will ultimately replace the existing BPS, STAS and AFM-DPI applications.

Chapter 24: Tire Speed Limit Takeoff Weight

Introduction

One factor that can limit the allowable takeoff weight of an airplane, less obvious than runway length or obstacles, is the ability of the tires to withstand the stresses and temperatures they experience during takeoff and landing.

In particular, a tire is sensitive to the rate at which it's rotating, because this rotation rate directly affects the stresses within the structure of the tire. Excessive stress can result in the failure of the tire's structure with possible consequences such as damage to the airplane and effects on directional control.

So we need to understand what causes the stresses within the tires, how much they can withstand, and how to calculate the effect these stresses can have on the allowable takeoff weight.

The assistance of Goodyear Aviation in the preparation of this chapter is gratefully acknowledged. Their document "Aircraft Tire Care and Maintenance" contains much useful information and is recommended reading. You may also find FAA Advisory Circular 20-97B, entitled "Aircraft Tire Maintenance and Operational Procedures" to be informative.

The Physics of Tires

It's natural to think of airplane tires as being essentially the same as automobile or truck tires, but in fact they're not the same at all. While the diameters are similar, the other characteristics of the tires are very different:

- Airplane tires are designed for much heavier loads than automobile tires. While the tires for an automobile are rated for loads between 1,000 and 2,000 pounds per tire, airplane tires are able to carry loads per tire many times that. Consider a fully loaded 747-400, for example, certified for takeoff weights in excess of 850,000 pounds, almost all of which is being supported by the 16 main landing gear tires.
- Tire pressures are markedly different. Conventional automobile tires are operated at pressures usually between 30 and 40 pounds per square inch; airplane tire pressures are adjusted according to weight and temperature, and are on the order of 200 pounds per square inch.
- The airplane tires used on today's commercial jet airplanes are designed for takeoff and landing speeds of 225 to 235 miles per hour, contrasted with a little over 100 miles per hour for automobile tires.
- Automobile tires are designed for relatively little *deflection*, compared to airplane tires. In this context, "deflection" means the amount of flexure of the body of the tire under load. It is designated in units of percent. You can think of it as the comparison of the radius of the tire where it's in contact with the ground compared to the radius of the tire at the top of its rotation where it's unloaded. Airplane tires operate at deflection percentages on the order of 32 to 35, roughly double the deflection of automobile and truck tires.

The physical limitations of a landing gear tire relate to the fact that wheel and tire rotation rates can become very high under some conditions. These high rotation rates will occur under conditions that demand high takeoff speeds, and they result in stresses and temperatures within the structure of the tire. These stresses and temperatures, when taken beyond their limits, can cause the tire to fail.

Tire failure can cause pieces of the tire to be thrown outward at high velocities. The high energies involved in these thrown pieces of tire can result in structural damage when their trajectories cause them to strike the airplane.

stresses within the tires

The stresses experienced within the body of the tire are complex and are due to a number of reasons. It's interesting and informative to examine those reasons.

due to centrifugal forces

Stresses internal to the structure of the airplane tire depend partly on the rate at which it's rotating.

At the heart of the problems resulting from high wheel and tire rotation rates is simply centrifugal force. We have talked previously about centrifugal force in other contexts such as the forces generated by an airplane flying in a banked turn. A centrifugal force can be calculated from:

$$\textit{centrifugal force} = m r \omega^2 \qquad \text{(eq. 1)}$$

where *m* is the mass being rotated
r is the radius of the arc of rotation
ω is the rotation rate in radians per unit of time

Since the centrifugal force is a function of the square of the rotation rate, and the rotation rate is a direct function of an airplane's velocity along the runway, the centrifugal force rises very rapidly with increasing speed.

Equation 1 can be written differently, to express the centrifugal effect in gees, rather than pounds or kilograms. Since Newton's law says that $a = \frac{F}{m}$, it follows that:

$$\textit{centrifugal acceleration} = \frac{m r \omega^2}{m} = r \omega^2 \qquad \text{(eq. 2)}$$

where *r* is the tire radius
ω is the rotation rate in radians per unit of time

For a tire radius in inches and *ω* in radians per second, equation 2 will yield an acceleration in inches per second per second.

If the diameter of an airplane tire is, for example, 30 inches, then its circumference is 30×π, or 94.25 inches. If the airplane is rolling at a velocity of 200 miles per hour, equal to 3520 inches per second, that means that the tires are rotating at 37.35 revolutions per second. Since one revolution per second is 2π radians per second, the tire's rotation rate will be 234.67 radians per second. Thus the acceleration is 826,026.7 inches per second per second.

How many centrifugal gees is that? One gee is 32.174×12 or 386.09 inches per second per second. Using a tire radius of 15 inches, that means that the tread of the tire is experiencing 2,139 gees!

due to tensile forces

As the centrifugal force increases, that outward force being experienced all the way around the circumference of the tire is, in effect, trying to increase the tire's radius by stretching it outward. Any tendency toward increasing the radius of the tire results in a tendency toward increasing its circumference. This creates a tensile force in the cross section of the tire.

The greater the rotation rate, the greater the tension. In fact, the tension force within the tread is a function of the square of the rotation rate and the square of the radius.

due to tire deflection

You'll recall that we mentioned above that the tire is deflected in the area where it's in contact with the ground – its radius is reduced as much as 35 percent compared with its unloaded radius.

In the illustration to the right[1], the heavy horizontal line represents the pavement. The wheel is rotating counterclockwise. The arc shown as a broken line would be the natural outline of the tire if it weren't compressed against the pavement by the airplane's weight acting downward on the wheel at its axle.

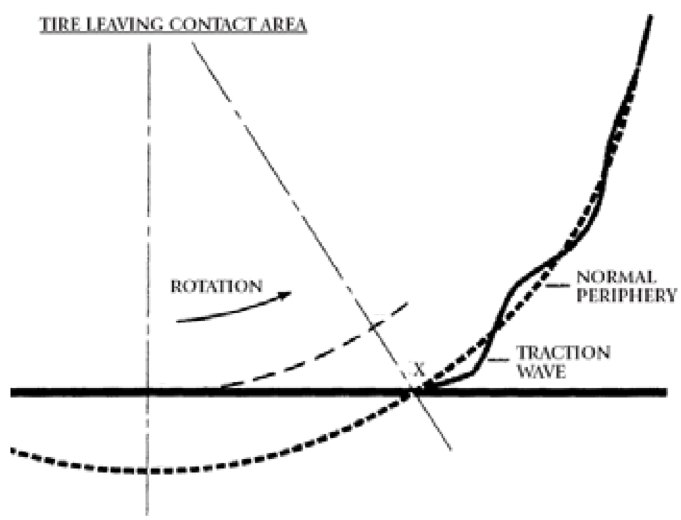


Figure 24-1

As the tire rotates away from the region of ground contact, it leaves the pavement at point X; at that point, its radius bounces back outward toward its usual shape. Because of the high rotation rate of the tire at high airplane speeds, this return toward its usual shape happens over an extremely short period of time.

Due to the high centrifugal force on the tire, and to inertia, the radius of the tire doesn't stop when it has regained its normal value but rather overshoots it. This induces a wave-like distortion of the

1. Illustration courtesy of The Goodyear Tire and Rubber Company, from their "Aircraft Tire Care and Maintenance" document, page 34, revised October 2004.

circumference of the tire, known as a *traction wave*. This wave continues for several cycles before damping out.

That very rapid cyclic change of radius due to the traction wave induces additional stress in the tire structure. Because of the high rotation rate of the tire at high rolling speed, this wave oscillation can occur at more than 10,000 cycles per second, at extremely high acceleration rates in the oscillation.

temperatures within the tires

Yet another contributor to the hardships endured by these tires is the heat generated by the cycles of flexure. Heat, in fact, has a more detrimental effect on the tire than the stresses resulting from its rotation rate. The heat is generated not only during the takeoff itself but also during taxiing, which can be distances of many miles at a large airport.

That heat depends on the taxi speed and also on the amount of tire deflection which in turn depends on the inflation pressure. Underinflated tires will experience much more temperature increase than overinflated tires; taxi speeds faster than recommended will also result in increased temperature rises.

Because the rubber of the tires is a good insulator and therefore a poor radiator of heat, the tire temperatures will steadily increase during taxi and takeoff, even with reasonable taxi speeds and proper inflation. Temperatures are greatest at the *bead* – the area where the wheel and tire join – and can increase by more than 150 °F during taxi.

This heat affects the strength of the nylon fabric that is molded into the tire cross-section to give it shape and strength. It also has an adverse effect on the strength of the rubber of the tire. These effects can result in damage to the tire.

tire stresses and temperatures during landing

Although the intent of this chapter is to discuss tires as they may affect takeoff, it’s worth pointing out here that the same sorts of stresses and temperatures will be experienced by tires during landing and taxi-in. This topic will be discussed in a later chapter.

Tire Speed Rating

In order to reduce or eliminate the chance of tire failure during takeoff or landing, all airplane tires have a rated maximum allowable speed. These ratings are provided by the tire manufacturer based on the design characteristics of the tire. Manufacturers are required to demonstrate the ability of their tires to withstand the stresses resulting from rotation of the tire at the maximum rated speed.

The tire speed ratings are in units of miles per hour (MPH). Let’s make this absolutely clear: the units are <u>statute</u> miles per hour rather than <u>nautical</u> miles per hour (knots). Divide tire speed limits in miles per hour by a factor of 1.15 if you want to know the speed limits in knots.

The earliest Boeing jet airplanes used tires rated for a maximum of 200 MPH. As time passed, airplanes and engines evolved permitting much higher takeoff weights. Despite advances in aerodynamic design, maximum takeoff speeds have necessarily increased somewhat. Because of those speed increases, tires having speed limits of 210, 225 and now 235 miles per hour have been developed. To date, no commercial jet transport airplane is certificated for tire speeds exceeding 235 miles per hour.

Since some airplane models have several options on tire speed ratings, when performing a takeoff analysis it's necessary to know which type of tire is fitted to the airplane in question.

The tire manufacturers are required to demonstrate these design tire speed capabilities on dynamometers as part of their test programs.

Tire Speed Limit Weight

So airplane tires have rated maximum speeds at which they're allowed to operate. How do these tire speed limits affect an airplane's takeoff weight?

You know that an airplane's takeoff speeds will be directly related to weight, pressure altitude, temperature, and flap setting. Thus it might sometimes be necessary to restrict the airplane's takeoff weight in order to avoid exceeding the tire speed limit.

But at what point during a takeoff are the airplane's tires rotating at their greatest rates? That's simple: the greatest tire rotation rate occurs at the liftoff speed. <u>Let's be more specific here: the greatest tire rotation rate occurs at the liftoff speed during an all-engine takeoff</u> (same V_R but a greater acceleration so a greater speed increase between rotation speed and liftoff speed) and hence we consider the all-engine takeoff case when calculating tire speed effects.

Following liftoff, the tire rotation rates decrease – first at a natural rate due to rotational friction losses, then because the wheel brakes are applied before retracting the landing gear. On Boeing airplanes and heritage McDonnell-Douglas airplanes, main landing gear retraction isn't begun until the wheels have stopped rotating. This sequence is followed in order to protect the wheel wells, and the components inside the wheel wells, from damage that could be caused by loose tire tread or a blown tire.

So what we need to do is to ensure that V_{LOF} doesn't ever exceed the rated tire speed limit. But here's the most important thing to remember about this topic: we are necessarily dealing with the <u>true ground speed</u> at liftoff, since tire rotation rates are dictated by the speed at which the airplane is rolling along the ground. When there's no wind, true groundspeed and true airspeed are equal; a tailwind, on the other hand, makes the true groundspeed greater than the true airspeed, and a headwind makes the true groundspeed less than the true airspeed.

What takeoff conditions cause the highest true ground speeds at liftoff?

- high takeoff weights

- high pressure altitudes and temperatures
- small flap deflections
- tailwinds

The first of those is obvious: high takeoff weights demand fast takeoff speeds to create the needed lift. The second item is the atmospheric conditions that produce a low value of the air density ratio σ and thus high true airspeeds; remember that TAS relates to equivalent airspeed by the equation:

$$\text{true airspeed } TAS = \frac{\text{equivalent airspeed } EAS}{\sqrt{\sigma}} = \frac{EAS\sqrt{\theta}}{\sqrt{\delta}}$$

(Remember also that for normal takeoff conditions, the EAS is essentially the same as the CAS shown on the airspeed indicators of most modern airplanes.)

High temperatures mean high values of θ since $\theta = \frac{T}{T_0}$, and high pressure altitudes (low air pressures) mean low values of δ, since $\delta = \frac{p}{p_0}$ so either – or, worse, both at the same time – results in higher true airspeeds for the same equivalent airspeed. Some of the better-known "high-hot" airports include:

- Mexico City, Mexico (elevation 7,349 feet). In April, the temperatures can be as much as 27°C, giving a value for σ of 0.7310.
- Albuquerque, New Mexico, United States (elevation 5,312 feet). Temperatures there can exceed 33°C, giving a value for σ of 0.7740.
- Lhasa, Tibet (elevation 11,975 feet). In June, the hottest month, at a QNH of 29.92 and the average temperature of 23°C, gives a value for σ of 0.6194.
- La Paz, Bolivia (elevation 11,942 feet). At their high temperature of 20°C, the density ratio will be 0.6257.

There are many cities having temperatures that are frequently above those quoted above, such as Phoenix, Arizona and Jeddah, Saudi Arabia, but their air density ratios are substantially higher than those listed above because of their lower pressure altitudes. Phoenix, for example, at an elevation of 1117 feet and a temperature of 45°C, has a density ratio of 0.8697.

Use of Unfactored Winds

You have noticed in earlier chapters the fact that when doing takeoff weight calculations including the effect of the wind, FAR Section 25.105(d) requires us to use <u>factored</u> winds:

> *The takeoff data must include, within the established operational limits of the airplane, the following operational correction factors:*

> *(1) Not more than 50 percent of nominal wind components along the takeoff path opposite to the direction of takeoff [i.e. headwinds], and not less than 150 percent of nominal wind components along the takeoff path in the direction of takeoff [i.e. tailwinds].*

Thus, when calculating for example a field length limited takeoff weight, if the reported wind is 10 knots headwind you are required to compute the weight based on a five-knot headwind – a 50% factor. For tailwinds, it's 150%, so for a reported 10 knot tailwind, the computed weight will correspond to a 15 knot tailwind.

<u>If you're using Airplane Flight Manual charts or AFM-DPI, enter the charts or AFM-DPI with the reported wind</u>; the chart and the software do the factoring for you invisibly. Thus, if the reported wind is 10 knots headwind, you should enter the chart using the line labeled for 10 knots headwind, or enter 10 knots into AFM-DPI – and the weight you will find will be valid for five knots headwind.

The only exception to this "factored winds" requirement that we are aware of is encountered when calculating tire speed limit takeoff weights. You'll observe in the AFM chart that we show you on the next page that the line of wind effect is a straight line, showing you that the winds aren't factored.

Why this difference? To be absolutely truthful, <u>we don't know</u>. If you know the reason, please share it with us. We have asked a number of knowledgeable people in the commercial jet transport industry, to date without success. We'll continue to look and will revise this book if we're successful in finding an answer. In the meantime, we'll go on assuming that the reason for this exception is simply lost in the sands of time.

Presentation of Tire Speed Limit Weight Data

the AFM

In the Boeing airplanes prior to those provided with the "digital flight manual" AFM-DPI, charts of tire speed limit takeoff weight are provided in the Airplane Flight Manual. An example is shown below.

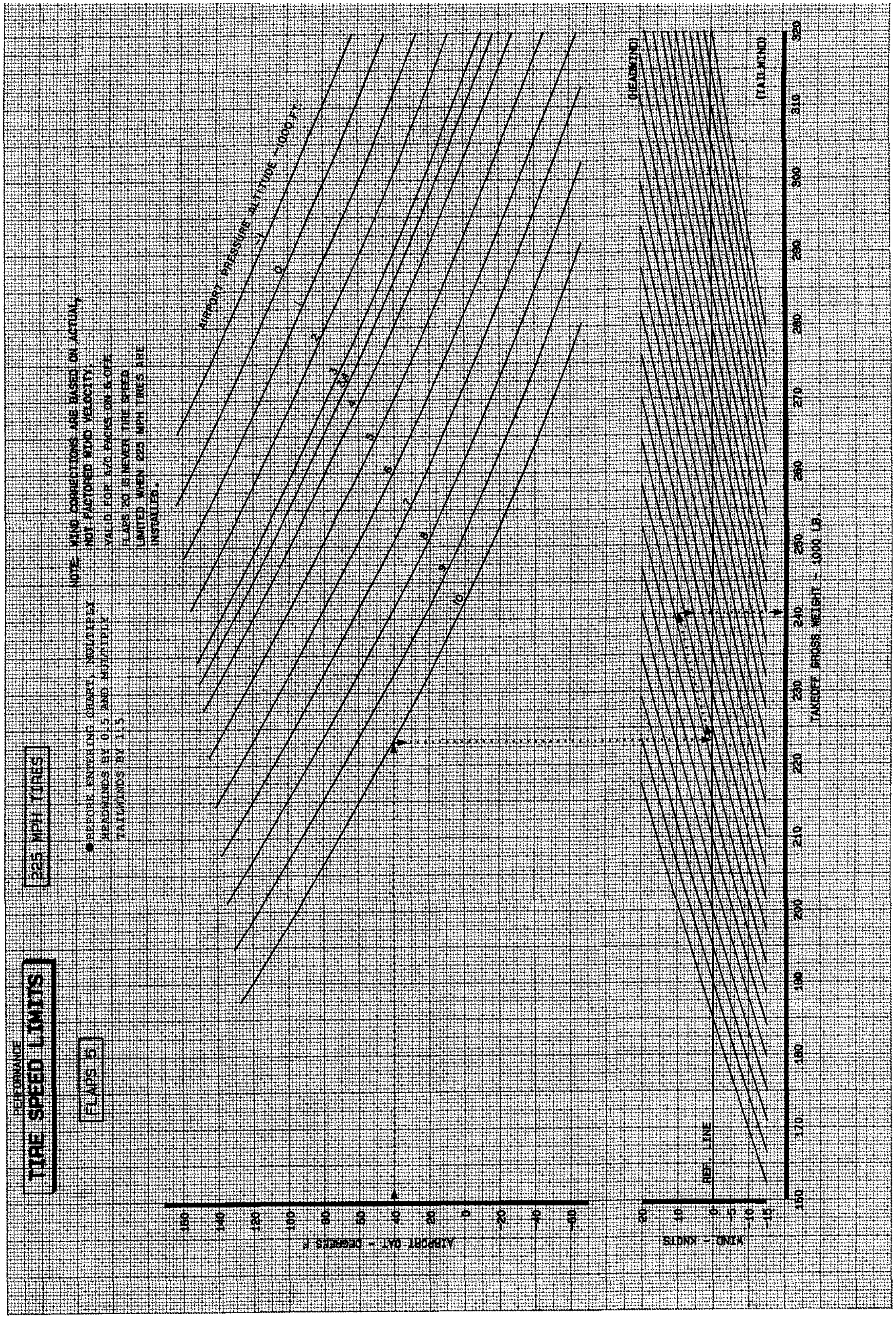

PERFORMANCE
TIRE SPEED LIMITS
225 MPH TIRES
FLAPS 5
AIRPORT PRESSURE ALTITUDE - 1000 FT
AIRPORT OAT - DEGREES F
WIND - KNOTS
(HEADWIND)
(TAILWIND)
REF. LINE
TAKEOFF GROSS WEIGHT - 1000 LB.

Figure 24-2

the FPPM

Tire speed limit weight is also provided in the Flight Planning and Performance Manuals, as shown here for one flap setting:

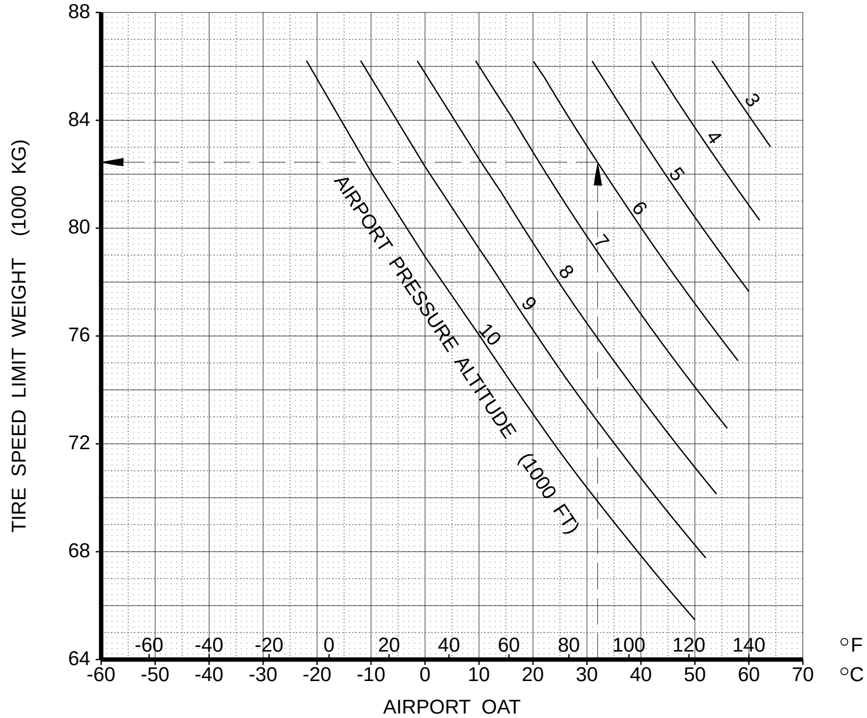


Figure 24-3

the software

The AFM-DPI for newer airplanes, and the digitized AFM chart databases for the takeoff analysis applications used for older airplanes, all incorporate tire speed limit takeoff weight data. All Boeing takeoff software applications will routinely check for possible weight limitations due to tire speed limits and will present that weight limitation where necessary.

Some airplane models are certified for more than one type of tire, having more than one tire speed limit. Some 747-400s, for example, are certified for both 225 and 235 MPH tires. In these instances, the user of the software must be sure to select the correct tire speed limit before running the takeoff analysis.

When Limited by Tire Speeds

When your takeoff weight will be limited by field length, or climb, or obstacles, there exist a number of performance options that may help to increase the limit weight: improved climb, a different flap setting, air conditioning bleeds off, range of V_1, and the like.

When your takeoff weight is limited by tire speeds, however, there is relatively little you can do to seek relief from this restriction. Operationally, the only options available will be to try a larger flap setting to decrease the liftoff speeds, or to schedule the flights for cooler times of day.

Chapter 25: Brake Energy Limit Takeoff Weight

Introduction

In the preceding chapter we examined the subject of the tire speed limit weight. You saw that it's sometimes necessary to restrict the takeoff weight in order to avoid excessively high takeoff speeds, in that case because of physical limitations of the airplane's tires.

There's another takeoff weight limitation that relates to the airplane's speed: the weight limitation that's due to the heating of the wheel brakes in the event of a high-energy rejected takeoff. Where the tire's limitations affect the maximum allowable liftoff speed, however, the brakes' limitations affect a different speed – the maximum allowable speed at which a rejected takeoff may be initiated, called V_{MBE}.

In this chapter, we'll examine the reasons why brake energy must be considered in calculating allowable takeoff weight, and how it's accounted for by the performance engineer and the pilot. But before we begin:

A General Comment About Brakes

Airplane brakes are a very interesting and complex subject. There are many different aspects to the subject of brakes: their energy absorption capabilities, their heat dissipation characteristics, residual brake energy as it may affect flight operations, and more.

In this chapter we'll be talking only about the ability of the brakes to absorb the kinetic energy of an airplane if a rejected takeoff becomes necessary, and how that may affect the allowable takeoff weight.

We'll be talking about brakes much more in the chapter entitled "Landing". There we'll discuss brake cooling times, "quick turnaround" times, residual heat energy considerations, fuse plug melting, and more. If you're looking for information on those topics, please look at the landing chapter.

The Physics of Brake Energy

It's accurate to think of brakes as mechanical devices designed to absorb kinetic energy and turn it into heat, and thereby to reduce an airplane's velocity.

brakes as friction devices

They accomplish this goal by creating high friction forces between the brake's rotors, which rotate with the wheel, and its stators, which are keyed to the brake housing and don't rotate.

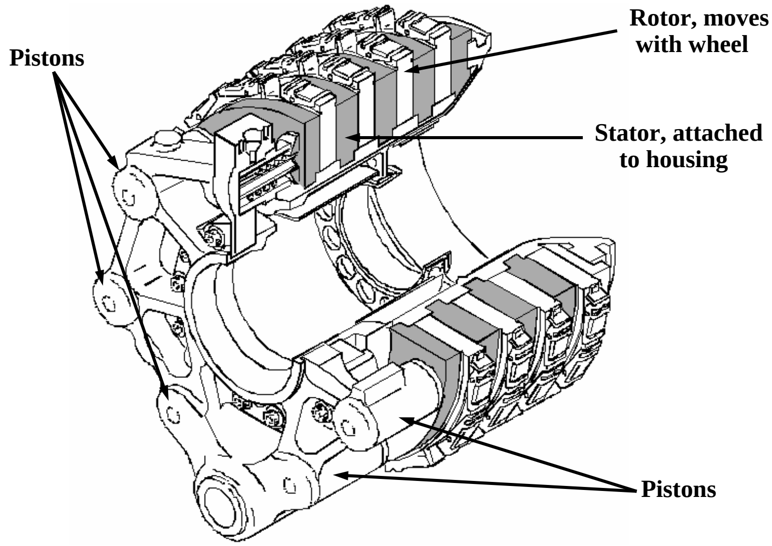


Figure 25-1

For all airplanes prior to the 787, the friction forces are generated by a number of hydraulic pistons that cause the rotors and stators to be pressed together, when actuated by the pilots' brake pedals. For the 787, the brakes are electrically actuated, but aside from that difference they're basically the same as hydraulic brakes.

During a rejected takeoff (RTO) or a landing, the brakes absorb a large amount of the airplane's kinetic energy – that's their function. The brakes convert that kinetic energy to heat, and the amount of heat depends on the speed at which the stop is initiated.

The heat, if allowed to exceed some safe limit, can result in tire deflation or even a brake fire with a potential for serious airplane damage. In the extreme case, brake integrity may be compromised.

Thus we find it necessary to determine by testing just how much energy a brake can safely absorb. Once that is known, we must ensure that a rejected takeoff is never initiated from a speed that is greater than the speed that would produce an energy equal to the brake's maximum capability.

the mathematics of stopping

Stopping an airplane when performing a rejected takeoff or a landing is largely the job of the brakes – but not exclusively so. During a stop, a number of forces are acting on the airplane:

- brake force
- aerodynamic drag
- thrust
- gravity, on a sloped runway

How much work is done by the brakes, and what will influence that?

Before the brakes are applied, some amount of energy will have been dissipated in the form or aerodynamic drag and airplane rolling friction drag. On the other hand, any residual engine thrust will be adding energy.

At the time when the brakes are applied at speed V_B, the airplane's kinetic energy is:

$$kinetic\ energy = \frac{1}{2} m V_B^2$$

where m is the airplane mass
V_B is the brakes-on true ground speed

That amount of energy must be absorbed by the brakes, drag, thrust and gravity. That can be expressed as:

$$KE = \int_0^{V_B} F_B\, ds - \int_0^{V_B} D\, ds + \int_0^{V_B} F_N\, ds - \int_0^{V_B} F_{slope}\, ds$$

where F_B is the retarding force from the brakes
D is the aerodynamic drag retarding force
F_N is the thrust force, positive for forward thrust, negative for reverse thrust
F_{slope} is the gravity force, when on a sloped runway

You'll recognize that the term $\int_0^{V_B} F_B\, ds$ is the energy absorbed by the brakes, and the other three terms are the energy absorbed by the drag, the engine thrust, and the slope component of weight.

From this, a number of conclusions are obvious:

- Decreasing the brake force, all other forces remaining the same, will increase the distance required to stop and decreasing the energy absorbed by the brakes;
- Decreasing the thrust by using reverse thrust, all other forces remaining the same, will decrease the distance required to stop, and the energy absorbed by the brakes;
- Uphill runway slopes create a retarding force, reducing the stopping distance, but downhill slopes create an accelerating force, increasing the distance;
- Increasing the brake force, other forces remaining the same, will shorten the distance, giving the drag, thrust and slope less time to act and increasing the kinetic energy absorbed by the brakes;
- Headwinds are helpful by decreasing the brakes-on ground speed for a given airspeed, decreasing the total kinetic energy to be absorbed and thus decreasing the energy absorbed by

the brakes. Tailwinds, on the other hand, significantly increase the energy that must be absorbed by the brakes.

Brake Certification

During the flight testing of any new airplane or any new brake installation, the brake energy absorption limitation must be established during a series of ground accelerate-stop test runs.

FAA Advisory Circular 25-7A, “Flight Test Guide For Certification of Transport Category Airplanes” has some informative wording about this:

> *(2) <u>Maximum Brake Energy Testing</u>. The following paragraphs describe regulatory requirements and acceptable test methods for conducting an accelerate-stop test run to demonstrate the maximum energy absorption capability of the wheel-brakes.*
>
> *(i) The maximum brake energy accelerate-stop demonstration should be conducted at not less than the maximum takeoff weight and should be preceded by at least a 3-mile taxi with all engines operating at maximum ground idle thrust, including three full stops using normal braking. Following the maximum brake energy stop, it will not be necessary to demonstrate the airplane’s ability to taxi.*
>
> *(ii)* [Federal Aviation Regulations (FAR)] *Section 25.735(h) requires the rejected takeoff brake kinetic energy capacity rating of each main wheel-brake assembly to be determined at the fully worn limits of its allowable wear range. The calculation of maximum brake energy limited takeoff weights and speeds, for presentation in the AFM performance section, must therefore be based on each airplane main wheel-brake being in the fully worn condition.*
>
> *(iii)* [FAR] *Section 25.109(i) requires a flight test demonstration of the maximum brake kinetic energy accelerate-stop distance to be conducted with not more than 10 percent allowable brake wear range remaining on each of the airplane wheel-brakes. The 10 percent allowance on the brake wear state is intended to ease test logistics and increase test safety, not to allow the accelerate stop distance to be determined with less than fully worn brakes. If the brakes are not in the fully worn state at the beginning of the test, the accelerate-stop distance should be corrected as necessary to represent the stopping capability of fully worn brakes.*
>
> *(iv) The maximum airplane brake energy allowed for dispatch should not exceed the value for which a satisfactory after-stop condition exists, or the value documented under the applicable Technical Standard Order (TSO) (or an acceptable equivalent), whichever value is less. A satisfactory after-stop condition is defined as one in which fires are confined to tires, wheels, and brakes, such that progressive engulfment of the rest of the airplane would not occur during the time of passenger and crew evacuation. The application of fire fighting means or artificial coolants should not be required for a period of 5 minutes following the stop.*

Several items in the above are noteworthy:

- First, that the testing for brake energy capacity must be preceded by a three-mile taxi including three full stops using the brakes. This means that the certified maximum brake energy capacity determined from the flight tests allows for some amount of normal ground maneuvering prior to the takeoff.
- Second, the Advisory Circular acknowledges that brake and tire fires may occur following a maximum energy stop. Its intent, however, is to ensure that any resulting fires won't endanger the safe evacuation of an airplane in such circumstances.
- Third, you see that the brake energy capacity determined from the tests must be based on fully worn brakes. Let's discuss that.

effect of brake wear

Following the installation of a new brake on an airplane, repeated use over time results in a reduction of the <u>mass</u> of the material in the brakes – that is, some of the brake material is worn away by the friction forces generated by each application of the brakes. This is normal.

The degree of brake wear will affect its ability to absorb heat. For the same amount of kinetic energy absorbed by the brakes during a stop, a brake having greater mass will exhibit a smaller temperature increase following the stop than a brake having less mass. Worn brakes therefore have a smaller brake energy capacity, and this fact is accounted for by the testing protocol required by the Advisory Circular.[1]

Maximum Brake Energy Speed V_{MBE}

As you've seen above, we determine by test the capacity of the brakes to absorb kinetic energy. In order to make this information available in a more readily usable form, we calculate and certify in the AFM a speed called the maximum brake energy speed, V_{MBE}. This speed is then published in the AFM (it's displayed with the other speeds in AFM-DPI for the later airplanes) and in the Flight Planning and Performance Manual (FPPM).

In the earlier chapter entitled "Speeds" we looked at V_{MBE} and defined it as follows:

> *The speed V_{MBE} is simply the maximum takeoff speed, for a given amount of airplane mass, at which the brakes may be applied in the event of a rejected takeoff without exceeding the brake energy absorption limitations. It assumes that the pilot applies the maximum manual brake pressure.*

---

1. A brief historical footnote: the requirement that the certified brake capacity must be based on fully worn brakes became effective as part of Amendment 25-92, dated March 1998. Prior to that date, although worn brake accountability had been included in the British certification rules, it had <u>not</u> been included in the FAA rules. As a consequence of a brake-related airplane accident in May, 1988, however, worn brake accountability for <u>all</u> airplanes became retroactively mandatory through the Airworthiness Directive process.

Allow us to emphasize this point: V_{MBE} assumes maximum manual braking – which is the greatest possible level of braking effort. For the sake of conservatism, no consideration is made of lesser degrees of braking effort, such as might be applied by pilots when stopping distance isn't critical. Also, no credit is taken for the energy absorbed by the application of reverse thrust.

factors affecting V_{MBE}

Think about kinetic energy:

$$\textit{kinetic energy} = \frac{1}{2} m V_{true}^2$$

where *m* is the mass of the object
V_{true} is the true velocity of the object

Thus the kinetic energy of an airplane making a stop, which must be absorbed by the brakes, depends on the airplane's mass and the square of the true ground speed of the airplane at the time the brakes are applied.

V_{MBE} is certified and published, however, in units of speed that are more readily usable. Takeoff speeds are calculated and published in terms of indicated or calibrated airspeed. Flight deck airspeed indicators don't show true airspeed, they display instead either the calibrated or indicated airspeed, depending on the airplane model.

You know that indicated or calibrated airspeed is virtually the same as equivalent airspeed for any takeoff within the certified environmental envelope. Further, you remember that:

$$\textit{true airspeed TAS} = \frac{\textit{equivalent airspeed EAS}}{\sqrt{\sigma}} = \frac{EAS\sqrt{\theta}}{\sqrt{\delta}}$$

Hence the published V_{MBE} values will depend on the pressure altitude (giving δ) and the air temperature (giving θ) in order to make the conversion from V_{true} to equivalent airspeed.

What else will affect V_{MBE}? Remember that kinetic energy depends on true ground speed. Thus the wind component along the runway at the time of takeoff will affect V_{MBE}.

V_{MBE} *as the maximum* V_1

During a takeoff, you'll recall, if a rejected takeoff becomes necessary then the brakes must be applied no later than V_1 in order to ensure that the accelerate-stop distance doesn't exceed the distance available. V_1 is thus the highest brakes-on speed permitted. And since V_{MBE} is defined as the maximum speed at which the brakes may be applied for brake energy reasons, it follows that the simple rule for avoiding brake energy exceedance is that V_1 may never exceed V_{MBE}.

Here's a rather obscure point which we feel we should mention, although it has no practical effect on what we've just said. Follow closely:

You may recall, from the chapter entitled "Calculating Takeoff Distances" that for airplanes certified to FAR 25.111 Amendment 25-42 and Amendment 25-92 there is a two second interval between V1 and the application of the wheel brakes. For the Amendment 25-92 airplanes, those two seconds are at constant velocity, V_1. For the Amendment 25-42 airplanes, however, those two seconds assume continued acceleration after V_1 until brake application. The two second interval is included in the distance calculations as a conservatism, recognizing that it's reasonable to expect some delay in pilot application of the brakes if the decision is made to reject the takeoff.

In view of that two second interval between V_1 and brake application, it's logical to ask if the statement we made just a moment ago – "V_1 may never exceed V_{MBE}" – still holds true for those Amendment 25-42 and Amendment 25-92 airplanes.

The answer is yes. The reasons?

- For the Amendment 25-42 airplanes, the value of V_{MBE} is computed including an allowance for the two seconds of continued acceleration after V_1. Although we continue to call it V_{MBE}, we might more accurately call it "V_1 for maximum brake energy".
- For all Boeing airplanes, for practical purposes we consider the brakes to have been applied at V_1. Pilot training emphasizes that to ensure a successful RTO maximum brake force must be applied at or before V_1.
- The certified accelerate-stop distances for all Boeing airplanes include two additional seconds of distance as an allowance for pilot reaction times. Only on the Amendment 25-42 and Amendment 25-92 airplanes is the allowance shown as being between V_1 and brake application. For those airplanes, we still consider V_1 to be a brakes-applied speed, with the two seconds as – you might say – an "invisible conservatism".

brake energy limited takeoffs

Figure 25-2 to the right is plotted for a 757-300 on a sea level runway at 86°F. It will help us to understand the relationship between VBME and field length limit weights.

The line marked V_{MBE} shows the maximum brake energy speed at each weight, and the accelerate-go and accelerate-stop distances for those combinations of weight and speed.

You see that the line representing the brake energy limit speed cuts across the higher weight - higher V_1 corner of the web chart.

Looking at this figure, it's easy to see the conditions at which this airplane's takeoff weight will be limited by brake energy for the given takeoff conditions.

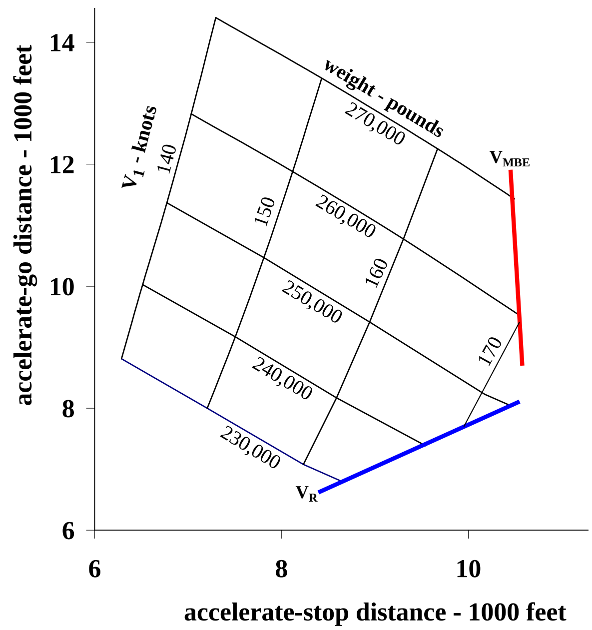


Figure 25-2

Let's say for example that the takeoff is being planned for an 11,000 foot runway with 500 feet of stopway.

Without the V_{MBE} limitation, you see that the field length limit weight would be in excess of 270,000 pounds. However, since the V_1 for those conditions would exceed VMBE, that weight is unacceptable.

What should you do in this case? Simply move to the left, along the line of 11,000 feet accelerate-go distance, until reaching the V_{MBE} line. That yields a new weight limit of approximately 268,000 pounds with a V_1 = V_{MBE} = 167 knots.

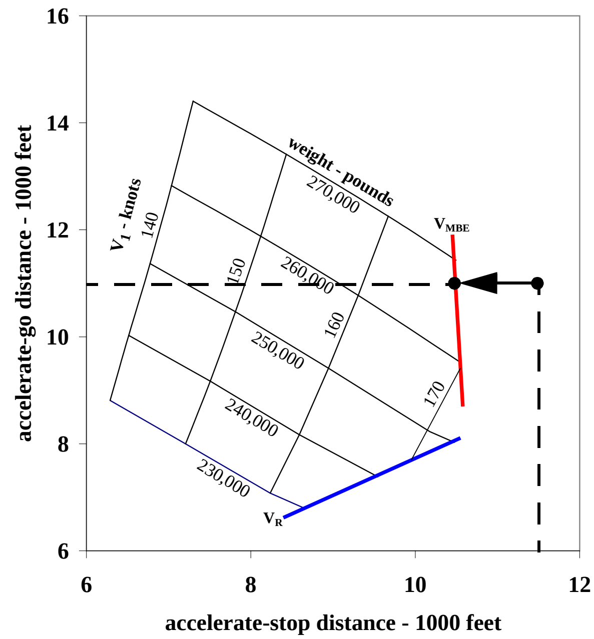


Figure 25-3

In so doing, you have ensured that your V_1 and brake energy limit takeoff weight produce takeoff distances that are within the available distances, with only a modest loss of weight capability.

Another possibility would be to try another flap setting for this takeoff. A larger flap setting would have the advantage of reducing the takeoff speeds and raising the field length limit weight. For the airplane we used in creating this example, the weight at the next larger flap setting would still be brake energy limited, but with a higher weight of about 282,000 pounds. Going to the largest flap setting, however, caused the weight to become climb limited at a weight only slightly above the 268,000 pounds you see in the illustration above.

Presentation of Brake Energy Limit Data

the AFM

The AFM does not present charts of brake energy limit weight. Instead, it presents brake energy data in the form of charts of speed: V_{MBE}. These are provided to enable the user to check the anticipated takeoff V_1 for brake energy considerations. In the event that the computed V_1 is greater than V_{MBE}, the user will need to find a solution to the problem.

A sample of an AFM chart of V_{MBE} is shown on the following page.

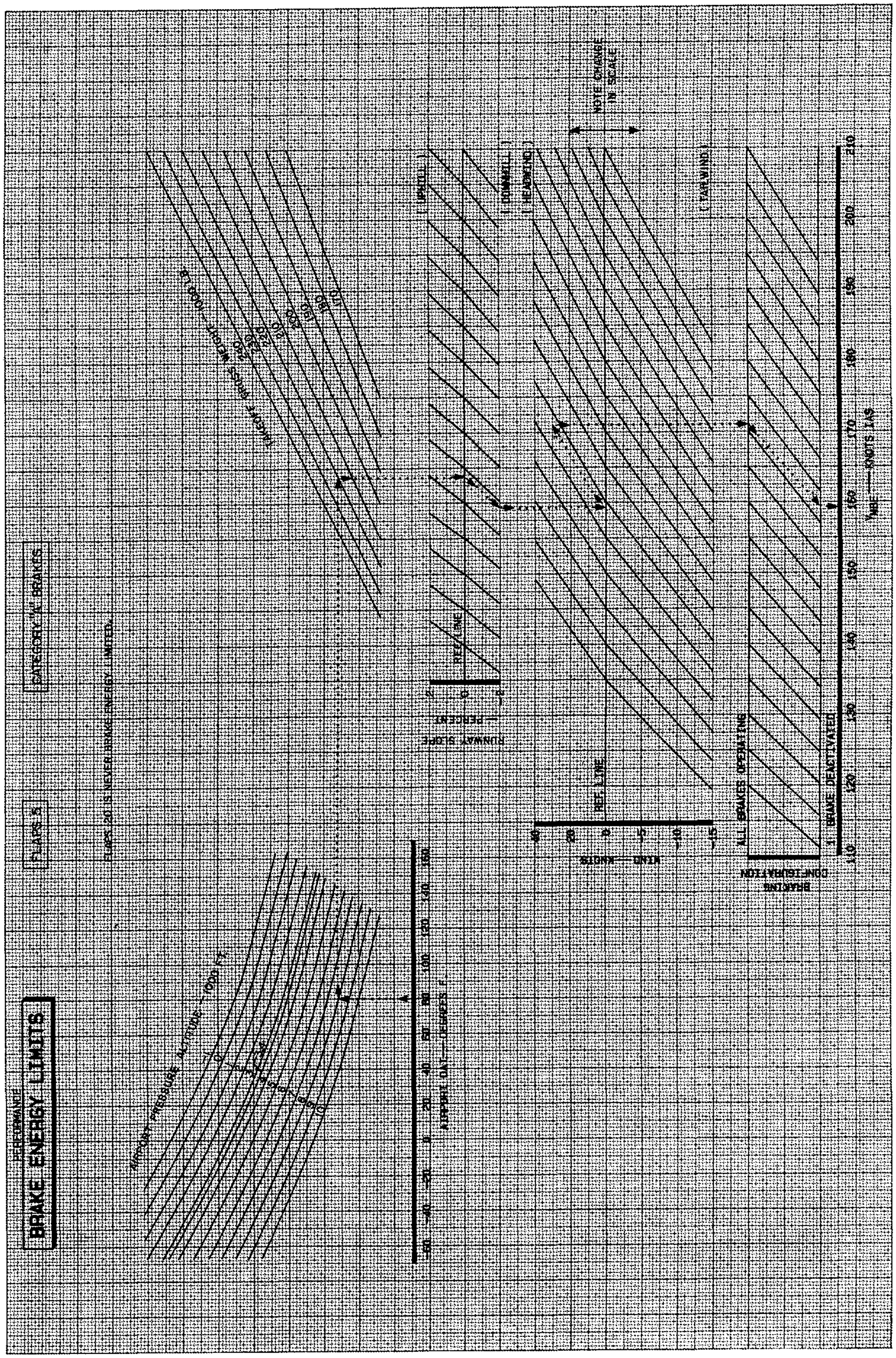

PERFORMANCE
BRAKE ENERGY LIMITS
FLAPS 5
CATEGORY "C" BRAKES
FLAPS 20 IS NEVER BRAKE ENERGY LIMITED
AIRPORT PRESSURE ALTITUDE — 1000 FT
AIRPORT OAT — DEGREES F
RUNWAY SLOPE — PERCENT
REF LINE
UPHILL
DOWNHILL
HEADWIND
TAILWIND
WIND — KNOTS
NOTE CHANGE IN SCALE
ALL BRAKES OPERATING
1 BRAKE DEACTIVATED
BRAKING CONFIGURATION
KNOTS IAS

Figure 25-4

the FPPM

The FPPM, like the AFM, presents simply a chart of V_{MBE}. It is incumbent on the user of the FPPM to check the V_1 for the expected takeoff against the V_{MBE} chart to ensure that the maximum brake energy speed isn't exceeded.

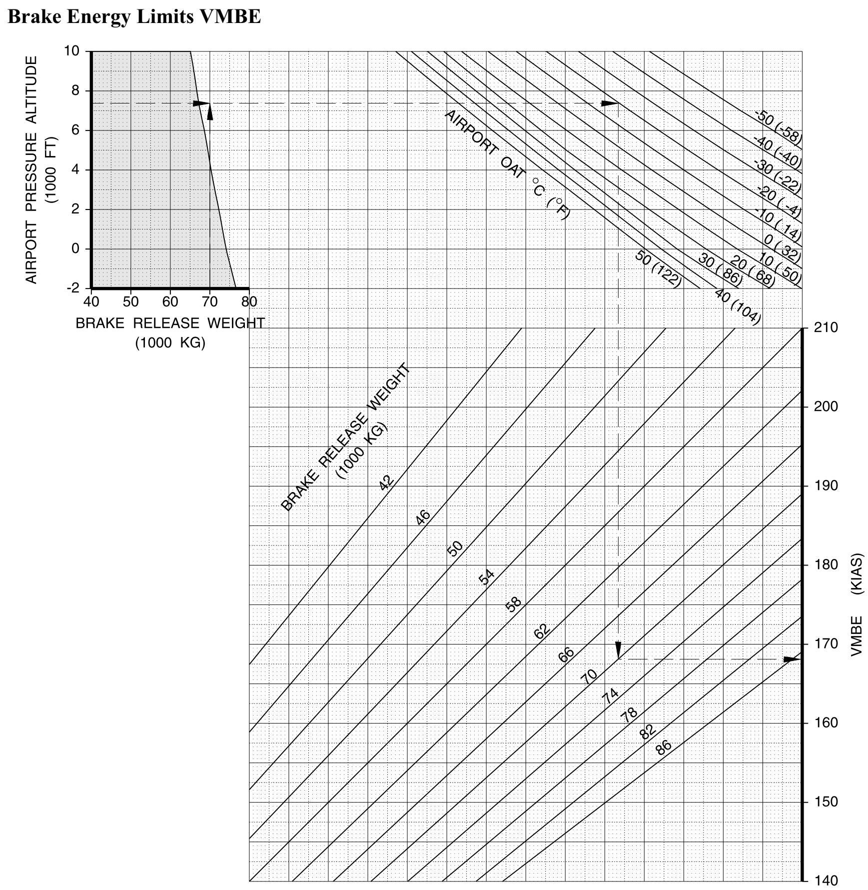


Figure 25-5

the takeoff software

The AFM-DPI for the newer airplanes, and the earlier software applications using the digitized AFM chart database, will check V_1 against V_{MBE} and will show the brake energy limited weight where appropriate. BPS, capable of preparing takeoff data for either AFM-DPI airplanes or earlier ones, will also ensure V_{MBE} compliance.

A Final Precautionary Note

One of the conservatisms in the brake kinetic energy absorption testing, as we pointed out earlier in this chapter, is that the brakes are not fully cool when the testing is done. The brakes will have undergone at least three miles of taxiing and three full stops. That means that the brakes will contain some amount of residual heat energy at the time the test is performed, and so the certified V_{MBE} values do allow for the fact that the brakes may not be fully cool when used for an RTO. But the energies of the three mile taxi and the three stops are relatively modest.

Let's consider a very possible operational scenario: an airplane makes a normal landing, taxies to the gate, taxies back out to the runway one hour later, and then conducts an RTO from a V_1 equal to V_{MBE}. Will the maximum allowable brake energy be exceeded in that RTO? The answer is probably “yes”.

Because brakes cool down quite slowly when on the ground, there will still be a substantial amount of residual heat energy in the brakes when the airplane taxies back to the runway and commences the takeoff.

Brake energy is cumulative, so it’s entirely possible that the sum of the residual energy in the brakes before the RTO plus the energy that’s absorbed during the RTO, will exceed the certified energy level.

If this scenario sounds like your sort of operation, you may want to consider some form of additional brake cooling while at the gate, such as electric fans; you may want to consider longer turn-around times. But you shouldn’t ignore residual heat as a very real consideration in airline operations.

We will be discussing this topic at greater length in the chapter entitled “Landing”.

possible consequences of a high energy RTO

High energy rejected takeoffs will almost certainly result in deflation of the main landing gear tires, and brake fires are a very real possibility. You have seen, however, that the brake testing protocol requires us to demonstrate that adequate time would exist for a safe evacuation of the airplane before the fires could progress to a dangerous point.

But: there are no known instances in which the brakes of a Boeing airplane have failed to bring the airplane to a full stop. You should find this fact comforting.

Chapter 26: Improved Climb Technique

Introduction

In an earlier chapter entitled "Calculating Takeoff Distances" we talked at length about how to compute the distances required for takeoff, for both the normal all engines operating case and for the engine failure case. In the next chapter, you saw clearly the relationship between takeoff distance available and the field length limit takeoff weight. From these chapters you saw how the takeoff speeds affect the distances required.

A little later, in the chapter entitled "Calculating Climb Angle and Rate of Climb", you saw how to calculate the climb gradient for any conditions. We showed you that the climb gradient was a function of the shape of the drag polar and that the ratio of drag to lift, one of the two primary factors used in calculating the climb gradient, depends on the climb speed at which the airplane is flying.

And after that, we examined the subject of the effect of the landing gear tires' speed limit on the allowable takeoff weight. Here too, the selection of the takeoff speeds will affect the tire speed limit takeoff weight.

So you have seen three different takeoff weight limitations that are influenced by the selection of the takeoff speeds V_1, V_R and V_2. The purpose of this chapter, "Improved Climb Technique", is to show you how – under some conditions – you will be able to increase the maximum allowable takeoff weight by increasing the takeoff speeds above their usual values. You'll also see how takeoff speed increases can also – sometimes – allow higher obstacle limited takeoff weights as well.

Note: the following discussion shows the improved climb methods as used on commercial jet transport airplanes designed and built by The Boeing Company. For those designed and built by the McDonnell Douglas Corporation, the flap system design and therefore the improved climb methodology is somewhat different. This will be discussed at the end of this chapter.

Let's begin by reviewing the effect of takeoff speeds on the first three of these takeoff weight limitations.

Effects of Increased Takeoff Speeds on Takeoff Weight Limits

In preceding chapters we have talked about the takeoff speeds and their relationship to weight, flap setting and atmospheric conditions. You saw in the chapter entitled "Speeds" that the speeds V_R and V_2 are determined from takeoff tests, and that they are at the lowest possible values that satisfy both the requirement for the minimum liftoff speed and also the requirement for the minimum speed at 35 feet.

Let's assume a set of conditions – just for example, sea level standard day, flaps 15, ten thousand foot runway, or whatever. Given these conditions, we can find the takeoff weight limitations and the corresponding takeoff speeds V_1, V_R and V_2.

For the purposes of the following discussion, the speeds that would be determined in the usual way for the given conditions will be referred to as the "normal" speeds.

In the following pages, we're going to demonstrate to you the effects of raising the takeoff speeds V_1, V_R and V_2 above their normal values. Then we'll show you how you can take advantage of these effects under some conditions.

effect on the field length limit takeoff weight

At our field length limit weight and the corresponding takeoff speeds, the takeoff distances will be just equal to the distances available.

What if now you were to increase the takeoff speeds slightly above their normal values, without changing the weight? The takeoff distances would now exceed the available distances. This isn't acceptable, so we must reduce the weight; only by doing so can we keep the distances with the new speeds within the available distances.

Thus it's easy to understand that the field length limit takeoff weight depends on the speeds being used. Any increase of the takeoff speeds above their "normal" values will necessitate a weight reduction.

For reasons that will become clear later, speeds that are increased above their normal values for performance purposes are usually referred to as *improved climb* speeds.

In the illustration to the right, we show a graph of field length limit weight without improved climb versus field length limit weight with improved climb.

The amount of speed increase is shown in "units" of speed increase. We'll be discussing the units of speed increase later; for now, we'll just say that in this example, one unit of speed increase is a little less than one knot.

You see that as the speed increase grows greater, the field length limit decreases, just as

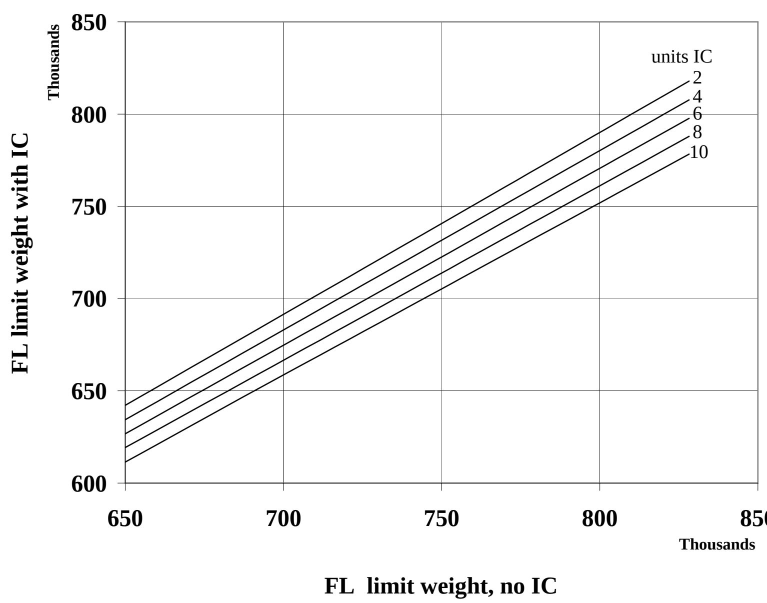


Figure 26-1

you would expect. As an example, if the field length limit weight is 700,000 pounds with normal takeoff speeds, it will be approximately 692,000 pounds with two units of speed increase.

effect on the climb limit weight

If you'll think back to the chapter entitled "Calculating Climb Angle and Rate of Climb", you'll remember the discussion of the variation of climb angle with climb speed V_2. You saw that the climb angle was a direct function of the ratio of drag to lift, which in turn is a function of the climb speed.

To the right, you see the climb limit weight with improved climb compared to the climb limit weight without improved climb.

You see that improved climb increases the climb limit weight as a function of the amount of speed increase. For example, if the climb limit weight is 750,000 pounds with normal takeoff speeds, it will become 760,000 pounds when using a speed increase of two units.

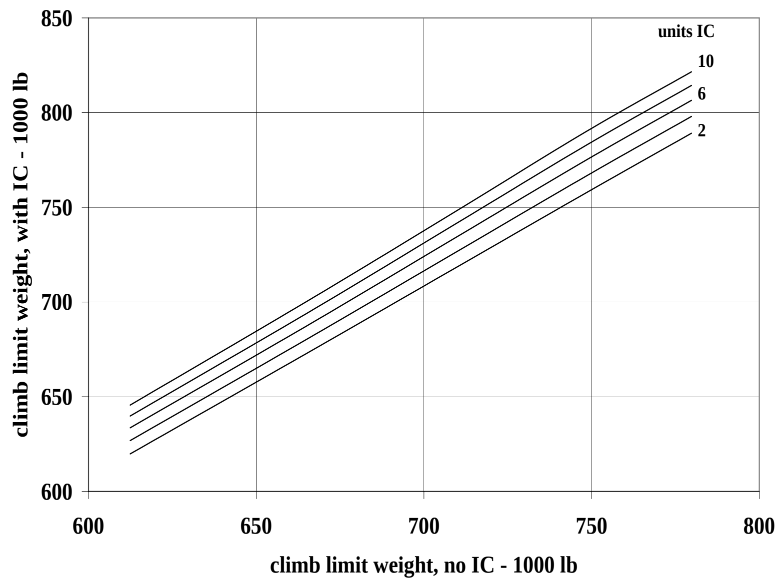


Figure 26-2

effect on the tire speed limit weight

Tire speed limit weight, you'll remember, is that weight at which the corresponding liftoff speed will be just equal to the rated tire speed limit. It is logical, then, that if one increases the takeoff speeds above their "normal" values, the tire speed limit weight must necessarily decrease to offset the speed increase.

In the illustration to the right you can see the relationship between the tire speed limit weight without any improved climb, compared to the tire speed limit weight with improved climb.

If, for example, the tire speed limit weight is 800,000 pounds when using normal takeoff speeds, it must decrease to 775,000 pounds when using two units of speed increase.

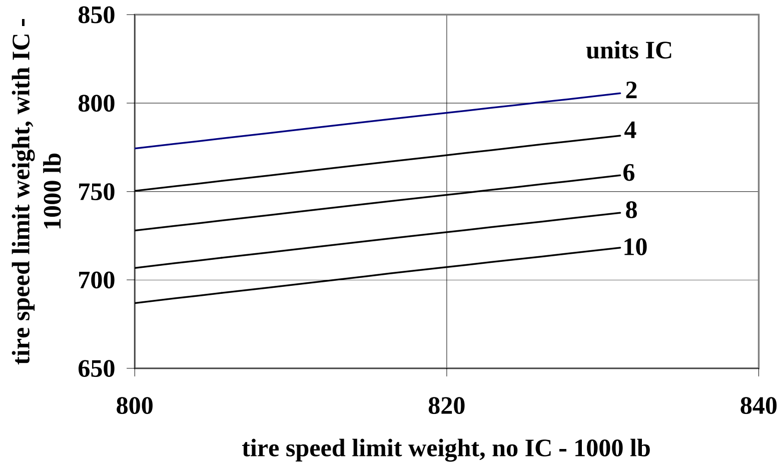


Figure 26-3

Notice in particular the rate of decrease of tire speed limit weight with the number of units of improved climb speed increase. That decrease of weight is greater than the decrease in weight due to the field length limits, which you've seen just above.

Should We Optimize V_R and V_2 For Climb?

Seeing how the climb limit weight improves with increased speed, it's logical to ask why the normal takeoff speeds aren't optimized for climb. Why, for example, isn't V_2 chosen to yield better climb performance?

The reason is simple: using faster takeoff speeds, optimized for climb, would have an adverse effect on the field length limit weights. In order to maximize the field length limit weight, two factors have the greatest effect: acceleration rate (in other words, the ratio of thrust to weight) and the takeoff speeds. Lower takeoff speeds will result in shorter takeoff distances – yielding more field length limit weight for a given runway length available.

Thus the normal takeoff speeds are optimized for takeoff distance rather than climb gradient – and improved climb performance data is provided for those cases where climb or obstacle performance is more restrictive than takeoff distance performance. This approach to the presentation of takeoff performance data offers performance engineers the tools for achieving the highest possible takeoff weights under any combination of conditions.

How the Improved Climb Technique Works

In preparing the charts above, we used the AFM-DPI software to compute the limit weights. Now we're going to show the data in a different manner to make it more useful: we're going to plot the limit weights versus the amount of speed increase.

For the conditions selected for this exercise, here's a table of the data from the AFM-DPI software.

You see that for normal speeds, the field length limit weight is 813,887 pounds and the climb limit weight is 747,219 pounds. The tire speed limit weight exceeds the maximum allowed in the software until the speed increase is at least six units.

Let's plot these numbers for the field length limit weight and the climb limit weight, then we'll illustrate how they can be used together to yield higher allowable takeoff weights.

units IC	fl lim	climb lim	tire spd lim
0	813887	747219	
1	808686	751709	
2	803543	756058	
3	798457	760288	
4	793425	764435	
5	788444	768682	
6	783512	772442	827304
7	778628	776131	815291
8	773795	779841	803669
9	769008	783369	792422
10	764267	786621	781532
11	759569	789956	770987
12	754913	792728	760770
13	750297	795520	750870
14	745714	798182	741272

Table 26-1

In the graph to the right we show the effect of speed on the field length limit weight.

As expected, the field length limit weight decreases with increasing takeoff speeds.

Thus for takeoff conditions yielding a field length limit weight of (in this example) of 813,887 pounds, one unit of speed increase decreases that to 808,686 pounds, two units decreases it to 803,543 pounds, and so on.

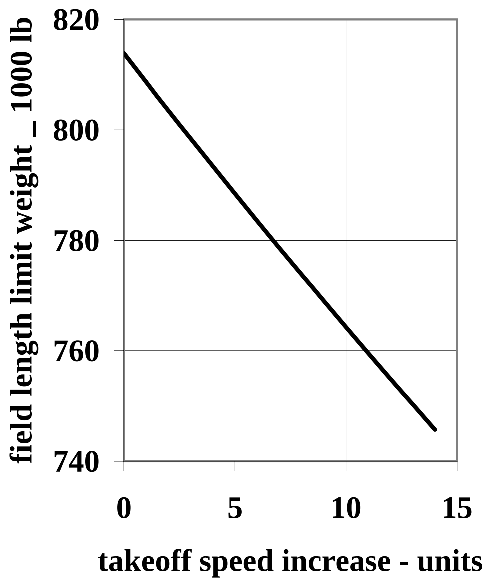


Figure 26-4

In the graph you see the effect of speed increase on the climb limit weight.

Here, as expected, you can easily see that any increase of the takeoff speeds above the normal values will produce higher climb limit weights because of the reduction of the ratio of drag to lift.

For conditions yielding a climb limit weight of 747,219 pounds with normal speeds, one unit of speed increase improves that to 751,709 pounds, two units increases it to 756,058 pounds, and so on.

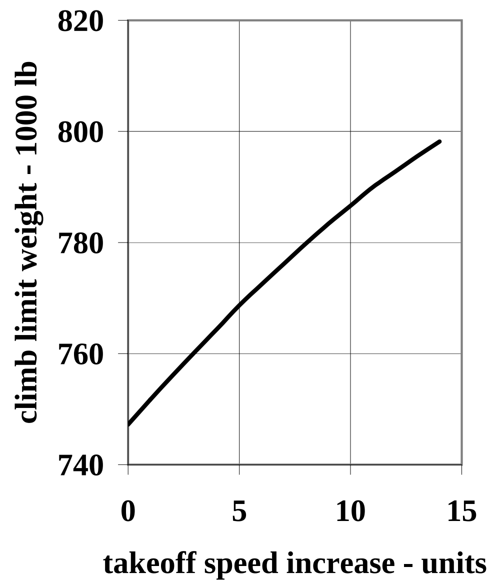


Figure 26-5

The two graphs just seen should give you an idea: for these example takeoff conditions, since the climb limit weight is less than the field length limit weight, why not use some speed increase for this takeoff, thereby getting a higher allowable takeoff weight? Runway length available clearly isn't a problem, at least not at the smaller amounts of speed increase, because the field length limit weight will still be greater than the climb limit weight and therefore isn't limiting.

Let's look at this example in a slightly different manner: here we're showing <u>both</u> the field length limit weight and the climb limit weight.

This chart shows you, in a single glance, just how the improved climb technique works: when for normal takeoff speeds the climb limit weight is less than the field length limit weight, simply <u>increase the takeoff speeds to the point where the two weight limits become equal</u>. The weight and speed increase at the intersection point are the solution to the improved climb limit condition. AFM-DPI calculated that the intersection for these conditions occurs at 777,200 pounds, with a speed increase of 7.3 units.

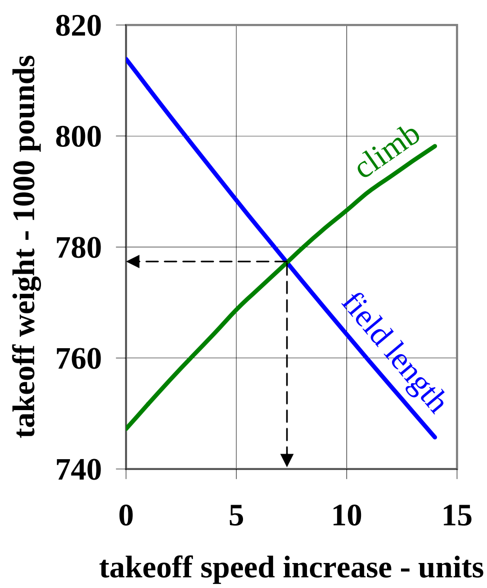


Figure 26-6

Could you increase the speeds above 7.3 units? Yes, but then your weight would become limited by field length performance at a weight less than 777,200 pounds. Therefore 7.3 units of speed increase is the optimum value.

How about the tire speed limit weight? Is that ever a factor?

Here is an illustration of the variation of tire speed limit weight with speed increase. Note two things in particular:

First, for the takeoff conditions used in this example, the tire speed limit weight isn't at all limiting at normal speeds and low amounts of speed increase.

Second, notice once again that the rate of decrease of tire speed limit weight with speed increase is much more rapid than is the decrease of the field length limit weight.

This fact is very significant. The following is a demonstration of why we say that.

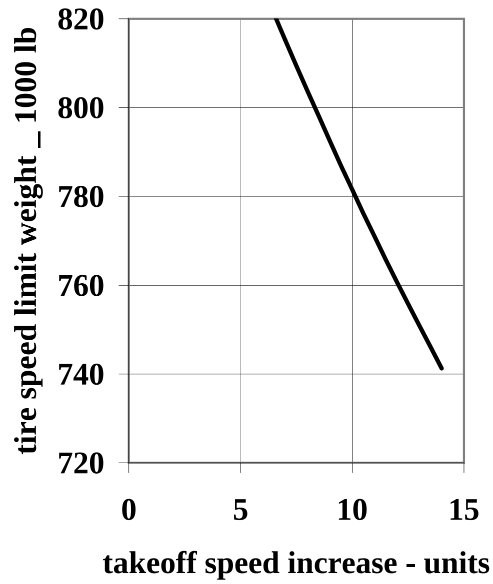


Figure 26-7

Look at the table to the right of the three limit weights. (These weights are for takeoff conditions that are different from those used in the first example.)

units IC	FL lim	cl lim	tire lim
0	635099	598208	658982
2	626637	604358	638081
4	618298	610149	618508
6	610055	615642	600177
8	601982	620633	582942
10	594068	624753	566747

Table 26-2

Notice that at normal speeds, that is, zero units of speed increase, the tire speed limit weight is substantially larger than the field length limit weight. It would be understandable if you were to believe that tire speeds couldn't become limiting, that the optimum weight and speed increase will come at the intersection of the field length and climb limit weights, just as they did in the example above. But that would be incorrect in this instance.

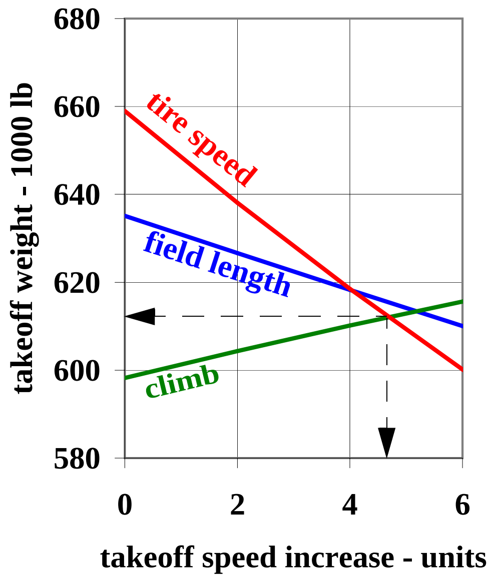


Figure 26-8

Here's a plot of the data in the table just above. You see that the tire speed limit weight starts – with no speed increase – well above the field length limit weight. But because of its greater slope, it will intersect the climb limit weight at a lower value of speed increase than the field length limit line will.

Thus, in this case the amount of improved climb that can be used is 4.7 units, for a new climb limit weight that is equal to the tire speed limit weight at approximately 612,000 pounds.

Had we not been careful, had we assumed that tire speeds wouldn't become limiting, we might have used the intersection of the field length limit line with the climb limit line, which would yield a slightly higher improved climb limit weight – <u>but which would have exceeded the tire speed limit weight</u>. This example illustrates clearly why it's always advisable to check the tire speed limit weights when using the improved climb technique. For most normal takeoff conditions, tire speeds will <u>not</u> become limiting with improved climb, but there are cases when it can, as you've just seen.

Presentation of Improved Climb Data in the AFM

The Airplane Flight Manual presents improved climb data in a more general manner that should, by now, be easy to understand. The figure to the right shows a sample of an AFM improved climb chart.

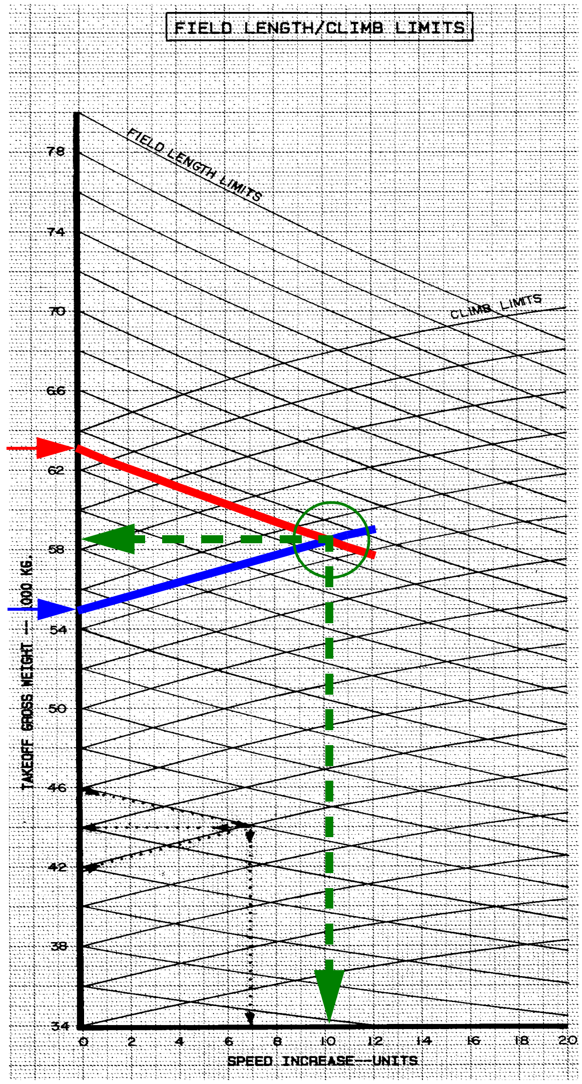


Figure 26-9

There are actually two charts: one, like the illustration to the right, is for the field length and climb limit weights; the other, identical in design, is for the tire speed and climb limit weights. Having the two charts allows the user to check all of the limit weights so as to avoid the possibility of accidentally exceeding the tire speed limit weight, as in the example shown above.

The first step in using these charts is to determine, from the other AFM charts, the field length limit weight, the climb limit weight, and the tire speed limit weight. If the takeoff weight is climb limited, then you can expect that some improvement will be available by using the improved climb technique. By using the improved climb charts, you can easily find the new climb limit weight and the amount of speed increase needed to achieve it.

In the chart, you see a “family” of field length limit lines and climb limit lines. The downward sloping lines show the decrease of field length limit weight with increasing speed. The upward sloping lines show the increase of climb limit weight with increasing speed.

To use this kind of improved climb chart is simple: enter with the field length limit weight at the weight scale on the left and follow the two nearest lines down and to the right; enter at the weight scale with the climb limit weight and follow the two nearest lines up and to the right. At the intersection of the two, read down to find the amount of speed increase, and read to the left to find the improved climb limit weight.

Follow the same procedure with the tire speed and climb chart, to be sure that the tire speed limit isn’t accidentally exceeded.

Improved Climb Technique and Obstacle Clearance

In an earlier chapter entitled “Obstacle Limit Takeoff Weight” we mentioned briefly how the use of the improved climb technique could – sometimes – yield higher obstacle limit takeoff weights. Let’s look at that subject in more detail here.

The heart of the subject of improved climb and obstacle clearance lies in this pair of facts:

- Use of improved climb speed increments will increase the takeoff distances required for acceleration and liftoff to the 35 foot point;
- Use of improved climb speed increments will increase the climb gradient available for the takeoff vertical profile commencing at the 35 foot point.

We have plotted below some data taken from AFM-DPI for a 777 showing, for constant weight, two profile choices: with no speed increase, and with 10 units of speed increase. We could have chosen more or less than ten units – this is just an example.

Notice that the vertical scale is obstacle height – that is, the height of an obstacle that can be cleared by the legally required amount – versus distance from brake release.

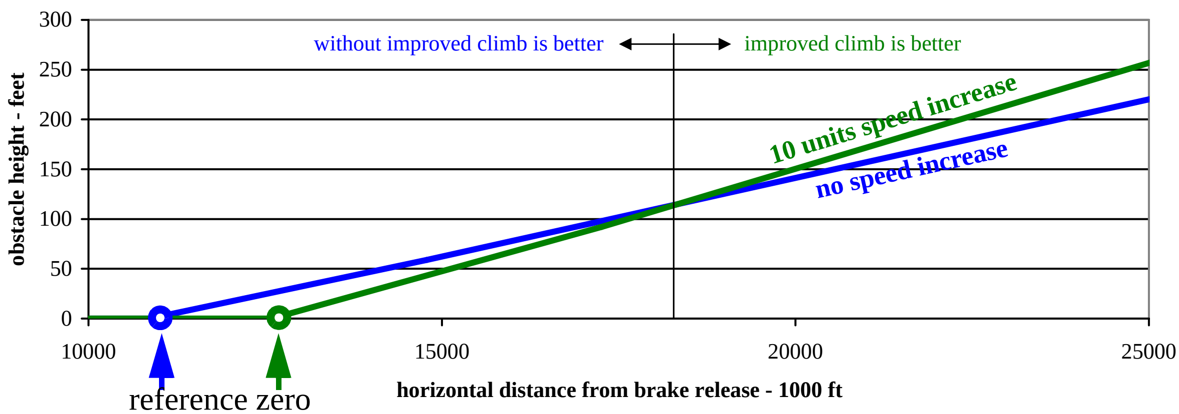


Figure 26-10

The point at which the climb profile begins is, of course, reference zero. You’ll see that the reference zero point for the profile without speed increase is at about 11,000 feet from brake release, whereas the reference zero point for the profile with speed increase occurs at about 12,500 feet. You’ll see also that the climb gradient with the increased speeds is greater than it is at the normal speeds.

For close-in obstacles, less than 18,400 feet from brake release, the profile without any speed increase is actually superior because the obstacle distance from reference zero is more important than the climb gradient. For obstacles farther out than 18,400 feet, some speed increase will result in better obstacle limit weights because of the greater gradient. Of course, that “crossover” distance isn’t always 18,400 feet – it will be different for other takeoff conditions.

Hence, when obstacles limit the allowable takeoff weight there may be extra runway length available to use for increased takeoff speeds, it's worth trying the improved climb technique to see if it will offer a better takeoff weight.

Units of Speed Increase

The first Boeing airplanes to utilize improved climb provided the speed increase in "percent". This was the amount of increase of the engine inoperative climb speed V2 expressed as a percentage.

Later airplanes provided the improved climb limit weight effects in terms of "units" of speed increase. The exact definition of "unit" is complex; it is chosen for engineering reasons when developing the improved climb weight data; basically, however, one unit of speed increase is one percent of a speed which is used as a reference speed.

In either case, the exact definition of the speed increments isn't particularly useful. For the convenience of the user, the flight manuals of the non-AFM-DPI airplanes provide charts showing the increased takeoff speeds; the AFM-DPI and BPS software provide the actual values for the increased speeds in units of knots.

Shown to the right is a small sample of the AFM chart for improved climb speeds for the earlier airplanes that used "percent" speed increase.

You can see, for example, that if a normal V_2 speed of 100 knots is subject to a 10 percent increase, the improved climb V_2 speed will be 110 knots.

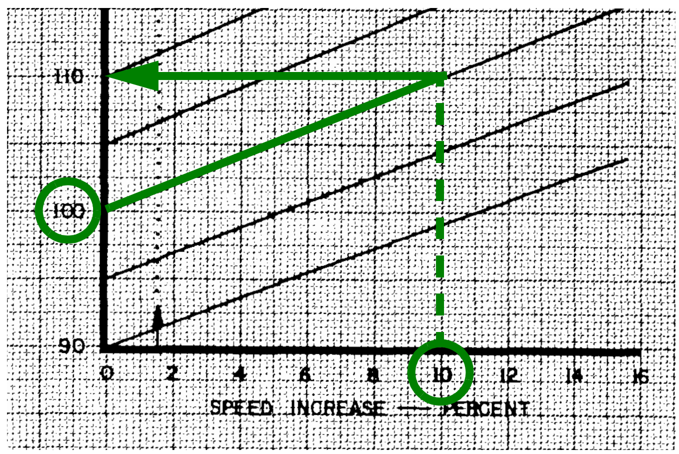


Figure 26-11

For airplanes using "units" of speed increase, the AFM chart is in the same format, as you see to the right.

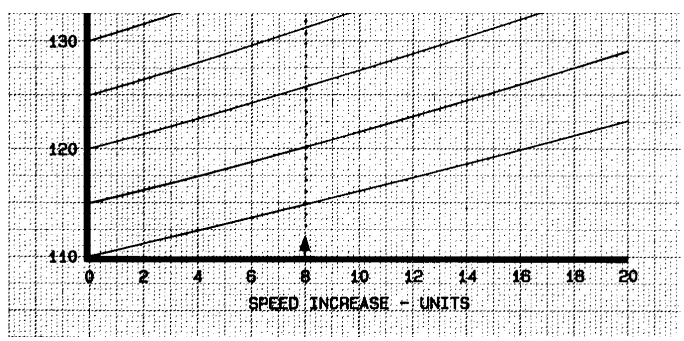


Figure 26-12

Optimizing Field Length and Climb Performance

Up to this point, you have seen the effect of increased takeoff speeds on the field length limit weight, the climb limit weight, the tire speed limit weight, and the obstacle limited weight. The first two of these limitations are the most frequently encountered performance limitations, and they behave in the same way at all airports. Let's look at how the field length limit weight and the climb limit weight behave for any takeoff. This will give us a better overall understanding of how the improved climb technique can benefit takeoff performance.

In the chart to the right, we have plotted the field length limit weights and the climb limit weights for the three flap settings of a 777. The vertical scale is runway length, the horizontal scale is weight.

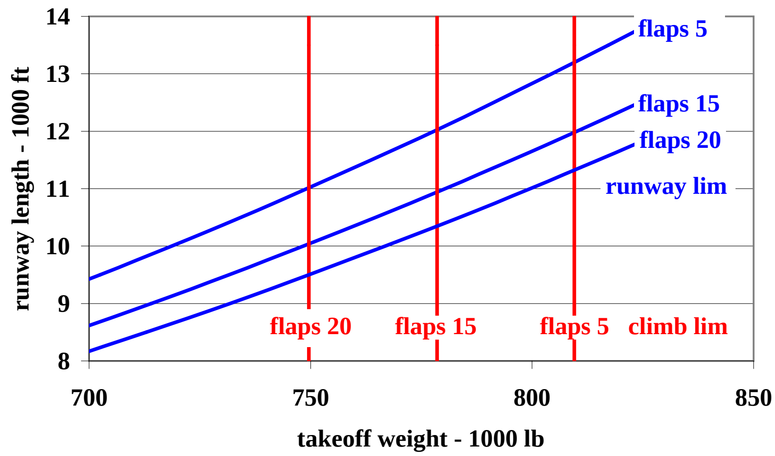


Figure 26-13

Observe the three upward sloping lines: these are the field length limit weight lines. Obviously, the largest flap setting, flaps 20, has the greatest field length limit weight for any given runway length. Observe also the three vertical lines: these are the climb limit weights for the three flap settings. Of course, they're not a function of runway length.

When you take the most limiting weights at each flap setting, you get a graph such as you see at the right.

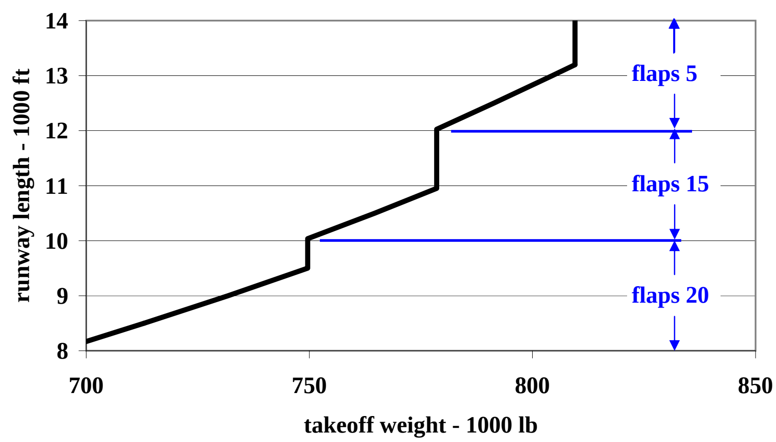


Figure 26-14

Any runway length up to approximately 10,000 feet, flaps 20 will provide the highest weight, first being field length limited and then climb limited. From 10,000 to approximately 12,000 feet, flaps 15 offers the best weights. Above approximately 12,000 feet, flaps 5 will be the best flap setting.

Now let's see how we can improve that plot by using improved climb technique.

To the right, we have taken a portion of the chart you just saw and enlarged it so that you can see more clearly the benefit of improved climb on the allowable takeoff weight. In this chart, we are considering only the field length and climb limit weights, there is no consideration of obstacles.

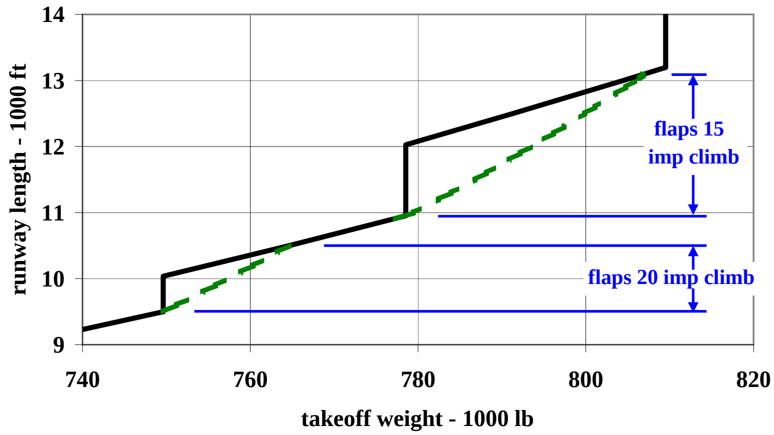


Figure 26-15

Observe the two broken lines: the lower one represents the allowable takeoff weight with flaps 20, using improved climb technique; the upper one shows the allowable takeoff weight with flaps 15, using improved climb technique.

As you can see, improved climb technique allows us to “fill in” the corners where the weight line is vertical because it is climb limited. The weight increases can be very substantial; you see in the chart above, for example, that for a runway length of 12,000 feet the improved climb benefit exceeds 14,000 pounds.

(You may have noticed that we’re not showing improved climb performance for flaps 5 in the above chart. That’s because the takeoff weight becomes brake energy limited for these conditions at a runway length of 13050 feet.)

The benefits of improved climb will vary with the airplane model and other takeoff conditions. Still, whenever you find that the takeoff weight will be limited by climb or by obstacles, it’s worth checking to see if improved climb technique will provide some additional takeoff weight.

For pilots, using the improved climb technique does not require any special procedures or skills – the takeoff procedure is the same as usual, with the only difference being higher takeoff speeds than normal. It’s the responsibility of the performance engineer to ensure that flight crews are provided with the correct takeoff speeds whenever improved climb technique is to be used, since the pilots’ usual sources of takeoff speed information probably display only the normal takeoff speeds for the given takeoff weight.

Improved Climb For McDonnell Douglas Airplanes

The wing flap system design philosophy used by the McDonnell Douglas Corporation on their commercial jet transport airplanes is somewhat different from that used by The Boeing Company on its models.

As you’ve seen, Boeing airplanes are certified for takeoff at any one of a number of different takeoff flap deflection settings, such as flaps 1, 5, 15 and 20 for the 757-200 and 767-200 airplanes. A smaller flap deflection such as flaps 1 offers superior climb limit takeoff weights but requires longer takeoff distances. A larger flap deflection such as flaps 20 offers superior field length limit takeoff weights but the climb performance suffers due to the greater drag.

Obstacle limit takeoff weights are also affected by the takeoff flap deflection. However, although a smaller flap deflection will always improve the climb limit takeoff weight, it won't always improve the obstacle limit takeoff weight. Sometimes it will, sometimes it won't – it depends on the location and height of the obstacle.

For optimizing takeoff performance on Boeing airplanes, the performance engineer's job is to select for takeoff that flap position that offers the best compromise between climb/obstacle performance and takeoff distance performance. The engineer may then be able to utilize the improved climb technique to gain some additional takeoff weight, as we have discussed in detail in this chapter.

McDonnell Douglas airplanes do not have a limited number of certified flap settings for takeoff as do the Boeing airplanes. Their airplanes are designed with an infinitely variable takeoff flap setting, allowing the selection of any desired deflection angle between the minimum and maximum values. The MD-11, for instance, can perform takeoffs at any desired deflection angle between 10 and 25. This design feature has come to be nicknamed "dial-a-flap".

Takeoff conditions vary widely from day to day and from location to location. Temperatures, elevations, runway lengths, obstacles: all of these are critical variables when determining the allowable takeoff weight. Some airports such as those at high elevations and high temperatures, but having very long runways, may lead the performance engineer to decide that the optimum flap deflection is the smallest one available in order to maximize climb performance. Other airports, especially those having short runways, may be such that the optimum takeoff flap deflection is the greatest one available, in order to shorten the required takeoff distances.

For many airports, however, the optimum flap setting is somewhere in between the minimum and maximum values. For these, the dial-a-flap design offers the ability to select for takeoff the exact deflection that makes the climb limits just equal to the takeoff distance limits. No additional optimization techniques such as improved climb will offer any improvement in the allowable takeoff weight.

Clearly, certification of takeoff performance for dial-a-flap airplanes will be different from the certification for Boeing airplanes. Boeing has in the past, and in the present as well, preferred to certificate a small number of takeoff flap settings, and to provide additional improved climb performance data for use in optimizing the takeoff weights when and where such optimization is required. Both approaches are valid, each has advantages and each has drawbacks.

Chapter 27: Reduced Thrust for Takeoff

Introduction

In all of our discussions of takeoff performance-limited weights up to this point, we have based our calculations on the use the maximum thrust available for the takeoff. You'll recall from the chapter entitled "Thrust Ratings" that the maximum allowable thrust for takeoff is the rating called – appropriately – "maximum takeoff thrust" and that the engine thrust setting parameters for maximum takeoff thrust are published in the Airplane Flight Manual.

However, when you think about your operations you'll probably see that for a large percentage of your flights the takeoff weight is less than the maximum allowable value: maybe the weight is below the limit because it's a shorter flight with a lighter fuel load, or maybe because it's a cool day at an airport having long runways and no obstacles.

So if your takeoff weight will be less than the maximum, do you need to use maximum thrust for takeoff? Are you required to use maximum takeoff thrust for all takeoffs?

The answer to both questions is "no". If your takeoff weight will be less than the maximum value, you don't need to use maximum takeoff thrust, and you are not required to do so. The regulatory agencies allow for the use of *reduced takeoff thrust* when the takeoff weight permits it; further, both airframe and engine manufacturers recommend the use of reduced takeoff thrust whenever possible. The Airplane Flight Manuals for all Boeing airplanes, and the AFM-DPI software for the more recent models, all include provision for operation at reduced thrust levels for takeoff.

Why use reduced thrust for takeoff? That's easy: the two principal benefits are reduced maintenance and improved engine reliability. The Boeing Company, and all of the companies providing engines for our airplanes, encourage operators to use reduced thrust in their operations where possible.

In this chapter we'll look at two different methods you can use to calculate reduced takeoff thrust settings, and you'll see that doing so will have substantial benefits to your operation. First, however, let's look at the benefits to an operator.

The assistance of GE Aviation in the preparation of this chapter is gratefully acknowledged.

NOTE: In the following discussion of reduced thrust, we will be showing you illustrations of data and formats taken from various Boeing documents, such as the Flight Planning and Performance Manual, the Airplane Flight Manual, the Flight Crew Operations Manual, and tabulations of takeoff data produced by the Boeing takeoff analysis software. These are useful for demonstrating the performance, methods and principles of reduced takeoff thrust.

Increasingly, however, reduced thrust as practiced by pilots on the flight deck relies more on electronic sources of data such as the Flight Management Computer System (FMCS) which gives pilots the appropriate thrust settings, stabilizer trim, and in some cases limited takeoff speeds, or some form of onboard performance computer such as the Boeing Onboard Performance Tool (OPT).

Performance engineers should take the time to familiarize themselves with both sources of information and should strive to ensure that the two forms as used in their operations provide the same information, regardless of the formats used. In a later chapter, entitled "Airplane Performance and the FMCS" we will be discussing the relationship between the printed and electronic sources of information.

Benefits of Using Reduced Takeoff Thrust

GE Aviation, manufacturer of many of the engines used on today's commercial jet transports, describes the benefits of reduced thrust in this way:

> *Three engine parameters that affect engine severity*[1] *are rotor speeds, internal temperature and internal pressure. Operating an engine at a lower thrust rating or at reduced thrust reduces the magnitude of these parameters, thus reducing engine severity.*
>
> *Less severe operation tends to lower exhaust gas temperature (EGT) deterioration. Since lack of EGT margin to redline is a major cause of scheduled engine removals, lowering the EGT deterioration rate increases the time on wing between shop visits.*
>
> *Fuel flow deterioration varies directly with EGT deterioration rate. For example, for CF6 engines, an EGT deterioration of approximately 10 degrees centigrade equates to a fuel flow deterioration of approximately 1%. Since reduced thrust reduces EGT deterioration rate, fuel flow deterioration rate is also reduced.*
>
> *Maintenance costs are reduced because of the longer time between shop visits and the lower labor and material costs of the shop visit to restore the engine to a specified condition.*

The text above emphasizes the financial benefits of reduced thrust operations, and they are substantial. But there's a second major benefit: safety. While it's harder to quantify the safety benefits, reduced thrust operations have been clearly shown to decrease the failure rate of some engine components; anything that improves engine reliability is certainly going to have a favorable effect on safety.

The benefits derived from a policy of reduced takeoff thrust depend on a number of factors. The number of flights per day, the average length of the flights, the amount of thrust reduction, the operating environment – all of these factors and more must be considered. We encourage airline staff to work closely with their engine manufacturer(s) when studying the potential benefits of a

1. The term "severity" in this context refers to the hardship experienced by engines operating at high thrust levels, affecting maintenance costs and engine reliability.

reduced takeoff thrust program or when implementing such a program. All of the manufacturers providing the engines installed on Boeing airplanes have done extensive work on reduced thrust and are ready to assist operators if necessary.

The Two Methods For Takeoff Thrust Reduction

The two methods for conducting takeoffs with decreased thrust settings are known as the *assumed temperature* method and the *derate* method. Both are widely used.

Let's be careful about terminology as we proceed through this chapter. In the relevant regulatory material, the FAA and EASA refer to the assumed temperature method as "reduced takeoff thrust" and to the derate method as "derated takeoff thrust". This has the potential to cause confusion, since both are methods for achieving a level of takeoff thrust that is less than the certified maximum takeoff thrust provided in the AFM. Boeing generally uses the term "reduced takeoff thrust" in a broader sense, to mean any method of achieving decreased thrust for takeoff, whether by the assumed temperature method or by the derate method.

the assumed temperature method

Let's say that we have done a takeoff weight analysis for a runway at one of our airports. We have checked all of the limit weights, and find that – in this example – the takeoff weight will be limited by either the climb requirement (at the lower and upper temperatures), or the field length requirement (at the intermediate temperatures). Obstacle, tire, and brake energy limit weights are not factors as they're greater than the climb and field length.

In Figure 27-1 to the right, you see the structural, climb, and field length limit weight lines.

In this example, the structural limit weight is never the smallest of the limit weights, so it's not a factor.

At some temperatures the climb limit weight is the smallest, at some other temperatures, the field length limit weight is the smallest.

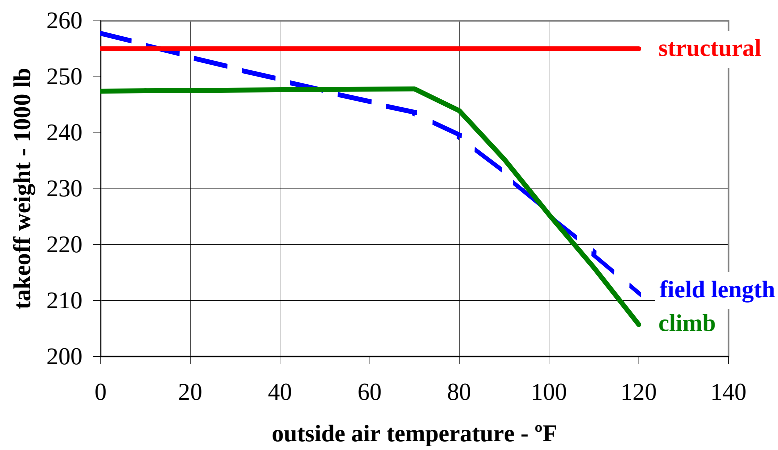


Figure 27-1

If we graph only the most limiting weight, as a function of temperature, it will look like the plot you see here. A plot such as this one will be different for every runway, since the weights depend on variables that are different for each runway – different runway lengths, different obstacles, different airport elevations, and so on. But for any given runway, you can make a chart like this one, showing the maximum allowable takeoff weight versus temperature. You could plot a family of lines on a chart like this for different winds, or perhaps for different altimeter settings.

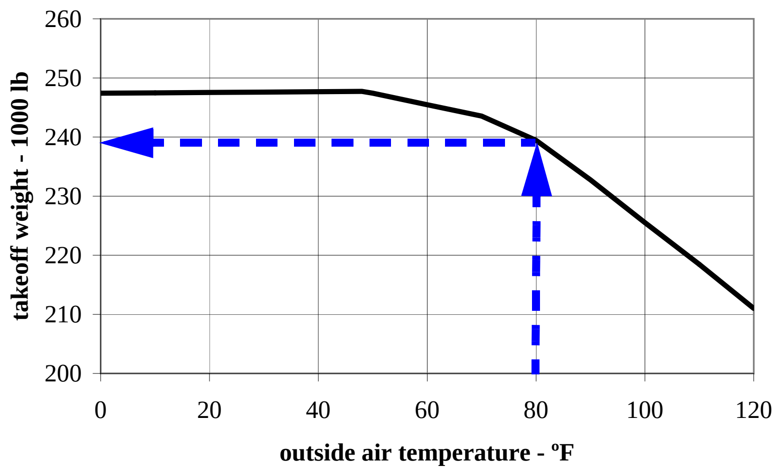


Figure 27-2

In ordinary operations work, this kind of chart would be used to determine the maximum allowable takeoff weight for this runway at a given temperature. If, for example, the expected takeoff temperature will be 80 °F, the allowable takeoff weight will be about 238,500 pounds.

But if we were to say that, for example, today's flight departing from that runway will have a takeoff weight of 230,000 pounds, then <u>we could use the takeoff weight chart backwards to find a maximum temperature for the given weight</u>.

In the illustration, you can see that the maximum allowable temperature for today's takeoff weight of 230,000 pounds would be 94 °F.

We're going to call this the *assumed temperature*, and <u>we're going to determine the takeoff power setting (EPR or N1) for today (80°) that would be result in the rated thrust for a 94° day</u>. We'll also find the takeoff speeds for that temperature.

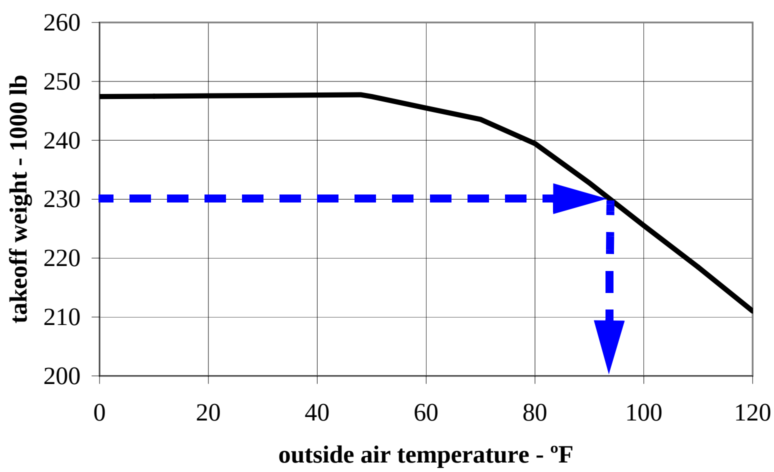


Figure 27-3

In the case of engines whose primary thrust setting parameter is N1, there is a special procedure for determining reduced thrust settings using assumed temperatures. That will be discussed later in this chapter.

For engines whose thrust setting parameter is EPR, it's necessary only to find the EPR corresponding to the assumed temperature and use that EPR for takeoff.

The method we've just described is what's called the assumed temperature method, sometimes abbreviated to just ATM. It's simple, and because we're determining the assumed temperature from a chart of the allowable takeoff weight for the runway to be used, it considers all of the factors that might limit the takeoff weight; hence there's no possibility of accidentally violating one of the takeoff weight limitations when we take off at the reduced thrust setting.

One advantage of the assumed temperature method is that it doesn't require the production of separate takeoff analysis tables for different thrust ratings; the existing takeoff weight data is used to find the assumed temperature. Contrast that with the derate method, which we'll discuss now.

derates

In an earlier chapter entitled "Thrust Ratings" we discussed the topic of derates.

You saw that a "derate" is an alternate thrust rating for an engine – a separate rating that yields less thrust than its normal maximum thrust rating. Because a derate is a thrust rating, even though it's a lower level of thrust than the engine is capable of, it is published in the Airplane Flight Manual and has legal status equal to the regular full thrust rating. The operator has the option of using either full rated thrust or derated thrust, depending on the amount of takeoff weight that will be needed for a flight. A lighter flight can possibly be operated at a derate, but a long flight needing a heavy takeoff weight may require the full rated thrust for takeoff.

Derates can take a number of forms. One such is a derate which is a fixed percentage below the full thrust capability of the engine. For example: the AFM for one of the 767s powered by the GE CF6-80 engine contains an appendix entitled "Operation at 10% and 20% Derated Thrust".

Another form of derate is to use the full thrust rating of a lower thrust version of an engine. For example, the AFM for a 737-800 fitted with CFM56-7B27 engines contains two derate appendixes, one of which is entitled "Operation of CFM56-7B27 Engines at 26K Derated Takeoff Thrust". The 7B27 engine designation indicates that it's operating at a 27,000 pound thrust rating. That engine is also certified to operate at lower thrust ratings of 26,000 pounds (26K) and 24,000 pounds (24K). Thus setting the N1s appropriate to the 26K or 24K rating on a 7B27 engine is operating it at a lower level of thrust – a "derate".

In the case of derates, the AFM provides all of the data needed to calculate the performance of the airplane at that lower thrust rating. For airplanes having the AFM-DPI software, if a derate is available it will be shown in the software as a user-selectable "alternate performance" option.

In the illustration to the right we show the takeoff weights at three different thrust levels for a 737: full thrust, and two derates.

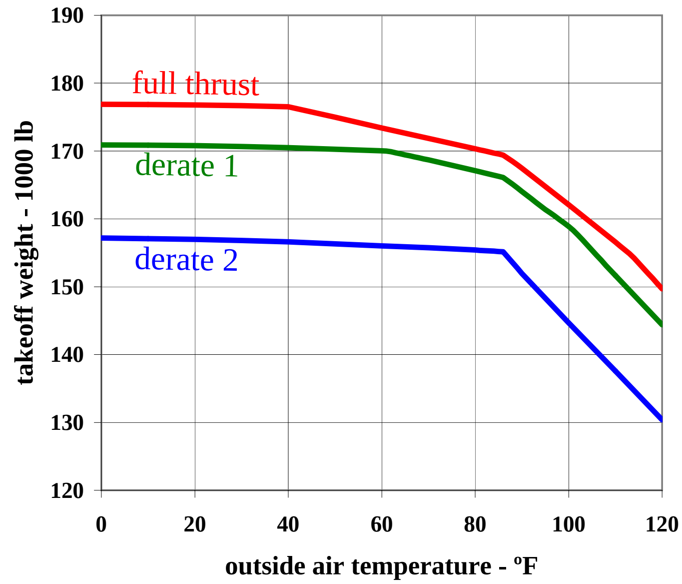


Figure 27-4

Practicing a reduced thrust policy by using derates is very simple. Let's say, for example, that the outside air temperature is 15°C (59°F).

Then if the takeoff weight expected for a flight is 172,000 pounds, you would need to use full rated thrust because the weight is greater than the weight allowed at derate 1 at the given temperature.

For any weight between 173,000 and 170,000 pounds, you must use full rated thrust. If it's between 170,000 and 156,000 pounds, you could use derate 1. If the expected weight is 156,000 pounds or less, you could use derate 2 thrust.

One disadvantage of the derate method is that it requires the operator to prepare and publish more than one takeoff weight analysis for the runway, whereas the assumed temperature method requires only one – the full thrust analysis. When accomplishing reduced takeoff thrust through the use of derates, the operator is obliged to prepare separate takeoff analyses for the full takeoff thrust rating and for each derate that may be used. This may or may not be seen as a serious disadvantage, depending on the operator's operational practices.

variable takeoff rating (VTR)

A newer variation on the idea of derates is a thrust-setting system called Variable Takeoff Rating, or VTR.

Available on the 747-400, 777, 717, 787, MD-11 and MD-90 airplanes, VTR is a derate thrust setting function provided by the Flight Management Computer System (FMCS). What makes VTR different from other derates is the fact that VTR allows an operator to select two derate thrust levels, called derate 1 and derate 2, <u>set to any two different percentages of thrust reduction desired by the operator</u>. On the ground, the operator can set the VTR derates 1 and 2 to any thrust reduction percentages (within a specified percentage range) that it thinks are optimum for its operation. The pilot can then select derate 1 or derate 2 (called TO1 and TO2) through the FMCS if desired for takeoff, in accordance with the airline's derate policy and the airplane's takeoff weight.

How the VTR function can be used to optimize the takeoff weight capability will become more evident when we discuss the effect of derated thrust on VMC-limited takeoffs, later in this chapter.

For VTR-equipped airplanes, the AFM or AFM-DPI allows the user to compute the takeoff performance at any percentage of thrust reduction, thus allowing the production of takeoff weight tables for any derate level.

Regulatory Background

The practice of using reduced thrust for takeoff has existed for many years. The FAA published Advisory Circular 25-13, entitled "Reduced and Derated Takeoff Thrust (Power) Procedures", in 1988.

The third paragraph of the AC sums it up nicely:

> *3. BACKGROUND. Takeoff operations conducted at thrust (power) settings less than the maximum takeoff thrust (power) available may provide substantial benefits in terms of engine reliability, maintenance, and operating costs....*

In the fifth paragraph, the AC sets out the rules for the use of reduced takeoff thrust. This portion of the AC is somewhat long, but it's worth quoting here because it contains a lot of useful information.

> *5. REDUCED THRUST: ACCEPTABLE MEANS OF COMPLIANCE. Under sections 25.101(c), 25.101(f), and 25.101(h) of the [Federal Aviation Regulations], it is acceptable to establish and use a takeoff thrust setting that is less than the takeoff or derated thrust if:*
>
> *a. The reduced takeoff thrust setting:*
>
> *(1) Does not result in loss of systems or functions that are normally operative for takeoff such as automatic spoilers, engine failure warning, configuration warning, systems dependent on engine bleed air, or any other required safety related system.*
>
> *(2) Is based on an approved takeoff thrust rating or derating for which complete airplane performance data is provided.*
>
> *(3) Enables compliance with the applicable engine operating and airplane controllability requirements in the event that takeoff thrust, or derated takeoff thrust (if such is the performance basis), is applied at any point in the takeoff path.*
>
> *(4) Is at least 75 percent of the takeoff thrust, or derated takeoff thrust if such is the performance basis, for the existing ambient conditions...*
>
> *b. Relevant speeds (V_{EF}, V_{MC}, V_1, V_R, and V_2) used for reduced thrust takeoffs are not less than those which will comply with the required airworthiness*

controllability criteria when using the takeoff thrust (or derated takeoff thrust, if such is the performance basis) for the ambient conditions...

c. The airplane complies with all applicable performance requirements, including the criteria in paragraphs a and b above, within the range of approved takeoff weights, with the operating engines at the thrust available for the reduced thrust setting selected for takeoff. However, the thrust settings used to show compliance with the takeoff flight path requirements of section 25.115 and the final takeoff climb performance requirements of section 25.121(c) should not be greater than that established by the initial thrust setting.

d. Appropriate limitations, procedures, and performance information are established and are included in the AFM.

e. A periodic takeoff demonstration is conducted using the airplane's takeoff thrust setting and the event is logged in the airplane's permanent records. An approved engine maintenance procedure or an approved engine condition monitoring program may be used to extend the time interval between takeoff demonstrtions.

f. The AFM states, as a limitation, that takeoffs utilizing reduced takeoff thrust settings:

(1) Are not authorized on runways contaminated with standing water, snow, slush, or ice, and are not authorized on wet runways unless suitable performance accountability is made for the increased stopping distance on the wet surface.

(2) Are not authorized when the antiskid system, if installed, is inoperative.

(3) Are not authorized unless the operator establishes a means to verify the availability of takeoff or derated takeoff thrust to ensure that engine deterioration does not exceed authorized limits...

g. The AFM states that:

(1) Application of reduced takeoff thrust in service is always at the discretion of the pilot.

(2) When conducting a takeoff using reduced takeoff thrust, takeoff thrust may be selected at any time during the takeoff operation.

h. Procedures for reliably determining and applying the value of the reduced takeoff thrust setting and determining the associated required airplane performance are simple (such as the assumed temperature method). Additionally, the pilot is provided with information to enable him to obtain both the reduced *takeoff thrust and takeoff thrust, or derated takeoff thrust if such is the performance basis, for each ambient condition.*

f. Training procedures are developed by the operator for the use of reduced takeoff thrust.

6. DERATED THRUST: ACCEPTABLE MEANS OF COMPLIANCE. For approval of derated takeoff thrust provisions, the limitations, procedures, and other information prescribed by Section 25.1581 of the FAR, as applicable for approval of a change in thrust, should be included as a separate Appendix in the AFM. The AFM limitations section should indicate that when operating with derated thrust, the thrust setting parameter should be considered a takeoff operating limit. However, inflight takeoff thrust (based on the maximum takeoff thrust specified in the basic AFM) may be used in showing compliance with the landing and approach climb requirements of Sections 25.119 and 25.121(d), provided that the availability of takeoff thrust upon demand is confirmed by using the thrust-verification checks specified in paragraph 5e above.

That's a lot of text we've quoted above. We'll be discussing the provisions of the AC as we proceed in this chapter.

Before we go any farther, let's discuss the safety implications of performing takeoffs at reduced levels of thrust. One might naturally think that taking off with less than full takeoff thrust would decrease the takeoff margins of safety; in the following discussion, we'll make it clear that this isn't the case.

Safety of Reduced Thrust Takeoffs

As an illustration of the inherent safety of the assumed temperature method, let's take a 757-200 powered by Rolls-Royce RB211-535E4 engines. Here are the conditions of the example:

- flaps 15
- sea level, OAT = 16°C (61°F)
- 7,200 foot runway
- field length limit weight = 244,200 pounds
- expected takeoff weight = 224,600 pounds

For these conditions, examination of the takeoff analysis for this runway would show that the expected takeoff weight of 224,600 pounds would permit an assumed temperature of 40°C (104°F).

In the table 27-1 below we compare the performance of this airplane at an actual temperature of 40°C to the performance at an actual temperature of 16°C but using an assumed temperature of 40°C as permitted by the expected takeoff weight.

Look at the first line: EPR. The EPR will be the same in both cases. If it's actually 40° we'll use the EPR for 40°, and if it's actually 16° but we're assuming 40°, we'll use the EPR for 40°.

Parameter	Actual temp is 16°C and assumed temp is 40°C	actual temp is 40°C	ATM Margin
EPR	1.654	1.654	
V_1 (knots IAS / TAS)	142 / 142	142 / 148	-6 knots
V_R (knots IAS / TAS)	145 / 145	145 / 151	-6 knots
V_2 (knots IAS / TAS)	149 / 149	149 / 155	-6 knots
Thrust at V_1, lb per engine	29,309	29,224	85 pounds
FAR field length, feet	6, 672	7,173	501 feet
Accelerate-**stop** distance (engine inop), feet	6, 672	7,173	501 feet
Accelerate-**go** distance (engine inop), feet	6, 672	7,173	501 feet
Accelerate-go distance (all engine), feet	5,676	6,086	410 feet
second segment gradient	3.48%	3.46%	+0.02%
second segment rate of climb, feet per minute	525	543	-18 ft/min

Table 27-1

Now look at the next three lines showing the takeoff airspeeds V_1 , V_R and V_2. We show the airspeeds both as indicated (calibrated) airspeed and as true airspeeds. The indicated airspeeds are the same for an actual 40° as they are for 16° assuming 40°, because whether we're actually at 40° or assuming 40°, we'll use the takeoff speeds for 40°. Notice, however, the difference in the true airspeeds.

For typical takeoff pressure altitudes and speeds, the equivalent airspeeds are essentially the same as the indicated (calibrated) airspeeds. Thus the true takeoff airspeeds are given by:

$$V_{true} = \frac{V_C}{\sqrt{\sigma}} = \frac{V_C \times \sqrt{\rho_0}}{\sqrt{\rho}}$$

where V_{true} is the true airspeed
V_C is the indicated (calibrated) airspeed

σ is the air density ratio $\frac{\rho}{\rho_0}$

ρ is the air density

ρ_0 is the sea level standard day air density

When we're assuming 40° but it's actually 16° the air density is greater than it is at an actual temperature of 40°, and hence the true airspeed, for the same indicated airspeed, will be slower since true airspeed is an inverse function of the square root of the air density.

Notice too that the engine thrust at V_1 is slightly greater when at 16° assuming 40° than at an actual 40°; this is due to the greater air density, although it's a very small effect. On some engines the difference is negligible.

Thus the true takeoff speeds, in the 40° assumed temperature takeoff case are slower than the true takeoff speeds if it's actually 40°. Since takeoff distances are a function of the <u>true</u> airspeeds to which the airplane must accelerate, (and because we have slightly more thrust) <u>the takeoff distances will be shorter at 16° assuming 40° than they are at an actual 40°</u>. You see in the table that the engine-inoperative distances are about 500 feet better (shorter) in the assumed temperature case!

Finally, you'll notice that the climb gradient is essentially unchanged. This is due to the fact that the thrust is only very slightly different for the two conditions. The rate of climb, however, is slightly better at the actual 40° because the true airspeed is slightly faster at the actual 40° than it is at an actual 16°.

From the above, then, you can see that the assumed temperature method is inherently conservative; the performance margins during an ATM takeoff are better than those during a takeoff at the limit takeoff weight.

How about the safety of derate takeoffs? While they don't have the inherent conservatism of the ATM, using derates maintains <u>at least</u> the normal regulatory performance margins. And unless the expected takeoff weight is right at the maximum allowed for the selected derate level of thrust, the performance margins will be better than the regulatory margins.

Determining the Reduced Thrust Power Settings

How are the takeoff thrust settings – EPR or N1, depending on your engines – determined for reduced thrust takeoffs? The answer depends on which of the two reduced thrust methods you're utilizing.

for the assumed temperature method

When using the assumed temperature method, the way you will determine the reduced thrust takeoff EPR is very different from the way you will determine the reduced thrust takeoff N1. It is essential that this be clearly understood, because treating them both the same way could, under some conditions, result in unintentionally exceeding engine limitations on an engine that is set by N1.

In either case, the method for determining the assumed temperature is the same.

Look at the illustration to the right. You see a typical line of maximum allowable takeoff weight versus outside air temperature.

Enter the horizontal axis with the actual temperature expected at the time of takeoff; move vertically upward to the weight line and then to the left, to find the maximum allowable takeoff weight for that runway at that temperature.

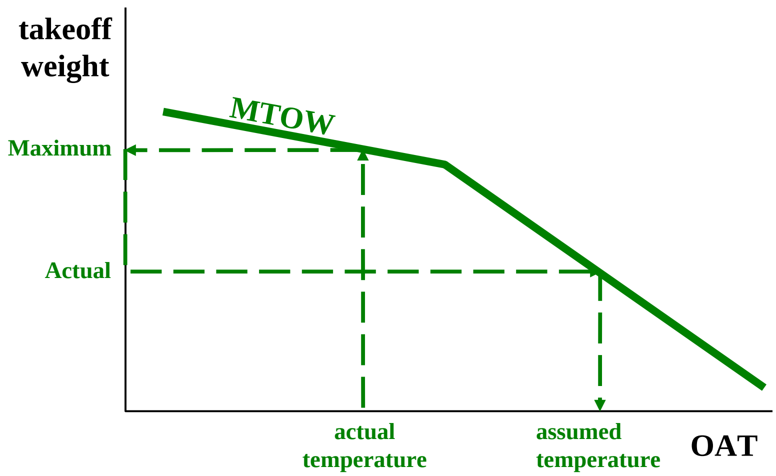


Figure 27-5

However, if the actual takeoff weight is expected to be less than the maximum, you can use the chart backwards: entering on the vertical axis with the actual takeoff weight, read to the right and then down to find the assumed temperature.

Now, what you do with that assumed temperature to find the reduced thrust takeoff setting depends on whether you're setting EPR or N1. Let's look at that carefully.

reduced thrust EPR

If we modify the previous illustration a bit by adding a line of takeoff EPR versus temperature and a scale of EPR on the right, you can see how to find the reduced thrust EPR: simply read upward from the assumed temperature to the EPR line, then to the right to the scale. The EPR you read for the assumed temperature is the correct EPR to use for the reduced thrust takeoff.

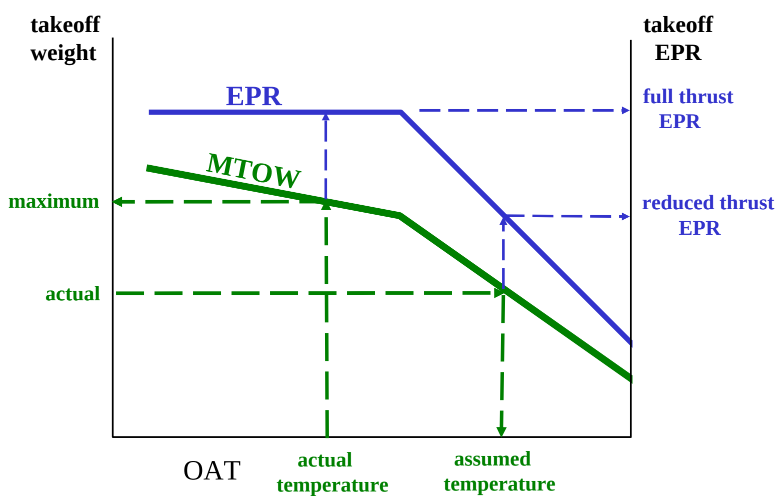


Figure 27-6

reduced thrust N1

Here's where you must be careful. You do NOT simply read the takeoff N1 corresponding to the assumed temperature – in fact, to do so can result in a serious exceedance of the engine RPM limitations. Yes, you first find the assumed temperature, just as you do for an EPR airplane, but that's where the similarity ends.

You may recall, from the chapter entitled "Jet Engine Fundamentals", that we defined a parameter called "corrected N1".

We defined corrected N1 as:

$$corrected\ N1\ = \ = \frac{gauge\ N1}{\theta_T^x}$$

where "gauge N1" is the actual RPM, in percent, of the engine low-pressure rotor
θ_T is the total air temperature ratio
x is a power to which θ_T must be raised; it is provided by the engine manufacturer

*Gauge N*1 is the name commonly given to the form of N1 that is suitable for pilots to use on the flight deck for setting thrust. That is, it's a true measure of the engine's low pressure rotor RPM, and it's sensed by a tachometer driven by the engine rotor.

Gauge N1 is NOT, however, a direct indicator of the amount of thrust being produced by the engine. For example, In the flat-rated region of takeoff thrust where the thrust is essentially independent of temperature, the gauge N1 decreases with decreasing temperature; EPR, however, is <u>constant</u> below the flat-rate temperature.

<u>Corrected</u> N1 on the other hand IS an indicator of the amount of thrust being produced. What this means is that if you were to use the same gauge N1 at two different temperatures, yielding two different values of corrected N1, you would have two totally different thrust levels.
Because of this fact, determining the value of gauge N1 for the pilot to set when using the assumed temperature method is as follows:

1. Find the gauge N1 at the assumed temperature;
2. convert the gauge N1 to corrected N1 using the <u>assumed</u> temperature in the equation above;
3. re-calculate gauge N1 from the corrected N1 found in step 2, using the <u>actual</u> temperature.

You now have a reduced takeoff thrust N1 setting properly computed for the actual takeoff temperature.

This process can be done manually, but it's obviously tedious, time-consuming and error-prone. For that reason, Boeing publishes tables that make the determination of reduced thrust N1 simple, given the assumed temperature.

for the derate method

Since a derate is considered to be a separate thrust rating and is published as a separate entity in the Airplane Flight Manual, it has its own thrust setting charts. If accomplishing thrust reduction by the derate method, then, since there's no assumed temperature, all that's needed is to determine the thrust setting – either EPR or N1 – from the thrust setting table for that derate at the actual takeoff temperature expected.

minimum assumed temperature

Let's go back to an earlier illustration, but let's change the conditions somewhat.

In Figure 27-7 you see a different set of conditions. In this case, we're taking off on a cold day, and the expected actual takeoff weight will be only slightly below the maximum allowable takeoff weight. Can you use reduced thrust for takeoff?

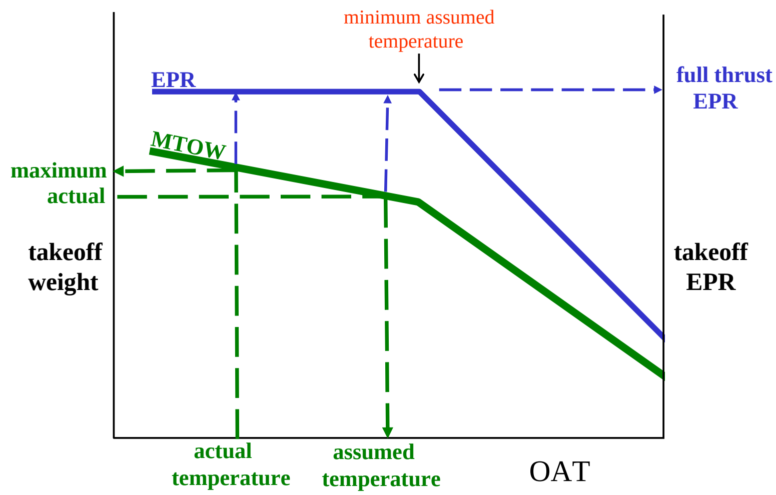


Figure 27-7

You see that while you might be able to set thrust to value corresponding to the assumed temperature, doing so wouldn't achieve any reduction of thrust. Why not? Because you're in the flat-rated region of thrust in this example, at both the actual and assumed temperatures.

You may remember from the discussion of "flat-rated thrust" in the chapter entitled "Thrust Ratings" that below some temperature referred to as the flat-rate temperature[1] the allowable takeoff thrust is limited by pressure and it becomes essentially constant regardless of outside air temperature. Therefore there is no thrust reduction if both the actual temperature and the assumed temperature lie within the flat-rate region of thrust.

For that reason, the flat-rate temperature is considered to be the minimum allowable assumed temperature. Reduced takeoff thrust is not possible when the assumed temperature is less than the flat-rate temperature.

maximum allowable thrust reduction

You may have noticed, when reading the Advisory Circular quoted above, the following phrase:

> *...it is acceptable to establish and use a takeoff thrust setting that is less than the takeoff or derated thrust if...the reduced takeoff thrust setting...is at least 75 per-*

---

1. This point is sometimes called the "break temperature" because it occurs at the transition from pressure-limited thrust to temperature-limited thrust, where a line of allowable takeoff weight has a "break" – that is, a sudden change of slope.

> *cent of the takeoff thrust, or derated takeoff thrust if such is the performance basis, for the existing ambient conditions...*

Phrased another way, this paragraph is saying that thrust may not be reduced by more than 25 percent below the maximum thrust available for the actual conditions.[1]

For the earlier airplanes, the Airplane Flight Manuals provide guidance on the maximum allowable thrust reduction.

Section 4 of the reduced thrust appendix to the 767 AFM, for example, shows the minimum allowable takeoff EPR as you see to the right.

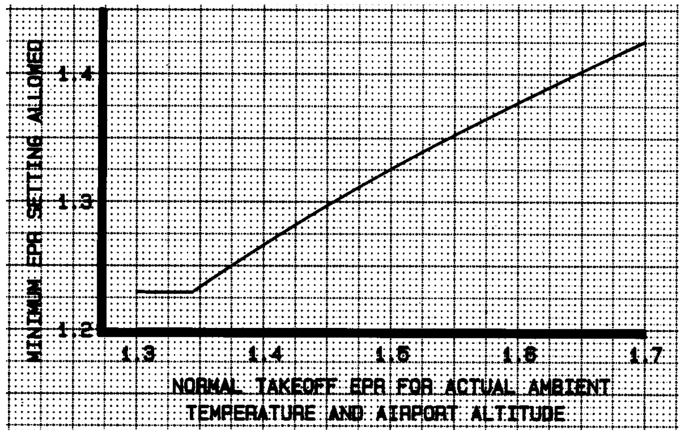


Figure 27-8

For the more recent airplanes, the AFM-DPI software will not permit data to be computed for a thrust reduction greater than 25 percent.

For all airplanes equipped with a Flight Management Computer (FMC) the system will not allow the thrust to be set to greater than a 25 percent reduction.[2]

Determining the Reduced Thrust Takeoff Speeds

When the takeoff thrust setting is reduced to a level less than the certified value, the takeoff speeds must be revised accordingly.

You will remember from the chapter entitled “Speeds” that the takeoff speeds V_R and V_2 for a given flap setting depend only on the thrust to weight ratio. The takeoff speed V_1 depends on the takeoff weight and the relationship between the accelerate-go and accelerate-stop distances.

1. It is worth noting here that a few airplane models <u>have</u> received authorization to extend this restriction beyond 25%.
2. For some more recent airplanes, such as the 777-200LR and -300ER, an option is available allowing 40% thrust reduction using the assumed temperature method. Note, however, that the 40% reduction can be applied to maximum takeoff thrust only. For derates, the maximum thrust reduction remains at 25%.

When taking off at a reduced level of thrust, the takeoff speeds must be determined for the new conditions. The conditions used to find the speeds will be the actual takeoff weight and the assumed temperature.

If the takeoff weight data is in the form of a tabulation of weight for different temperatures and winds, such at the one shown below, finding the takeoff speeds is simple. Let's say, for example, that today the temperature is 20 degrees, no wind, dry runway. We'll say that today's expected takeoff weight will be 70,000 kilograms. Following the procedure of finding the assumed temperature and speeds using the highest temperature at which the allowable takeoff weight is still greater than the expected takeoff weight, we would see that the assumed temperature is 44°C and the takeoff speeds V_1, V_R, and V_2 will be 144, 145, and 151 knots. Since those speeds are actually for 70,400 kilograms rather than 70,000 kilograms, they're not precise but are sufficiently accurate.

737-800 TAKEOFF PERFORMANCE KBFI RWY 13R LENGTH 10000 FT
CFM56-7B26 STANDARD CONFIGURATION BOEING FIELD ELEV 17 FT
FLAPS 5 FULL RATED (26K) RWY COND DRY

MAXIMUM ALLOWABLE TAKEOFF WEIGHT (100 KG) / TAKEOFF SPEEDS

TEMP (C)	WIND (KT) -10	0	5	10
50	647*/38-40-45	669*/41-42-47	673*/42-43-48	677*/42-43-48
48	658*/39-41-46	681*/42-43-48	685*/43-44-49	688*/43-44-49
46	669*/39-42-47	692*/43-44-50	696*/44-45-50	700*/44-45-50
44	680*/40-43-49	704*/44-45-51	707*/45-46-51	711*/45-46-51
42	691*/41-44-50	715*/45-46-52	719*/46-47-52	723*/46-47-53
40	702*/42-45-51	727*/46-47-53	731*/46-48-53	735*/47-48-54
38	713*/43-46-52	738*/47-48-54	742*/47-49-54	746*/48-49-55

Figure 27-9

There are many different formats of tabulated takeoff data. In the sample shown above, the takeoff speeds are included in the data; in many others, the takeoff speeds are not shown in the tabulation. If the assumed temperature is determined using a tabulation that doesn't include the takeoff speeds, then the speeds can be found from a takeoff speeds table such as this one:

737-800
CFM56-7B

BOEING

737 Operations Manual

Performance Inflight - General **Chapter PI Section 10**

Takeoff Speeds - Dry Runway
V1, VR, V2 for Max Takeoff Thrust

WEIGHT (1000 KG)	FLAPS 1			FLAPS 5			FLAPS 10			FLAPS 15			FLAPS 25		
	V1	VR	V2	V1	VR	V2	V1	VR	V2	V1	VR	V2	V1	VR	V2
84	162	164	169	155	158	163									
80	158	160	166	151	154	160	151	152	158	148	149	155			
76	154	156	163	147	150	157	147	148	155	144	145	152	141	142	149
72	149	151	159	143	145	154	142	144	152	139	140	149	137	138	146
68	145	146	156	139	141	150	138	139	148	135	136	145	132	133	143
64	140	142	152	134	136	146	133	134	145	130	131	142	128	129	139

V1, VR, V2 Adjustments*

TEMP		V1						VR						V2					
		PRESS ALT (1000 FT)						PRESS ALT (1000 FT)						PRESS ALT (1000 FT)					
°C	°F	-2	0	2	4	6	8	-2	0	2	4	6	8	-2	0	2	4	6	8
60	140	4	4	5	6			3	4	5	6			-2	-3	-3	-4		
50	122	2	3	4	5	6	7	2	3	4	5	6	7	-2	-2	-3	-3	-4	-5
40	104	1	1	3	3	5	6	1	1	2	4	5	6	-1	-1	-2	-2	-3	-4

Figure 27-10

Reading this tabulation for a weight of 70,000 kilograms and an assumed temperature of 44°C would yield speeds of 143, 145 and 151 knots.

minimum V_1 and V_R when using assumed temperatures

One of the more obscure paragraphs in the Advisory Circular reads as follows:

> *...it is acceptable to establish and use a takeoff thrust setting that is less than the takeoff or derated thrust if...the reduced takeoff thrust setting...enables compliance with the applicable engine operating and airplane controllability requirements in the event that takeoff thrust, or derated takeoff thrust (if such is the performance basis), is applied at any point in the takeoff path.*

It goes on to say:

> *Relevant speeds (V_{EF}, V_{MC}, V_1, V_R, and V_2) used for reduced thrust takeoffs* [may not be] *not less than those which will comply with the required airworthiness controllability criteria when using the takeoff thrust (or derated takeoff thrust, if such is the performance basis) for the ambient conditions...*

The meaning of this? If a problem were to occur during the reduced thrust takeoff and the pilot elected to advance the thrust levers from their reduced thrust setting to the setting for full thrust (or the derate maximum thrust for the actual conditions) it's essential that the airplane remain fully controllable when that thrust increase is applied.

Let's say, for example, that an engine fails one second before V_1 and the pilot decides to continue, rather than reject, the takeoff. While there is no need to advance the thrust levers to the maximum available thrust setting, since the performance is computed assuming that the throttles remain at their reduced thrust setting, neither is it prohibited to advance the thrust levers at the pilot's discretion if additional thrust is desired. Thus, if the V_1 were less than the minimum control speed corresponding to the maximum available thrust, airplane directional control could possibly be compromised under some conditions.

In other words: the V_1 used for the reduced thrust takeoff may not be less than the minimum V_1 corresponding to the actual – not the assumed – temperature. Similarly, the V_R used for the reduced thrust may not be less than the minimum V_R corresponding to the actual temperature.

If either the V_1 or the V_R computed for the assumed temperature (or both) is/are less than the minimum value for the maximum available thrust, it or they must be increased to equal the minimum value.

If V_R is increased to make it equal to the minimum value, then the V_2 speed must be increased by the same amount.

minimum V_1 when using derates

There is an important difference between the assumed temperature method and derates when looking at the minimum value of V_1.

We know that the minimum value for V_1 is V_{1MCG} or, on the older models, V_{MCG}. This is simply for reasons of lateral control in the event an engine fails at a slower speed, at which rudder control in yaw is relatively less than at a higher speed. For this reason, pilots are expected to reject a takeoff if an engine failure occurs prior to reaching V_1.

When conducting a takeoff using the assumed temperature method, you have just learned that the minimum control speed, and thus the minimum V_1 speed, must be kept at the value corresponding to the actual temperature rather than the assumed temperature even though a lesser amount of thrust is being used. This is not true when taking off using derated thrust.

Allow us to quote once again the relevant paragraph from AC 25-13:

> *Relevant speeds (V_{EF}, V_{MC}, V_1, V_R, and V_2) used for reduced thrust takeoffs* [may not be] *not less than those which will comply with the required airworthiness controllability criteria when using the takeoff thrust (or derated takeoff thrust, if such is the performance basis) for the ambient conditions...*

We underscored to the last four words to emphasize once again the difference between assumed temperature and derate takeoffs:

- A takeoff made at a reduced thrust level using the assumed temperature method must, when determining the minimum V_1, use the "ambient" – the actual – temperature. If the temperature is 15°C assuming 40°C, the minimum V_1 must be found using a temperature of 15°C in the full thrust minimum V_1 speed table, rather than for the assumed temperature.
- A takeoff made using a derate level of thrust should, when determining the minimum V_1, also use the ambient temperature, but using the minimum V_1 speed table for the derate level of thrust. The minimum V_1 at a derated level of thrust is less than the minimum V_1 for the same ambient temperature at full thrust.

When taking off using a derate level of thrust, it's still physically possible to manually advance the thrust levers to the full thrust setting even though doing so isn't necessary or encouraged. Thus, if taking off using derated thrust, advancing the thrust levers at the minimum V_1 value could possibly lead to loss of directional control in an engine-inoperative situation, because the minimum V_1 for a derate takeoff is the value corresponding to the lesser amount of thrust.

The Boeing Flight Crew Training Manuals address that possibility in the following way, when discussing reduced takeoff thrust:

> *The fixed derate is considered a limitation for takeoff. Takeoff speeds consider ground and in-air minimum control speeds (VMCG and VMCA) at the fixed derate level of thrust. Thrust levers should not be advanced beyond the fixed derate limit unless conditions are encountered during the takeoff where additional thrust is needed on both engines, such as windshear. A thrust increase, following an engine failure could result in loss of directional control.*

By specifying "where additional thrust is needed on both engines", the Flight Crew Training Manual is reminding pilots that a symmetric increase of thrust below the full thrust takeoff minimum V1 speed, as could occur on a derated thrust takeoff, will not cause any directional control problems, where advancing the thrust levers of the operative engine(s) following an engine failure – an asymmetric condition – could cause a problem at low speeds.

Combining Derate and Assumed Temperature

From the discussions above about the assumed temperature method and the derate method, it might occur to you to ask whether or not it's permitted to combine the two – that is, to find an assumed temperature based on a derate level of performance.

The answer is "yes, you can" if the weight is low enough to permit this. If you have read the wording of Advisory Circular 25-13, quoted above, very carefully, you will have seen this:

> *"it is acceptable to establish and use a takeoff thrust setting that is less than the takeoff or derated thrust if... the reduced takeoff thrust setting...is based on an*

> *approved takeoff thrust rating or derating for which complete airplane performance data is provided."*

In that quoted phrase, the term "reduced takeoff thrust setting" is meant in the sense of an assumed temperature thrust reduction.

Therefore, if data is available giving the allowable takeoff weights at a derate, it's permitted to apply the assumed temperature method to that weight data.

Restrictions on the Use of Reduced Thrust

There are times when you can't use reduced thrust for takeoff for performance reasons; there are other times when, even though the airplane performance would permit it, you're not permitted to use reduced thrust for other reasons.

First, recall the wording of Advisory Circular (AC) 25-13, quoted above:

> *It is acceptable to establish and use a takeoff thrust setting that is less than the takeoff or derated thrust if... the reduced takeoff thrust setting...does not result in loss of systems or functions that are normally operative for takeoff such as automatic spoilers, engine failure warning, configuration warning, systems dependent on engine bleed air, or any other required safety related system.*

This particular requirement has had an effect on the use of reduced thrust on several different Boeing models. On the 707, for example, Section 1 of the AFM appendix "Operation at Reduced Takeoff Thrust" contains the following paragraph:

> *Do not use reduced thrust procedures below actual ambient temperatures of 0 deg F (-18 deg C) for a 10 percent thrust reduction or 33 degrees F (1 degree C) for a 25 percent thrust reduction unless the 12.5 degree thrust lever quandrant switch has been installed during airplane manufacture, or by Boeing Service Bulletin 3208 or 3209.*

The reason for that particular prohibition was that at the smaller thrust lever angles that might be used for a reduced thrust setting, the takeoff configuration warning system would not be armed and hence the protection against a takeoff with an unsafe configuration (e.g. flaps not set for takeoff, or spoilers extended) would be rendered inoperative.

There was a similar restriction on the use of reduced thrust on the 727 airplanes.

Later in the AC appeared these words:

> *...takeoffs utilizing reduced takeoff thrust settings...are not authorized when the antiskid system, if installed, is inoperative.*

Accordingly, Section 1 of every reduced takeoff thrust AFM appendix has stated that “use of reduced thrust procedures is not allowed with anti-skid inoperative.”

The AC also states that:

> *...takeoffs utilizing reduced takeoff thrust settings...are not authorized on runways contaminated with standing water, snow, slush, or ice, and are not authorized on wet runways unless suitable performance accountability is made for the increased stopping distance on the wet surface.*

AFM appendixes for all Boeing airplane models prior to the 747-400 contain the words “Use of reduced thrust procedures is not allowed when the takeoff runway is contaminated with water, ice, slush, or snow.” These words are, you’ll observe, more restrictive than the quoted sentence from the AC. A regulatory agency person might well take the position that since the AFM prohibits takeoff on “...runways contaminated with water...”, this wording overrides the wording of the Advisory Circular. At present, Boeing is considering seeking relief from the regulatory agencies from this restriction, based on the fact that the AC does allow reduced thrust operations on wet runways provided that the loss of stopping performance on the wet runway is accounted for.

For the 747-600 through -900, 747-400, 757-300, 767-400 and 777 airplanes, the AFMs prohibit reduced thrust takeoffs on runways that are “...contaminated with standing water, ice, slush, or snow...” but they go on to say that “...use of reduced thrust procedures is allowed on a wet runway if suitable performance accountability is made for the increased stopping distance on the wet surface”, thus making them consistent with the Advisory Circular.

Here’s one of the most important restrictions imposed by the AC:

> *...it is acceptable to establish and use a takeoff thrust setting that is less than the takeoff or derated thrust if...a periodic takeoff demonstration is conducted using the airplane’s takeoff thrust setting and the event is logged in the airplane’s permanent records...*[and if]*...the AFM states, as a limitation, that takeoffs utilizing reduced takeoff thrust setings...are not authorized unless the operator establishes a means to verify the availability of takeoff or derated thrust to ensure that engine deterioration does not exceed authorized limits...*

The intent of the text quoted here is clear: regulatory agencies are willing to accept the use of reduced takeoff thrust as a means of reducing maintenance costs and improving engine reliability by reducing the stresses and temperatures imposed on an engine in service; they are not, however, willing to accept the use of reduced thrust for takeoff as a means of avoiding necessary engine maintenance actions.

In other words: if an engine is incapable of normal operation within limits at its rated thrust, it is not permitted to extend its time in service on the airplane through the use of reduced thrust techniques.

There is no single means of satisfying the requirements for periodic full-thrust takeoffs and verifying that their engine deterioration doesn't exceed the authorized limits. Different airlines have taken different approaches to meeting these requirements, and that's a matter for negotiation and agreement between the airline and its regulatory agency. That topic will not be discussed here.

Effect of Derated Thrust On V_{MC} Limited Takeoff Weight

Here's an interesting question for you: can you think of any set of conditions in which you can achieve a higher allowable takeoff weight by using derated thrust for takeoff?

Intuitively, you might say "No. Less thrust means less takeoff weight." That's true – usually. But it's also true that when using derated thrust for takeoff, the minimum control speed to be used is that for the derated level of thrust – which will be less than the minimum control speed at the full thrust rating. We can sometimes take advantage of that fact. Here's how:

Look at the illustration to the right. It shows the relationship between runway length and the field length limit takeoff weight, for a given set of conditions.

Look at the way that the field length limit weight varies with runway length. Starting with a long runway, as the runway length decreases, the field length limit weight and the V_1 both decrease.

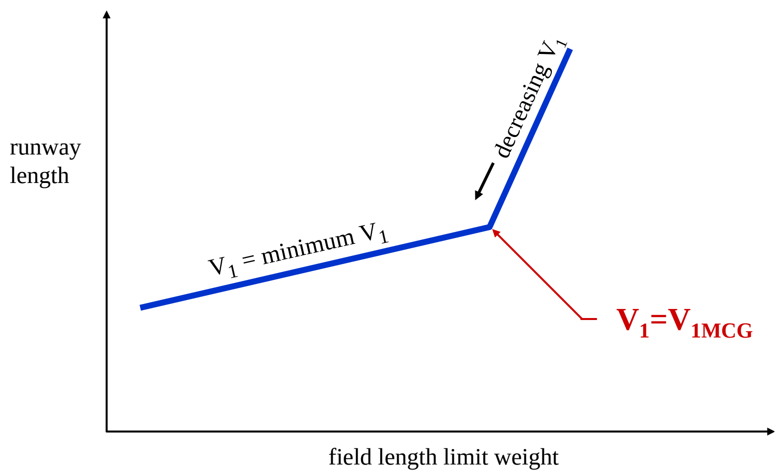


Figure 27-11

At some value of runway length, the weight has decreased to the point at which the corresponding V_1 has become equal to the minimum value V_{1MCG} (or V_{MCG} for the earlier airplanes).

As the runway length decreases further, the V_1 must remain constant at its minimum value; for that reason, as the runway length decreases the field length limit weight must decrease at a more rapid rate, hence the change in the slope of the line.

Now let's add another line to the chart, this one for takeoff at derated thrust.

Initially, as you would expect, the field length limit weight is less at the derated thrust than it is at full rated thrust. Also, as runway length decreases, the field length limit weight decreases in the same manner as for the full thrust case.

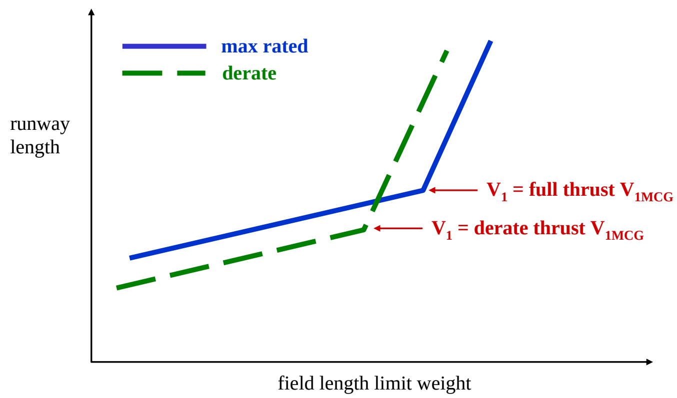


Figure 27-12

Notice, though, that the break in the line – where the V_1 has just become equal to its minimum value – occurs at a shorter runway length <u>because the minimum V_1 is a slower speed in the derate case</u>. And from that value of runway length on downward, the minimum control speed-limited field length limit weight line for the derate is to the right of the corresponding line for full thrust.

The end result of the relationship that you see in the illustration just above is that for shorter runways, on which the weight is minimum control speed limited, it is possible to get a higher field length limit weight by using a thrust derate.

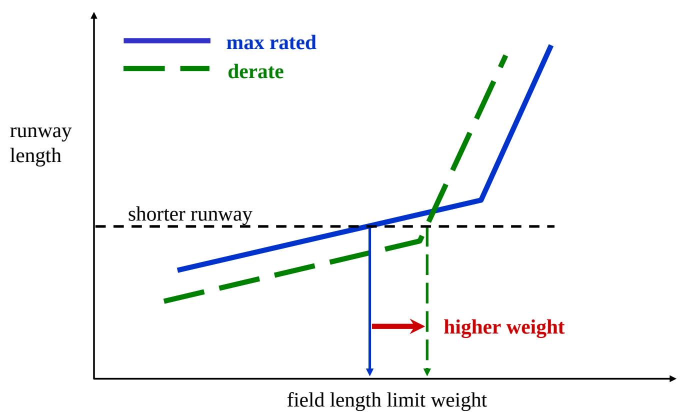


Figure 27-13

To give a real-world illustration of the principle discussed above, here's a chart showing some takeoff weights for a 767 with and without derates.

On a 4300 foot runway, for example, you can see that using a 10% derate offers a weight improvement of about 31,000 pounds!

Derated thrust as a means to improve the takeoff weight may be useful in only a small percentage of takeoffs, but it's still a good thing to keep in mind for those circumstances in which you find that your allowable takeoff weight is limited by the minimum control speed.

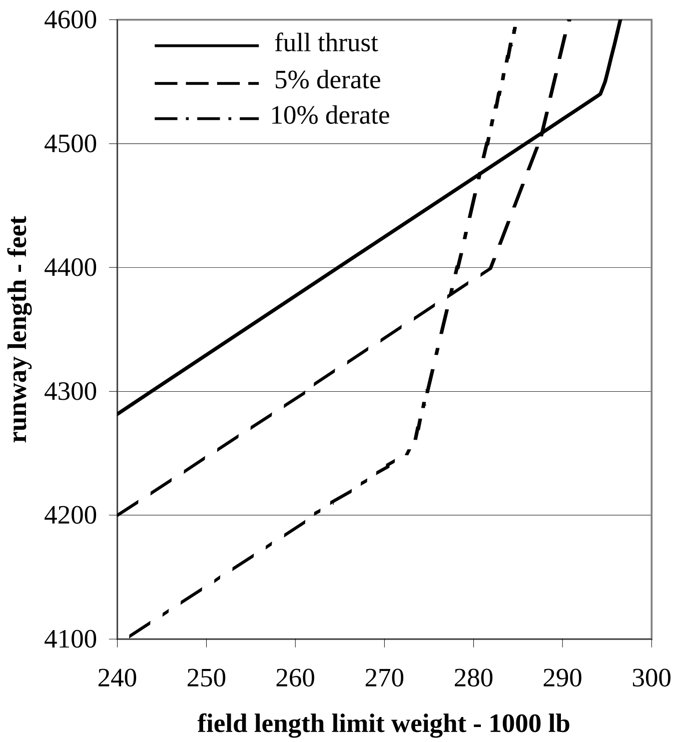


Figure 27-14

Takeoff Weight Benefits of VTR

Earlier in this chapter we discussed the Variable Takeoff Rating feature of the Flight Management Computer System (FMCS) that's offered on the 747-400, 717, 777 and 787, and on the MD-11 and MD-90 models. You saw that the VTR feature allows the operator to select any two percentages of thrust reduction for derate thrust levels TO1 and TO2.

An operator who understands well the minimum V_1 speed effect on takeoff weight that we discussed just above can analyze its takeoff operations with an eye to selecting the two thrust reduction percentages that will offer the greatest possible benefits.

As examples, refer back to Figure 27-14 above. For the takeoff conditions used in that chart, if taking off from a 4400-foot runway then a derate of five percent thrust would offer the best takeoff weight. Any other percentage would yield a lower allowable weight. Ten percent derate would be optimal for a 4250-foot runway.

Therefore, careful selection of the TO1 and TO2 derate percentages can offer substantial benefits on takeoff weight for operators in takeoff conditions that are limited by minimum control speeds.

Other Examples of Derated Thrust Benefits

There are a few other instances in which the use of derated thrust for takeoff can offer a benefit:

- Weight permitting, derated thrust takeoffs are permitted on non-dry runways, while assumed temperature takeoffs are not (with the exception of wet runways). Operators can use this fact to permit thrust reduction as part of an overall policy of reduced thrust for its maintenance and reliability benefits, or to increase the allowable weight when V_{MC} limited on a non-dry runway.

- When a takeoff must be conducted with the brake anti-skid system inoperative, the V_1 will be considerably reduced, increasing the likelihood of a V_{MC} limited takeoff. Here too a derated thrust takeoff may offer weight benefits.

Effect of Reduced Thrust On Stabilizer Trim

Before every takeoff, the stabilizer position will be set according to the airplane's weight and center of gravity. Setting the stab trim according to the takeoff conditions ensures that the pilots will use a consistent and comfortable force on the control column for rotation and initial climb.

Shown in Figure 27-15a to the right is a typical stab trim setting table for a 737. This table shows the effect of the weight and the CG.

Stab Trim Setting
Max Takeoff Thrust
Flaps 1 and 5

WEIGHT (1000 KG)	C.G. (%MAC)						
	6	8	9	11	16	21	26
85	8 1/2	8 1/2	8 1/2	8 1/4	7	6 1/4	5 1/2
80	8 1/2	8 1/2	8 1/4	7 3/4	6 3/4	6	5 1/4
75	8 1/4	8	7 3/4	7 1/2	6 1/2	5 3/4	5
70	8	7 3/4	7 1/2	7 1/4	6 1/4	5 1/2	4 3/4
65	7 3/4	7 1/4	7	6 3/4	6	5 1/4	4 1/2
60	7 1/4	7	6 3/4	6 1/2	5 3/4	5	4 1/4
55	6 3/4	6 1/2	6 1/2	6 1/4	5 1/2	4 3/4	4 1/4
50	6 1/2	6	6	5 3/4	5	4 1/2	3 3/4
45	6	5 3/4	5 1/2	5 1/2	4 3/4	4	3 1/2
40	6	5 3/4	5 1/2	5 1/2	4 3/4	4	3 1/2

Figure 27-15a

Because the engines of Boeing airplanes are mounted below the airplane's center of gravity[1], the thrust will have a nose-up pitch effect. The stab trim table shown as Figure 27-15a is computed for full rated takeoff thrust.

When preparing for a reduced thrust takeoff using the assumed temperature method (ATM), the pilots will calculate the stab trim setting using the table for full rated thrust. It follows that a reduced thrust takeoff using stab trim calculated for full thrust would result in slightly different control forces. The Flight Crew Training Manual addresses that issue:

> ***Note:*** *An increase in elevator column force during rotation and initial climb may be required for ATM takeoffs.*

1. except, of course, for the Boeing 717 and the McDonnell Douglas (now Boeing) DC-9, MD-80 and MD-90 series airplanes.

For derated thrust takeoffs, the pilots will use stab trim tables calculated for the lower level of thrust. Figure 27-15b to the right is an example of such a table, in this case for a 22K derate for the same airplane as Figure 27-15a above. Comparing the two, you see that the differences in stabilizer trim setting are less than one unit of trim.

Stab Trim Setting (22K Derate)
Flaps 1 and 5

WEIGHT (1000 KG)	C.G. (%MAC)					
	6	8	10	16	21	26
85	8 1/2	8 1/2	8 1/2	7 1/2	6 3/4	6
80	8 1/2	8 1/2	8 1/4	7 1/4	6 1/2	5 3/4
75	8 1/4	8	7 3/4	7	6 1/4	5 1/2
70	8	7 3/4	7 1/2	6 3/4	6	5 1/4
65	7 3/4	7 1/2	7 1/4	6 1/2	5 3/4	5
60	7 1/2	7 1/4	7	6 1/4	5 1/2	5
55	7 1/4	7 1/4	6 3/4	6	5 1/2	4 3/4
50	7	6 3/4	6 1/2	5 3/4	5	4 1/4
45	6 3/4	6 1/2	6 1/4	5 1/2	4 3/4	4
40	6 3/4	6 1/2	6 1/4	5 1/2	4 3/4	4

Figure 27-15b

An increase in thrust from the derate setting during the takeoff can be expected to increase the nose-up pitching tendency of the airplane and pilots should be prepared for that to occur it they elect to increase the thrust.

Effect of Reduced Takeoff Thrust On Noise

Since the community noise levels generated by commercial jet airplanes are of great concern in many noise-sensitive areas, questions are frequently asked about the effect on noise of operation with reduced takeoff thrust.

The answer is, simply, that the using reduced thrust for takeoff depends on where the sound is being measured.

- “Takeoff noise” – that noise measured on the ground directly beneath the takeoff flight path – is increased, because the airplane’s height above the ground will be reduced due to the decrease in the amount of thrust being used.
- “Sideline noise” – that noise measured on the ground along a line parallel to but laterally separated from the takeoff ground track by some distance – may increase or decrease, depending on the location of the noise monitors.

At the time of this writing (December 2009), the Boeing ClimbOut Program (BCOP) – a software application already available to facilitate calculations of the vertical and lateral climbout profiles following takeoff – is being enhanced to include the capability to calculate the community noise levels

The new Performance Engineers Tool (PET) software application just now (2009) coming into use will also be enhanced to permit noise calculations.

Readers interested in knowing the status and availability of these software applications should address any questions to the Boeing Flight Operations Engineering Group.

Chapter 28: Alternate Forward CG

Introduction

In preceding chapters discussing takeoff weight, we've seen a number of special methods that can be used by the knowledgeable performance engineer to obtain increases in the allowable takeoff weight and thereby improve the allowable payload for the flight. In the chapter entitled "Obstacle Limit Takeoff Weight", for example, we discussed the use of an increased V_1 when the takeoff weight is limited by a more distant obstacle. To use that method when the conditions permit it costs the airline nothing more than the effort of preparing the necessary data and making it available to the dispatchers and pilots.

Here's another method that can be used – sometimes – to obtain increases in the field length, climb, and obstacle limit weights. It's called "alternate forward CG". Unlike some methods such as increased V_1, however, this method requires an airline to purchase a revision to the Airplane Flight Manuals for the airplanes that will utilize this special technique.

The data that is used to generate takeoff weights in the AFM or AFM-DPI is proprietary. Since the weight benefits of alternate forward CG operation can be derived only from an operator's AFM or AFM-DPI, we're not going to use this chapter to display the algorithms used to derive the weights. Instead, the intent of this chapter is to familiarize the reader with the principles of operation at alternate forward centers of gravity, the potential benefits, and how an operator may assess whether or not alternate forward CG operation is feasible in its operating environment.

Airplane CG Limits

Every airplane delivered – whether a commercial jet transport airplane or a single-engine light airplane – has a precisely defined "envelope" within which the airplane's weight and center of gravity must always be located. An example is shown to the right.

You see that the weight limitations are shown vertically, and the CG limitations are shown horizontally. The CG limitations are shown in terms of *percent MAC;* this unit of measurement is discussed in the chapter entitled "Mass, Weight, and Center of Gravity".

It's not within the scope of this document to explain in detail the reasons for all of the limits prescribed by the envelope. Briefly, however, let's talk about the forward and aft CG limits.

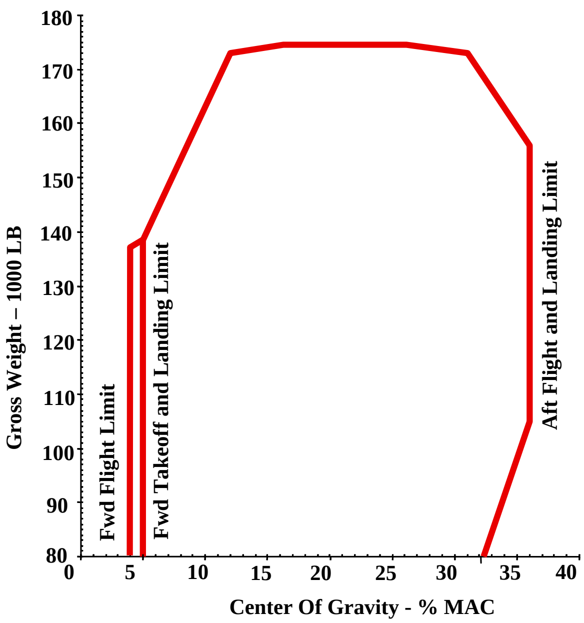


Figure 28-1

The more forward limits, designated by the lower %MAC numbers, are necessary restrictions to the CG location for any one or more of a number of reasons: the load placed on the airplane's nose landing gear, or the structural strength of the fuselage in bending, or the amount of aerodynamic force that can be generated by the horizontal tail of the airplane. These are all possible reasons for defining the forward limit of the airplane CG's allowable range of position, and there are others.

The aft limit of an airplane CG's allowable range of position (the higher %MAC numbers) may be the result of stability and control requirements, or loads on the main landing gear, or nose gear steering effectiveness, or other requirements.

The CG limits are different for every airplane model, according to its structural characteristics, its flight handling characteristics, and other factors.

For reasons that will become clear as you progress through this chapter, the location of an airplane's center of gravity affects its lift and drag. Due to these effects, more forward CGs will produce airplane performance capabilities that are different from those at more aft CGs. In some regards, a more forward CG position is more adverse. <u>Because of that fact, for conservatism an airplane's takeoff performance is always based on the assumption that the CG is at the certified forward limit</u>.

Some aspects of takeoff performance will always improve as an airplane's takeoff CG moves farther aft. Some other aspects of takeoff performance may or may not improve at more aft CGs. For <u>some</u> conditions, then, performance benefits could be achieved by moving the forward CG limit somewhat more aft, and using that <u>new</u> limit as the basis for computing the takeoff performance.

Alternate Forward CG Limits

From the above discussion, you've seen that moving the forward CG limit somewhat more aft may produce performance benefits. That fact gives rise to the "alternate forward CG limits" technique for achieving higher takeoff weights.

As you see to the right, two alternative forward limits can be chosen by the customer. If these selected limits replace the original forward limit, the takeoff performance will then be based on whichever limit is now selected for use – and since the alternate limit is more aft, higher takeoff weights may be available.

Let's look now at the aerodynamic reasons for the CG's effects on takeoff performance.

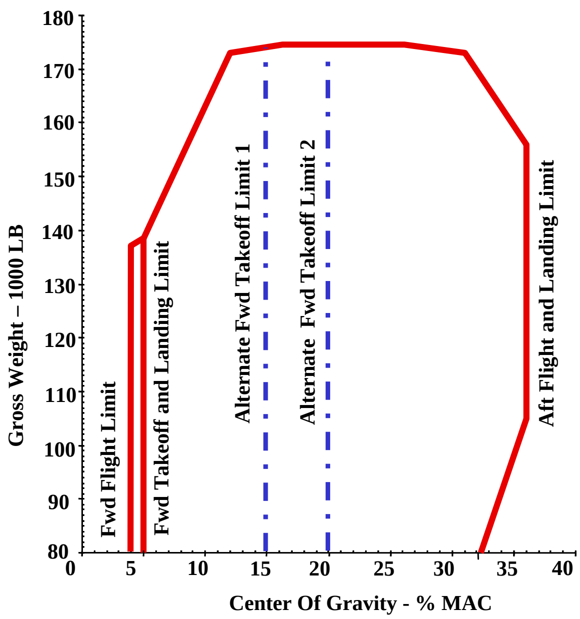


Figure 28-2

The Aerodynamics of Alternate Forward CG

There are many vertical forces acting on an airplane in flight. For simplicity in calculations, however, we can group them together into three separate vertical forces that are equivalent in their effects to the many actual forces. Thus:

The weight acts downward, at the airplane's center of gravity. This is discussed in detail in the chapter entitled "Weight, Mass, and Center of Gravity"

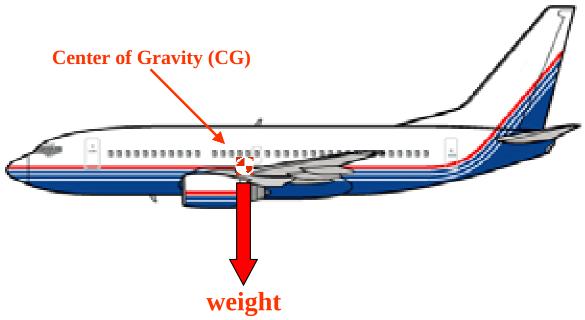


Figure 28-3a

The wing, and to a much smaller extent the body, both produce aerodynamic lift force upward. Taken together, we'll just call that the lift, acting at the point called the center of lift.

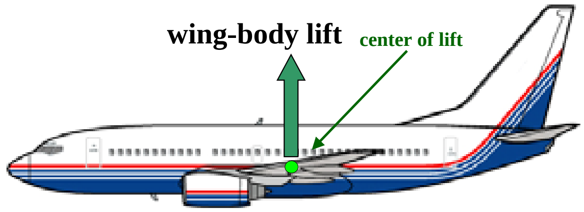


Figure 28-3b

The horizontal tail generates a downward aerodynamic force, acting at the tail's center of lift.

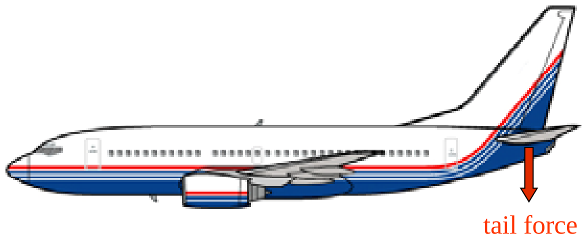


Figure 28-3c

For a conventional airplane having the horizontal stabilizer and elevators located at the tail of the airplane, the airplane's center of lift is always aft of (behind) its center of gravity. This configuration imparts static pitch stability: if the airplane pitches nose up about the center of gravity the lift force of the wing will increase, creating a restoring nose-down pitch moment about the CG.

Because of this configuration, the airplane's weight creates a nose-down pitching moment about the center of lift. Since the center of lift is like the point from which the airplane is suspended in the air, for the airplane to remain in steady-state flight we need to keep the moments around that point at a net value of zero – in "equilibrium". That's the task of the horizontal tail.

For simplicity, let's look at an analogy.

To the right, you see a simple balance beam, acted on by three forces. This is a workable analogy to the airplane and the three forces acting on it that we just looked at above.

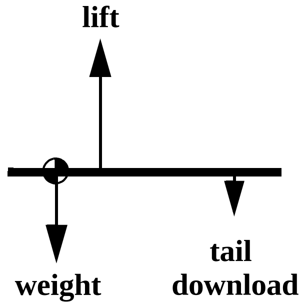


Figure 28-4a

The beam is suspended from above; that suspension creates an upward force that we'll call lift. It acts at a fixed location called the center or lift.

The beam has weight, acting downward, ahead (to the left) of the lift, at the beam's center of gravity. The center of gravity is movable.

To counter the effect of the weight, there must be a download behind (to the right of) the suspension point. The location of the download force is fixed.

For the beam to remain in balance, two conditions must be met:

- the sum of the three vertical forces must be equal to zero. Thus the upward force must be equal to the sum of the two downward forces:
- the two moments, generated by the weight and the tail download, acting about the suspension point, must be equal and opposite.

In this example, let's say that the beam weighs 30 pounds. If the CG of the beam is exactly at the suspension point, as shown in Figure 28-4b, the beam experiences no moment. No tail download is necessary. Then for balance in the vertical forces, the upward force would have to be equal to the downgrade force, or 30 pounds.

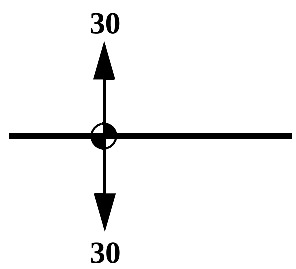


Figure 28-4b

But what happens if the beam's center of gravity isn't at the suspension point?

Let's say that it's now one foot to the left of the suspension point. That creates a counterclockwise moment of 30 foot-pounds about the suspension point. The beam isn't in balance. One way to restore the balance is to place a tail force downward and to the right of the suspension point, creating an equal and opposite moment. For the sake of our example, let's say that the point at which the counterbalancing force will be applied is three feet to the right of the suspension point.

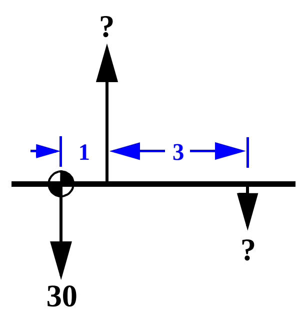


Figure 28-4c

How many pounds of force must be applied at the counterbalance point? And how will that affect the vertical balance of forces?

To keep the beam in equilibrium in moment, the counterbalancing force would have to be ten pounds, thus creating a clockwise moment of 30 foot-pounds about the suspension point, equal and opposite to the 30 foot-pounds of moment created by the weight of the beam.

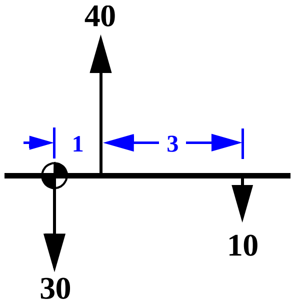


Figure 28-4d

To keep the beam in balance vertically, the upward force must be equal to the sum of the downward forces, or 40 pounds.

The sum of the 40-pound upward force and the 10 pound downward force is 30 pounds upward – equal to the weight.

Let's make one more change to the example. Let's say now that the CG of the beam has moved more to the left; it's now 1.5 feet to the left of the suspension point. We'll keep the distance from the suspension point to the downward counterbalancing force the same – three feet.

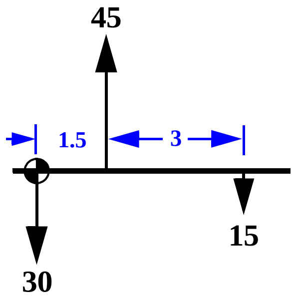


Figure 28-4e

To keep the beam in equilibrium, the counterbalancing force must now be 15 pounds. The upward force must now be 45 pounds.

The sum of the 45-pound upward force and the 15 pound downward force is again 30 pounds upward – equal to the weight.

Let's summarize what you've learned:

- As the CG moves more to the left, the lift must increase and the tail download must increase.
- The sum of the lift (upward) and the tail force (downward) is constant, equal to the weight.

An airplane behaves in exactly the same way as the beam we used as an analogy. Thus you see that for steady-state flight, if the CG moves more forward (toward the nose) the upward wing/body lift force must increase and the tail downward force must increase by an equal amount; if the CG moves more aft, the lift force will decrease and the tail downward force will decrease correspondingly. But in all cases, the sum of the lift force and the tail download force is constant, equal to the weight.

The consequences of these facts will affect an airplane's performance in these ways:

- As an airplane's CG moves forward, the wing must increase its lift, and the horizontal tail must increase its download. An increase in these two aerodynamic lift forces (one upward, one downward) will increase the drag forces as well. But the sum of the two aerodynamic forces remains constant, equal to the weight.
- As an airplane's CG moves forward, the wing-body lift must increase. That can only be accomplished by increasing the airplane's angle of attack. Thus an airplane's angle of attack depends in part on the CG position.

Let's talk about these two effects.

effect of CG on the drag polars

Let's define "net lift" as the sum of the aerodynamic upward lift force and the aerodynamic downward tail force. You have already learned that this sum must always be equal to the airplane's weight, regardless of the CG position.

An airplane's lift coefficient is calculated from the net lift force, which is equal to weight, thus:

$$C_L = \frac{net\ lift}{\frac{1}{2}\rho S V_{true}^2} = \frac{W}{\frac{1}{2}\rho S V_{true}^2}$$

where W is the airplane's weight
ρ is the air density
S is the airplane's reference wing area
V_{true} is the airplane's true airspeed

Thus in steady-state flight the lift coefficient doesn't depend on the CG position.

From discussions earlier in this book, however, you know that aerodynamic lift forces – whether upward (wing and body) or downward (horizontal tail) – are never achieved without causing drag, resulting primarily from the effects of viscosity, pressure distribution, and wingtip vortices. It follows that an increase in the two lift forces necessitated by a forward CG movement will cause a corresponding increase in drag. Thus we can see that for constant lift coefficient, the drag coefficient will be greater for a more forward CG.

Figure 29-5 shows the example airplane's drag polar. We have arbitrarily selected two CG positions at which we show the CG effect: 9% MAC to illustrate a more forward CG, 30% MAC to show a more aft CG.

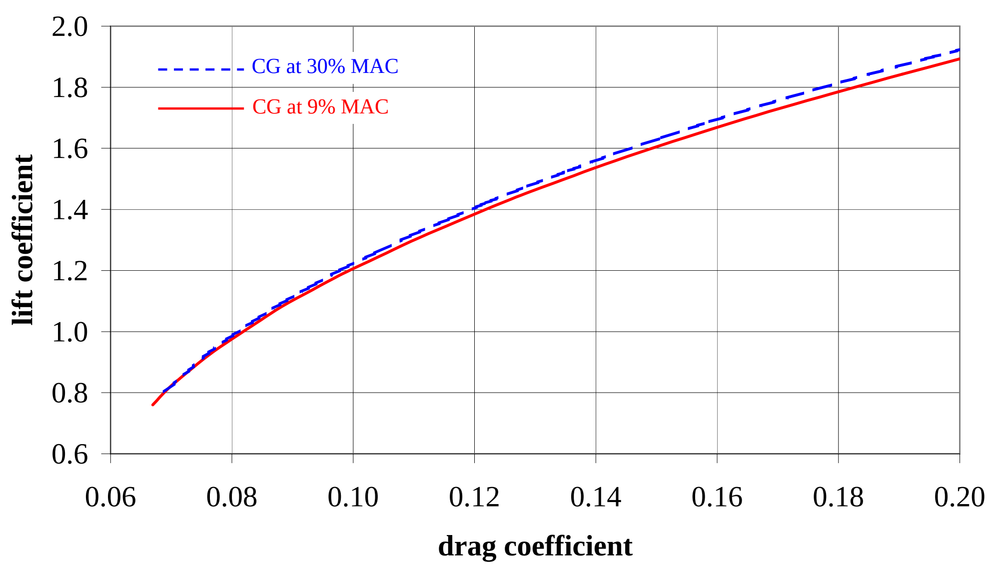


Figure 28-5

As you see in Figure 28-5, for the same lift coefficient you see that the drag coefficient at 9% MAC is greater than it is at the more aft position of 30% MAC. While the effect is not large, it's not insignificant.

What conclusion can we draw from this drag polar? Simply that for any given flight conditions, <u>at a more aft CG position there will be less drag</u>. This will influence the thrust required and thus the fuel flow. We'll be discussing this more in the chapter on cruise performance.

effect of CG on the lift curves

You saw that moving an airplane's CG more forward necessitates an increase in the wing-body lift force which can only be accomplished by increasing the airplane's angle of attack. But <u>this increase of angle of attack is done at constant lift coefficient</u>, because the airplane let lift force is constant, equal to the weight at all times.

In Figure 28-6 to the right you see the lift coefficient on the vertical axis as a function of angle of attack on the horizontal axis, for each of the two center of gravity positions.

You see that as the CG moves more forward, from 30% to 9%, the angle of attack increases.

Alternatively, you could say that for any given angle of attack, the lift coefficient is greater at the more aft CG position.

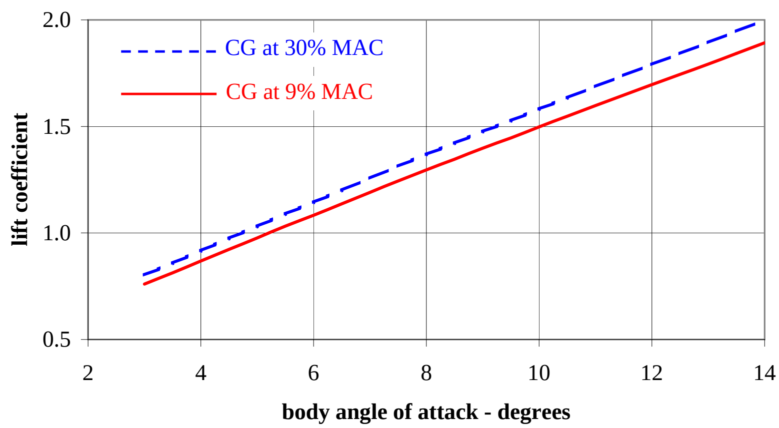


Figure 28-6

Effects of CG on Takeoff Performance

Now that you understand the effects of CG location on the drag polar and the lift curve, let's move on to see how these effects will influence an airplane's performance during takeoff and how we can utilize these effects to gain an improvement in allowable takeoff weights.

In the example data shown below, we're using a 737 at flaps 10 on a sea level standard day.

effect on takeoff speeds and distances

Looking at the lift curve chart above, we can make a logical prediction: if, at a given attitude such as a takeoff attitude, the lift coefficient is greater at a more aft CG then it follows that, for the specified conditions, <u>at a more aft CG the takeoff speeds will be somewhat slower</u>.

The derivation of the takeoff speeds V_R and V_2 are discussed in detail in the chapter entitled "Speeds", if you wish to review that subject.

As an example, for the same 737 as we show in the lift curve chart above, here are the takeoff speeds and distances at the two different CG positions:

weight (pounds)	CG	V_R (knots)	V_2 (knots)	distance (feet)
130,000	9% MAC	122.1	127.4	4997
130,000	30% MAC	119.0	124.0	4746

Table 28-1

You see from the table that there's a change in V_R and V_2 of approximately three knots between the most forward and most aft CG positions, and the takeoff speeds are slower for the more aft CG position, as predicted.

Notice also that the takeoff distance with the more aft CG is about 250 feet less.

effect on field length limit takeoff weight

It stands to reason that if a more aft CG allows shorter takeoff distances, as you've just seen, then for a given runway length the field length limit weight should be greater at the more aft CG.

Continuing the same example as above, here are the field length limit weights for a five thousand foot runway. You see that there's an increase in the field length limit takeoff weight of more than 3,400 pounds at the most aft CG location.

CG	field length limit weight (pounds)
9% MAC	130,045
30% MAC	133,488

Table 28-2

effect on climb gradient

This one is a little more complex. First, how does the climb gradient for a constant weight vary with CG position? You know that gradient is influenced by the ratio of drag to lift, and you know that drag varies with CG for a given net lift. Thus the ratio of drag to lift depends on CG.

The chart to the right shows the second segment climb gradients for a range of climb speeds, for the two different CG positions. To illustrate the effect of climb speed on the gradient, we have chosen a typical weight of 130,000 pounds. The trends that you see in the chart would, however, be the same at any weight.

But we have to be <u>very</u> careful when we talk about this chart, because it would be very easy to misunderstand what it is telling us.

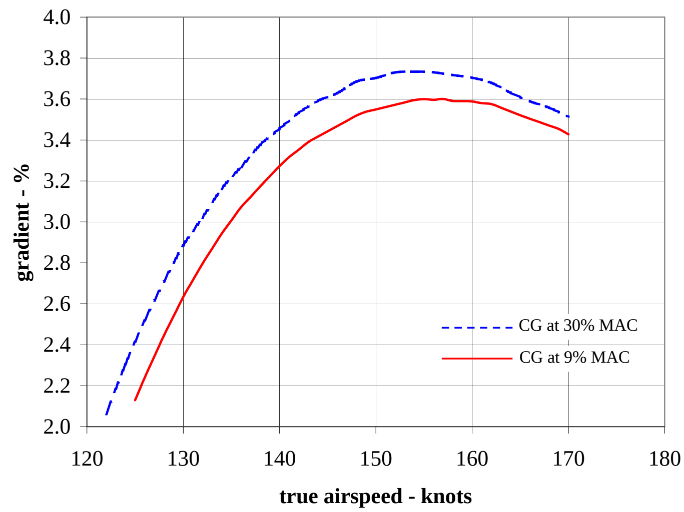


Figure 28-7

The chart implies that climb performance is better at the more aft CG position. If the climb speed at both CG positions were the same, that would be true. But remember that the V_2 speeds – which are, as you know, the speeds at which the second segment climb gradients are calculated – are different for the two CGs, as you saw in the first of the two tables above.

Here's the same chart with the lower left part enlarged. We have added the V_2 speeds for the two different CGs. Now you can see the truth: at the V_2 speeds at which the climbs are made, <u>the climb angle at the more aft CG is actually less than the climb angle at the more forward CG</u>.

(However, you shouldn't assume that this same characteristic applies to all airplane models at all flap settings and at all speeds. As you'll soon see, the effect of CG position on climb capability varies from airplane to airplane, and sometimes even from one flap position to another.)

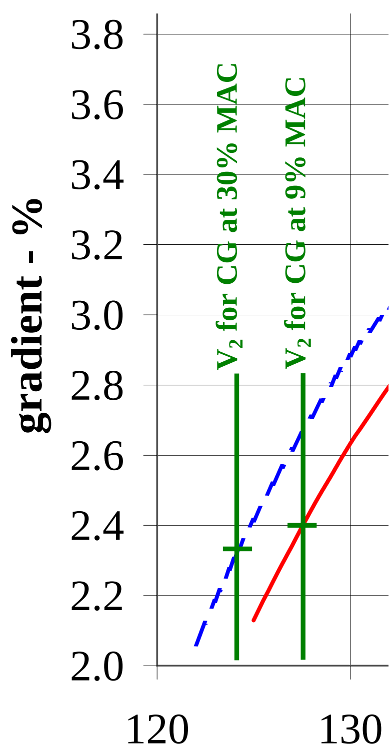


Figure 28-8

effect on the climb limit takeoff weight

From the chart just above, you can now understand that the climb limit weight at the more forward CG will actually be better than it is at the more aft CG.

The climb limit weights for the two CGs used in our example are as shown in the table to the right. Here, you see that the most forward CG position yields a higher second segment gradient, and hence a higher climb limit weight.

CG	V_2 (knots)	climb limit takeoff weight (pounds)
9% MAC	127.5	130,297
30% MAC	123.7	129,422

Table 28-3

We sometimes say that alternate forward CG is like a negative improved climb, and that's exactly what you have just seen. Refer back to the chapter entitled "Improved Climb" if you want to refresh your memory; in brief, it states that increasing the V_2 above its normal value will yield a higher climb limit weight. Here, we're doing just the opposite: we're decreasing the V_2 below its normal value – and you see that the climb limit weight has decreased.

So one might ask: is alternate forward CG ever beneficial to climb limit weight? The answer is "yes – some of the time".

For our 737 example airplane, there is a weight benefit to alternate forward CG at high amounts of improved climb as shown in the chart to the right.

If the takeoff runway is long enough to permit this much speed increase, then there's an increase in the climb limit weight of approximately 450 pounds.

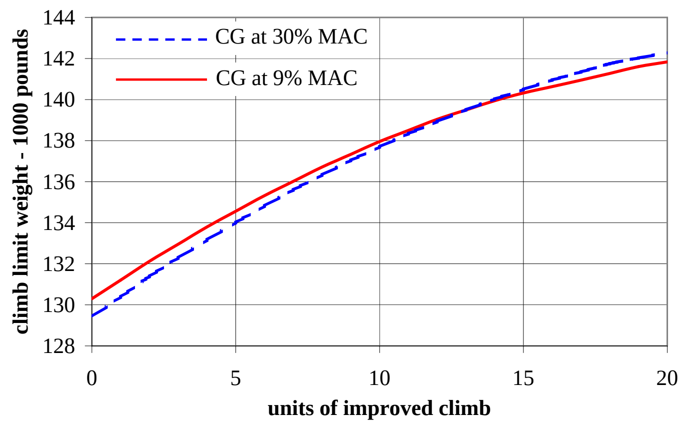


Figure 28-9

The benefit of alternate forward CG varies from airplane to airplane, in accordance with each airplane's aerodynamic characteristics. For the 737 that we've been using for all of the examples up to this point, the improvement in the climb limit weight was quite small.

The chart to the right shows a very different climb limit weight improvement for a 757 using flaps 20 for takeoff. Here, the more aft CG gives a superior climb limit weight even without any improved climb speed increase.

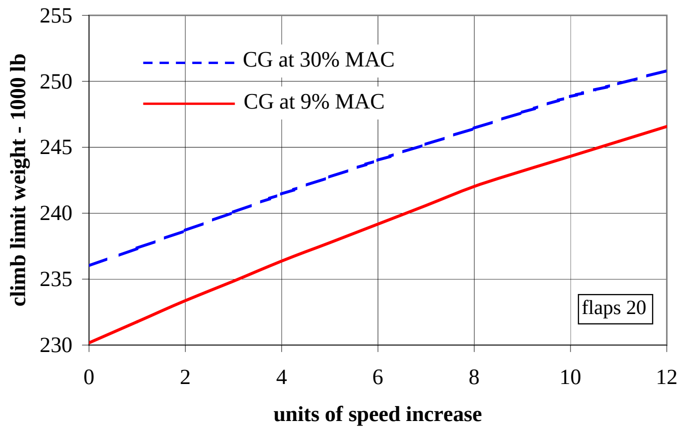


Figure 28-10

But notice in the chart to the right that, for the same airplane and the same takeoff conditions, at flaps 15 an aft CG has a benefit on climb limit weight only with some improved climb speed increase.

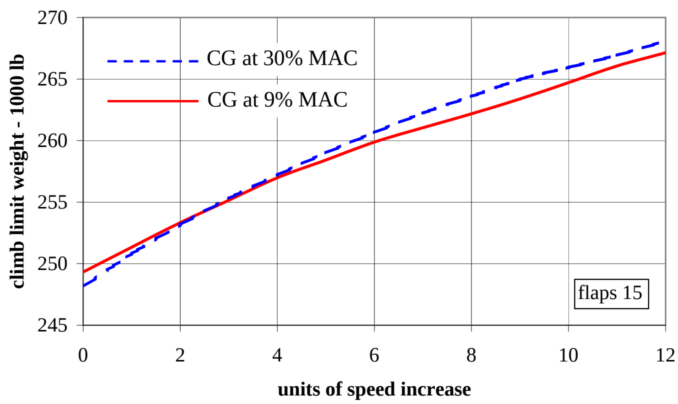


Figure 28-11

effect on the obstacle limit weight

Obstacle-limited weights are sensitive to a number of variables, principal among which are the climb gradient available and the location of the obstacle.

You have seen that the use of alternate forward CG may or may not improve the climb gradient available depending on the airplane model, the flap position and other factors. It's reasonable to assume that a better climb gradient might provide a higher obstacle limit weight, depending on the obstacle's location.

Here's an interesting tabulation, for a 757:

flaps	CG	climb limit weight (pounds)	obstacle limit weight (pounds)	field length limit weight (pounds)	engine-out takeoff distance (feet)
20	9% MAC	219,893	206,499	256,772	6388
20	30% MAC	225,690	211,160	259,585	6461
15	9% MAC	238,482	215,375	246,294	7562
15	30% MAC	237,493	216,750	251,526	7282

Table 28-4

First, notice that at flaps 20, the more aft CG offers the better climb limit weight, the better obstacle limit weight, and the better field length limit weight. The engine-out takeoff distance at the obstacle limit weight increases slightly when going from the more forward to the more aft CG position. So at flaps 20, all of the weight limits benefit from the more aft CGs.

At flaps 15, however, the situation is different: in this case, the more aft CG offers the better obstacle limit weight and the better field length limit weight, but has the worse climb limit weight. Notice that the engine-out takeoff distance at the obstacle limit weight decreases when going from the forward CG to the aft CG.

You saw previously that for a 757 at flaps 20 there is a generous increase in the climb limit weight with no need for any improved climb speed increase. For flaps 15, though, you saw that with no improved climb speed increase, the climb limit weight decreased at the aft CGs. Still, the decrease in the engine-out takeoff distance, due to the decrease in the takeoff speeds at the aft CG, increased the distance to the obstacle from reference zero. That had a greater effect on the obstacle limit weight than the slight loss of gradient at the more aft CG.

From your knowledge of the different types of obstacles (obstacles in second segment, obstacles in third segment, and so on) you'll understand that some obstacles may benefit from alternate forward CG operations, others may not.

effect on the tire speed limit weight

The use of alternate forward CG techniques offers reduced takeoff speeds and hence improved field length limit weights. Here's another possible benefit of the reduced takeoff speeds: tire speed limit weights.

It's logical to expect that lowering the takeoff speeds by using an alternate forward CG limit will also increase the tire speed limit weight under those conditions where the tires may be limiting.

Here's an example: for a 757 equipped with 210 miles per hour tires, taking off at Denver, Colorado, on a 75 °F day, at the more forward CG, the takeoff weight is tire speed-limited to 219,886 pounds; at the more aft CG, it is tire speed-limited at 230,941 pounds.

summary of alternate forward CG benefits

From all of this discussion, you've seen that alternate forward CG operation can offer substantial increases in field length, obstacle, and tire speed limit weights; it may or may not offer an improvement in the climb limit weights.

You've seen also that the benefits of alternate forward CG operation are model-dependent and for a given airplane model they may depend on the flap setting. Thus, for some operations the purchase of the alternate forward CG authorization may be a profitable decision. For some other operations, it might not.

Any operator considering the use of alternate forward CG technique for its airplanes should assess carefully the effects of alternate forward CG limits in its operating environment before purchasing the option for any or all of its fleet.

Determining Takeoff Weights With Alternate Forward CG

Operators using airplanes delivered with "paper AFMs" will need to purchase an appendix that authorizes the use of alternate forward CG limits and provides the necessary performance data.

To the right is an example of an AFM appendix chart showing the takeoff climb limit weight increase available at different forward CG limits.

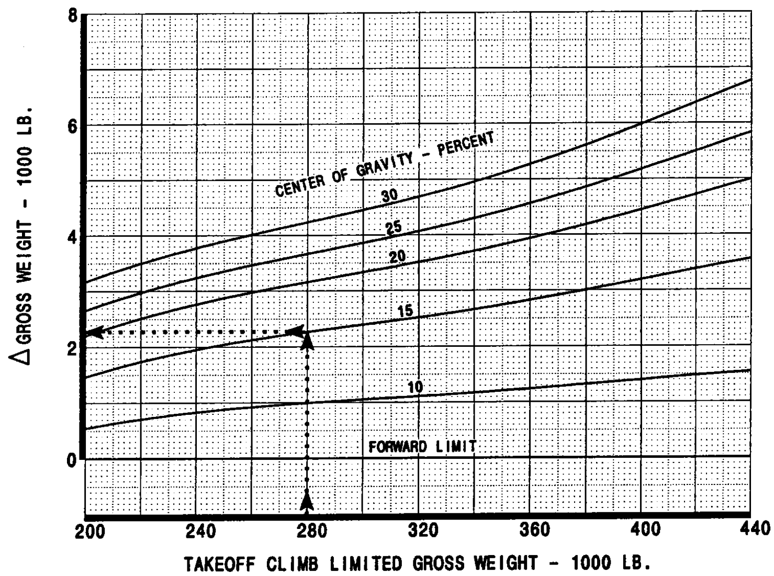


Figure 28-12

For airplanes operating with a paper Airplane Flight Manual, the operator will need to purchase an appendix to their AFM that provides the revised takeoff CG limits and the necessary charts providing the weight increments.

Operators flying AFM-DPI airplanes will find that all AFM-DPIs already include the ability to calculate alternate forward CG performance.[1] This can be very useful in evaluating the desirability of purchasing the option. While the data is included in all AFM-DPIs, <u>it can not be used legally for dispatch before purchasing the option</u>.

1. For the 737NG airplane, alternate forward CG benefits for landing are also available.

Who Can Use Alternate Forward CG Technique?

You've seen that benefits of the alternate forward CG technique are available only when an operator can ensure that the takeoff centers of gravity of their airplanes remain aft of some forward limit that is more restrictive than the normal forward limit. This means that an operator must assess its operating environment to see whether or not the alternate forward CG technique is practical for them.

If an operator wants to take advantage of alternate forward CG operation, it is Boeing practice that the operator may choose any two alternate forward CG limits that are suited to its operations. Those two selected alternate forward centers of gravity are then published in the operator's Airplane Flight Manual and Weight and Balance Manual; only when these two selected alternate forward CG limits appear in the operator's documentation is it legal for the operator to take the weight increases made available from more aft takeoff centers of gravity.

Shown to the right is an example of the CG limits chart for a 767 from one operator's AFM, showing the normal forward CG limit and the two selected alternate forward limits:

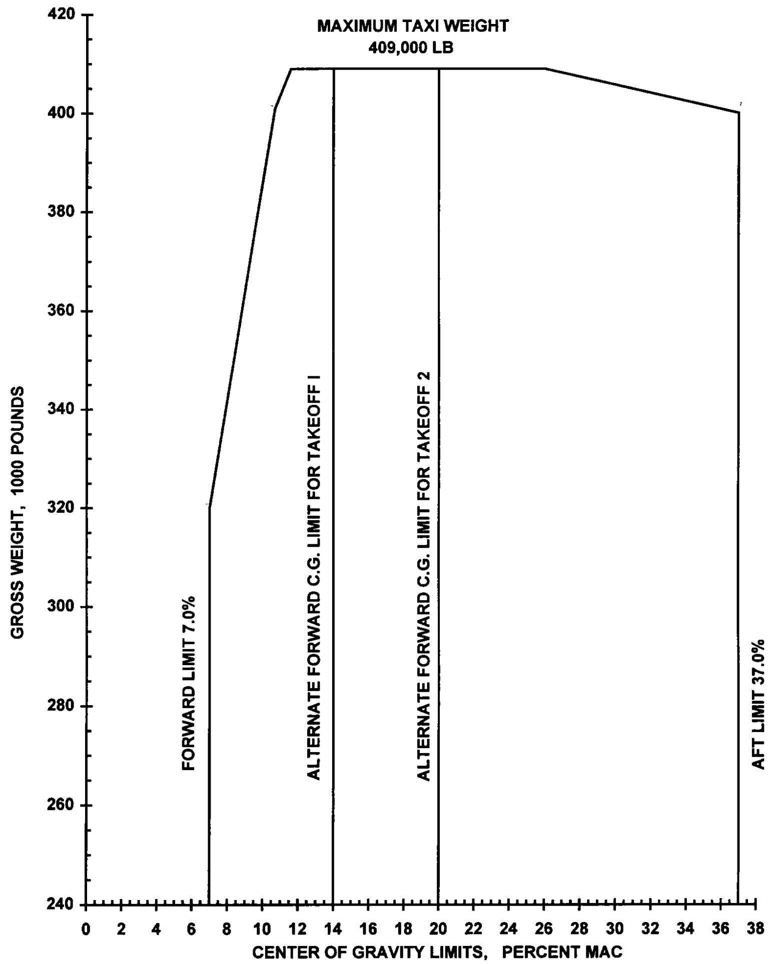


Figure 28-13

As you see, for this airplane the normal forward CG limit is seven percent MAC up to a gross weight of 320,000 pounds, then moving more aft as the gross weight increases, to a final limit of 11.6 percent at the maximum taxi weight of 409,000 pounds.

This operator has selected the first alternate forward CG limit at 14 percent, and the second one at 20 percent.

For this operator, if his takeoff weight were, for example, 320,000 pounds and his CG were at 12% MAC, then it would be necessary to use the normal forward CG performance. If, on the other hand, the CG were at 17% MAC, the operator could use the data for a 14% forward limit; were the CG aft of 20%, they could use the data for a 20% forward limit.

a vital precautionary note

At this point in our discussion, it's essential that we emphasize that the CG limits shown above are the certified limits. CG limits appearing in the AFM and Weight and Balance Control and Loading Manual issued to the operator are always the certified values.

To illustrate the importance of this emphasis, let's say that you want to take off at a weight of 320,000 pounds and the takeoff CG is computed to be 7.1 percent MAC. Would that takeoff be legal? The chart above might be misunderstood as saying "yes", but the real answer is "no".

Why? Because if you take off at 7.1 percent MAC, the retraction of the landing gear and flaps after takeoff will cause the CG to move forward of the seven percent forward limit – that is, outside the certified limit envelope – because the CG moves forward with gear and flap retraction. This takeoff would therefore be in violation of the AFM CG limitation of seven percent MAC at this weight.

Other operational variables could also contribute to violation of the certified limits: inflight movement of passengers and crew members, fuel usage, and other effects.

For that reason, the CG limits used in daily operation are more restrictive than the certified limits; only by doing this can the operator be sure that the certified limits are not accidentally violated due to operational variables. These more restrictive operational CG limits are usually referred to as "curtailed" or "constrained" CG limits.

Just as the normal forward CG limit must be curtailed to allow for operational effects, the alternate forward CG limits must also be curtailed. Thus, while the chart seen above speaks of alternate forward CG limits of 14 and 20 percent, the curtailed alternate limits will be aft of those values; the difference between the certified and curtailed limits depends on a number of factors that differ from one operator to another.

It's not within the scope of this document to discuss weight and balance methods. Boeing offers assistance and training on this subject. If you aren't familiar with the principles of operational weight and balance, we urge you to take the time to learn them before attempting to evaluate your operation for the feasibility of alternate forward CG operation.

assessing the desirability of alternate forward CG operation

An operator's assessment of its daily operations, with an eye toward using alternate forward CG limits, will logically start with a review of its statistics on the takeoff CG values experienced in service. These statistics could be for system-wide operations; if the alternate forward CG technique is attractive only at certain airports where weight increases are frequently desirable, the statistics might be only for operations at those locations.

Takeoff centers of gravity depend on a large number of variables: the airplanes' interior configuration, takeoff fuel loads, number and distribution of passengers, amount and distribution of cargo, and more. An assessment should consider both its typical takeoff CGs without using any special load control methods, and also whether or not changes to the operational practices might be feasible: altering the cargo loading policies, for example, to produce more aft CGs.

If an operator sees from the statistics that his takeoff CGs are never forward of – just for example – 17 percent MAC, then that might determine the first alternate forward CG limit after allowing for the difference between operational and certified CG limits. Let's say that the operator calculates that the difference between its operational and certified forward limits are – again, we're just making up numbers here for example purposes – three percent, then the operator could say that a certified alternate forward CG limit of 14 percent MAC would be acceptable.

Further, let's say that the operator calculates that by selective loading of cargo and passengers the takeoff CG can be kept six percent more aft than when using the normal loading policies. It could then logically select 20 percent MAC as its second certified alternate forward CG limit.

Only a careful assessment of its operational statistics and possible changes to its weight and balance practices can allow an operator to decide whether or not the use of alternate forward CG limits is feasible for obtaining increased takeoff weights.

Chapter 29: Cost Index

TBS

Chapter 30: Climb to Altitude

Introduction

This phase of flight is sometimes called “enroute climb”, meaning simply climb along the route toward the cruise altitude. We won’t use that term here, since the regulations use it to refer to climb capability with one or more engines inoperative as might occur in the event of engine(s) failure along the route during climb to altitude, cruise, or descent from altitude[1]. We’ll be discussing that subject in a chapter appearing later in this book.

For flight planning purposes, we need to be able to calculate the time, fuel burnoff, and distance required to climb to altitude. We may want to do this for any of a number of different speeds: speed for minimum climb time, speed for minimum climb distance, speed for minimum fuel consumption, or any other speed selection dictated by the flight environment.

In an earlier chapter entitled “Calculating Climb Angle and Rate of Climb” we showed how to calculate climb gradients and rates of climb. We’ll be using those methods again in this chapter. The technique of “step integration” that we introduced you to in the chapter entitled “Calculating Takeoff Distances” will appear again to enable us to calculate climb parameters over a range of altitudes – such as from sea level to 33,000 feet – rather than just instantaneous values at specific altitude/temperature/speed/weight conditions. You may want to go back to those chapters to review before continuing in this chapter. Where they’re needed, we’ll repeat the equations from those chapters for convenience in this discussion.

Let’s begin by discussing the speeds for which we’ll typically be calculating climb performance.

Climb Speed Schedules

The speed schedules for climb to the cruise altitude are customarily shown as a combination of calibrated airspeed at the lower altitudes, changing to Mach number at the higher altitudes. Calibrated airspeed (or, on the older airplanes not equipped with air data computers, indicated airspeed) and Mach number are the speeds available to pilots on the flight deck, hence we always state the climb speed schedules in those units.

An example would be a climb speed schedule shown as 290/.78, meaning 290 knots CAS at the lower altitudes, then Mach 0.78 at the higher altitudes. That’s one of the published climb speed schedules for the 757-200 airplane.

Let’s illustrate that with a simple chart.

1. See, for example, FAR 121.191 and 121.193. These are discussed in the chapter entitled “Cruise - Engine Failure and Driftdown”.

To the right, you see a graph of Mach number versus altitude for a climb speed schedule of 290/.78.

At altitudes below 30,875 feet (you'll see why it's that value in just a minute) you see a sloping line of a constant 290 knots CAS as it varies with altitude following the equation:

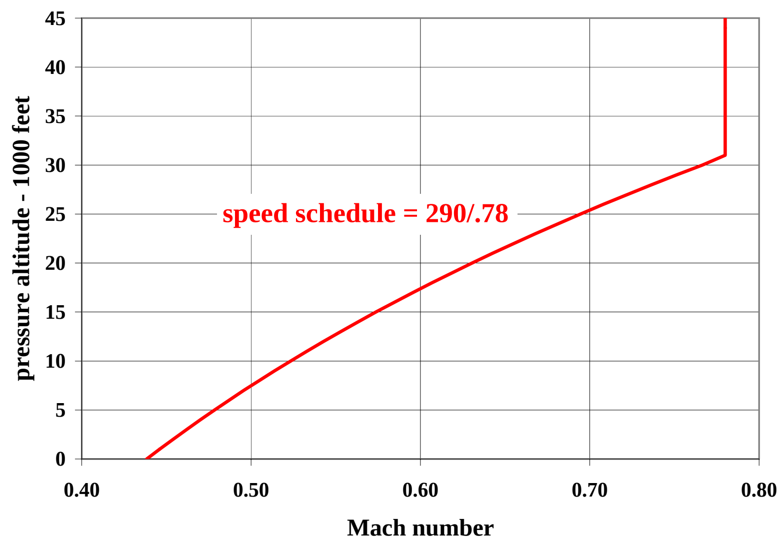


Figure 30-1

$$M = \sqrt{5\left[\left(\frac{1}{\delta}\left\{\left[1 + 0.2\left(\frac{V_C}{661.4786}\right)^2\right]^{3.5} - 1\right\} + 1\right)^{\frac{1}{3.5}} - 1\right]}$$

where V_C is the calibrated airspeed, in knots
δ is the atmospheric air pressure ratio

At the *crossover altitude* of 30,875 feet, at which 290 knots CAS is exactly equal to Mach 0.78, the speed schedule changes from being one of constant 290 knots CAS to being one of constant Mach 0.78; for the remainder of the climb, the airplane speed will be kept at that Mach number.

Why are we using a speed schedule of 290/.78? There are valid reasons for selecting any one of a number of different possible speed schedules:

- minimum time to altitude (maximum rate of climb)
- minimum distance to altitude (maximum angle of climb)
- minimum fuel consumption
- minimum trip time or cost

In the earlier chapter entitled "Calculating Climb Gradient and Rate of Climb" we discussed the speeds for maximum angle of climb and maximum rate of climb. You saw that the speed for maximum angle was somewhat slower than the speed for maximum rate of climb. Let's look at them in the context of climb to altitude.

speed for maximum angle of climb

This speed might be of particular importance when air traffic control constraints dictate that the airplane should reach a specified altitude within a distance less than the distances resulting from other speed schedules. It might be necessary if required to pass a specified climb waypoint at an altitude greater than those resulting from other speed schedules.

This would usually be for reasons of traffic avoidance, particularly in navigation areas having a high volume of traffic and a number of different routes. The flight clearance for such a flight might include a requirement such as "cross waypoint LACRE at or above five thousand feet" – in such a case we might want to achieve the greatest possible altitude gain in the least possible distance, hence the need for a speed that will offer the best angle of climb.

Looking at a graph of climb gradient versus calibrated airspeed for four different altitudes, in this example for a 757-200 taking off at a weight of 220,000 pounds. You see that the speed for best angle is almost the same across the altitude range shown. We could use a speed schedule at these altitudes of approximately 210 to 220 knots to achieve the best angle of climb.

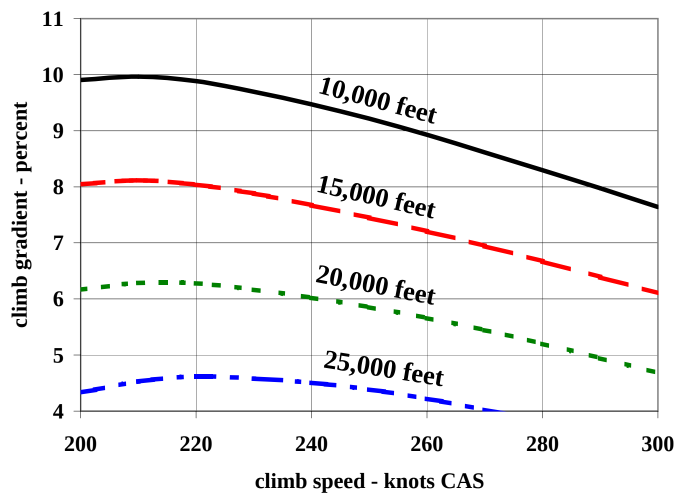


Figure 30-2

If we calculate and plot the distance to an altitude of – for example – 30,000 feet as a function of the climb airspeed, we would see the results shown in the graph to the right. You can see that, as expected from the graph of climb gradient versus speed, shown just above, a speed of about 220 knots will yield the maximum climb angle.

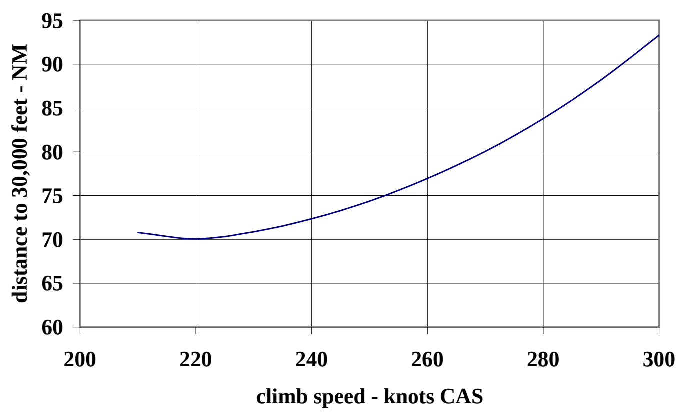


Figure 30-3

speed for maximum rate of climb

Air traffic control requirements will sometimes require an airplane to climb to a specified altitude within a specified time interval – again, usually for reasons of traffic avoidance. For that reason, a pilot might on occasion want to climb at the speed that will offer the best rate of climb.

For the maximum rate of climb, you see in the chart to the right that the best speed would be approximately 250 to 265 knots CAS, again for a 757-200 taking off at 220,000 pounds.

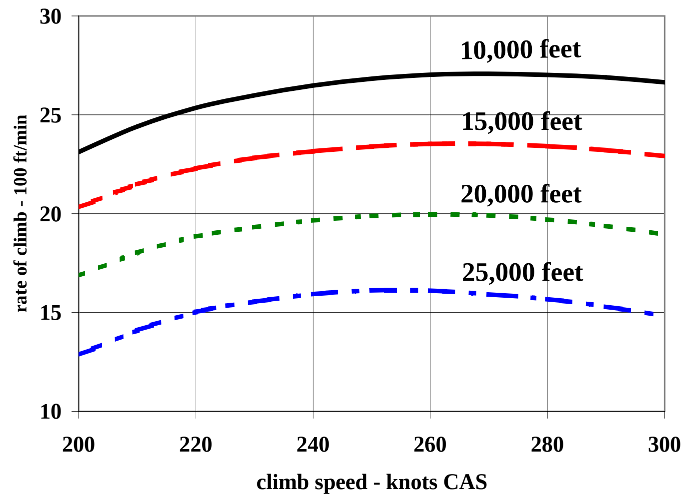


Figure 30-4

Graphing the time to climb to 30,000 feet as a function of the climb speed shows that – again, as expected – a climb speed of about 255 knots yields the minimum time to altitude.

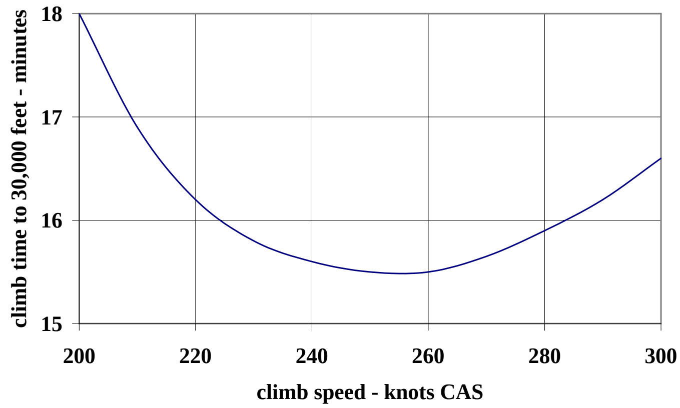


Figure 30-5

speed for minimum fuel consumption

When fuel conservation is the primary concern, another speed schedule will be necessary, but this one isn't quite as easy to compute.

First of all, how about the fuel flow rate during climb? How does it vary with climb speed? In the graph to the right, you see the climb fuel flow rate at four different altitudes as a function of climb speed, for the same 757 example airplane.

You see that fuel flow rate steadily increases with climb speed. Does that mean that the slowest climb speed is the most fuel efficient?

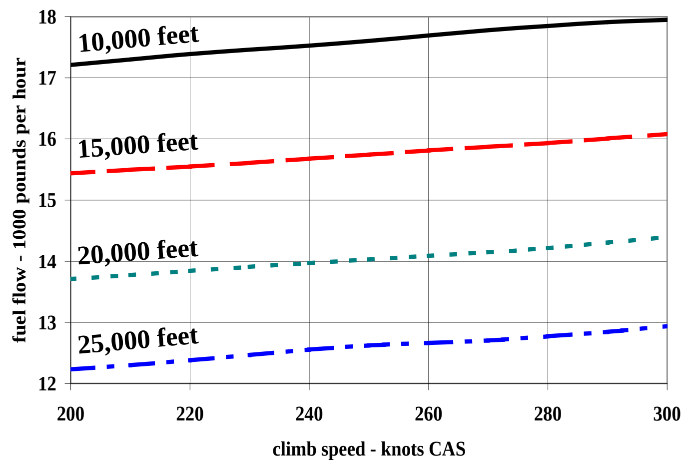


Figure 30-6

No. Although the fuel flow rates are at their least at the slowest climb speed, we must also consider the climb time because that also affects the total fuel consumption to the top of climb – since fuel consumed is equal to fuel flow rate multiplied by time. Since the time to top of climb depends directly on the airplane's climb speed, you might suspect that there is a speed for minimum fuel consumption to top of climb. If we calculate and graph the fuel to 30,000 feet, as we did just above for the time and distance, here's what you'd see:

You'll notice that the climb speed that will yield the minimum fuel to (in this example) 30,000 feet is about 245 knots CAS or so.

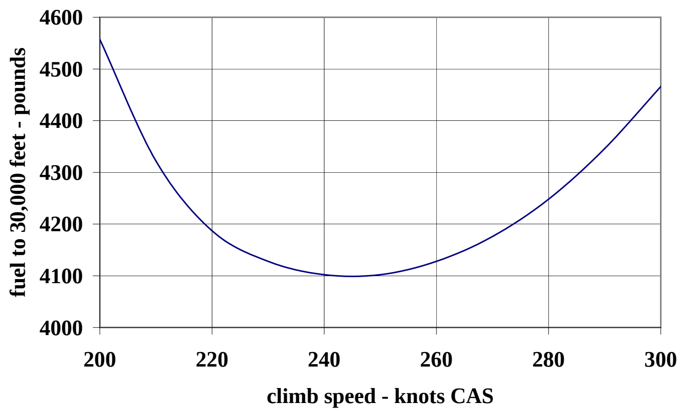


Figure 30-7

If you are concerned only about the fuel consumed during the climb, then that would be the correct speed for minimum fuel consumption. However, if your desire is to select a climb speed that will yield the minimum trip fuel consumption – that is, the total quantity of fuel burned from takeoff to touchdown – then the fuel burned from takeoff to top of climb is only half of the story. Here's the rest of the story:

By reference to the climb distance graph above you can see that if you climb to 30,000 feet at a speed of 245 knots, the climb distance would be about 73 nautical miles. If, on the other hand, you were to climb to 30,000 feet at a speed of – for example – 270 knots, the distance to 30,000 feet would be about 80 nautical miles.

If you're going to find the climb speed that will yield the least trip fuel consumption, then it is not meaningful to use the speed for the minimum climb fuel burnoff, because at that speed the climb distance will be less and you must therefore fly farther in cruise. To make a truly meaningful choice of climb speed, we need to consider the fuel burned not only during the climb, but during the cruise after climb. We need to talk for a minute about *fuel to a common point*.

The sketch to the right, while not really an accurate drawing of the actual flight profiles, gives the idea of fuel to a common point.

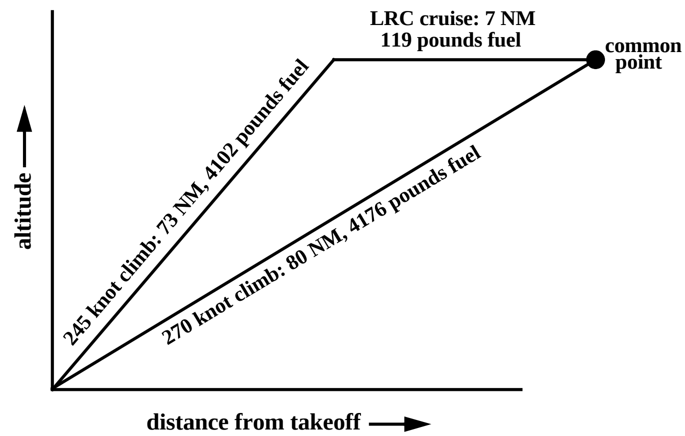


Figure 30-8

You see that if the airplane climbs to the common point at 270 knots, it consumes 4176 pounds of fuel while traveling 80 nautical miles to top of climb.

If, on the other hand, the climb is made at 245 knots, it will burn 4102 pounds of fuel in the climb, and will travel 73 NM. To fly to the common point it must then cruise for a distance of seven NM. Those seven miles flown at LRC speed require 119 pounds of fuel, so for a climb at 240 knots the fuel to the common point is 4221 pounds, which is 45 pounds more than the fuel for the climb at 270 knots.

If we calculate and plot the fuel to a common point 200 NM from takeoff for a variety of climb speeds, we'll get the graph shown to the right.

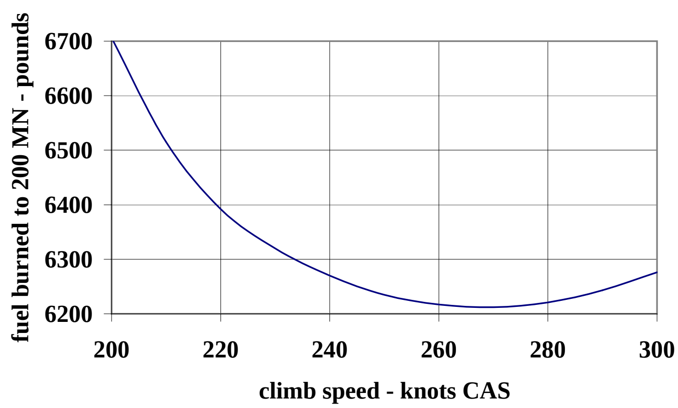


Figure 30-9

So there's the final answer to the question "what's the best speed for minimum trip fuel consumption?" At this takeoff weight, it's approximately 270 knots CAS.

effect of takeoff weight on climb speeds

All of the preceding charts are based on a 757-200 taking off at a weight of 220,000 pounds. How much does the takeoff weight affect the climb speeds for maximum rate of climb, maximum angle of climb, and minimum trip fuel consumption?

Let's re-do the calculations for a takeoff weight of 180,000 pounds, and graph the results.

Here's the comparison of the distance to 30,000 feet at two weights: a heavier takeoff weight of 220,000 pounds and a lighter takeoff weight of 180,000 pounds. You can see that the speed for the best angle of climb is about 15 knots less at the lighter weight.

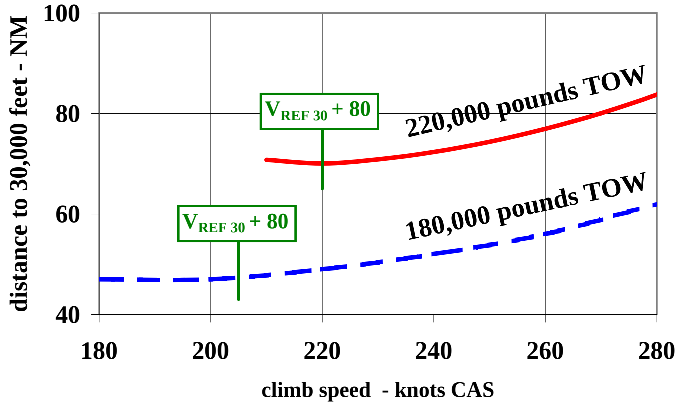


Figure 30-10

Notice that we have added two markers to the chart labeled "V_{REF30} + 80". That speed is called the "flaps up maneuvering speed" for the 757 – that is, the minimum speed for maneuvering the airplane after takeoff with the flaps fully retracted. As you'll learn in the chapter entitled "Approach For Landing", V_{REF} is the landing approach reference speed, equal to 123% of the 1-g stalling speed (or 130% of the FAR stalling speed, for the earlier airplanes).

Thus, for the best angle of climb, the pilots may elect to climb at the flaps up maneuvering speed; that will yield approximately the best angle of climb at any weight.

How about the speed for the best rate of climb?

To the right you see the graph of time to 30,000 feet versus climb speed, again for the two different weights of 220,000 pounds and 180,000 pounds.

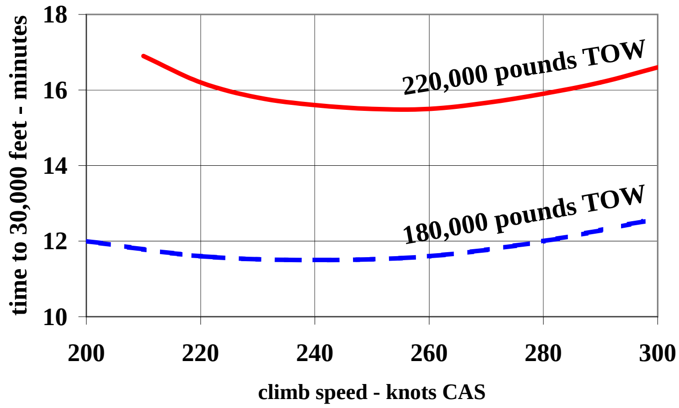


Figure 30-11

For increasing weight the speed increases, as it did for the best angle of climb. However, the difference in time to altitude changes only slightly with speed, so it will be acceptable to use a single speed schedule for all weights.

Finally, how about the speed for the minimum trip fuel consumption?

The graph to the right shows the effect of climb speed of fuel to a common point, for two different takeoff weights.

As you see, small changes of speed make only a small difference to the angle of climb, rate of climb, or fuel consumption. It's therefore common practice to select a single speed schedule for climb that will serve a range of weights with acceptable accuracy, rather than tailor the climb speed schedule to the exact takeoff weight.

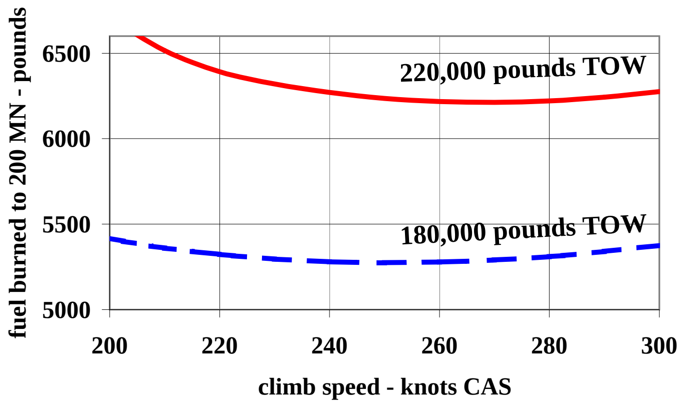


Figure 30-12

constant Mach number climb at the higher altitudes

Up to this point, we have been concentrating the discussion on the constant CAS portion of the climb speed schedule – that is, the portion of the climb profile below the crossover altitude. However, as you saw at the beginning of this section, climbing at constant CAS means climbing at increasing Mach number as the altitude increases. At some point, the Mach number will reach a value that we don't want to exceed, for one reason or another.

For that reason, the standard climb speed schedules specify a Mach number at which the climb will be flown at higher altitudes. For example, we looked at a climb speed schedule of 290/.78 and saw that the climb will be flown at 290 knots CAS up to 30,875 feet, above which it will be flown at a constant Mach 0.78.

How is the Mach number portion of the climb speed schedule selected? There are several different factors to consider:

- margin of speed from buffet
- fuel conservation
- certified maximum Mach number M_{MO}
- climb thrust available

The principal considerations, for today's airplanes and engines, are the first two of those listed above.

margin of speed from buffet

In the following chapter, entitled "Altitude Selection" we're going to be discussing the subject of high-altitude buffet in considerable detail. We're going to show you the "buffet boundary" for an airplane, which is an important factor in deciding the altitude to be flown in cruise.

You have already seen a discussion of stalling and "low-speed buffet" in the chapter entitled "Speeds". You know that as an airplane decelerates, it will reach a speed at which the airplane begins to buffet as the airflow over the wing becomes more turbulent, the predecessor to stalling.

In the next chapter, we're going to discuss the condition known as "high-speed buffet". That phenomenon is another form of airplane buffet occurring at higher speeds, at which the airflow velocity over the wing exceeds Mach 1, causing shock waves to form resulting in turbulent airflow behind the wing and consequently some airplane buffet.

Clearly, an airplane in flight should always respect these two buffet speeds, remaining within the band of airspeed that lies between low-speed buffet and high-speed buffet. This band of airspeed between the two buffet speeds becomes narrower as altitude increases. Hence, at higher altitudes it becomes increasingly important to fly at a carefully selected Mach number that has an acceptable margin of speed from both low- and high-speed buffet.

You'll see in the next chapter that, for the 757 airplane we've been using as an example in this chapter, the middle of the band between the buffet speeds occurs at approximately Mach 0.78. Thus M.78 is a good choice for climb Mach number, at least from the standpoint of buffet margin.

fuel conservation

As much as possible, we'd like the climb Mach number to be one that's efficient from the standpoint of fuel consumption, as well as one that provides good buffet margins.

Very soon, in the chapter entitled "Normal Cruise", we'll show you the fuel mileage characteristics of a typical jet airplane at its cruising altitudes. As you see in the chart to the right, as the cruise Mach number increases, initially the fuel mileage will increase – more miles flown per unit of fuel. At some Mach number, the fuel mileage will reach a maximum, then at higher Mach numbers the fuel mileage will again decrease. Therefore, there is some Mach number which will offer the very highest possible fuel mileage for that weight and altitude.

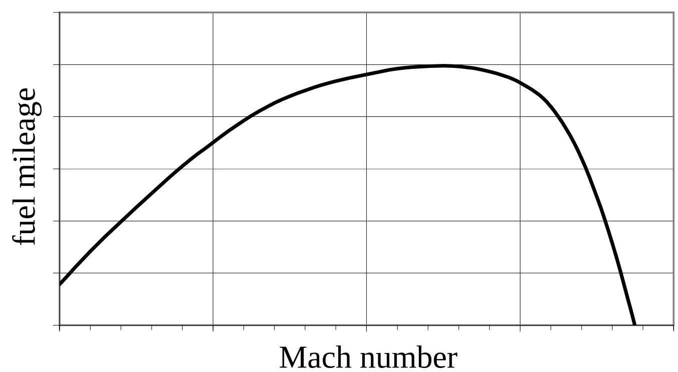


Figure 30-13

For the 757 that we're using as our example airplane, the peak of the fuel mileage curve, when the airplane is being flown at its most fuel-efficient altitudes, is approximately Mach 0.78. Thus you see again that M.78 is a good choice for the climb Mach number. Then, when the airplane reaches its cruising altitude, it will already be flying at its optimum speed for fuel conservation, making for an easier transition from climb to cruise.

effect of Mach number on fuel to a common point

We've shown you that the climb calibrated airspeed can be chosen for minimum fuel to a common point. The same is true of the climb Mach number above the crossover altitude. However, the effect of the Mach number on fuel consumed to a common point is smaller than the effect of the calibrated airspeed selection. Since the Mach number at the higher altitudes may be selected to maintain a good margin of speed from buffet, it's comforting to know that doing so doesn't result in a substantial loss of fuel efficiency. The difference in fuel burn to a common point between 290/.76 and 290/.80, for example, is only 42 pounds. The airspeed below the crossover altitude has a greater effect since the greatest amount of time in climb is spent below the crossover altitude.

Calculating the Time, Fuel, and Distance For Climb

All of the foregoing data was computed using the Boeing INFLT/REPORT software, and in today's flight operations environment computers are usually available that can readily utilize the INFLT/REPORT and the other performance software applications. Performance Engineers are encouraged to become familiar with the Boeing software; courses are available that teach the use of all of the software applications available to our customers.

But computers can't do anything that <u>we</u> can't do with paper and pencil and a simple calculator – they just do it effortlessly, and <u>much</u> more quickly. So it will be instructive to lead you through the process that can be followed in order to calculate climb performance manually. Even if you never need to do it, knowing how it's done will help you to understand the data you receive from the computer.

These calculations can be done in a manner very similar to that which we used for calculating takeoff distances: a process of step integration. We divide the altitude range through which we're climbing into discrete steps; we then calculate the time, distance and fuel consumed to climb each step, and finally add together the results for the individual steps to arrive at the sums for the entire climb to altitude.

Suppose, for example, that we're going to take off from a sea level airport and then climb to a cruise altitude of 33,000 feet. First, let's decide where the takeoff ends and the climb to the cruise altitude begins.

the starting point for the climb

That decision is really up to the person doing the calculations. Boeing standard practice is to define the end of the takeoff as 1500 feet above the airport elevation. We provide operators with data for the time, fuel and distance consumed during the takeoff from brake release to 1500 feet, and consider the climb to altitude to begin at that point.

For our example, we'll adhere to that standard practice; what we demonstrate here could be used equally well for any other assumed climb starting point.

the step integration step sizes

In the chapter entitled "Calculating Takeoff Distances", you saw that we calculated the takeoff distance by establishing steps – increments – of speed, and then determining the distance traveled as the speed changes from the initial value to the final value of that step. For example, we could define a step as five knots, such as 100 to 105 knots, and we could then calculate the distance the airplane travels as it accelerates from 100 to 105 knots, knowing the airplane's aerodynamic drag, thrust, friction forces, and so on.

For calculation of the time, fuel and distance for climb to altitude, we will begin by step-integrating steps of altitude. The step increment is arbitrary, but if the step size is too large there will be some loss of accuracy, and if the step size is very small there is little gain in accuracy at the expense of additional computing effort.

A suggested step size for climb to altitude calculations is one thousand feet. Later, we'll show how much accuracy is gained by using smaller step sizes.

As an airplane approaches its cruise altitude, its rate of climb decreases. Because of this, keeping a one thousand foot step means that the time in that step can become quite long – depending on weight, of course – and as a result some accuracy will be lost. When the airplane's rate of climb is low, it is more accurate to use time steps. It is Boeing's standard practice to transition from 1000-foot altitude steps to two-minute time steps when the rate of climb drops below 500 feet per minute.

the calculation process

The following paragraphs will demonstrate a workable step-by-step process for calculating the time, fuel and distance for one altitude step of the step integration process.

Clearly, given that computers are now so readily available, and given also that Boeing provides operators with INFLT and BPS software for their Boeing airplanes at no charge, few people will attempt such a calculation without computer assistance. Still, it can be useful to understand the process.

Obviously, computer software will determine some of its information by reading tables of data – for example, to find the climb thrust, the drag, the fuel flow, or whatever. Were we doing the calculations by hand, we would probably determine these parameters from graphs rather than from tabulations. In this chapter, however, we're not going to demonstrate the methods for reading the graphs – it's tedious and not particularly instructive, especially since chart formats vary from airplane model to model.

The Performance Engineer's Manuals (PEMs) for each Boeing model contain the relevant charts and tables of performance data that can be used for either manual or computerized calculations. Also, Boeing offers a variety of Performance Engineer courses to assist customer personnel in learning more about the airplanes and about the manual methods and software used for various tasks.

To illustrate the step integration process used for calculating time, fuel and distance to altitude, we'll use as an example the first step of altitude following the completion of the takeoff at 1500 feet. This step will be a 500-foot step rather than the normal 1000-foot step because of its starting point. The next altitude step would be from 2000 to 3000 feet, and so on, until transitioning to time steps at a much higher altitude.

For the following calculation process we will assume that the following parameters are provided:

takeoff airport elevation
takeoff weight
time, fuel and distance from takeoff to 1500 feet
climb speed

1. At the mid-step altitude – that is, the altitude half-way through the step – find the values of θ and δ:

$$\theta = \frac{288.15 - (0.0019812 \times h_p) + \Delta ISA°C}{288.15\ °K}$$

where h_p is the pressure altitude at the mid-step altitude, in feet
ΔISA is the ISA deviation, in degrees C

and:

$$\delta = \left(\frac{288.15 - 0.0019812 \times h_p}{288.15} \right)^{5.25588} = (\theta_{ISA})^{5.25588}$$

Recall from the chapter entitled “The Atmosphere” that δ must be calculated for the standard day value of θ. Thus if ΔISA is non-zero, the value of θ calculated above may not be used to calculate δ – instead, θ_{ISA} must be used.

2. With the θ and δ from step 1 and the known climb calibrated airspeed, calculate the equivalent airspeed, the true airspeed and the Mach number:

$$V_e = 1479.1 \sqrt{\delta \left[\left(\frac{1}{\delta} \left\{ \left[1 + 0.2 \left(\frac{V_C}{661.4786} \right)^2 \right]^{3.5} - 1 \right\} + 1 \right)^{\frac{1}{3.5}} - 1 \right]}$$

where V_C is the climb calibrated airspeed in knots
V_e is the climb equivalent airspeed in knots

and:

$$V_{true} = 1479.1 \sqrt{\theta \left[\left(\frac{1}{\delta} \left\{ \left[1 + 0.2 \left(\frac{V_C}{661.4786} \right)^2 \right]^{3.5} - 1 \right\} + 1 \right)^{\frac{1}{3.5}} - 1 \right]}$$

where V_{true} is the climb true airspeed in knots

and:

$$M = \sqrt{5\left[\left(\frac{1}{\delta}\left\{\left[1 + 0.2\left(\frac{V_C}{661.4786}\right)^2\right]^{3.5} - 1\right\} + 1\right)^{\frac{1}{3.5}} - 1\right]}$$

where M is the climb Mach number

3. At the mid-step altitude, determine the engine thrust and fuel flow. This may be accomplished by reference to either charts or tabulations of propulsion data.

4. Make an estimate of the airplane's weight at the mid-step altitude. This estimate will be refined to an exact value at a later point in this calculation process. For this first pass through the process, assume that the lift force required will be equal to the weight. (For subsequent passes through the calculations, we will <u>not</u> assume that lift equals weight, but rather that lift is equal to the component of weight perpendicular to the flight path.)

5. At the mid-step weight, for the calculated equivalent airspeed, determine the airplane's lift coefficient, assuming that the lift is equal to the weight:

$$C_L = \frac{295.369 \times L}{V_e^2 S}$$

where L is the lift
S is the reference wing area

Assuming that the airplane's lift force is equal to its weight is an approximation; later in the process the airplane's weight will be corrected for the angle of the airplane's flight path in climb.

6. From the airplane's drag polar, determine the drag coefficient corresponding to the calculated lift coefficient. For greatest accuracy, the drag coefficient should be corrected for Reynolds number effects as described in the chapter entitled "Drag Analysis":

$$\Delta C_{D_{RE}} = B \times 10^{-4} \log_{10}\left[\frac{\frac{1}{M}\left(\frac{RE}{ft}\right)}{\frac{1}{M}\left(\frac{RE}{ft}\right)_{nom}}\right]$$

where $\left(\frac{1}{M}\frac{RE}{ft}\right) = 5.13384\times10^{6}\left(\frac{\theta + 0.38312}{\theta^{2}}\right)\times\delta$

B is a model-unique value, found in the “table of B values for Reynolds number correction to high-speed drag” found in Appendix 1 to this document.

For the nominal Reynolds number in the denominator, θ is the temperature ratio for standard day, δ is the pressure ratio for the reference altitude. These are available in the PEM.

7. Knowing the thrust, weight, drag and lift coefficients, calculate the climb path angle:

$$climb\ path\ angle\ \gamma = sin^{-1}\left(\frac{\frac{T}{W}-\frac{C_D}{C_L}}{1+\frac{V}{g}\frac{dV}{dh}}\right)$$

where $\left(1+\frac{V}{g}\frac{dV}{dh}\right)$ is the acceleration factor discussed in the chapter entitled “Calculating Climb Angle and Rate of Climb” and shown in Appendix 1.

8. Calculate the rate of climb:

$$rate\ of\ climb\ R/C = 101.268\times V_{true}\sin\gamma$$

where *R/C* is the rate of climb in feet per minute
V_{true} is the true airspeed in knots
γ is the climb path angle in degrees

9. Knowing the rate of climb and the altitude step size, calculate the step time by dividing the step size in feet by the rate of climb in feet per minute to arrive at the step time in minutes.

10. Knowing the step time, calculate the distance traveled in the step and the fuel burned in the step by multiplying the fuel flow rate and true groundspeed by the step time.

11. Knowing the fuel consumed in the step, calculate the mid-step weight and the weight at the end of the step.

If this is the first calculation of mid-step weight, the results will contain some inaccuracies. First, the results will be based on an estimated mid-step weight. Second, the results will be based on lift and drag coefficients based on the assumption that the required lift is equal to the weight.

While assuming that lift equals weight isn’t a bad approximation, for the greatest accuracy it’s better to say that the lift is equal to the component of weight perpendicular to the flight path. (Remember that an airplane’s lift and drag forces are defined as being perpendicular to and parallel to the flight path respectively. This is discussed in the chapter entitled "Lift and Drag".) Thus accounting for the climb path angle yields a lift following the equation:

$$lift = W \cos \gamma$$

where W is the weight
γ is the climb path angle

Because of the inaccuracy of the first pass through the calculations, it's necessary to repeat the calculations, using the newly calculated mid-step weight found in step 11 above. This time, we have a climb path angle, found in step 7, so we can use the weight corrected for the climb path angle in calculating the new lift coefficient. We thus loop back to step 5 with this new weight.

Continuing through steps 5 through 11, we find a new mid-step weight. It should be closer to the one previously calculated. Repeat this process until the mid-step weight calculated in step 11 is acceptably close to the value calculated in step 11 of the previous pass through the calculations. This iterative looping will yield the most accurate results.

Having calculated the time, fuel and distance for the step from 1500 feet to 2000 feet, those values will be added to the values at 1500 feet to arrive at the total time, fuel and distance from brake release to 2000 feet above the airport elevation.

For the next altitude step of 2000 to 3000 feet, we would then estimate the weight at 2500 feet, and repeat the process of steps 1 through 11 shown previously.

With the previous discussion in mind, you're now in a position to understand the following excerpt from a climb calculation performed using the BPS software:

	PRESSURE ALTITUDE FT	WEIGHT LB	FUEL LB	DIST NM	TIME H:M	IAS CAS KTS	EAS TAS KTS	MACH	GRND SPD KTS	FUEL FLOW LB/HR	CL CD	D D/DELTA LBS	FN FN/DELTA LBS	EPR1 EPR2	ACC FACT	GRAD BODY ANGLE	R/C FPM
*	0	200000	0.0	0.00	0:00.0												
*	1500	199350	650.0	3.90	0:01.8												
	1750	199326	48.1	0.59	:00.1	250.0	249.7	.3897	253.8	20595	0.4793	10786	40448	1.243	0.0822	.1388	3568.0
						250.0	256.2				.02619	11495	43105			11.70	
*	2000	199302	698.1	4.49	0:01.9												
	2500	199254	95.9	1.21	:00.3	250.0	249.6	.3949	256.6	20327	0.4798	10785	39876	1.248	0.0843	.1359	3531.1
						250.0	259.0				.02621	11814	43680			11.53	
*	3000	199206	794.0	5.70	0:02.2												

Figure 30-14

You see that this calculation starts at sea level, at a takeoff weight of 200,000 pounds. Fuel, distance and time are set to zero at the beginning of the takeoff.

When the airplane arrives at 1500 feet, it has burned 650 pounds of fuel, traveled 3.9 nautical miles, and taken 1.8 minutes from brake release. Those are default values for this airplane model, not rigorously calculated by the software.

Notice the next line, showing a pressure altitude of 1750 feet. This is the mid-step altitude for the first step from 1500 to 2000 feet. After several iterations, the software has calculated that the step fuel is 48.1 pounds, the step distance is 0.59 nautical miles, and the step time is 0.1 minute.

You can also see the indicated and calibrated airspeeds, the Mach number and TAS, the groundspeed and fuel flow at the mid-step point. You see also the calculated lift and drag coefficients, the thrust and drag, the engine thrust setting EPR, the acceleration factor, the gradient and airplane body angle, and the rate of climb.

Adding the fuel, distance and time increments from the step from 1500 to 2000 feet to the values at 1500 feet yields the fuel, distance and time from sea level to 2000 feet, as you see on the line of data for 2000 feet pressure altitude.

The process then continues with a series of calculations for the step from 2000 to 3000 feet, based on a mid-step altitude of 2500 feet. And so on, repeating the process of step integration up to any desired altitude.

effect of using smaller altitude increments

We mentioned earlier that the default altitude step for the step integration process is 1000 feet. As you saw before in the discussion about step integration for the takeoff distances, in the chapter entitled “Calculating Takeoff Distances”, changing the step size from twenty knots to one knot made a difference in the calculated takeoff distance of only five feet – a negligible difference.

Similarly, in the climb calculation changing the step size from one thousand feet to a smaller step has essentially no effect. For example, changing from 1000 to 100 foot steps changed the computed fuel by only two pounds, the distance by only about a tenth of a nautical mile, and the time was unchanged.

Climb With An Engine Inoperative

Earlier, we mentioned that the aviation regulations require us to consider the failure of an engine (or two engines, in some circumstances) at the most critical point during the flight[1], following which the airplane’s path must meet requirements for terrain clearance during the remainder of the flight as it proceeds toward an airport where the airplane can land.

It thus may be necessary to calculate the gradients of climb with one or two engines inoperative. This has already been discussed in the chapter entitled “Climb Angle and Rate Of Climb”.

It may also be necessary to calculate the flight path of the airplane following engine(s) failure, and that will require a step integration process similar to what we’ve demonstrated above. In that case, the process is based on rates of descent rather than rates of climb.

1. It’s important to emphasize that the “most critical point” could be during climb, cruise or descent, not only in cruise, although that’s the case which is most frequently considered.

The flight path of an airplane following the failure of one or more engines, however, may include not only descent to the altitude it's capable of maintaining with engine(s) inoperative, but a climbing path as the weight decreases due to fuel consumption following the level-off.

The Boeing software is capable of such calculations including the case of "driftdown and climbing cruise". The calculation process, whether by computer or by hand, will be discussed in detail in the chapter entitled "Cruise – Engine Failure and Driftdown". It will be simply an extension of what you have already learned in this and preceding chapters.

Chapter 31: Altitude Selection

Introduction

Selection of the best altitude at which to cruise during a flight is a complex subject with a number of equally important factors to consider. The best altitude for cruise depends on the airplane's weight, its design characteristics, the engine's thrust and fuel flow characteristics, the air temperature, the flight direction, the wind direction and velocity at the cruise altitude, air traffic constraints, the length of the flight, and the phase of the moon.[1]

Selection of the best cruise altitude for any given flight will have a significant effect on the flight's fuel efficiency. It can also affect the airplane's ability to maneuver at the cruise altitude. It can affect the cruise speed that can be flown. And it can effect the airplane's margin of speed from aerodynamic buffet.

In this chapter, we're first going to ask the question "what is the most fuel-efficient altitude at which to cruise?" Obviously, fuel costs are a major consideration in airline operations. This discussion introduces you to the concept of *optimum altitude*.

We'll next ask "to what altitude is it possible to climb, given the relevant operational factors such as weight and air temperature?" This will lead us to the concept of *altitude capability*. We'll then check to see if we can maintain cruise at that altitude, given the amount of thrust available.

We'll ask "at the selected cruise altitude, does the airplane maintain a suitable margin of speed from aerodynamic buffet?" Here, we're going to be talking about *maneuver capability*.

Finally, we'll look at the sort of altitudes you might select for a short flight. Climbing to 35,000 feet doesn't sound too practical if the flight is only 100 nautical miles.

This chapter will deal only with the selection of the <u>initial</u> cruising altitude for a flight. In the next chapter, entitled "Normal Cruise", we'll discuss altitude changes – "step climb" – during cruise for the purpose of maximizing fuel efficiency on longer flights.

Optimum Altitude

The word *optimum* comes from the Latin word "optimus", which simply means "best". When we're speaking of selecting an altitude for cruise, we take *optimum altitude* to mean the altitude at which the best possible fuel efficiency can be achieved in cruise.

In an automobile, we're accustomed to thinking of *fuel mileage* as a good measure of fuel efficiency. Fuel mileage in that context means simply the number of miles or kilometers that can be driven while using a specified amount of fuel. Determining automotive fuel mileage is simply a

1. Okay, forget about the phase of the moon. It's such a small effect that we usually neglect it.

matter of measuring how many miles were driven and how much fuel was consumed over that distance. Dividing the miles or kilometers by the gallons or liters yields the fuel mileage in miles per gallon or kilometers per liter.

The concept of fuel mileage is equally applicable to commercial jet transport airplanes. There too it means the distance that can be traveled while using a specified amount of fuel. It is customary at Boeing to show fuel mileage in units of nautical air miles (NAM) flown per 1000 pounds of fuel.

For an airplane, we determine fuel mileage this way:

$$\textit{fuel mileage} = \frac{\textit{nautical miles flown}}{\textit{pounds fuel burned}} = \frac{\textit{nautical miles}}{\cancel{\textit{hour}}} \times \frac{\cancel{\textit{hour}}}{\textit{fuel burned}} = \frac{\textit{true airspeed}}{\textit{fuel flow rate}}$$

The last term in that expression is the answer: <u>an airplane's fuel mileage is simply its true airspeed divided by its total fuel flow rate</u>. If we measure fuel flow in pounds per hour, the fuel mileage thus obtained will be in units of nautical air miles per pound of fuel.

Let's look at a typical graph of fuel mileage for a given cruise speed as it varies with altitude and weight. In that graph we're going to use fuel mileage units of nautical air miles per 1000 pounds of fuel burned. This would be obtained from:

$$\textit{NAM per 1000 pounds} = \frac{\textit{TAS (knots)}}{\textit{total fuel flow (lb/hr)}} \times \textit{1000}$$

As you'll see, fuel mileage depends on the cruise altitude. Low altitudes are inefficient, and very high altitudes are also inefficient. At some intermediate altitude, the fuel mileage in cruise is at its maximum achievable value. That is what we call the optimum altitude.

To the right, you see a 757-200's fuel mileage for several weights over a range of altitudes. As you see when you look at any of the four lines of airplane weight, the fuel mileage at the low altitudes is relatively low. As the altitude increases, so does the fuel mileage – up to a point. Above that point, the airplane has passed its most fuel-efficient altitude and the fuel mileage again decreases.

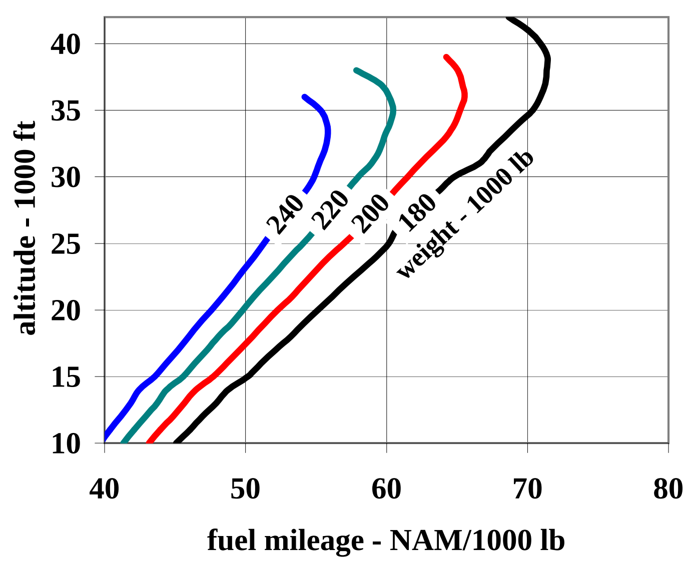


Figure 31-1

At the heaviest weight of 240,000 pounds, the fuel mileage reaches a peak value around 33,500 feet. At the lightest weight of 180,000 pounds, the fuel mileage is considerably greater, and it reaches its peak a little above 38,000 feet.

The altitude at which the maximum fuel mileage occurs on each line is the optimum altitude. Thus you see that the optimum altitude is a function of the airplane's weight.

The trends shown in the chart above are very typical for any commercial jet airplane. Only the magnitudes of the fuel mileage and optimum altitudes vary from airplane to airplane.

optimum altitude as a function of weight

You saw above in the graph of fuel mileage versus weight and altitude that the optimum altitude varied with weight, increasing as the weight decreases. The relationship between the optimum altitude and the weight looks like a smooth and simple one. Let's graph it:

To the right you see a graph of optimum altitude at Long Range Cruise speed as a function of weight, for our example 757-200.

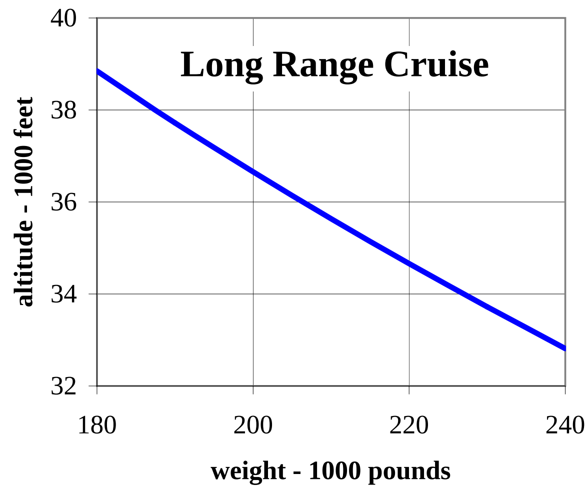


Figure 31-2

optimum altitude as a function of temperature

Temperature has very little effect on optimum altitude. While it's true that both true airspeed and fuel flow are functions of air temperature, it turns out that the increase in true airspeed with air temperature at a given Mach number is virtually the same as the increase in fuel flow with increasing air temperature. And since:

$$\textit{fuel mileage} = \frac{\textit{true airspeed}}{\textit{fuel flow}}$$

A similar increase in both airspeed and fuel flow means that the fuel mileage is almost unaffected.

A quick check for ISA and ISA+20°C shows that for the 757-200 the optimum altitude changes by less than 50 feet for that 20 degree change in air temperature.

ground fuel mileage

Up to this point, we've been talking about fuel mileage in terms of nautical miles air distance per 1000 pounds of fuel. We've gotten those fuel mileage numbers by dividing the airspeed by the fuel flow (and multiplying the result by 1000). As a measure of fuel efficiency, nautical air miles per pound is a perfectly acceptable parameter.

In the real world, though, an airplane doesn't usually fly in "still air" – air having no velocity over the ground. That's just another way of saying that an airplane is usually flying in some amount of wind: it might be a headwind or a tailwind, it might have low velocity or high velocity. And when an airplane is flying in a wind, its speed and path over the ground are different from its speed and path through the air.

An airplane's speed over the ground (its *ground speed*) and the direction of its path over the ground (its *track*) depend on its airspeed and the direction in which it's flying (its *heading*) and on the speed of the wind it's experiencing and the direction from which the wind is coming. Refer to figure 31-3 to the right to see how these three speeds and directions relate to each other.

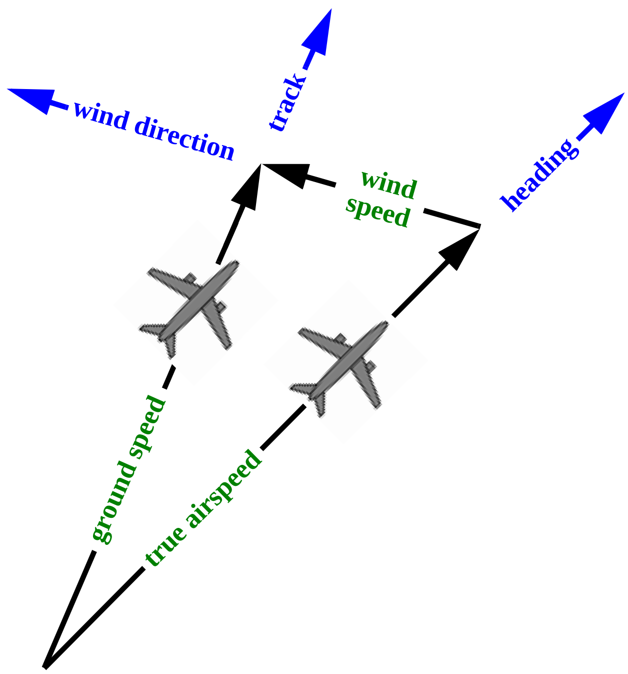


Figure 31-3

Notice that as the airplane flies along its track, it is yawed relative to the track: the airplane is maintaining its heading, and the wind is causing the angle of the track – the path over the ground – to be different from the heading.

In correct engineering terminology, we would say that an airplane's ground vector is the vector sum of the its flight vector (its speed and direction) plus the wind's vector (its speed and direction). In vector notation:

$$\overrightarrow{V_{gs}} = \overrightarrow{V_{air}} + \overrightarrow{V_{wind}}$$

When you stop to think about it, if you're looking to select a cruise altitude that will offer you the least possible trip fuel burnoff, it's not nautical air miles per pound of fuel burned that's important, it's the nautical ground miles per pound. Why? Because the amount of cruise fuel burned when flying from waypoint A to waypoint B along the route is equal to the ground distance divided by the ground fuel mileage:

$$\textit{fuel burned from A to B} = \textit{ground distance A to B} \div \frac{\textit{nautical ground miles}}{\textit{pound}}$$

which is the same as:

$$fuel_{A-B} = \frac{pounds}{NGM} \times NGM_{A-B}$$

and we find the ground fuel mileage from:

$$ground\ fuel\ mileage = \frac{NGM}{pound} = \frac{ground\ speed}{fuel\ flow}$$

A tailwind improves ground fuel mileage, and a headwind reduces it. A headwind is customarily designated as a negative value for that reason. (For takeoff, on the other hand, a headwind is helpful and is denoted as a positive value, a tailwind is negative.)

Suppose now that the winds at all of the cruise altitudes are the same magnitude of headwind or tailwind. How would that affect the optimum altitude?

It wouldn't. If the wind component is the same at all altitudes, the altitude offering the best fuel mileage is the same as the zero-wind optimum altitude.

Where the winds are important is in the case when the winds are different at different altitudes. That leads us to the subject of:

wind-altitude trades

It's quite common when planning a flight to see that the winds at different altitudes are markedly different. It's not unusual to see more than a fifty-knot wind change between different flight levels. This condition is due to high-altitude wind conditions, the best-known of those being the *jet stream*, a fast-moving relatively narrow current of air most commonly found between 30,000 and 40,000 feet altitude.

Let's say, for example, that the wind at your optimum altitude is expected to be a 60-knot headwind. Let's say also that at an altitude 8000 feet below optimum the wind is only a 20-knot headwind. Which altitude would produce the better ground fuel mileage?

Obviously, the air fuel mileage 8000 feet below the optimum altitude would be considerably less than it would be at the optimum altitude. But when you consider the winds in this example, it's very possible that the ground fuel mileage would be better at the lower altitude.

If we use a 757-200 cruising at Long Range Cruise speed at a weight of 220,000 pounds, its optimum altitude is approximately 35,000 feet. At that altitude its true airspeed is 459 knots and its fuel flow per engine is 3801 pounds per hour. The ground fuel mileage, considering the 60-knot headwind, will be:

$$ground\ fuel\ mileage = \frac{(459 - 60)}{(2 \times 3801)} \times 1000 = 52.5\ NGM\ per\ 1000\ pounds$$

Eight thousand feet lower, the true airspeed is 446 knots and the fuel flow is 3999 pounds per hour per engine, giving a ground fuel mileage for the 20-knot headwind case of

$$\frac{(446 - 20)}{(2 \times 3999)} \times 1000 \; = \; 53.3 \text{ NGM per 1000 pounds}$$

From that, you see that for these example conditions it would be more fuel-efficient to cruise 8000 feet below the optimum altitude.

What you have just seen is an example of what's called a *wind-altitude trade*. By giving up some altitude, you gain on fuel burned because of the wind difference – you "traded" deviation from optimum altitude for a more favorable wind.

To the right, you see a graph which may help you to see how wind-altitude trades work. The green (solid) line represents ground fuel mileage at optimum altitude as a function of the wind. (For this chart, we're using the same 757-200 data as we've shown above.)

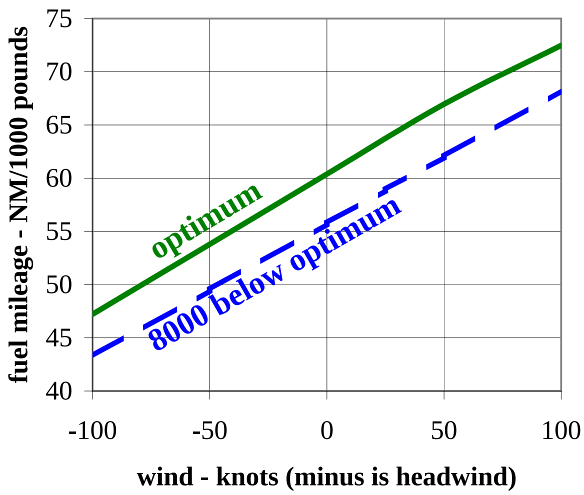


Figure 31-4

The blue (broken) line shows the ground fuel mileage at a cruise altitude 8000 feet below optimum.

You see that at optimum with a wind of minus 60, the ground fuel mileage is approximately 52.5, just as we calculated above; for 8000 feet below optimum with a minus 20 wind, the ground fuel mileage is approximately 53.3.

We can use a chart such as you see in figure 31-4 to solve wind-altitude trade problems. Here we'll look at a tailwind example.

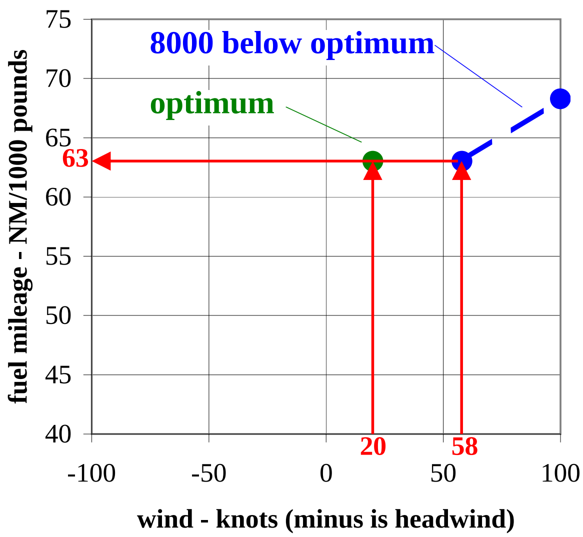


Figure 31-5

For example, let's say that the wind at optimum altitude is expected to be 20 knots tailwind. That sounds pretty good. Would there ever be any advantage to cruising 8000 feet below optimum?

At optimum altitude with a 20-knot tailwind, there's a ground fuel mileage of 63 nautical miles per 1000 pounds. <u>If the wind 8000 feet below optimum is more favorable than 58 knots tailwind, then flying at the lower altitude</u>

offers fuel mileage greater than 63. "More favorable" in this context of course would mean a lesser headwind or a greater tailwind.

We tend to think of cruising at the optimum altitude as being the best possible condition; from what you've just seen, however, you now understand that cruising below or above the optimum altitude can reduce your trip fuel consumption under some wind conditions. Careful examination of the enroute winds when planning a flight can often yield substantial fuel benefits by enabling the planner to make the best possible selection of the cruise altitudes.

penalty for off-optimum operation

Clearly, cruise operation at the optimum altitude will yield the best possible fuel efficiency. It follows logically then that for any cruise operation conducted away from the optimum altitude, there will be a "penalty" for that off-optimum operation, in the form of additional fuel consumption. (Note, please, that for now we're not considering any possible wind-altitude trade benefits. In effect, we're saying that the winds are essentially the same at all of the altitudes.)

OFF-OPTIMUM CONDITION	FUEL MILEAGE PENALTY %	
	LRC	.80M
2000 FT ABOVE	1	1
OPTIMUM ALTITUDE	0	0
2000 FT BELOW	1	1
4000 FT BELOW	3	4
8000 FT BELOW	12	14
12000 FT BELOW	18	24

Table 31-1

The table shown to the right is a typical fuel penalty table as published in the Flight Planning and Performance Manual (FPPM). It's not precise because the exact magnitude of the penalty depends on a number of variables; still, it's a useful guide for someone wanting an approximate sense of the penalty for off-optimum operation. More precise values of the off-optimum penalty can be calculated from the true airspeed and fuel flow data for the condition of interest.

You'll notice that the off-optimum penalty increases more rapidly for constant-Mach cruise, such as Mach 0.80 shown in the table above, as compared to Long Range Cruise (LRC) speeds. We will be discussing LRC and constant-Mach cruise much more in the next chapter, "Normal Cruise".

flight above optimum altitude

From a fuel conservation standpoint, it would be nice if an airplane could simply fly at optimum altitude at all times. This isn't practical, however – because as an airplane burns fuel and becomes lighter, the optimum altitude moves upward. Maintaining optimum altitude at all times would thus entail a climbing cruise procedure.

As a result, it is a common operational practice to begin a flight somewhat above its optimum altitude, knowing that as the airplane cruises (at constant altitude) it will be getting closer to optimum. That is, while the airplane maintains level flight, the optimum altitude is rising beneath it, approaching the airplane's cruise altitude. At some point, if the cruise continues long enough, the cruise altitude will become equal to the optimum altitude; flight continuing beyond that point means that the airplane's altitude will gradually become farther and farther below the optimum value.

On a flight having a long cruise distance, it is standard practice to increase the cruising altitude in "steps" in order to keep the cruise altitude close to the optimum altitude. This is referred to as *step climb*, and this subject will be discussed at length in the next chapter.

Fuel mileage decreases rapidly above optimum altitude. As an initial cruise altitude, one or two thousand feet above optimum is fine, since the fuel mileage will then gradually improve as the airplane weight decreases. More than a few thousand feet above optimum is unnecessary and quite inefficient. As you saw in the off-optimum fuel penalty table above, flying more than 2000 feet above or below optimum will incur a penalty that rises rapidly with increasing separation from optimum.

We're sometimes asked about flight at altitudes well above optimum in order to overfly adverse weather conditions, or because a better cruise altitude isn't available because of heavy traffic. The answer to such questions is that such flight is probably not advisable, for two reasons that we're going to discuss: thrust limitations at higher altitudes and "buffet margins". First, let's look at the way that thrust limitations at higher altitudes will affect our selection of a cruise altitude. That's the topic we refer to as altitude capability.

Altitude Capability

An airplane's altitude capability is really two engineering problems in one package:

- The thrust available from the engines in climb steadily decreases with increasing altitude. That limits the maximum altitude to which the airplane can climb.
- The thrust available from the engines in cruise decreases with increasing cruise altitudes. That limits the maximum cruise altitude at which the airplane will be capable of maintaining the desired cruise speed.

Let's examine the first of those two.

ability to climb to cruise altitude

An airplane's ability to climb depends directly on the thrust available from the engines, and on its drag.

You'll recall from an earlier chapter that rate of climb is calculated using this equation:

$$\text{rate of climb} = 101.268 \times V_{true} \times \left[\frac{\frac{T - D}{W}}{\left(1 + \frac{V}{g}\frac{dV}{dh}\right)} \right]$$

where V_{true} is the airplane's true airspeed in knots
T is the thrust available
D is the airplane drag
W is the airplane weight

You'll recall also that the denominator of the equation is the "acceleration correction" arising from the airplane's very gradual increase in true airspeed as it climbs.

From the equation, you see that the airplane's rate of climb depends directly on the difference between the thrust available and the drag, $(T - D)$.

Thrust available depends directly on the density of the air passing through the engines. Increasing altitudes mean decreasing air density. That in turn means that the engine thrust available decreases as altitude increases. The drag, on the other hand, remains essentially constant, although it does show some small variation at altitudes above approximately 30,000 feet. The drag is frequently referred to as the *thrust required*.

As altitude increases, therefore, $(T - D)$ – the thrust available minus the thrust required – decreases. As a result, the rate of climb decreases until ultimately the airplane reaches an altitude at which the rate of climb is near zero.

Let's look at some numbers. As an example, we'll go back to the 757-200 that we were looking at in the preceding chapter on climb to altitude. We'll keep the takeoff weight at 220,000 pounds. For these conditions, we'll make a graph of thrust required and the thrust available for two different air temperature conditions.

To determine the thrust required, we first calculated the lift coefficient at each altitude from the weight, altitude, speed, and air temperature. Then from the drag polar we found the corresponding drag coefficient; this was corrected if necessary for Reynolds number effects and converted back to a drag force.

To the right, you see two sloping lines of thrust available versus altitude; one is for a standard day, the lower one is for a hotter day at ISA+20°C.

You see a single vertical line of thrust required. In fact, the thrust required does change with temperature, but it is a very small effect. For example, at 10,000 feet the thrust required is only 110 pounds greater at

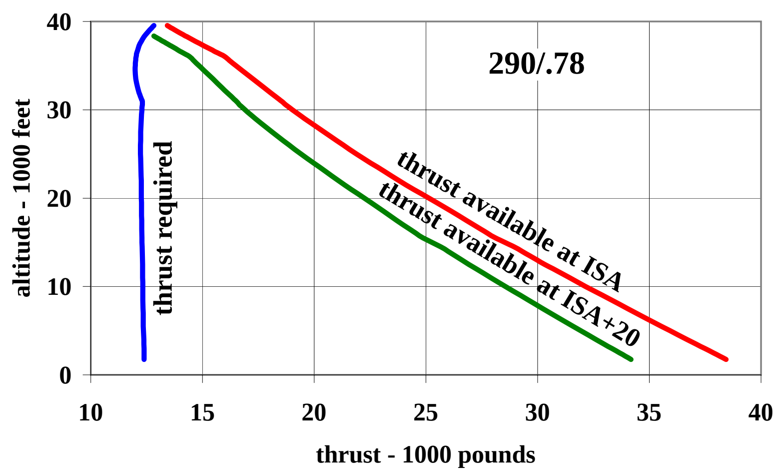


Figure 31-6

ISA+20 than it is at ISA. For practical purposes, we can neglect the effect of air temperature on thrust required.

In the chart above, you see that the quantity (thrust available minus thrust required) decreases very rapidly as the airplane climbs. Approaching 40,000 feet it becomes very small.

Here's a graph of rate of climb versus altitude for both ISA and ISA+20 conditions.

Notice how the rate of climb changes instantaneously when the airplane speed transitions from a constant calibrated airspeed to a constant Mach number at the crossover altitude of approximately 31,000 feet.

You see also that the rate of climb changes slightly when the airplane passes through the tropopause at 36,089 feet.

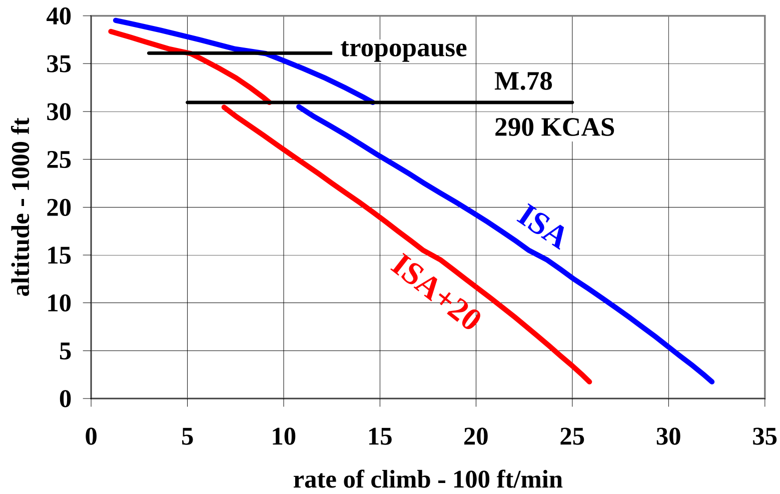


Figure 31-7

absolute ceiling and service ceiling

The altitude at which zero rate of climb remains is called the *absolute ceiling*.

It is an industry standard practice to define an airplane's *altitude capability* as that altitude at which there remains 100 feet per minute rate of climb, called a *residual rate of climb*. That altitude is sometimes referred to as the airplane's *service ceiling*.

The reason for including a small residual rate of climb in an airplane's published altitude capability is to avoid placing an airplane in a climb environment in which the rate of climb is extremely low, requiring the airplane to spend considerable extra time and fuel achieving its target altitude. The following chart will illustrate that clearly.

In the graph to the right, you see a side view of the airplane's climb path after takeoff. (We've omitted the portion of the climb from takeoff to 30,000 feet.)

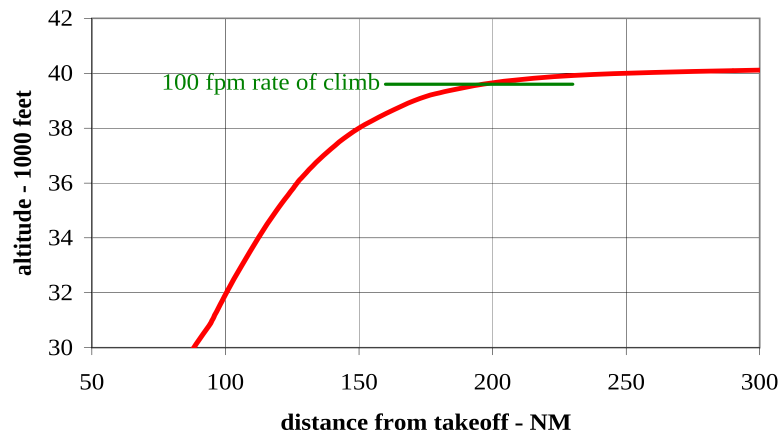


Figure 31-8

You see clearly how the rate of climb decreases rapidly as it begins to approach its altitude capability. We have marked the point at which the airplane's rate of climb has decreased to 100 feet per minute, which occurs at approximately 39,650 feet.

It's interesting to look also at the <u>time</u> to climb to altitude. You can see in the figure to the right that the rate of climb is initially quite high, but extending the climb above the service ceiling greatly prolongs the climb time.

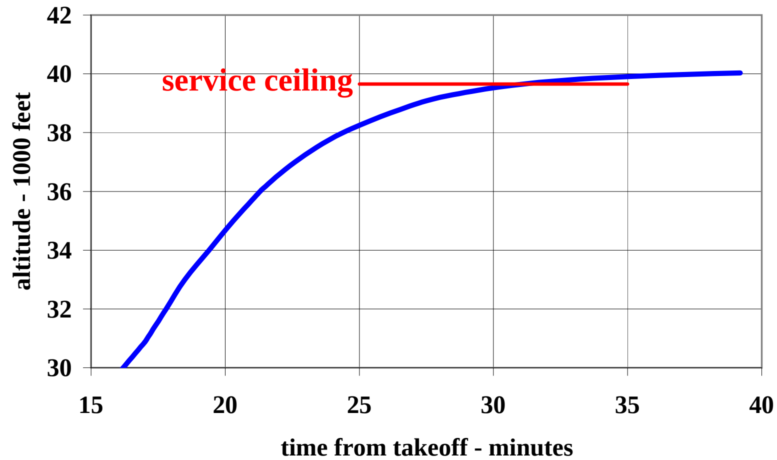


Figure 31-9

You see that extending the climb above the service ceiling is a very prolonged process. In fact, examining the computer data for this climb analysis shows that the airplane never truly reaches zero rate of climb. Instead, it stabilizes at a rate of climb of about 12.5 feet per minute, which it is capable of maintaining almost indefinitely. That condition exists because there is enough thrust available to enable the airplane to maintain a very low but stable climb rate as its weight gradually decreases due to fuel burnoff.

From the preceding discussion, you see that "service ceiling" is a useful concept. It provides a reasonable maximum cruise altitude considering rate of climb and time to climb, avoiding the excess time and distance required to climb to the airplane's absolute ceiling. Today's commercial transport airplanes generally have service ceilings sufficiently above their optimum altitudes. Thus <u>there is no real performance benefit in exceeding the service ceiling for cruise.</u>

Let's tabulate the service ceiling of a 757-200 as a function of weight and temperature as an example.

Here's a table showing the altitude capabilities, as a function of ISA deviation. These values are based on a climb speed of 290/.78 using maximum climb thrust.

WEIGHT pounds	ISA	ISA + 10°C	ISA + 15°C	ISA + 20°C
180,000	42,822	42,772	42,101	41,254
200,000	40,751	40,705	40,061	39,256
220,000	37,874	38,830	38,208	37,439
240,000	37,156	37,114	36,512	35,676

Table 31-2

The first thing you'll probably notice is that the altitude capability is essentially the same for ISA and ISA + 10°C. Why? Because the engines powering this example airplane are to ISA + 10°C at their maximum climb thrust rating. Above ISA + 10°C the climb thrust decreases with increasing air temperatures.

cruise speed capability at altitude

At the beginning of this discussion, we said that altitude capability had to consider two different factors: first, what altitudes could we climb to, and second, having reached an altitude, could we sustain cruise speeds at that altitude.

So what we might call *cruise speed capability* is simply the ability of the airplane to fly at the desired cruise speed at an altitude without exceeding the maximum <u>cruise</u> thrust rating of the engine. That's really quite similar to the calculation of altitude capability, but we're going to compute it a little differently: we're going to calculate <u>the highest altitude at which the airplane can maintain zero rate of climb at maximum cruise thrus</u>t while flying at the desired cruise speed. Think about it: zero rate of climb means that thrust required is equal to thrust available. If we say that the thrust available is the maximum cruise thrust, then saying zero rate of climb is the same as saying that the cruise thrust available is adequate to maintain the desired cruise speed, but not to accelerate. And <u>that</u> is the definition of the maximum altitude at which the desired cruise speed can be flown.

To create the table you see to the right, we used the Boeing software to compute the maximum altitude at which zero rate of climb can be maintained at the maximum cruise thrust at the desired speed, in this example the Long Range Cruise speed. (We'll be discussing LRC in detail in the next chapter.)

WEIGHT pounds	ISA	ISA + 10°C	ISA + 15°C	ISA + 20°C
180,000	42,314	42,269	41,334	40,192
200,000	40,264	40,223	39,336	38,234
220,000	38,404	38,366	37,520	36,458
240,000	36,702	36,666	35,724	33,690

Table 31-3

summary: altitude capability

You have now seen two tables of altitude capability: first, table 31-2 for maximum altitude at the climb speed at Maximum Climb Thrust, with 100 feet per minute residual rate of climb; second, table 31-3 for the maximum altitude LRC speed at Maximum Cruise Thrust with zero residual rate of climb.

By comparing the two tables, you see what we mean when we say that altitude capability is two engineering considerations in one: (a) to what altitude can you climb, and (b) at what altitude you can maintain the desired cruise speed. Tables 31-2 and 31-3 clearly illustrate why both factors must be considered.

Table 31-2 is giving us valid information as to the maximum altitudes to which the airplane can climb. However, the altitudes shown in table 31-2 don't consider the fact that after arriving there the airplane would be incapable of maintaining the desired cruise speed. Thus you have a choice: fly at the altitude shown in table 31-2 and accept a lower cruise speed capability, or cruise at the altitude shown in table 31-3.

Bear in mind that while the maximum altitude for speed capability is lower than the altitude capability in climb for this airplane with this engine at this speed, that is not necessarily always the case. It depends on the relationship between climb and cruise thrust, and what cruise speed has been selected.

Remember also that at the altitude at which the maximum available thrust is required in order to maintain the desired cruise speed, there is zero surplus thrust. That means that the airplane can be flown only in straight and level flight. If it became necessary to enter a bank angle, the airplane could not then maintain its desired cruise speed – because banking the airplane would increase its drag. The thrust required would then exceed the thrust available, and the airplane would decelerate. Flight at this altitude thus appears to be rather impractical since it doesn't allow for any maneuvering in cruise without incurring a speed decrease.

Altitude Selection For Buffet Margin

So far, we've examined the questions of "how high an altitude can we climb to?" and "what is the most fuel-efficient altitude for cruise?" But there's another major consideration that should never be left out of the altitude selection process: how much "maneuver margin" will there be at the selected cruise altitude. That's going to take some explaining.

review: low- and high-speed buffet

low-speed buffet

In an earlier chapter entitled "Speeds" we spent a lot of time discussing low-speed buffet and stall.

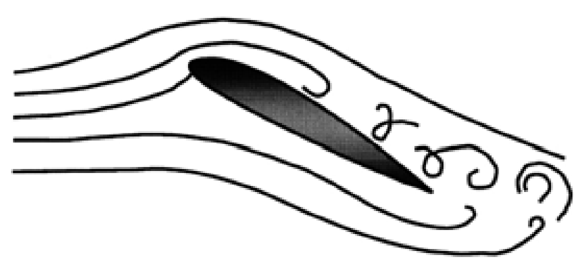


Figure 31-10

We mentioned that as an airplane decelerates to slower and slower airspeeds, the stall will be preceded by a condition called "low-speed buffet"; this is a condition in which the development of turbulent flow over the top of the wing, while not yet a fully stalled condition, will cause the airplane to shake as the turbulent flow strikes the horizontal tail and the aft fuselage. That condition of shaking is called “buffeting”.

Hence *low-speed buffet* is, you might say, a warning to the flight crew that the airplane’s airspeed is becoming unacceptably close to the stalling condition. And while stalling an airplane isn’t necessarily dangerous, recovery from a stall incurs a loss of altitude, and if the airplane is at takeoff or approach altitudes insufficient height may exist above the ground to permit a stall recovery, leading to a ground impact.

Flight at speeds below that speed at which the low-speed buffet begins are therefore to be avoided.

high-speed buffet

In the chapter entitled “Wings” we discussed the characteristics of the airflow over a wing at increasing airspeeds.

We showed that although today’s commercial jet transport airplanes always fly at subsonic speeds, a region of supersonic flow will develop on a wing’s upper surface, and later on its lower surface, at higher speeds.

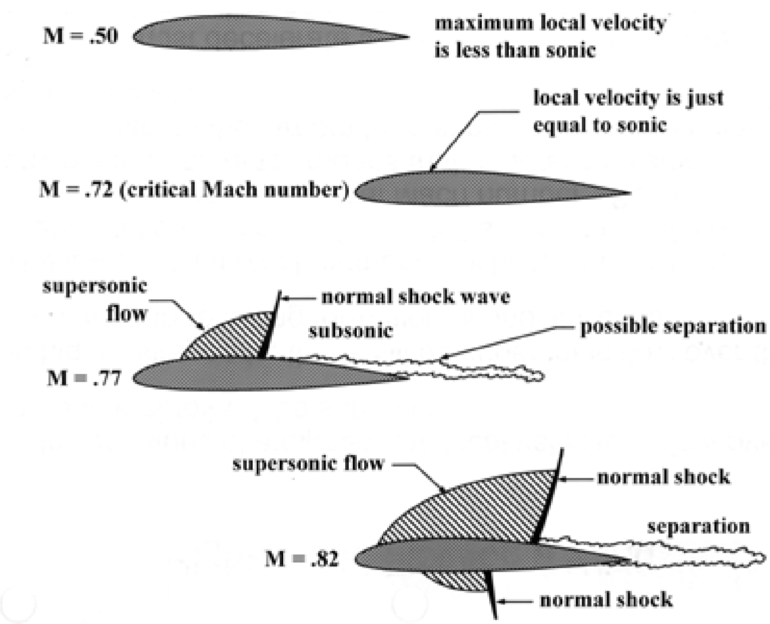


Figure 31-11

At some point, the airflow velocity in the supersonic flow zone must decelerate back to subsonic speeds; this deceleration is done through a “shock wave” – a very thin region of extremely rapid deceleration from supersonic to subsonic velocities, with rapid change of air pressure and air temperature across it.

Shock waves on a wing have the characteristic of causing separation of the airflow from the wing behind the shock; this separation causes the flow to become turbulent and that turbulence can cause airplane buffet in a manner similar to low-speed buffet. This is the condition referred to as *high-speed buffet*.

While high-speed buffet doesn’t have the potential for stalling as does the low-speed buffet condition, it’s still a natural aerodynamic warning of the development of shock waves on the wings and the onset of rapid drag increase; flight at speeds above the speed at which high-speed buffet begins are thus to be avoided, just as speeds below the low-speed buffet onset are to be avoided.

buffet boundary

The speeds at which low- and high-speed buffet will begin – the *initial buffet speeds* – are established during the flight testing of any new airplane. From those speeds, the initial buffet lift coefficients can be determined over a wide range of Mach numbers.

Ordinary approach-to-stall tests are conducted to determine the buffet C_L values for the low Mach numbers. To determine the buffet lift coefficients for the higher Mach numbers, a series of "wind-up turns" are conducted: the airplane is trimmed for each test Mach number and altitude, and then the bank angle is gradually increased in a coordinated turn until a moderate buffet level is experienced. The bank angle, weight, altitude and Mach number allow a buffet C_L to be calculated.

The flight test results can be shown graphically, as you see in the figure to the right.

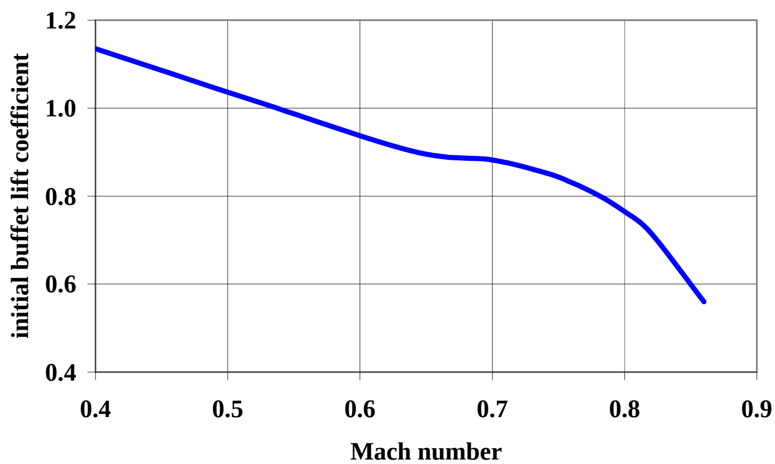


Figure 31-12

The chart shows what is referred to as the *initial buffet boundary.* To the right you see the initial buffet boundary for the 757-200 airplane. For any given Mach number, it shows the lift coefficient at which buffet will begin.

At the low Mach numbers, the buffet will be stall-onset buffet – low-speed buffet. At the high Mach numbers, the buffet will be shock wave-induced buffet – high-speed buffet.

By itself, this buffet boundary isn't intuitively obvious. Let's try to make it easier to understand.

You may remember that the equation for lift coefficient as a function of weight, air pressure ratio δ, Mach number, and wing area is:

$$C_L = \frac{L}{1481.4\, M^2\, \delta S}$$

where L is the airplane lift force
M is the Mach number
δ is the air pressure ratio
S is the reference wing area

First of all, in steady-state level flight the airplane lift force is equal to the airplane weight, so we'll use W instead of L. Second, we're going to create a new variable we'll call "weight over delta" which is simply $\frac{W}{\delta}$ where δ is the air pressure ratio. From these two items, we get that:

$$C_L = \frac{\frac{W}{\delta}}{1481.4\, M^2 S}$$

Thus the lift coefficient C_L can be seen to be a function of only $\frac{W}{\delta}$ and the Mach number M.

To illustrate how this relationship can be useful, let's take a 757-200 at a weight of 200,000 pounds flying at an altitude of 42,000 feet (δ=0.1681). The value of $\frac{W}{\delta}$ for those two values will be 1,189,578. Since δ is dimensionless, being a ratio of pressures, the units of $\frac{W}{\delta}$ are therefore also in pounds.

We can then find the lift coefficient over a range of Mach numbers for that combination of weight and altitude, using the above equation. The following chart shows the results.

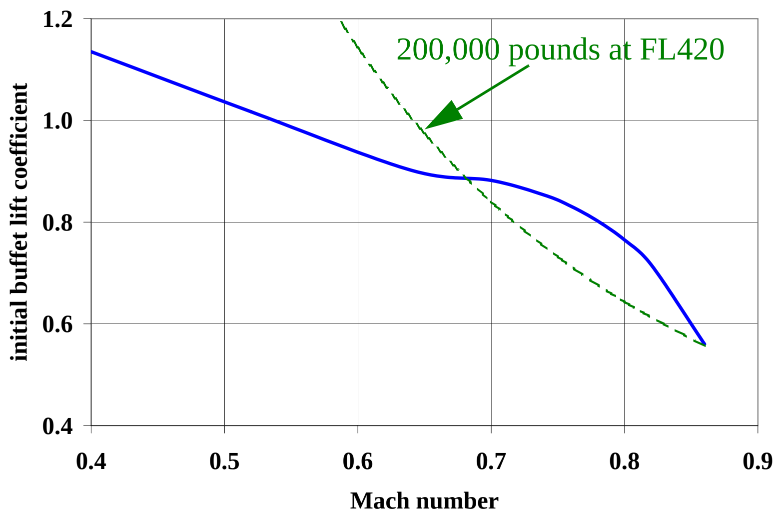


Figure 31-13

To the right, you see the buffet boundary line we introduced you to previously, but now we have added a broken line showing the lift coefficient at a weight of 200,000 pounds at a altitude of 42,000 feet, as a function of the Mach number.

What does this new line show us? Simply this: for these conditions, the low-speed buffet will begin at Mach 0.68, and high-speed buffet will begin at Mach 0.86. Hence <u>any speed between Mach 0.68 and 0.86 is free of buffet</u>; at speeds slower than 0.68 the airplane will be in low-speed buffet or stall; at speeds faster than 0.86 the airplane will be in high-speed buffet.

In the chart to the right, we've added one more line: a weight of 200,000 pounds again, but at an altitude of 30,000 feet. Now you see that the low-speed buffet will begin at approximately 0.465. The high-speed buffet will begin at a speed greater than Mach 0.86, which is the 757's maximum certified operating Mach number, M_{MO}.

Thus you see that as the altitude decreases the buffet-free speed range becomes wider.

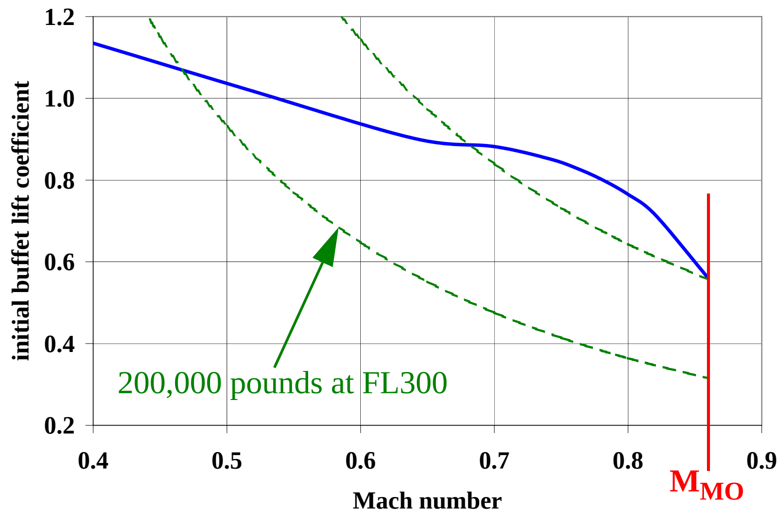


Figure 31-14

coffin corner

That raises a very interesting question: what if you were to fly right at the very peak of the buffet boundary?

Figure 31-15 to the right illustrates a condition known as *coffin corner*. This occurs at very high values of $\frac{W}{\delta}$.

In this condition, the airplane is simultaneously at the low-speed buffet boundary and the high-speed buffet boundary. <u>The airplane can thus neither accelerate nor decelerate without entering buffet</u>. Nor can the airplane climb to a higher altitude.

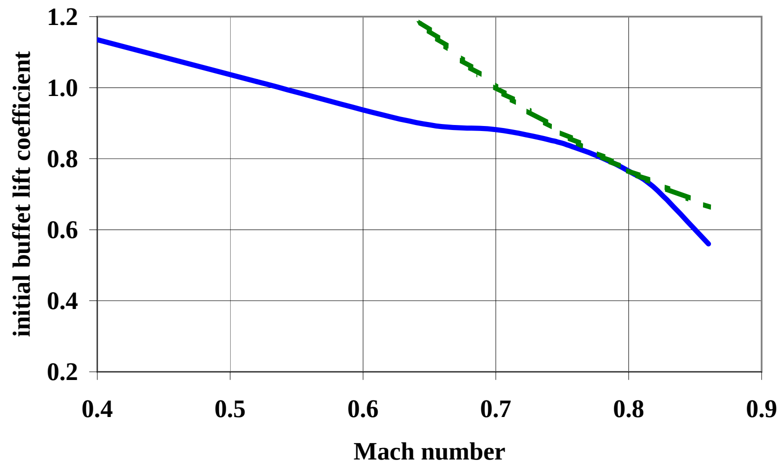


Figure 31-15

Even a slight amount of turbulence could result in a change of airspeed adequate to put the airplane into buffet or – possibly – to stall at high altitude. This is clearly a very undesirable condition in which to attempt to fly.

Some well-known airplanes such as the Lockheed U-2 high-altitude reconnaissance airplane could and did frequently fly at or near to coffin corner. This took considerable flying skill and was potentially hazardous.

You'll be relieved to hear that today's commercial jet transport airplanes don't have enough thrust to place themselves at the very high values of $\frac{W}{\delta}$ at coffin corner.

altitude-speed-weight buffet boundary

Now that you understand the idea of buffet boundary, we're going to show low- and high-speed buffet Mach numbers as a function of altitude. The following diagram shows the same buffet information as we have been discussing, but we have re-drawn it to show the buffet boundary in more useful terms of weight, altitude and Mach number rather than buffet lift coefficient and $\frac{W}{\delta}$.

Here's the altitude-speed buffet boundary for a weight of 200,000 pounds. You see, for example, that at 30,000 feet altitude the airplane will be free of buffet at any speed between Mach 0.465 and the maximum operating Mach number of 0.86, and at 42,000 feet the range is from 0.68 to 0.86, as we saw in the previous chart.

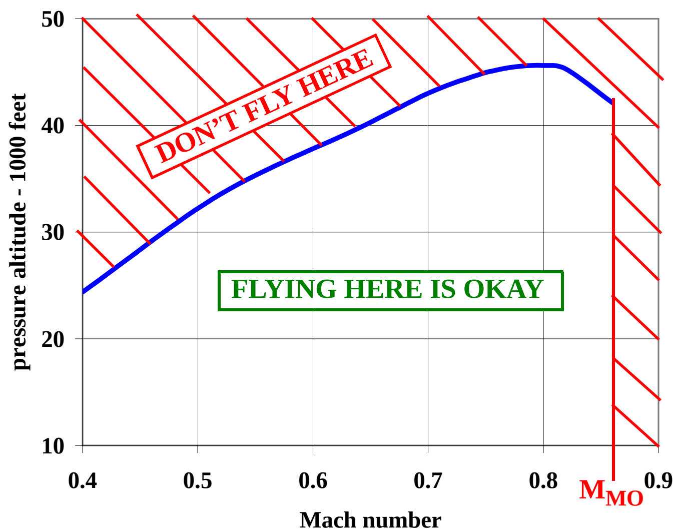


Figure 31-16

This buffet boundary line establishes the border between the altitudes and speeds where the airplane is buffet-free, and the "no-fly zone" where the airplane will be either in buffet or exceeding M_{MO}.

Here's the same chart, but with some additional weights. You see that as the weight increases, the buffet boundary moves down in altitude – or, for any given altitude, as the weight increases the band of acceptable flight speeds becomes narrower, more restrictive.

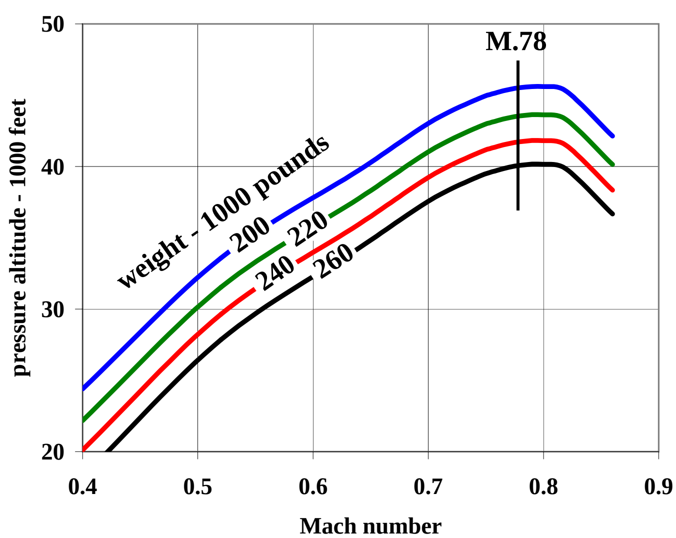


Figure 31-17

Here's a question for you: when cruising at Mach 0.78 at a weight of 260,000 pounds, what is your "buffet-limited altitude"?

You see at a glance that for a cruise speed of Mach 0.78 at a weight of 260,000 pounds, <u>flight at any altitude higher than 40,000 feet would put the airplane outside of its buffet envelope</u> into the "don't fly here" area. For those conditions, then, we would say that the airplane's buffet-limited altitude is 40,000 feet. Similarly, at weights of 240,000 pounds, 220,000 pounds and 200,000 pounds, the buffet-limited altitudes would be approximately 42,000 feet, 44,000 feet and 46,000 feet respectively. So you see that as weight decreases, the buffet-limited altitude increases.

Allow us now to make a quick change to the way we look at the buffet boundary.

Let's agree that an airplane weighing 260,000 pounds is aerodynamically the same as an airplane that weighs 200,000 pounds but is experiencing a normal acceleration of 1.3 gees. This normal acceleration could come from a turn having a bank angle of 40 degrees, or from turbulence. (Normal acceleration can also, of course, be the result of rapid pitch changes such as a rapid recovery from a rate of descent, but that would be unusual in normal flight.)

Similarly, an airplane weighing 240,000 pounds is aerodynamically the same as an airplane weighing 200,000 pounds when it is experiencing 1.2 gees of normal acceleration. A weight of 220,000 pounds is the same as a weight of 200,000 pounds at 1.1 gees, and, finally, a weight of 200,000 pounds is the same as 200,000 pounds at 1.0 gees.

Now we're going to repeat figure 31-17, keeping the lines exactly the same. However, we're going to use the weight-gee equivalences we've just shown you to change the way we label those lines.

The entire chart now is shown as being for a weight of 200,000 pounds. Notice that the four lines are now lines of normal acceleration, from one gee to 1.3 gees.

The line that was previously labeled "260,000 pounds" is now labeled "1.3 gees". The previous 240,000-pound line is now labeled 1.2 gees, and so on.

Aerodynamically, figures 31-17 and 31-18 to the right are exactly the same.

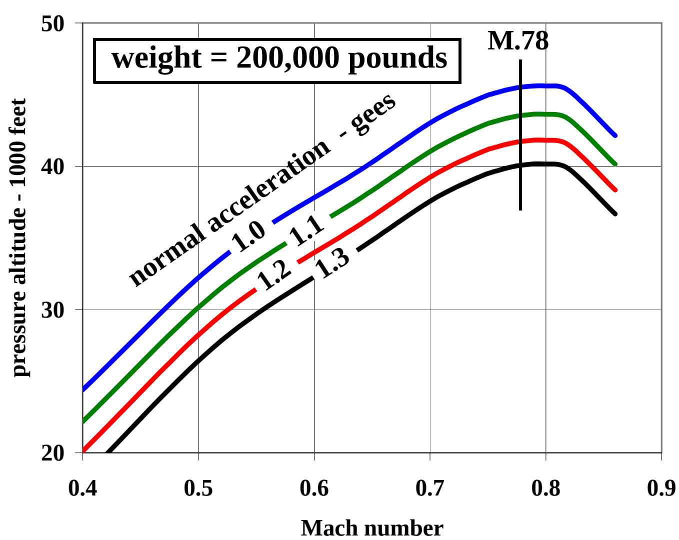


Figure 31-18

maneuver capability

The ability of an airplane to withstand some amount of normal acceleration in flight without entering buffet is referred to as its *maneuver capability.*

Thus, in the chart above, you can see that at a weight of 200,000 pounds when cruising at Mach 0.78, altitudes of 40,000 feet, 42,000 feet, 44,000 feet, and 46,000 feet will offer maneuver capabilities of 1.3, 1.2, 1.1 and 1.0 respectively.

regulatory requirements for maneuver capability

At the time of this writing, there are no known regulatory requirements for minimum maneuver capability at cruise altitude.

Many airlines have a policy of requiring a maneuver capability of 1.3 gees or better at the selected cruise altitude, and more if turbulence is expected. Operators are free to require more conservative margins if they wish – we understand that there are some operators who do require maneuver capabilities greater than 1.3.

Flight at maneuver capabilities less than 1.3 should not be thought of as inherently dangerous, but rather that the speed margins to buffet are less. Flight in light buffet isn't dangerous, but it <u>is</u> a natural aerodynamic reminder that the airplane is approaching its operating limits.

Bank Angle Capability to Maximum Cruise Thrust

There's one more condition to be considered when determining the maximum cruise altitude.

Previously, we've spoken quite a lot about the ability of an airplane to withstand some amount of normal acceleration – gees – before encountering initial buffet. However, there's another kind of maneuver capability: the ability of the airplane to conduct normal cruise operations, including bank angles, without exceeding the maximum cruise thrust limit!

Think about this scenario: you have just leveled off into cruise at your assigned altitude. You have set the throttles to maintain the desired cruise speed. Your maximum available thrust now decreases from Maximum Climb Thrust, MClT, to Maximum Cruise Thrust, MCrT. Unless you are right at the cruise thrust limited altitude, then the required thrust setting will be less than MCrT.

Let's say now that for reasons of traffic or weather avoidance or changes to the direction of your assigned route of flight it's necessary to bank the airplane. You know what happens when the airplane banks: the bank angle imposes a normal acceleration equal to $\frac{1}{cos(bank\ angle)}$, so the wings have to generate more lift than usual. When the <u>lift</u> increases, so does the <u>drag</u>. Therefore, <u>when banking, to maintain the cruise speed the thrust must be increased</u>. What if the bank angle becomes sufficiently great that the necessary thrust exceeds the maximum cruise thrust limit?

That isn't permitted: automatic throttle systems will not allow the thrust to exceed maximum cruise thrust when the maximum cruise thrust limit is selected manually or by the flight management computer. If setting thrust manually then it might be <u>possible</u> (depending on the airplane model) to exceed the limit thrust, but that would be a violation of the engine manufacturer's limitations and warranty conditions. Thus, <u>if the bank exceeds the angle at which the thrust required is just equal to the maximum thrust available, the airplane can not continue to maintain the desired cruise speed at the desired altitude – it must either decelerate or descend</u>.

It is uncommon, in cruise flight, to bank more than five or ten degrees, perhaps fifteen. Heading changes are usually small, and maneuvering for traffic avoidance at cruise altitudes is infrequent. Also, bank angles are seldom continued for any length of time. Thus it's not often that an airplane in cruise will exceed the angle to maximum thrust: but it's possible and does occasionally occur. Performance engineers should be aware of this possibility and should plan accordingly if the conditions may warrant it.

Since the maximum cruise thrust limit decreases on hotter days, the bank angle to maximum cruise thrust will be less on a hotter day. The following will give two examples of the bank angle to MCrT for the 757-200 we've been using as an example.

In the graph to the right, the solid lines are lines of optimum altitude, optimum+1000 and optimum+2000 feet.

The broken lines show the altitudes at which a bank angle of the amount shown will place the airplane just at its cruise thrust limit condition.

This chart is for standard day (ISA) conditions.

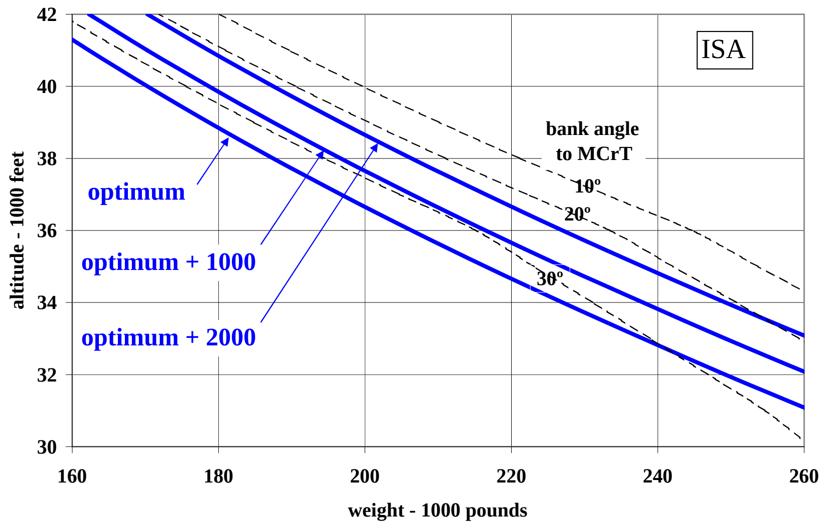


Figure 31-19

You see that even when flying 2000 feet above optimum altitude, the airplane will be able to bank more than 20 degrees before becoming cruise thrust-limited.

Compare the graph above for ISA with the graph to the right for ISA+15.

While the optimum altitude lines are the same at both temperatures, the maximum cruise thrust is less at ISA+15, and thus for any given weight, the thrust-limited bank angle will be less.

Notice that for ISA+15, flight 2000 feet above optimum altitude will allow bank angles between 10 and

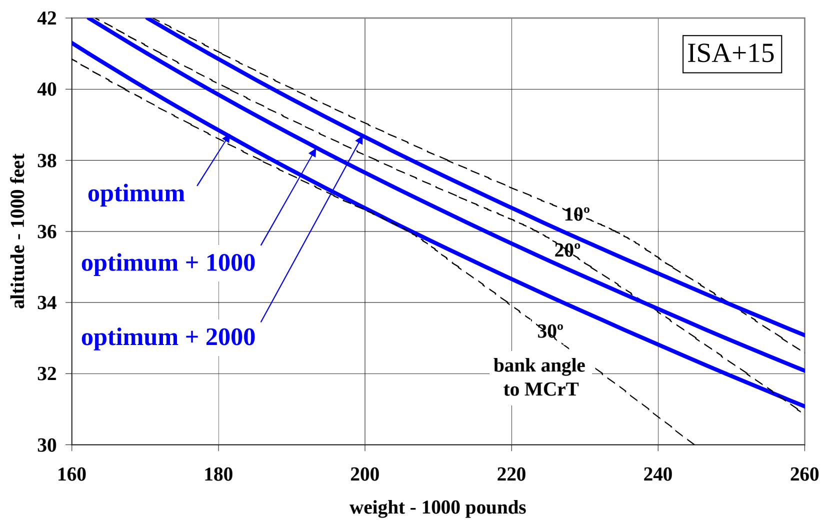


Figure 31-20

20 degrees (and less than 10 degrees at the heaviest weights) before becoming cruise thrust-limited.

standard practice at Boeing

The altitude capability tables published by Boeing in the Flight Planning and Performance Manuals and elsewhere make an attempt to consider three factors we have just discussed: the airplane's service ceiling, the ability to maintain the cruise speed in level flight, and the ability to maneuver without speed loss.

For the 737NG, for example, the published altitude capability is based on level flight at maximum cruise thrust with a 100 foot per minute residual rate of climb. The FPPM text notes that:

> *Flying above these altitudes with sustained banks in excess of approximately 15° may cause the airplane to lose speed and/or altitude.*

For the 757 and 767, the published altitude capability is based on either climb thrust with 100 feet per minute rate of climb, or cruise thrust with zero rate of climb. For these airplanes, flight with sustained bank angles greater than approximately 12° of bank may cause loss of altitude or speed.

For the 777, the altitude capability is based on level flight at maximum cruise thrust with a 300 feet per minute residual rate of climb. This is more conservative than the other models: the 777 is somewhat more likely to encounter speed loss at cruise altitude when maneuvering. With a minimum residual rate of climb of 300 feet per minute at max cruise thrust, sustained bank angles greater than approximately 21° may cause speed or altitude loss.

By tailoring the published altitude capability data in this manner, Boeing attempts to show altitudes that will satisfy all three considerations.

Connecting Them All Together

Now that you understand each of the principal considerations in cruise altitude selection, it might be wise to see how they relate to each other for any given airplane. Let's ask ourselves "at any given airplane weight, what is the airplane's altitude capability, what is its optimum altitude, and how much maneuver margin will exist at each of those? If we fly 2000 feet above optimum, how much maneuver margin will exist, and will we have an acceptable bank angle capability to maximum cruise thrust?

The answers to these questions, of course, vary between airplanes, although not widely. Let's summarize the key altitude selection factors in a table, to illustrate typical relationships. We'll use the 757-200 as our example again. We show the parameters for two different weights:

altitude selection criterion	200,000 pounds	240,000 pounds
optimum altitude	36653 feet	32811 feet
altitude for 1.3g margin to initial buffet	40133 feet	36339 feet
service ceiling, ISA (100 ft/min at max climb thrust)	40751 feet	37156 feet
service ceiling, ISA+15 (100 ft/min at max climb thrust)	40061 feet	36512 feet
service ceiling, ISA+20 (100 ft/min at max climb thrust)	39256 feet	35676 feet
max cruise thrust limit, ISA (0 ft/min at max cruise thrust)	40264 feet	36702 feet
max cruise thrust limit, ISA+15 (0 ft/min at max cruise thrust)	39336 feet	35724 feet
max cruise thrust limit, ISA+20 (0 ft/min at max cruise thrust)	38234 feet	33690 feet
normal accel to initial buffet at optimum altitude + 2000 feet	1.40 gees	1.40 gees
bank angle to max cruise thrust at opt alt + 2000 feet, ISA+15	15.3 degrees	13.8 degrees

Table 31-4

This airplane is capable of easily reaching optimum altitude plus 2000 feet at ISA+15 degrees. Observe, though, that although there is still a good buffet margin of 1.4 gees at optimum plus 2000 feet, the bank angle capability to max cruise thrust at that altitude and ISA+15 is only around 14 to 15 degrees. This could affect altitude selection should a higher bank angle capability be desired.

Further, notice that for ISA+20, although there is still adequate climb thrust to achieve optimum plus 2000 feet, you'll find that the max cruise thrust limit altitude at that temperature is less than optimum plus 2000. Flight at this cruise speed (Long Range Cruise) at temperatures greater than one or two degrees warmer than ISA+15 is thus impossible. It might be possible at a slower speed, but such operation would probably result in a loss of fuel efficiency.

Keep in mind that other airplanes may have somewhat different altitude selection characteristics, although the general principles apply to all models.

Short-Distance Cruise Altitude

Many flights are scheduled for busy city pairs that are separated by relatively short distances. Examples of that might be New York City to Boston, or Amsterdam to London.

Since climb distances to optimum altitudes are typically greater than 100 nautical miles, and descent distances are similarly in excess of 100 nautical miles, it follows that a trip distance of, for example, 150 NM (Seattle to Portland, maybe) isn't well suited for cruise at the airplane's optimum altitude. Even if the descent began immediately after reaching the cruise altitude, the descent distance would be greater than the distance remaining to the destination.

For a short-distance flight, the highest altitude to which the airplane should climb would be that altitude which would result in zero cruise distance – that is, a flight which transitions directly from climb to descent, the two segments together traveling the exact distance of the trip. If a short cruise segment is desired, however, then a cruise could be scheduled for an altitude somewhat below that for zero cruise distance. There will be very little effect on the trip fuel consumption.

For example: if a 757-200 takes off at 200,000 pounds, climbs to 25,000 feet and transitions directly to descent, then when it lands it will have traveled 139 nautical miles and it will have burned 3901 pounds of fuel. If instead a cruise segment at 20,000 feet is included, then the cruise segment will be approximately five minutes in length, and the trip fuel consumption will be 3905 pounds over the same 139 nautical miles. Further deviation from the “no-cruise” altitude would lead to a gradual increase in the trip fuel consumption.

Another example, using a different altitude: if the same airplane takes off at the same weight, climbs to 30,000 feet this time, and transitions directly to descent, it will travel 179 nautical miles and will consume 4604 pounds of fuel. Scheduling a cruise sector for this flight at 20,000 feet would offer 11 minutes of cruise and would consume 4664 pounds of fuel over the same 179 nautical miles – only a 60 pound increase in fuel consumption.

Therefore, as you’ve seen from the above numbers, on short flights one needn’t be obsessive about choosing the best cruise altitude, because the effect on trip time and fuel is minimal. Other factors such as passenger comfort, cabin service requirements, traffic, and weather may be more compelling considerations than fuel consumption.

Boeing standard practice

Boeing publishes a chart in the Flight Planning and Performance Manual that offers suggested altitudes for short distance flights as a function of trip length and takeoff weight. The altitudes obtained by using this chart are based on a flight profile that includes one minute in cruise.

The reason for using that particular criterion is that the Flight Management Computer Systems on Boeing airplanes provide short-distance cruise altitudes based on that same rule.[1] The FPPM short-distance cruise altitudes are computed for the same flight profile as the FMCS for the sake of consistency.

1. It’s worth mentioning, however, that this one minute in cruise is the FMCS default value; airlines can opt to enter a different value of cruise time into their FMCS if desired.

Chapter 32: Normal Cruise

Introduction

In this chapter, we'll be discussing "normal" cruise, by which we mean ordinary cruise flight with all engines operating. In later chapters, we'll address the subject of cruise during which one or two engines might fail, and also we'll look at cruise with a loss of pressurization in the cabin, necessitating an emergency descent to a lower altitude.

We're going to begin by looking at the amount of thrust that is required in cruise at a given altitude, as a function of speed, weight, and air temperature.

We'll then look at the engine characteristics of thrust, fuel flow rate, and power settings. We'll also find out how much thrust is available in cruise at the maximum cruise thrust setting.

All of this information gives us the ability to solve many types of cruise-related performance questions. We'll be able to find the airplane's fuel mileage as a function of cruise speed. That will show us which cruise speeds yield good fuel mileage, and which ones don't. As part of this topic, we'll be defining the speed called maximum range cruise (MRC) and another speed called long range cruise (LRC).

We'll also be able to determine the airplane's speed capability – what cruise speeds are possible considering the cruise thrust limitations of the engines.

We're also going to look at the subject of *step climb*, a process by which the flight will stay as close as possible to its optimum altitude during the cruise segment of flights over long distances.

Finally, we'll discuss the performance of an airplane at a special kind of cruise speed: holding.

You can see that this is going to be an interesting chapter, so let's get started.

Thrust Required In Cruise

With what you already know from preceding chapters, you're able to calculate how much engine thrust is required in cruise, for given conditions of speed, altitude, air temperature and weight. Let's go through one sample calculation, and then we'll show you a completed chart of thrust required for a range of cruise speeds and weights, at a specified altitude and air temperature.

Let's use these conditions for the demonstration calculation:

- 757-200 with PW2040 engines
- cruise at FL350 (35,000 feet pressure altitude)
- Mach number 0.80
- airplane weight of 220,000 pounds

- standard day conditions

The first step is to calculate the lift coefficient for those conditions. Remember that:

$$lift\ coefficient = \frac{L}{1481.4\,M^2\,\delta\,S}$$

where L is the airplane lift
M is the Mach number
δ is the air pressure ratio, which at FL350 is 0.2353
S is the reference wing area, which for the 757-200 is 1951 square feet.

In steady-state cruise, the airplane lift will be equal to the weight, so L = 220,000 pounds.

For our example conditions, then, the lift coefficient will be

$$C_L = \frac{220000}{1481.4 \times 0.8^2 \times 0.2353 \times 1951} = 0.5055$$

Next, remember that the lift coefficient and the drag coefficient for the same conditions are related to each other by the airplane drag polar.

For the lift coefficient we have just computed, and the given Mach number, the drag polar for the 757 tells us that the drag coefficient for this C_L is equal to 0.02923. The Reynolds number correction $\Delta C_{D_{RE}}$ for these conditions is -0.00017, so the corrected standard day drag coefficient is 0.02906.

Now that we know the drag coefficient, it's easy to calculate the drag:

$$drag = 1481.4 C_D M^2 \delta S = 12{,}648\ pounds$$

In steady-state cruise, you know that the drag is equal to the thrust. For that reason, we will refer to the drag as the "thrust required".

Now that you've seen how to calculate a single point of thrust required, here's the data for a range of Mach numbers.

First, here's the lift coefficient:

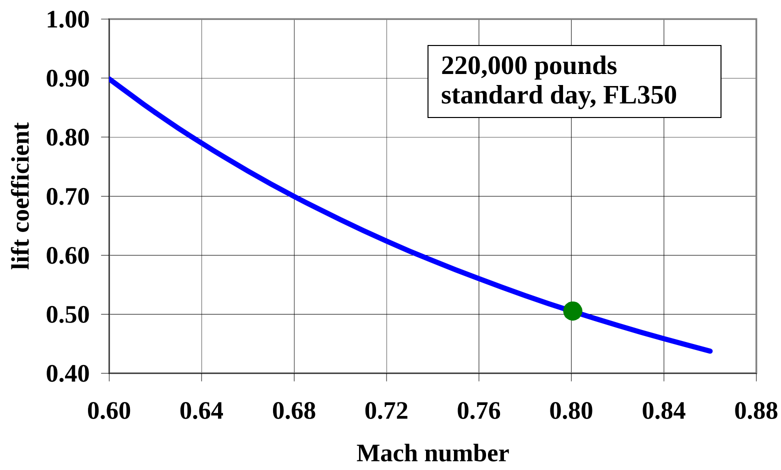


Figure 32-1a

Next, the drag coefficient:

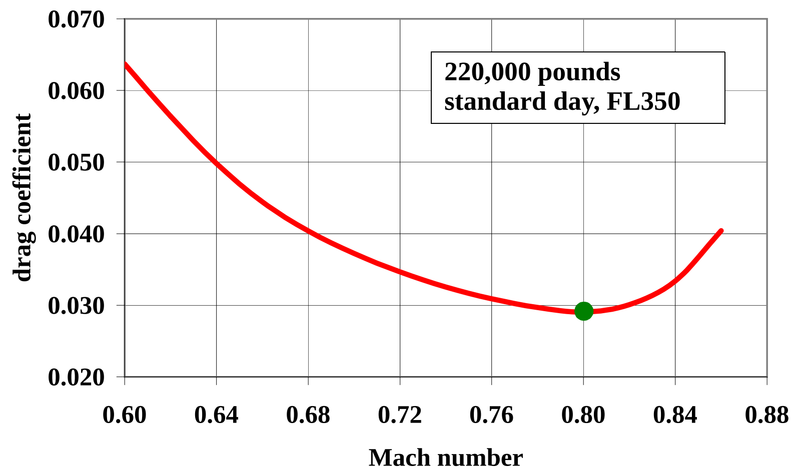


Figure 32-1b

Finally, the thrust required:

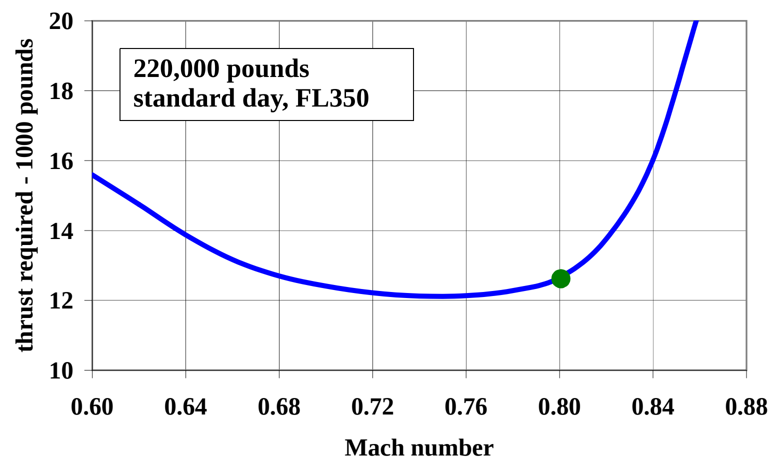


Figure 32-1c

air temperature effect on thrust required

Temperatures warmer than standard day will increase the thrust required, temperatures colder than standard day will decrease it. This effect results from the air temperature's effect on the Reynolds number correction to the drag coefficient, $\Delta C_{D_{RE}}$. An increase of the air temperature results in an increase of the Reynolds number correction and therefore an increase in the airplane's drag coefficient.

You saw just a few paragraphs previously that for a Mach number of 0.8 at FL350 and 220,000 pounds weight, the drag coefficient was 0.02923. For a standard day at that altitude, $\Delta C_{D_{RE}}$ was a negative 0.00017, so the corrected drag coefficient was 0.02906. For that drag coefficient, you saw that the thrust required was 12,648 pounds.

Repeating that calculation for a temperature of ISA+20, the $\Delta C_{D_{RE}}$ is now +0.000078. That means that the corrected drag coefficient is now 0.02931, and the thrust required increases from 12,648 to 12,756 pounds. While this effect on thrust required is relatively small, it isn't negligible.

Cruise Fuel Flow

The engine manufacturers provide us with engine performance data in several forms. One of these is fuel flow data. Boeing uses tabulations of fuel flow as a function of thrust and Mach number. In these tabulations, fuel flow is given in the form of "corrected fuel flow", and thrust is given as $\frac{F_N}{\delta}$, thrust divided by the atmospheric pressure ratio δ.

These specific forms of fuel flow and thrust are used in order to generalize the chart, allowing a single chart or table to include a <u>range</u> of pressure and temperature conditions, instead of requiring a different one for each different temperature and altitude.

For the known conditions of flight, it's an easy matter to convert $\frac{F_N}{\delta}$ to thrust or vice versa, and corrected fuel flow to actual fuel flow or vice versa. It only requires us to know the Mach number, the air temperature, and the altitude.

You'll recall that in one of the propulsion chapters we defined a parameter called *corrected fuel flow*. You saw that it is defined as:

$$\textit{corrected fuel flow} = \frac{\textit{fuel flow}}{\delta_T \theta_T^x}$$

where δ_T is the total pressure ratio
θ_T is the total temperature ratio
x is a power to which the temperature ratio is raised

The parameter x is discussed in the chapter entitled “Jet Engine Fundamentals”.

Remember that the total pressure and temperature ratios are defined by:

$$\delta_T = \delta\,(1 + 0.2\,M^2)^{3.5} \quad \text{and} \quad \theta_T = \theta\,(1 + 0.2\,M^2)$$

Combining and simplifying yields:

$$\textit{corrected fuel flow} = \frac{\textit{fuel flow}}{\delta_{amb}\,\theta_{amb}^{x}\,(1 + 0.2\,M^2)^{3.5 + x}}$$

or

$$\textit{fuel flow} = \textit{corrected fuel flow} \times \delta_{amb}\,\theta_{amb}^{x}\,(1 + 0.2\,M^2)^{3.5 + x}$$

where δ_{amb} is the ambient atmospheric pressure ratio $\frac{p}{p_0}$

θ_{amb} is the ambient atmospheric temperature ratio $\frac{T}{T_0}$

Given the engine manufacturer’s fuel flow data and the above equations, a little data manipulation allows us to graph the fuel flow required as a function of Mach number and thrust.

Having a chart such as this allows us to find the fuel flow for any given condition of flight because we can always calculate the thrust required for a known airplane speed.

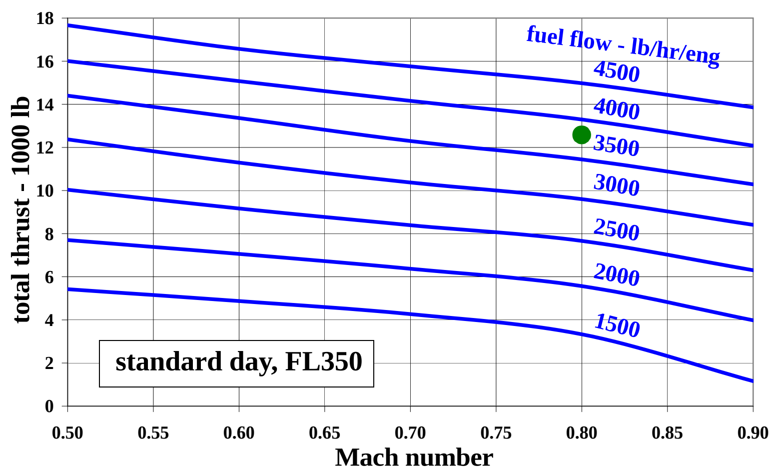


Figure 32-2

You'll remember the example that we were working on earlier in this chapter of a 757-200 at a weight of 220,000 pounds at FL350.

When we take the above fuel flow data and apply it to that example, we'll get a graph such as you see to the right.

We find that the fuel flow per engine for these conditions is 3829 pounds per hour.

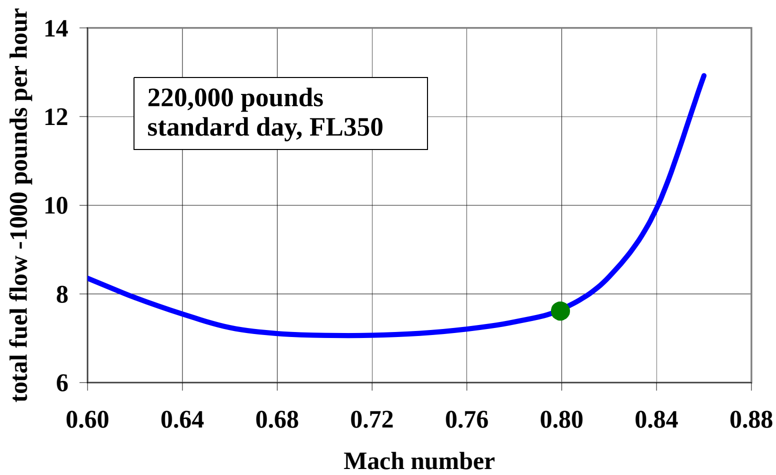


Figure 32-3

temperature effect on fuel flow

For a given condition of weight, altitude and Mach number, temperature actually has two different effects on the airplane's fuel flow rate.

First, you've already seen that temperatures above ISA will increase the airplane's drag – and hence its thrust required – due to the Reynolds number correction to the drag coefficient. You saw, in the example that we showed previously, that a temperature increase from ISA to ISA+20°C increased the drag from 12,648 pounds to 12,756 pounds.

Second, for a given amount of thrust required at a given altitude and Mach number, the corrected fuel flow is not a function of temperature, but the actual fuel flow does depend on temperature. Just a few paragraphs ago, we showed you the equation relating fuel flow to corrected fuel flow:

$$\textit{fuel flow} \;=\; \textit{corrected fuel flow} \times \delta_{amb}\,\theta^{x}_{amb}\,(1 + 0.2\,M^{2})^{3.5 + x}$$

From this equation, you see that in increase in θ causes an increase in the fuel flow.

To illustrate approximately how much the fuel flow would increase for a reasonable temperature increase, let's say that we have, for example, an ISA fuel flow in Mach 0.80 cruise at FL350 of 3800 pounds per hour. The standard day fuel flow of 3800 pounds per hour would thus increase to 4008 pounds per hour, an increase of 208 pounds per hour for a 20°C increase in air temperature.

Now consider the combined effect of temperature on fuel flow. Let's look at an actual example.

Earlier, we did some calculations for a 757-200 at a weight of 220,000 pounds at FL350. For a Mach number of 0.80, we saw that for a standard day the airplane's thrust required was 12,648 pounds per engine.

On an ISA day, for a thrust required of 12,648 pounds, the corrected fuel flow was 11725 pounds per hour.

For the engines on our example airplane, the value of x is 0.61. At ISA, the value of $\delta_{amb}\,\theta_{amb}^{x}\,(1 + 0.2\,M^{2})^{3.5 + x}$ would be 0.32636, and for ISA+20°C it would be 0.34424.

For the standard day, then, the actual fuel flow is 11725×0.62636, or 3826 pounds per hour per engine.

On an ISA+20°C day, for a thrust required of 12,756 pounds per engine, the corrected fuel flow would be 11815 pounds per hour. If it were a standard day, the fuel flow would then be 3856 pounds per hour, but because we're actually calculating the fuel flow now for ISA+20 conditions, that becomes 4067 pounds per hour per engine.

Thus, for our example a 20-degree increase in the air temperature caused a total fuel flow increase of 241 pounds per hour per engine. The increase in fuel flow is thus 6.25%, or approximately 3% per 10°C increase in temperature.

Cruise Thrust Settings

Another form in which engine manufacturers give us engine performance data is sometimes referred to as *generalized thrust* data. This form of data provides us with information on engine thrust produced for a given thrust setting – whether EPR or N1 – at a given Mach number.

For our example engine, the graph to the right illustrates the relationship between EPR, Mach number, and thrust produced.

Since we know that the thrust required for our example of 220,000 pounds at FL350 is 12,648 pounds, or 6324 pounds per engine, it's now a simple matter to find the EPR required for that case.

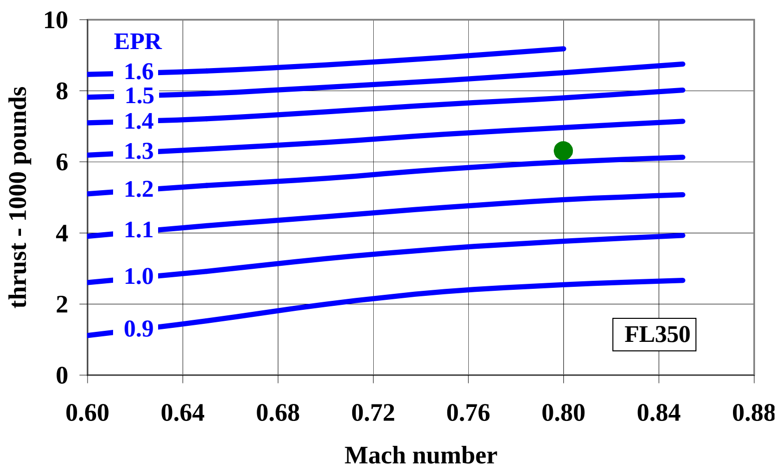


Figure 32-4

For our example conditions, on an ISA day, we would find that the cruise EPR would be 1.236.

We could continue the process and produce a chart such as you see to the right, showing EPR required versus speed for a given weight and altitude.

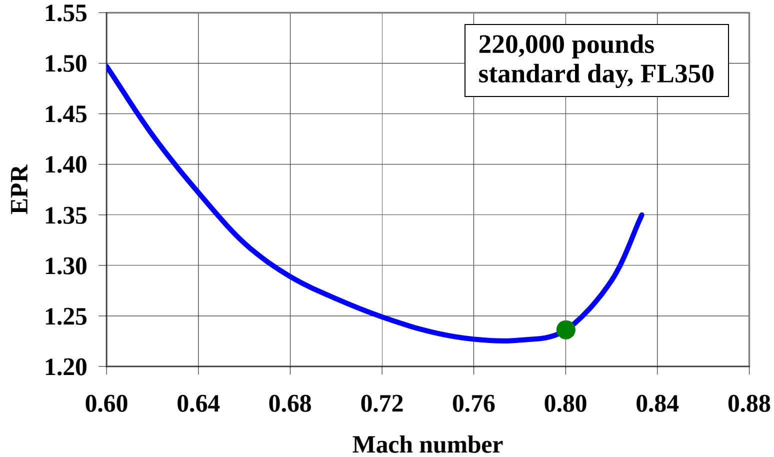


Figure 32-5

Cruise Fuel Mileage

Now you have all the tools that are needed to enable you to calculate cruise fuel mileage.

You have seen how to calculate the thrust required for any given cruise conditions. You have also seen how to calculate the fuel flow rates. So now let's look at the procedure for calculating fuel mileage. As an example for this exercise, we'll use the 757-200 again, at a cruise weight of 220,000 pounds at FL350 on a standard day. For these conditions, we have already calculated the fuel flows for a range of Mach numbers. So what's next?

calculating fuel mileage

First, remember the basic equation for fuel mileage:

$$\textit{fuel mileage} = \frac{\textit{true airspeed}}{\textit{fuel flow}}$$

Let's begin, then, by calculating the true airspeeds corresponding to Mach numbers from 0.60 to 0.86.

The relationship between Mach number and true airspeed is:

$$\textit{true airspeed} = Ma_0\sqrt{\theta}$$

where M is the Mach number

a_0 is the sea level standard day speed of sound, 661.4786 knots

θ is the temperature ratio $\dfrac{T}{T_0}$

As an example, the true airspeed for a Mach number of 0.80 on a standard day at 35,000 feet would be:

$$true\ airspeed\ =\ 0.80 \times 661.4786 \times \sqrt{\frac{(288.15 - 0019812 \times 35000\,)}{288.15}}\ =\ 461.1\ knots$$

If you repeat this calculation for other Mach numbers over the range of 0.60 to 0.86 and then graph the results, you'll have the graph you see to the right.

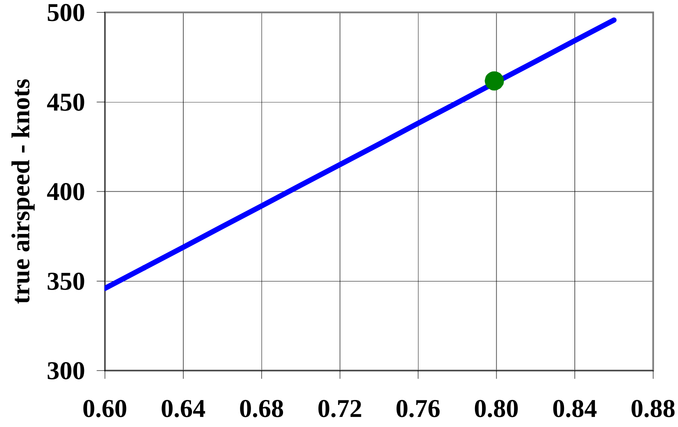


Figure 32-6

Now we have the true airspeeds, and we have the fuel flows. For each Mach number, then, the fuel mileage will be equal to the true airspeed divided by the fuel flow.

It's Boeing's standard practice to show fuel mileage as nautical air miles flown per thousand pounds of fuel consumed. Thus the equation for fuel mileage following that convention would be:

$$fuel\ mileage\ (NAM\ per\ 1000\ pounds\ of\ fuel)\ =\ \frac{NAM\ per\ hour}{fuel\ flow,\ pounds\ per\ hour} \times 1000$$

Since we use NAM per hour, true airspeed, this fuel mileage definition doesn't consider the effect of wind.

So now we will combine the two charts we have already seen, the chart of true airspeed and the chart of fuel flow:

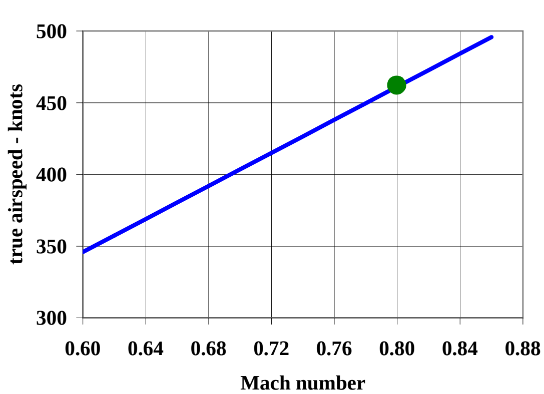


Figure 32-7a

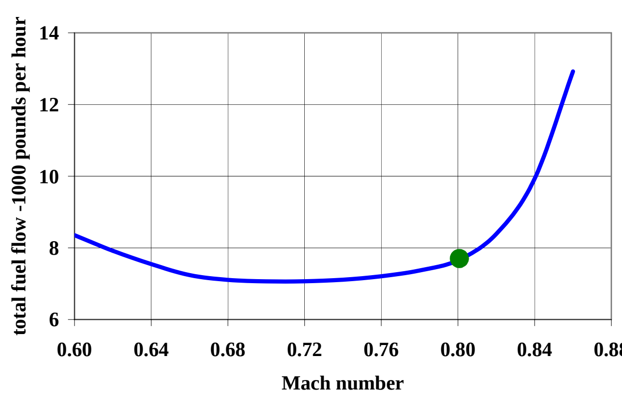


Figure 32-7b

Combining these two charts yields the fuel mileage chart, at last:

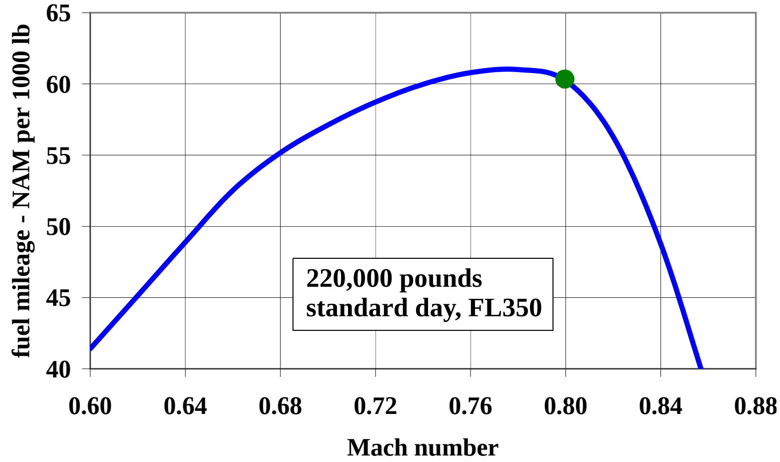


Figure 32-7c

Now you can see the fuel mileage as it varies with Mach number, and you can understand why it looks as it does.

At the lower Mach numbers, the fuel flow rate changes only a small amount as Mach number increases. As the airspeed increases with an almost constant fuel flow, the airspeed is the more dominant factor and we can understand that the fuel mileage (true airspeed divided by fuel flow) should increase, as you see in the chart.

At the higher Mach numbers, however, the rapidly rising drag results in a rapidly rising fuel flow, thus the fuel flow predominates and the fuel mileage shows a rapid decrease above Mach 0.80.

For our example conditions, with a true airspeed of 461.1 knots and a total fuel flow of 7658 pounds per hour, the fuel mileage is 60.21 nautical miles per 1000 pounds of fuel consumed.

summary: cruise parameters

We have gone through the calculations of the cruise parameters for a typical cruise condition. Given the manufacturers' aerodynamic data and propulsion data, it's a simple matter. Here's a summary of our calculations:

For a 757-200 at a weight of 220,000 pounds at FL350 and ISA, cruising at Mach 0.80:

- the lift coefficient is 0.5055
- the drag coefficient is 0.02906
- the thrust required is 12,648 pounds, or 6324 pounds per engine
- the fuel flow is 3829 pounds per hour per engine, and the total fuel flow is 7658 pound per hour
- the true airspeed is 461 knots
- the fuel mileage is 60.21 nautical miles per 1000 pounds of fuel
- the EPR required is 1.23.

MRC and LRC

The chart above is a typical fuel mileage curve. Using the curve we can define two special cruise speeds: *Maximum Range Cruise* speed (MRC) and *Long Range Cruise* speed (LRC).

Maximum Range Cruise is, just as the name suggests, the speed that yields the best possible fuel mileage, thus allowing cruise for the longest possible distance on a given amount of fuel. For these conditions of weight, altitude and temperature, the value of MRC will be just under Mach 0.78, with a fuel mileage of approximately 61 NM per 1000 pounds of fuel.

Long Range Cruise is something a little different. LRC is a speed that offers a somewhat faster cruise, but with only a small sacrifice in fuel mileage. The standard aviation industry definition is that LRC is the speed which offers 99% of the maximum possible fuel mileage.

LRC has long served as an accepted compromise between a desire for higher fuel mileage and a desire for shorter cruise times. No, it's not the best fuel mileage, but neither is it the slowest cruise speed.

To illustrate these two speeds, let's look at just one small section of the fuel mileage curve, centered on its peak.

Exact calculation shows that for the conditions of our example, MRC occurs at Mach 0.7766, with a fuel mileage of 61.03 NAM per 1000 pounds.

99% of 61.03 is 60.42. Calculation shows that this will occur at Mach 0.7979.

The difference between the two Mach numbers is a true airspeed difference of 12.3 knots.

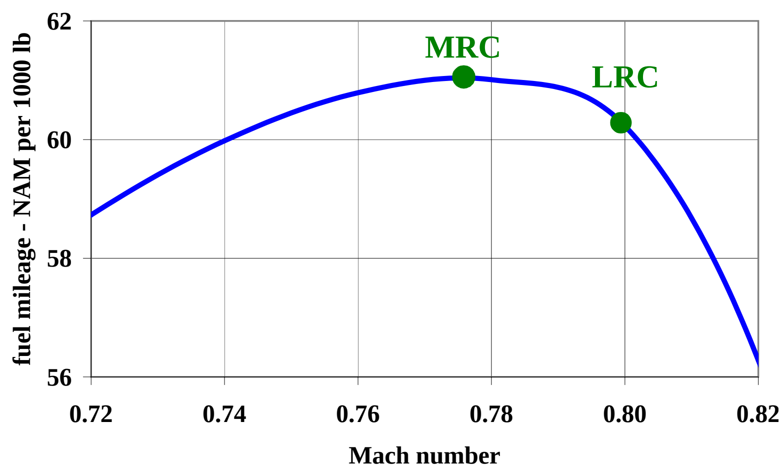


Figure 32-8

Over a cruise distance of 1000 NM, for example, the difference in cruise time between MRC and LRC would be about four minutes. The fuel consumption at LRC would be approximately 165 pounds greater than at MRC. Is LRC a good compromise? You be the judge. Many airlines prefer LRC to MRC.

Another advantage of LRC over MRC is that it is higher on the curve of thrust required, and hence it is slightly more speed-stable. For a discussion of speed stability, refer to the chapter entitled "Miscellaneous Topics".

To the right you see that we have added two more weight lines. Also, we have drawn a green line connecting the points of maximum fuel mileage.

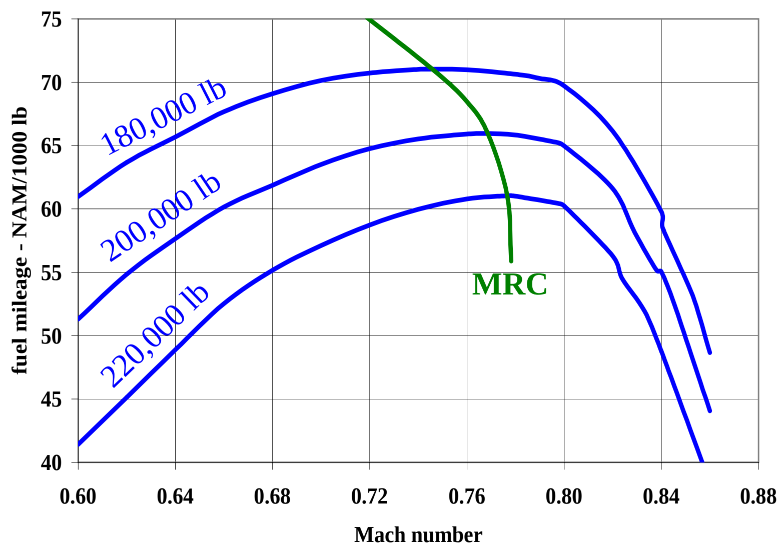


Figure 32-9

In the chart to the right, we show the same three lines of weight as in the chart above. Here, however, we have added a line connecting the points corresponding to LRC.

The fuel mileage chart shown is typical of all fuel mileage charts. Fuel mileage charts for all of the Boeing airplanes are published in the Performance Engineer's Manual (PEM).

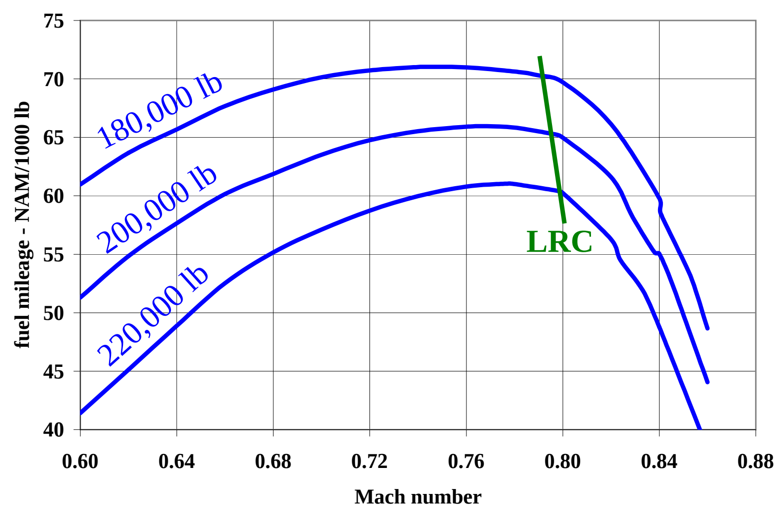


Figure 32-10

effect of temperature on fuel mileage

We've already demonstrated that temperatures warmer than ISA affect the cruise fuel flow in two ways: by increasing the thrust required due to the Reynolds number change, and because the fuel flow for a given amount of thrust required increases as a function of θ_T^x. It's natural, therefore, to expect that fuel mileage is strongly influenced by air temperature. But that would only be true if you forget that increasing air temperature also increases the true airspeed of the airplane in cruise.

Thus, since fuel mileage is equal to true airspeed divided by fuel flow, and since both of those are increasing as temperature increases, it follows that the temperature effect on fuel mileage will be less than its effect on either true airspeed or fuel flow.

Continuing the example we've been looking at in this chapter, for a 757-200 at FL350 weighing 220,000 pounds and cruising at Mach 0.80:

- On a standard day, the true airspeed is 461.1 knots, the fuel flow is 3829 pounds per hour per engine, so the fuel mileage is 60.21 NM per 1000 pounds of fuel.
- At ISA+20, the true airspeed is 481.7 knots, the fuel flow is 4068 pounds per hour per engine, so the fuel mileage is 59.21

Thus, when the air temperature increased by 20 degrees, the true airspeed increased by 4.5%, and the fuel flow increased by 6.2%, but the fuel mileage decreased by only 1.7%.

From this you can see that fuel mileage is less affected by air temperature than are the true airspeed and the fuel flow.

a different view of fuel mileage

The chart shown above is the traditional way of graphing fuel mileage, and the fuel mileage charts published by Boeing in the Performance Engineer's Manuals are presented this way. But there is a different way of viewing fuel mileage, and it's useful in visualizing the relationship of fuel mileage to thrust required.

You'll remember from a discussion earlier in this chapter that the engine manufacturers provide us with fuel flow data in the form of fuel flow as a function of speed and thrust required.

When you think about that, you realize that fuel mileage is speed and fuel flow combined. So with a little manipulation of the manufacturer's data, we can transform it into a different format: fuel mileage versus thrust required.

To the right, you see how fuel mileage for this engine varies with Mach number and thrust required. You can think of this chart as being a graph of engine fuel mileage, not airplane fuel mileage, because it's independent of the airplane the engine is mounted on.

But how does this chart relate to a specific airplane? This chart would be equally applicable to any airplane powered by this engine, wouldn't it?

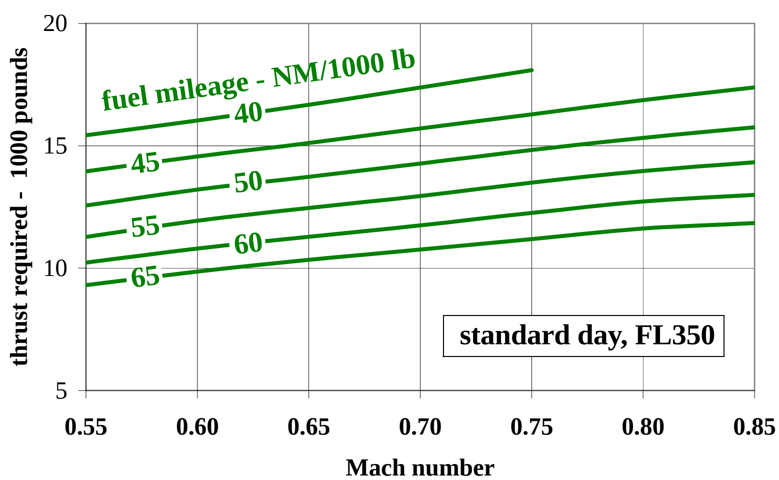


Figure 32-11

What makes the engine fuel mileage chart relevant to a specific airplane is when you add in the data showing the airplane's curve of thrust required. In the graph to the right, you see that we have overlaid the line of thrust required for a weight of 220,000 pounds that we calculated previously.

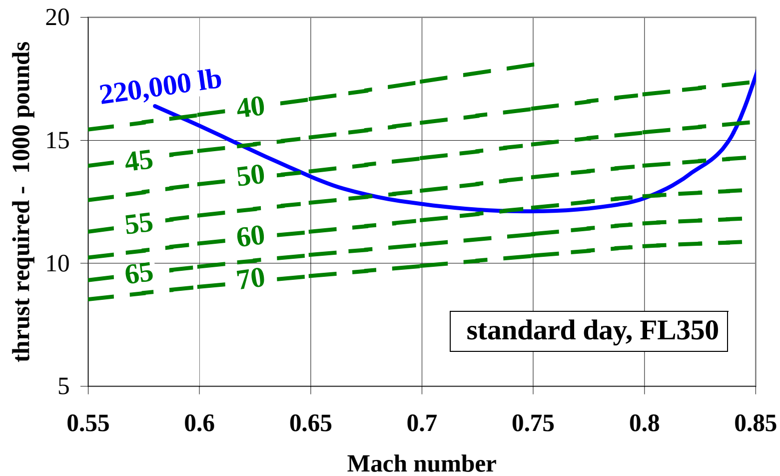


Figure 32-12

Now we can see exactly how the fuel mileage will vary for this specific airplane as the speed changes. You can see, for example, that the maximum fuel mileage will occur at approximately Mach 0.78 at a value of about 61 nautical miles per 1000 pounds. That agrees exactly with the values for MRC at this weight that we saw previously in the other format of fuel mileage chart.

In the figure you see to the right, we have drawn the thrust required lines for three different weights. We have also drawn the fuel mileage lines that are tangent to the thrust required lines.

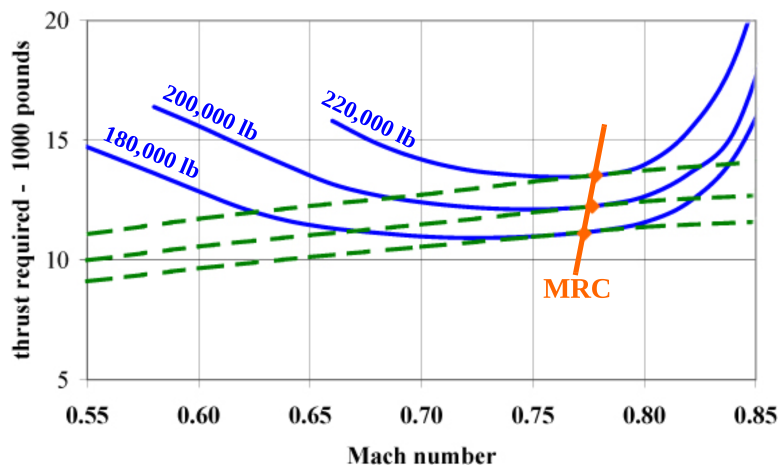


Figure 32-13

The point of tangency: that's MRC, isn't it? It's the point at which the highest possible fuel mileage occurs, and that's MRC, by definition. Thus it's possible to see the variation of MRC Mach number with weight. We've drawn in a line of MRC. Here you see clearly that the speed for maximum range cruise <u>doesn't</u> occur at the point for minimum thrust required, but rather at a slightly faster speed.

Maximum Thrust Available In Cruise

In cruise, the maximum allowable thrust is the engine rating called Maximum Cruise Thrust, or MCrT, as discussed in the earlier chapter entitled "Thrust Ratings". Maximum cruise thrust is a function of the airplane's speed, altitude, and air temperature.

As you see in the graph to the right, for temperatures of ISA+10 and colder the thrust is constant, varying only with Mach number. At air temperatures warmer than ISA+10, the thrust decreases with increasing temperature.

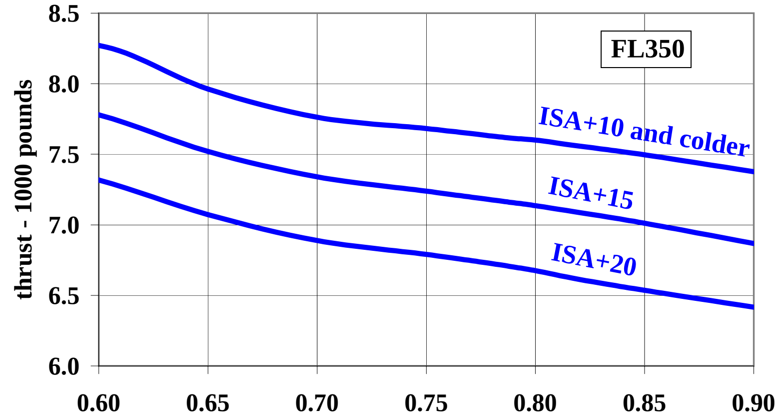


Figure 32-14

We're showing here the maximum cruise thrust per engine, so to relate it to the thrust required charts you've seen, you'll need to multiply the thrust from this chart by the number of engines, which is two for our 757 example.

Maximum Cruise Speed

The maximum cruise speed is the slower of two different limitations: M_{MO}, or the speed that occurs when the engines are set to their Maximum Cruise Thrust (MCrT) rating.

If we superimpose the line of maximum cruise thrust available onto the chart of cruise thrust required, it enables you to see how MCrT restricts the speed that can be achieved in cruise for a given set of conditions.

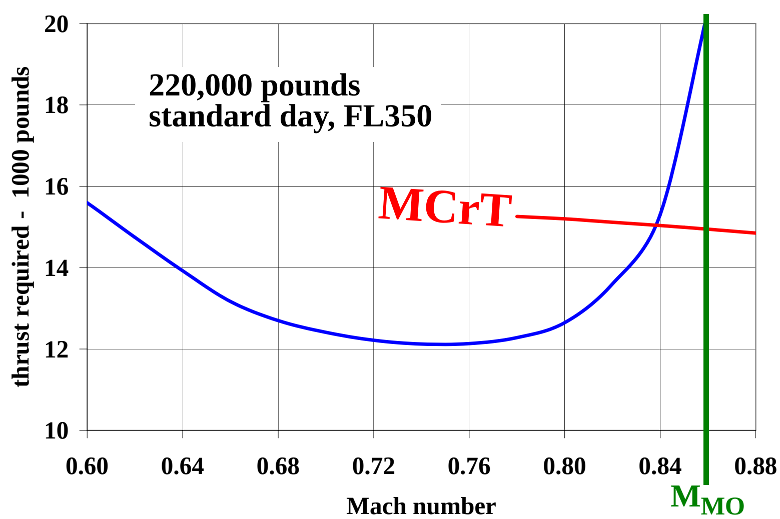


Figure 32-15

For the 757, the maximum operating Mach number, M_{MO}, is 0.86. Depending on air temperature and weight, the maximum speed cruise may be faster or slower than M_{MO}. If the maximum cruise speed is faster than M_{MO}, of course, M_{MO} becomes the maximum cruise speed.

Minimum Cruise Speed

You have just seen the maximum cruise speed. What about the minimum cruise speed?

You know from the discussion in the previous chapter, entitled "Altitude Selection", that the buffet boundary defines the slowest cruise speed. Attempting cruise slower than the low-speed initial buffet speed will result in airframe buffet.

In the graph to the right, you can see that the initial buffet speeds for our three example cruise weights at FL350 are all between Mach 0.50 and 0.60.

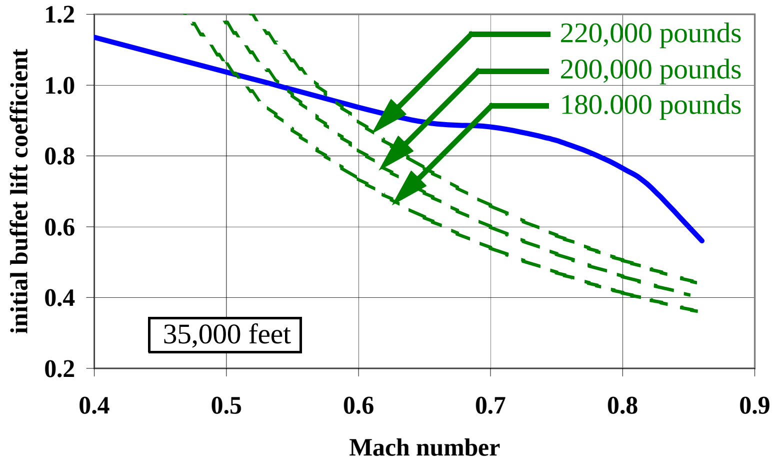


Figure 32-16

Step Climb In Cruise

Consuming fuel in cruise causes the airplane's weight to decrease steadily. The weight decrease over time in turn causes the airplane's optimum altitude to increase as the flight progresses. Since the optimum altitude is steadily rising, a cruise segment that might have begun at or even above the optimum altitude for the top-of-climb weight will sooner or later fall below the optimum altitude – and that means that the airplane's fuel efficiency will decrease.

Although it would be possible to cruise at a steadily increasing altitude so as to keep the airplane always at its optimum altitude, air traffic control constraints make a climbing cruise flight path undesirable. You know that a small amount of deviation from optimum altitude results in a relatively minor fuel consumption penalty – around a one percent penalty for a two thousand foot deviation from optimum – so if we can keep the cruise altitude within a few thousand feet of optimum, we'll be doing a good job of fuel conservation.

Since we're talking now about longer flights, let's use a 747-300 as an example, flying eastbound. We're going to say that the takeoff weight will be 770,000 pounds. In climb we'll burn about 20,000 pounds of fuel, so we'll arrive at cruise altitude at about 750,000 pounds. We know that for that weight, the optimum altitude is 31,000 feet.

So what altitude should we select for cruise?

For the purposes of this example, to make it a little simpler, we're going to use the older cruise altitudes, which were separated by 2000 feet between tracks in opposite directions, 4000 feet between tracks in the same direction. Flight levels 290, 330, 370 and 410 were available for eastbound flights, 310, 350, 390 and 430 were available for westbound flights. (Yes, in the current air traffic system altitude separations are (in most cases) 1000 feet between opposite direction tracks.)

So since we're eastbound we can choose between flight levels 290, 330, 370 and 410. Let's plan ahead – let's select FL330 as our initial cruise altitude. That's two thousand feet above optimum at top of climb. Looking at the airplane's performance data, we can see that for the 747-300, buffet margin isn't a problem at FL330. We can see also that altitude and speed capability at that altitude aren't limiting either. So FL330 is acceptable.

In the graph to the right, we show a line of the optimum altitude. You see that as the flight proceeds along its route the optimum altitude is steadily increasing, as expected.

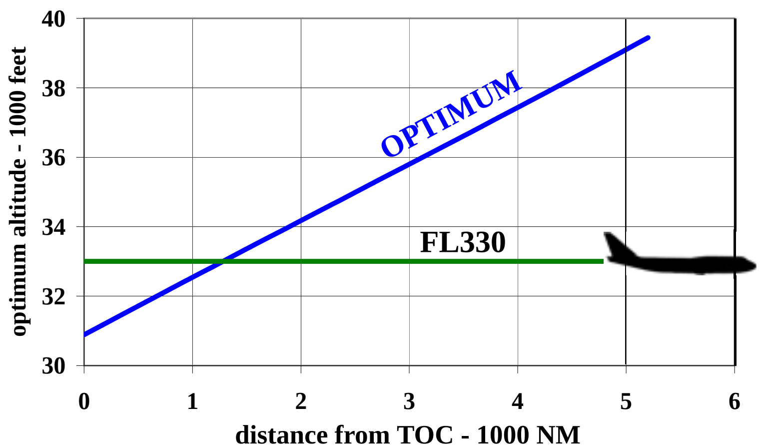


Figure 32-17

The horizontal line represents flight at a constant cruise altitude of FL330.

Initially, the flight is 2,000 feet above optimum. That's fine, that's only a one percent fuel mileage penalty, and it's getting better as we cruise and approach the optimum altitude line.

After we've flown approximately 1250 miles from top of climb, now the airplane is at its optimum altitude, and from this point on the fuel mileage begins to decrease again.

If we have, for example, a 4000-mile cruise distance (Tokyo to Seattle, or Seattle to London, for example) and if we don't change our cruise altitude, by the time we reach top of descent we will be more than 4000 feet below optimum altitude. At that point, we will be achieving a fuel efficiency that is much less than the best we can do.

How do we improve on this situation? By climbing to the next higher available altitude at the appropriate time. So how should we know when or where is the appropriate time, then?

Here's a quick way to answer that: climb when the flight level half-way between your present cruise flight level and the next available one is the optimum altitude. For example, if climbing from FL330 to FL370, do the climb when FL350 is optimum. For our example 747-300, that will be at a weight of 620,000 pounds.

Here's a more precise way to answer that:

To the right, you see a new kind of chart. Here we plot fuel mileage versus weight. In this chart, we are showing the fuel mileage for FL330 and FL370.

You see that we start cruise at FL330 at a weight of 750,000 pounds. As we consume fuel, the fuel mileage increases, as expected.

At some point, however, the fuel mileage line for FL330 will cross the line of fuel mileage for FL370. <u>If we continue to cruise at FL330 beyond that point, we will be incurring a fuel mileage penalty</u>.

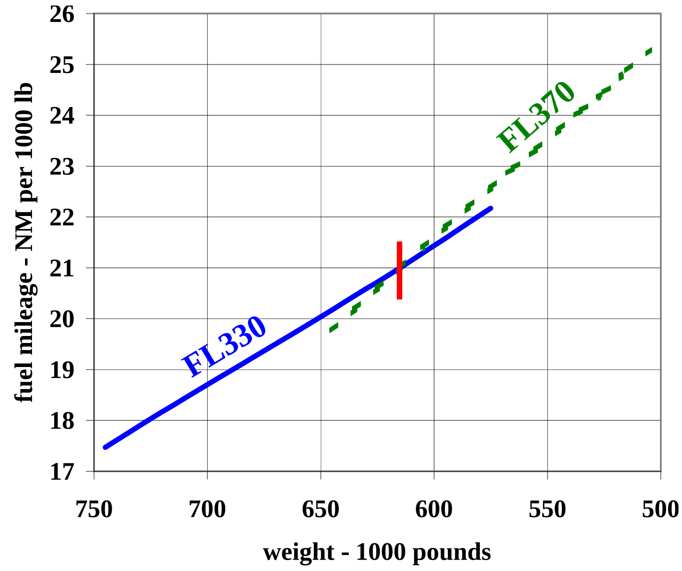


Figure 32-18

The solution, then, is to climb at the weight at which the two fuel mileage lines cross. In this chart, that can be seen to occur at a weight of about 615,000 pounds. So the quick method of saying "climb when the half-way-between altitude is optimum" is pretty accurate.

The flight profile now looks like this.

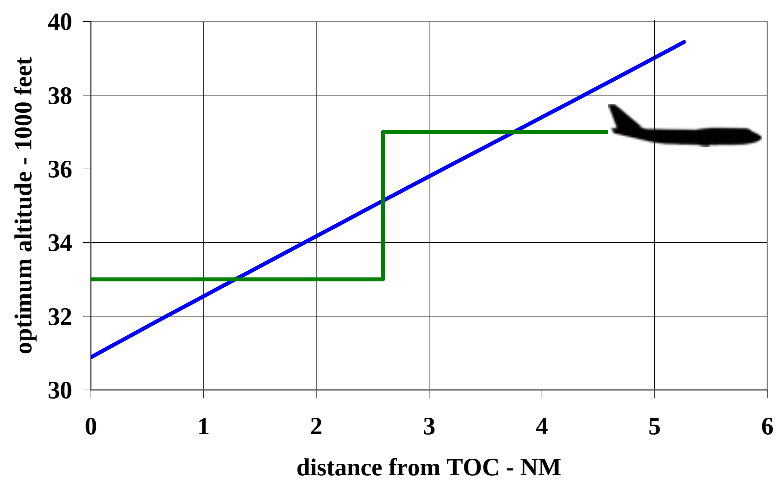


Figure 32-19

If your flight were longer than 5,000 miles or so, a second step climb might be in order. In that case, you would want to step to FL410 somewhere around 5000 miles after top of climb, at which point FL390 is optimum.

But here's something to keep in mind about step climb: yes, step climb offers fuel mileage improvements. However, performing a step climb also has a cost: the extra fuel that is burned at climb thrust during the step. Thus, if you're not going to be at the new altitude long enough for the benefit to outweigh the cost, then a step climb wouldn't be justified.

To illustrate the benefit of step climb on the trip fuel consumed for the example we've just been examining, we ran the Boeing software for the following two cases: for a 4400 nautical mile trip, no wind, ISA, cruise at FL330 only, and for the same trip but with a step climb to FL370 at 615,000 pounds.

Here are the results:

- For cruise at FL330 only, the trip fuel consumption was 228,474 pounds.
- For cruise at FL 330 then FL370, the trip fuel consumption was 227,633 pounds.

In this example, then, the fuel saving is 841 pounds. While that may not sound like a lot, if this is a daily flight then in a year the savings would be more than 300,000 pounds of fuel. Under some conditions, it might also allow 841 pounds of payload increase.

Holding – a Special Kind of Cruise

Holding, as we all know it, is a kind of cruise: it's flight at a constant altitude, at a constant speed. The significant difference between holding and conventional cruising flight is that an airplane in a holding pattern is going nowhere except around and around in a racetrack pattern.

In regular cruise, the emphasis is usually on either speed, in order to minimize the time in cruise and arrive at the destination sooner, or – more usually, in these days of high fuel costs – the emphasis is on fuel mileage. Maximizing the fuel mileage means minimizing the amount of fuel consumed during the cruise segment of the flight.

In the holding pattern, however, while the emphasis is still on fuel efficiency in this case we want to minimize the rate of fuel flow. Minimizing the fuel flow rate means minimizing the amount of fuel consumed while holding.

"All right," you say, "we'll just calculate the fuel flow at the speed for minimum drag. Minimum drag means minimum fuel flow, right?"

Sorry, no. It's a little more complicated than that. Actually, the minimum flow will occur at a speed slightly slower than the speed for minimum drag. Let's see why that's true. Let's return to our 757 airplane example.

Earlier in this chapter, when we were discussing fuel flow, we showed you this graph. It shows us thrust versus Mach number for specified fuel flow rates. We can use it to predict the thrust available at a given fuel flow rate or – more probably – we can use it to predict the fuel flow at a given speed and thrust required.

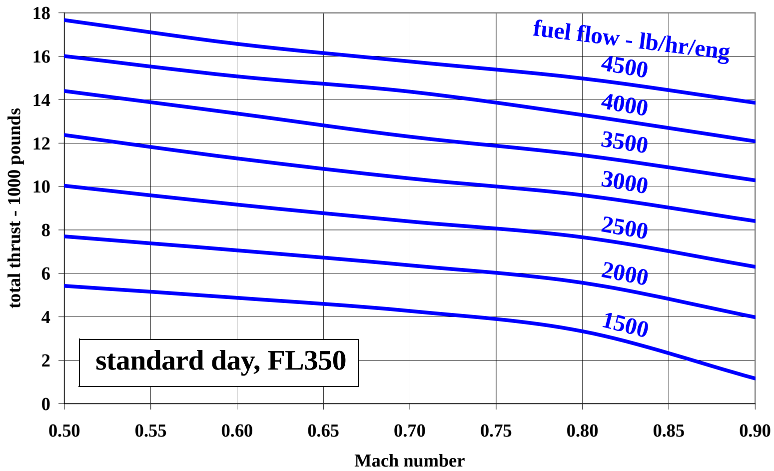


Figure 32-20

Let's now combine this graph of fuel flow with a line of thrust required. We'll change the scales a little bit to enlarge the area of interest so you can see more clearly what's happening.

In the graph to the right, the dashed lines are the lines of fuel flow rate. The solid line is the line of thrust required versus Mach at a weight of 220,000 pounds at FL350 on a standard day.

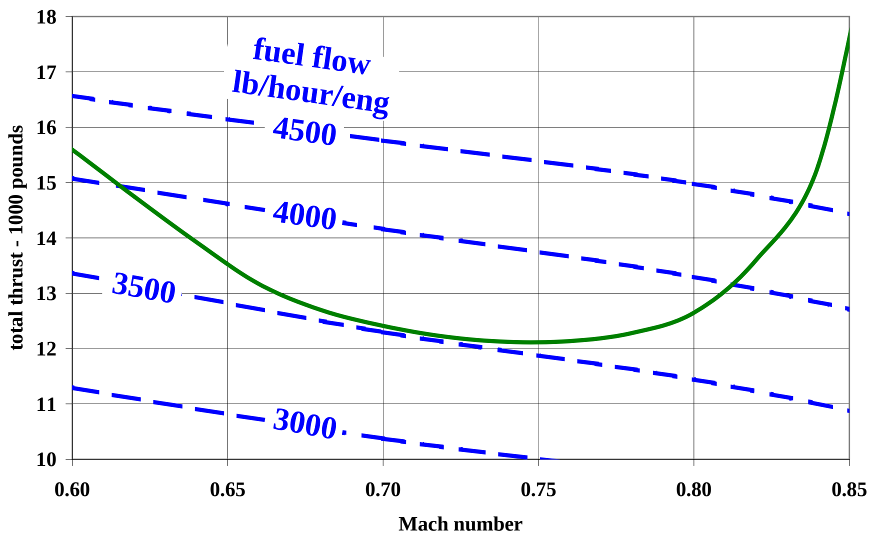


Figure 32-21

So where would the minimum fuel flow occur, for this weight?

It will occur at the point where a fuel flow line is tangent to the thrust required line. In the example you see here, the fuel flow at the tangent point is slightly more than 3500 pounds per hour per engine. And notice the speed at the point of tangency: about Mach 0.71. What's the speed for minimum drag in this graph? Approximately Mach 0.75, which is 0.04 Mach or 23 knots true airspeed faster than the speed for minimum fuel flow.

Let's draw that same graph again for three weights and their tangent fuel flow lines:

In the graph to the right you can see clearly the effect of weight on the speed for minimum fuel flow. The weights shown are 200,000 pounds at the bottom, then 220,000 pounds in the middle, and the highest weight line is 240,000 pounds.

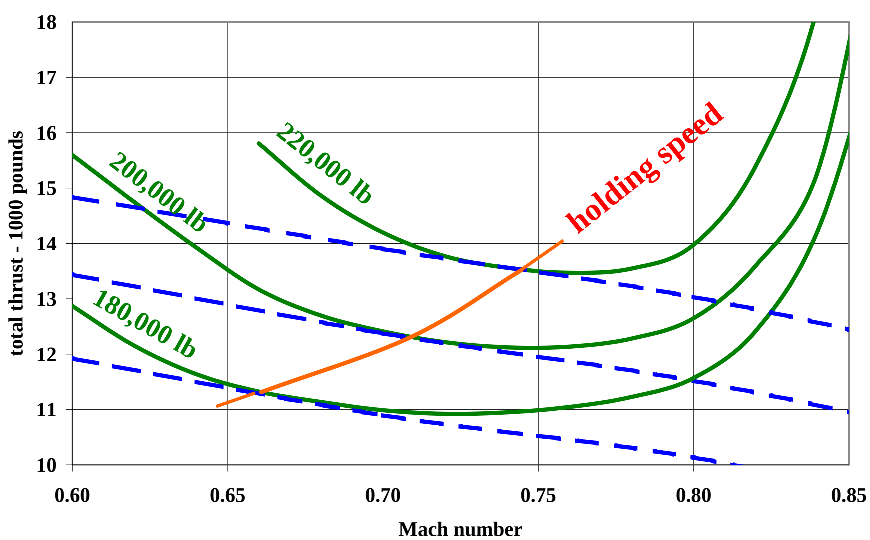


Figure 32-22

Engine-inoperative Holding and Fuel Mileage

Some interesting things can be seen when one examines the performance of an airplane in cruise with an engine inoperative as compared to the condition with all engines operating.

Fuel mileage is a useful indicator of cruise performance, so let's look at the fuel mileage at LRC for a given weight at a range of altitudes.

In the figure to the right, we compare the fuel mileage at LRC for all-engine and engine-inoperative conditions, on a standard day at a weight of 180,000 pounds.

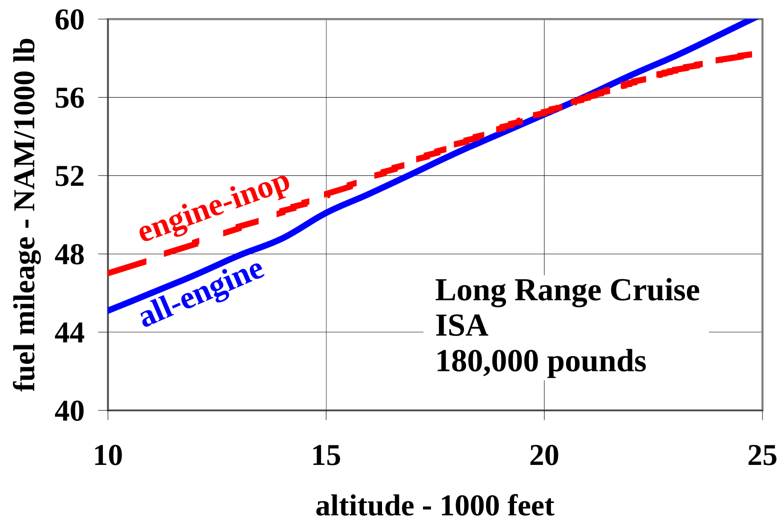


Figure 32-23

You observe that the fuel mileage with an engine inoperative is actually superior at altitudes below approximately 20,000 feet.

You know from earlier chapters that an airplane's drag in the engine-inoperative condition is greater than for the all-engine condition, for the same speed, weight and altitude, because in an engine-out condition there is additional drag resulting from the thrust asymmetry. So it's natural to wonder why the fuel mileage is <u>better</u> for the engine-out case, despite the greater drag.

Another way to view this difference is by looking at fuel mileage versus Mach number, at a given altitude and weight.

Here too you see that the fuel mileage in the engine-inoperative case is better than it is for the all-engine case, at Mach numbers up to approximately 0.61.

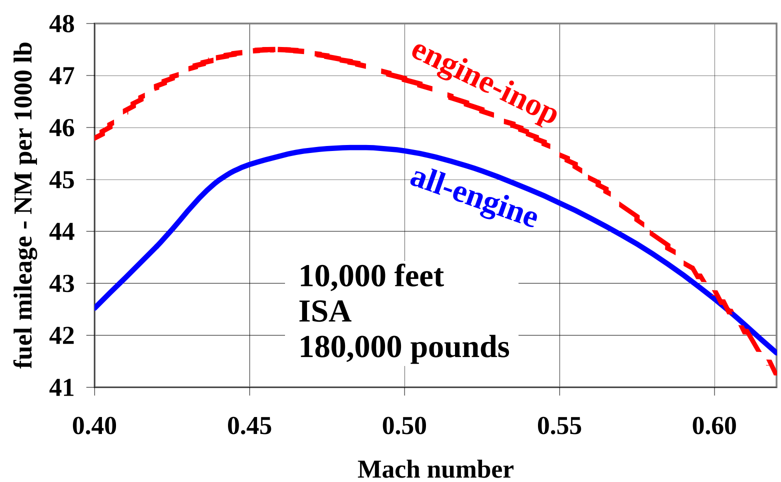


Figure 32-24

Why is this happening?

The answer lies in the fuel flow characteristics of the engine. Let's look at that.

One useful way in which we can look at the fuel flow characteristics of an engine is by looking at a parameter called *thrust specific fuel consumption*, or TSFC. That is defined as the rate of fuel flow divided by the thrust – for example, pounds per hour per pound of thrust.

In the chart to the right, you see the TSFC for the engine that we are using as an example in our calculations, on a standard day at 10,000 feet.

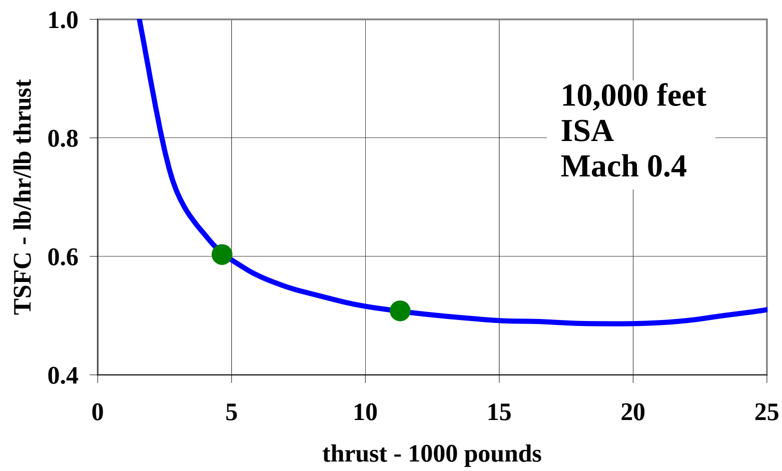


Figure 32-25

Notice that the TSFC rises sharply at the lower values of thrust. It's this rise that causes the superiority of the engine-inoperative fuel mileage compared to the all-engine case.

If you do the calculations of thrust required for both all engines operating and engine-inoperative conditions at Mach 0.4 and 180,000 pounds at 10,000 feet, standard day, you'd find that for the all-engine conditions, the airplane drag is 9799 pounds; for the engine-inoperative case, the airplane drag is 10,796 pounds. The difference between the two is the sum of the control drag and the windmilling drag that occur in the engine-inoperative case.

For all engines operating on this two-engine airplane, each engine would have to produce about 4900 pounds of thrust. The chart says that for 4900 pounds of thrust, the TSFC is about 0.61 pounds per hour per pound of thrust, giving a fuel flow of 4900×0.61 or 2989 pounds per hour. The total fuel flow is therefore just under 6000 pounds per hour.

For the engine-inoperative case, the operative engine has to produce 10,796 pounds of thrust. The TSFC is then about 0.52, giving a fuel flow of 10796×0.0.52 or 5614 pounds per hour. The fuel flow of the one engine in the engine-inoperative case is less than the fuel flow of the two engines together in the all-engine case.

In the flight conditions of lower speeds, altitudes and weights, the relatively low thrust required from the engines in the normal all-engine condition puts the engines up onto the rising portion of the TSFC curve; this increase in the TSFC for the all-engine case is greater than the increase in airplane drag in the engine-inoperative case, so as a result the all-engine case burns more fuel than the engine-inoperative case for the same conditions.

This characteristic can also be seen when looking at all-engine holding fuel flows compared to engine-inoperative holding fuel flows at the lower altitudes.

The superiority of the engine-inoperative performance under conditions of relatively low thrust is frequently seen in ETOPS critical fuel scenario calculations.

Boeing Standard Practice

Throughout this chapter, we have completely ignored several other factors that – in the real world – have an effect on cruise fuel consumption. These factors will also affect the climb and descent fuel consumption as well, but since climb and descent are usually much shorter than cruise times, the effects there are considered to be negligible. In cruise, these factors become more significant, although their effect is relatively small.

We refer to:

- The energy content of the fuel being used.
- The effect of the airplane's center of gravity.
- Electrical generator loads.
- Gravitational effects

fuel energy content

Airplane engines produce thrust by changing the combustion energy contained in the fuel to thrust energy. Not all fuels are the same. Different sources of the crude petroleum from which the fuel is refined, and different refining methods result in fuels that vary in the amount of combustion energy per unit of weight. This topic was discussed briefly in the chapter entitled "Jet Engine Fundamentals". You may recall that in that chapter we defined the fuel combustion energy content, calling it the "lower heating value", or LHV.

When Boeing conducts flight tests to determine the drag polar of an airplane, it samples the fuel being used for the flights and tests the samples to determine the amount of combustion energy per pound of fuel. We use units of British Thermal Units (BTUs) per pound of fuel.

Through flight test measurement of fuel flow as a function of speed, altitude and weight we are able to deduce the airplane's drag characteristics. But here's the problem: <u>the fuel flow depends on the combustion energy of the fuel</u>. If the fuel used has a high energy content, an engine doesn't have to burn as much of it to achieve the needed thrust, thus resulting in lower fuel flow rates. Fuel having a <u>low</u> energy content, on the other hand, will result in <u>higher</u> fuel flow rates for a given amount of thrust.

When analyzing the flight test data, we account for the fuel LHV by correcting the data for its effect by correcting the deduced drag data back to what it <u>would</u> be at a specified standard LHV of 18,580 BTUs per pound of fuel. The published fuel flow and fuel mileage data, then, are truly precise only at the standard LHV.

center of gravity effect

You have seen in earlier chapters that the airplane's drag is influenced by the position of the airplane's center of gravity. More forward CGs result in higher drag coefficients for a given lift coefficient. This effect, you'll remember, is due to the differing amounts of horizontal tail download that are required for pitch equilibrium at different CGs.

In flight test, the airplane CG is known precisely, and is considered when producing the published drag data. The drag calculated from the flight test data is standardized by converting it to the amount of drag that <u>would</u> have existed at a standard CG. That standard CG is a value that is representative of typical cruise conditions, somewhere in the middle of the allowable CG range. The published cruise data – thrust settings, fuel flow, and fuel mileage, therefore represent typical cruise performance.

electrical generator loads

The rotors of the generators which provide power to an airplane's electrical systems are rotated by (for all airplanes prior to the 787) by the rotation of the shaft of the engine on which the generator is mounted. <u>The energy for the work done by the generator is therefore provided by the fuel that the engine burns</u>. Greater electrical output means higher engine fuel flow, lesser electrical output means lower fuel flow. The electrical output that is demanded of the generators in flight is highly variable, depending on what electrical systems are being powered at the moment.

The published fuel flow and fuel mileage data are based on a standard generator load that is representative of typical load demanded during flight.

gravitational effects

If you don't yet have a headache from thinking about all of these corrections, here are some more:

<u>*speed of flight*</u>

In the chapter entitled "Mass, Weight and Center of Gravity", we discussed the effect of an airplane's velocity inflight on the gravitational acceleration it experiences. The published data is based on a nominal cruise speed.

<u>*latitude of flight*</u>

In the same chapter, we addressed the variation of gravitational acceleration with latitude. The published data is based on a nominal latitude of 45°.

<u>*direction of flight*</u>

We also talked about the fact that the direction in which an airplane is flying has a small effect on the gravitational acceleration that the airplane experiences in flight. An airplane flying toward the west will be slightly heavier than an airplane flying toward the east, assuming that other conditions are the same in both cases. The published cruise data assume that the airplane is flying north or south, for which there is no effect on the gravitational acceleration.

<u>*altitude of flight*</u>

You know that altitude has a direct effect on the gravitational acceleration and hence on the airplane's performance. For the same mass, an airplane that is at a higher altitude is effectively slightly less heavy than an airplane at a lower altitude. The published cruise data is based the gravitational acceleration g at a nominal altitude that is a function of the airplane's $\frac{W}{\delta}$.

cross-isobar effects

Here's an effect that will really give you a headache: consider the effect of the atmospheric conditions through which the airplane flies – in this case, the atmospheric pressure. Imagine an airplane that's flying across a region of high pressure or low pressure (crossing the isobars), meaning that the atmospheric pressure along the route is constantly changing. How does this affect the airplane's cruise performance?

Remember that in cruise an airplane is flying at a constant altimeter height – which is a constant pressure altitude. Now consider this: suppose you're flying from an area of lower atmospheric pressure toward an area of higher atmospheric pressure. If the airplane were to maintain a constant true altitude, it would experience a gradual increase in air pressure.

In cruise, however, the airplane's autoflight system is designed to maintain a constant pressure altitude, not a constant true altitude. If the atmospheric pressure is increasing along the route of flight, the autoflight system will think that the airplane is descending, so it will compensate by increasing the airplane's true height as it seeks to maintain constant pressure height. That is, the airplane will be climbing in terms of true height. If the atmospheric pressure is decreasing along the route of flight, the autoflight system will think that the airplane is climbing so it will compensate by decreasing the airplane's true height – so the airplane will be descending in terms of true height.

Causing the airplane to climb will demand more thrust from the engines than is needed for level flight. The fuel flow will increase correspondingly. Since the published cruise data doesn't consider this effect, the published data will show slightly lower fuel flow or fuel mileage than the airplane is actually achieving.

Causing the airplane to descend will demand less thrust, so the published data will show slightly higher fuel flow or fuel mileage than the airplane is actually achieving.

why should you care?

At this point, you may be thinking "okay, that's all understandable, but so what? Your published cruise data doesn't show us the magnitude of the effects of LHV, center of gravity, generator loads, direction of flight, or whatever, so why should we care?"

This is more than simply an intellectual exercise. While each of these effects is small, they are not negligible and when the greatest possible data precision is desired, they should be considered.

Airlines that are conscientious about fuel conservation monitor the fuel burnoff condition of the airplanes in their fleet, with two goals: first, to enable the airline to calculate trip fuel loads that truly account for each airplane's condition, and second to help identify airplanes that are in need of remedial maintenance action. This fleet performance monitoring is based on cruise data recorded during its regular revenue service.

Boeing makes available to its customers a software application called Airplane Performance Monitoring (APM) that enables the airline to analyze recorded cruise data and from that to deduce the drag condition of the airplane and the fuel flow condition of the engines. Because the recorded data is usually collected during a number of flights that have different LHVs, CGs, generator loads and so on, the results of the APM calculations typically will show an amount of “scatter”.

APM allows the operator, if desired, to include LHV, CG, generator load, and gravitational effects in its analyses of recorded data. While this requires additional data to enable the analysis, by doing so the operator reduces the data scatter and thus gains a more precise knowledge of the airplane and engine condition.

APM does <u>not</u> include any corrections for errors induced by cross-isobar flight. These are very difficult to know with any real precision because the atmospheric pressure conditions are constantly changing and it’s difficult or impossible to know the atmospheric pressure pattern along the route accurately; without that knowledge, no correction can be determined.

Chapter 33: Cruise - Engine Failure and Driftdown

Introduction

In the preceding chapter, we discussed “normal” cruise, which – we’re happy to say – represents more than 99% of all cruise operations.

However, both the aviation regulations and our own natural desire to protect against the unforeseen lead to this chapter. Here we will examine the performance of an airplane subsequent to the failure of one or – in the case of airplanes having three or four engines – two engines in flight.

Obviously, an engine may fail during any portion of a flight. In previous chapters, we’ve discussed in great detail the subject of engine failure during takeoff. We have placed this discussion in the cruise chapter simply because most cases of engine failure and driftdown analysis involve some portion of the route under the cruise segment of the flight, which could take place over mountainous terrain or other obstructions. The discussion in this chapter is equally applicable to engine failure during climb to altitude, cruise, or descent.

We’re going to start by discussing the case of enroute failure of one engine. We’ll be looking at the regulatory requirements relating to the inflight failure of an engine and its descent path following the failure. Then we’ll examine the performance of an airplane following a single engine failure.

Later we’ll take look at the case of simultaneous failure of two engines.

Failure of One Engine in Cruise

Failure of a single engine on a three- or four-engine airplane is unlikely to result in any operational penalties. This is true simply because the operating engines are adequate to provide sufficient altitude capability to comply with the regulatory requirements, except possibly on routes having very high terrain clearance requirements.

Failure of one engine on a two-engine airplane, however, is more likely to cause operational constraints, although even in this case most flights will not be penalized.

Let’s look at the regulations related to engine failure during a flight:

regulatory requirements

FAR Part 121 contains the following regulation. Some of the language in this section deals with airplanes certificated before August 29, 1959. We have omitted those portions of the text, assuming that none of our readers are operating airplanes certified before that date.

Sec. 121.191

Airplanes: Turbine engine powered: En route limitations: One engine inoperative.

(a) No person operating a turbine engine powered transport category airplane may take off that airplane at a weight, allowing for normal consumption of fuel and oil, that is greater than that which (under the approved, one engine inoperative, enroute net flight path data in the Airplane Flight Manual for that airplane) will allow compliance with paragraph (a)(1) or (2) of this section, based on the ambient temperatures expected en route:
(1) There is a positive slope at an altitude of at least 1,000 feet above all terrain and obstructions within five statute miles on each side of the intended track, and, in addition... there is a positive slope at 1,500 feet above the airport where the airplane is assumed to land after an engine fails.
(2) The net flight path allows the airplane to continue flight from the cruising altitude to an airport where a landing can be made under Sec. 121.197, clearing all terrain and obstructions within five statute miles of the intended track by at least 2,000 feet vertically and with a positive slope at...1,500 feet above the airport where the airplane lands after an engine fails.
(b) For the purposes of Paragraph (a) (2) of this section, it is assumed that--
(1) The engine fails at the most critical point en route;
(2) The airplane passes over the critical obstruction, after engine failure at a point that is no closer to the obstruction than the nearest approved radio navigation fix, unless the Administrator authorizes a different procedure based on adequate operational safeguards;
(3) An approved method is used to allow for adverse winds;
(4) Fuel jettisoning will be allowed if the certificate holder shows that the crew is properly instructed, that the training program is adequate, and that all other precautions are taken to insure a safe procedure;
(5) The alternate airport is specified in the dispatch or flight release and meets the prescribed weather minimums; and
(6) The consumption of fuel and oil after engine failure is the same as the consumption that is allowed for in the approved net flight path data in the Airplane Flight manual.

For operators following JAR-OPS, FAA Section 121.191 quoted above has its equivalents in JAR-OPS 1.500, with minor differences that will be explained where appropriate.

Let’s look more closely at this Section. We’ll paraphrase it to make it a bit easier to understand.

- An airplane’s takeoff weight must meet the requirements of <u>either</u> paragraph (a)(1) or (a)(2) For JAR-OPS operators, the equivalent paragraphs are (b) or (c).

    Paragraph (a)(1) requires that there must be a positive flight path slope at an altitude of at least 1000 feet above all terrain and obstructions within five statute miles on each side of the intended flight track.

Paragraph (a)(2) says that if (a)(1) can't be met, then the flight path of the airplane during descent after engine failure must be able to clear vertically by at least 2000 feet all terrain and obstructions within five statute miles on each side of the intended track.

- In either of the above two cases, the net flight path must also be capable of a positive net slope at 1500 feet above the airport where the airplane is assumed to land after an engine fails.
- The driftdown path calculations must use net flight path data. For enroute flight paths (as opposed to takeoff flight paths) FAR Section 25.123 defines the difference between gross and net gradients as 1.6, 1.3 and 1.1 percent for four-, three- and two-engine airplanes in that order.
- The engine failure must be assumed to occur at the most critical point.
- The calculations must assume adverse winds as appropriate.
- Taking credit for weight reduction through normal consumption of fuel and oil, and/or by means of jettisoning fuel is allowed.
- The driftdown calculations must be based on the ambient temperatures that are expected.
- The alternate airport is listed in the dispatch release and meets weather minima.

Note: JAR-OPS 1 has a slightly different requirement for the width of the path within which all obstacles must be considered: instead of five statute miles each side of the intended flight track, it requires a width of five nautical miles on each side of the intended track.

With this material in mind, let's look at these regulations in more detail.

requirements of FAR 121.191(a)(1)

the first requirement

The first of the two requirements contained in FAR 121.191(a)(1) can be shown as you see to the right. JAR-OPS 1.500(b) is its equivalent.

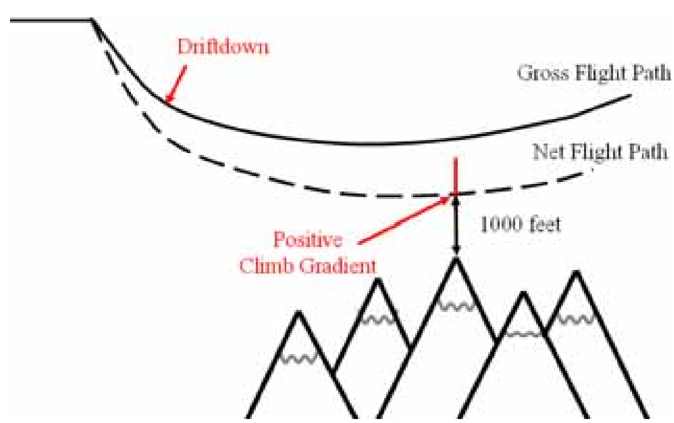


Figure 33-1

From the point at which the engine fails, the airplane will most probably need to descend to a lower altitude. The path followed as the airplane descends and then stabilizes at a new altitude is referred to as *driftdown*.

At the point of engine failure, the pilots will set thrust on the remaining engine(s) to the Maximum Continuous Thrust rating. You will remember that MCT is a special thrust rating usable only in case of emergency.

What speed should they maintain during the descent? It makes sense to use a descent speed that will minimize the descent gradient, thus keeping the descent flight path as high as possible.

In the chapter entitled "Climb Angle and Rate of Climb" we talked about the effect of speed on an airplane's climb gradient. There, you saw that the speed for the best possible gradient is approximately the speed at which the ratio of drag to lift is at its minimum value. For planning the descent performance of an airplane following engine(s) failure, we will base our calculations on the use of the speed for the best gradient, which is referred to in the AFM as the "enroute climb speed". Don't let the use of the term "climb" in that name confuse you: the climb referred to can be either a positive gradient or a negative one. In either case, the speed is the same.

Knowing the thrust and the descent speed, it's a relatively simple matter to calculate the gradient available at any given weight, altitude, and temperature.

In Section 121.191, you saw the requirement that the net flight path must have a "positive net gradient" at a specified height above all relevant obstacles and also above the airport where the airplane intends to land. That raises the question: how do you define "positive net gradient"? Is it + 0.1%? Is it + 0.001%? Is it + 0.0000001%? All of those are positive net gradients.

Since the term "positive net gradient" is unusably vague, we take it to mean the same thing as "non-negative gradient". Hence, a positive net gradient is a value of zero percent or any positive value. That is the accepted interpretation of this ruling.

Knowing the gradient capability as it varies with altitude, we can find the altitude at which the net gradient is just equal to zero. That altitude, then, is the altitude at which the flight path must be able to clear by 1000 feet vertically all obstructions within a ten mile wide path centered on the intended track.

the second requirement

Figure 33-2, to the right, shows pictorially the second requirement of FAR 121.191(a)(1), or JAR-OPS 1.500(b).

This requirement is very similar to the first one, except in this case the requirement is that the net flight path must have a zero percent gradient or better at an altitude 1500 feet above the airport where the airplane intends to land.

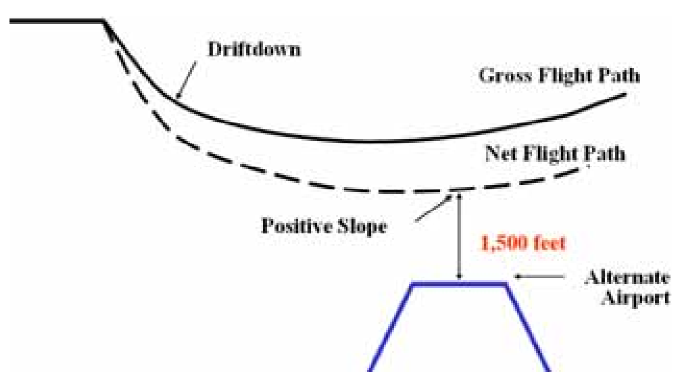


Figure 33-2

requirements of FAR 121.191(a)(2)

In the event that the first requirement of 121.191(a)(1) can't be met, the regulation provides an alternative, in 121.191(a)(2). The JAR-OPS equivalent is 1.500(c).

Paraphrasing the regulation:

- The net flight path must allow the airplane to continue flight from the cruising altitude, following engine failure, to an airport that meets the requirements for landing including weather minima. That net flight path must be capable of clearing vertically all terrain and obstructions, located within five statute miles on each side of the intended flight track, by at least 2000 feet.

- The net flight path must also have a positive net gradient at least 1500 feet above the airport at which the airplane will land.
- The accuracy of the navigational equipment with which the airplane is equipped and which exists along the route must be considered.
- Any adverse winds that exist must be considered.
- Weight reduction by fuel jettisoning, and through normal consumption of fuel and oil, may be taken into account.

the first requirement

You saw, in the discussion of paragraph 121.191(a)(1) that it establishes a requirement for obstacle clearance in level flight ("positive net gradient") at the airplane's altitude capability with an engine inoperative.

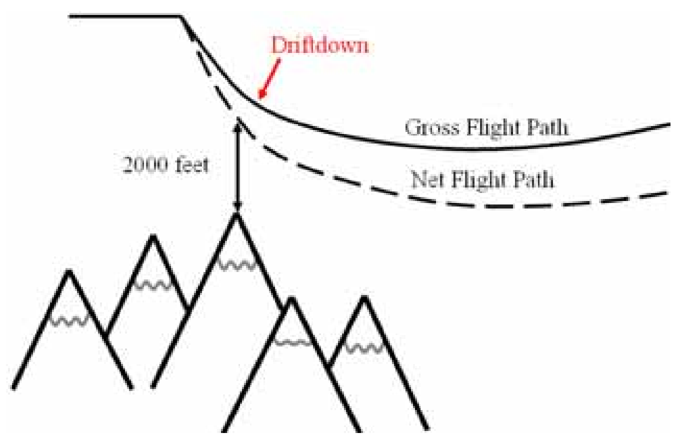


Figure 33-3

Paragraph (a)(2) recognizes that the airplane may not be capable of level flight at least 1000 feet above all terrain with an engine inoperative. For that case, it provides an alternative: if the airplane isn't capable of level flight above the terrain, then *driftdown* over the terrain is permitted, provided that the net flight path clears vertically all of the relevant terrain by at least 2000 feet.

the second requirement

The second requirement of 121.191(a)(2) is identical to the second requirement of (a)(1), namely the requirement that the airplane must be capable of level flight at least 1500 feet above the airport at which it will land.

calculating the driftdown flight path

For a performance engineer, establishing compliance with 121.191(a)(1) isn't particularly difficult. It only requires the determination of the weight/altitude at which level flight is possible and demonstration that the level flight altitude is at least 1000 feet above all relevant terrain. However, it's more difficult for the performance engineer to establish compliance with 121.191(a)(2). The engineer in this instance needs to compute the net flight path during the driftdown, and establish accurately the height of the path above all of the relevant terrain.

In the chapter entitled "Climb To Altitude" we discussed in detail the method for calculating the flight path of the airplane in climb. You saw that it uses a process of "step-integration" – that is, adding together a series of altitude steps, for each one of which the engineer calculates average gradient and rate of climb, leading to the time, the distance, and the fuel consumed during the step.

Why should driftdown be computed any differently? The only differences between normal climb and driftdown are that in driftdown one engine is inoperative, and the gradient is negative rather than positive. The gradient equation we used for calculating an engine-inoperative climb gradient is equally applicable to the driftdown gradient:

$$\gamma = sin^{-1}\left[\frac{\dfrac{T-D}{W}}{\left(1+\dfrac{V}{g}\dfrac{dV}{dh}\right)}\right] = sin^{-1}\left[\frac{\dfrac{T}{W} - \dfrac{(C_D + \Delta C_{D_{WM}} + \Delta C_{D_\Psi})}{C_L}}{\left(1+\dfrac{V}{g}\dfrac{dV}{dh}\right)}\right]$$

where T is the thrust of the operating engine(s)
W is the weight
C_D is the drag coefficient with all engines operating
$\Delta C_{D_{WM}}$ is the windmilling drag coefficient of the inoperative engine(s)
ΔC_{D_Ψ} is the "control drag" coefficient increment due to the yaw

In the case of the driftdown, though, the term $\dfrac{T}{W} - \dfrac{(C_D + \Delta C_{D_{WM}} + \Delta C_{D_\Psi})}{C_L}$ is negative. This is because the loss of thrust, the windmilling drag coefficient increment of the failed engine $\Delta C_{D_{WM}}$ and the drag increment ΔC_{D_Ψ} due to the yaw resulting from the asymmetric thrust condition all combine to make the thrust term $\dfrac{T}{W}$ smaller than the drag term $\dfrac{(C_D + \Delta C_{D_{WM}} + \Delta C_{D_\Psi})}{C_L}$.

The driftdown flight path normally begins with a short deceleration segment in level flight. Following engine failure, the crew will set the remaining engine(s) to maximum continuous thrust, then they will maintain the cruise altitude and allow the airplane to decelerate to its optimum driftdown speed. Only then will the descent be initiated.

Once established in the driftdown, the method for calculating the flight path is the same as for climb.

To save you the effort of doing the those calculations manually, the Airplane Flight Manuals provide charts called "enroute climb weight for positive net gradient" for both one engine inoperative and also, for airplanes having three or four engines, with <u>two</u> engines inoperative. The AFM-DPI software, for airplanes for which one is provided, also offers this capability.

These charts allow the user to determine either the maximum weight at a given altitude, or the maximum altitude at a given weight, at which the airplane's net flight path will meet the positive net gradient requirement.

The AFM also contains charts giving the enroute climb net gradient as a function of weight, altitude and temperature. From these gradient charts it's possible to construct a driftdown flight path.

Certainly the <u>easiest</u> way to produce a driftdown profile for the known conditions is by running the Boeing software. Here's a brief excerpt from a driftdown analysis that can be obtained by using either the Boeing INFLT program or the BPS software. You'll recognize the output format as being identical to that which you saw previously in this document for computing a climb profile.

PRESSURE ALTITUDE FT	WEIGHT LB	FUEL LB	GRND DIST NM	AIR DIST NAM	TIME H:M	IAS CAS KTS	EAS TAS KTS	MACH	GRND SPD KTS	FUEL FLOW LB/HR	CL CD	D D/DELTA LBS	FN FN/DELTA LBS	EPR1 EPR2	ACC FACT	GRAD BODY ANGLE	R/C FPM
* 35000	230000	0	0.0	0.0	0:00.0												
* 35000	229874	126	10.5	10.5	0:01.4												
34500	229837	74	5.7	5.7	:00.8	250.1	238.3	.7338	423.8	5463	0.6127	14025	9200	1.617	0.1885	-.0287	-1230.4
						250.1	423.9				.03740	58206	38182			2.32	
* 34000	229800	200	16.2	16.2	0:02.2												
33500	229759	83	6.1	6.1	:00.9	250.0	238.8	.7184	416.7	5655	0.6096	13921	9570	1.617	0.1883	-.0269	-1137.0
						250.0	416.9				.03695	55113	37889			2.50	
* 33000	229717	283	22.3	22.3	0:03.1												
32500	229670	94	6.6	6.6	:01.0	251.0	240.3	.7062	411.5	5890	0.6018	13826	10001	1.621	0.1885	-.0250	-1042.5
						251.0	411.6				.03624	52235	37787			2.61	
* 32000	229623	377	28.9	28.9	0:04.0												
31500	229567	112	7.4	7.4	:01.1	253.7	243.3	.6986	408.9	6170	0.5869	13747	10484	1.626	0.2742	-.0222	-917.6
						253.7	409.0				.03515	49587	37815			2.69	
* 31000	229511	489	36.3	36.3	0:05.1												

Figure 33-4

In this analysis, a gradient decrement of 1.1% is included, so the results are those for a <u>net</u> descent flight path.

In this computer run, a 757-200 is cruising initially at FL350 at a weight of 230,000 pounds. You see that the first segment is a deceleration at FL350, which takes 1.4 minutes, 10.5 nautical miles, and 126 pounds of fuel.

Now the driftdown begins: between 35,000 and 34,000 feet, at the average step altitude of 34,500 feet, you can see that the gradient is -2.87%, the rate of climb is -1230.4 feet per minute, the true airspeed is 423.9 knots, and the fuel flow is 5463 pounds per hour. The step from 35,000 feet to 34,000 feet therefore takes 0.8 minutes, 5.7 miles, and 74 pounds of fuel.

This process of step integration continues until an altitude is reached at which the net gradient is equal to zero.

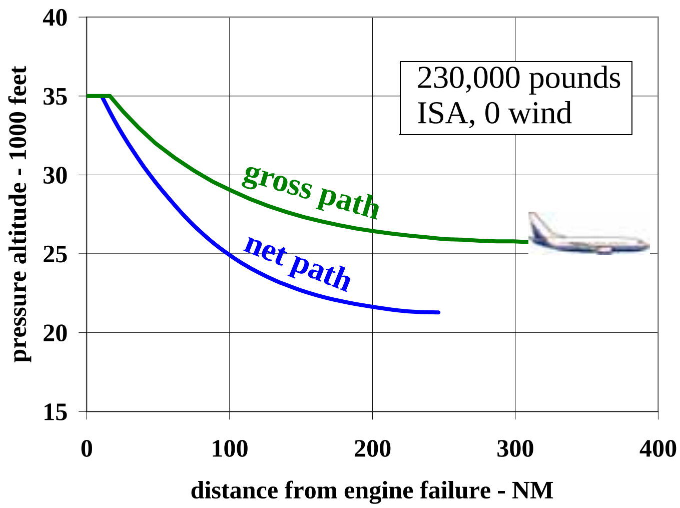


Figure 33-5

Graphing the outputs of altitude and distance from the computer data for the driftdown shows a view of the profile as seen from the side.

Notice that the gross driftdown path is substantially higher than the net driftdown path, a difference of almost 5000 feet in this example.

Of course, the vertical scale greatly exaggerates the steepness of the initial descent. Remember that the gradient when beginning the driftdown at FL350 is only -2.78% – and, in fact, the body attitude was still 2.32 degrees nose up, despite being in a driftdown. Clearly, a driftdown isn't a big dramatic maneuver. (In the next chapter, dealing with emergency descent in the event of a loss of pressurization – there you're going to see a dramatic maneuver!)

In this example, the driftdown net path ends almost 250 nautical miles from the point at which the engine failed, where the net flight path has achieved "positive net gradient" status. This occurs at an altitude of approximately 21,000 feet. The driftdown has taken about 41 minutes and approximately 5300 pounds of fuel have been burned.

If the enroute terrain or obstructions along the route are at no point higher than 20,000 feet, then the requirements of 121.191(a)(1) would be met. If, however, there are higher obstacles along the route, then it may be necessary to show compliance with 121.191(a)(2) and the performance engineer should conduct a more detailed analysis of the route.

the path following leveloff

After drifting down to the altitude for positive net gradient, the flight crew can choose from several options.

- They can decide to leave the engines at their Maximum Continuous Thrust rating and perform a gradual climbing cruise.
- They can decide to continue flight at the leveloff altitude, maintain MCT to allow acceleration to engine-out LRC, then maintain that speed and gradually reduce the thrust setting as the weight decreases.
- If terrain isn't a consideration, they could choose to descend to an altitude lower than the altitude for positive net gradient; they could then continue the flight in level flight at a speed such as engine-out LRC.

In the graph to the right, we show the flight path computed using the INFLT software for a 757-200.

Following engine failure, the airplane descends at the speed for best angle of descent. It drifts down to the positive net gradient altitude of approximately 21,300 feet. Maintaining MCT and the best angle of climb speed, the airplane gradually drifts back upward as the fuel is consumed.

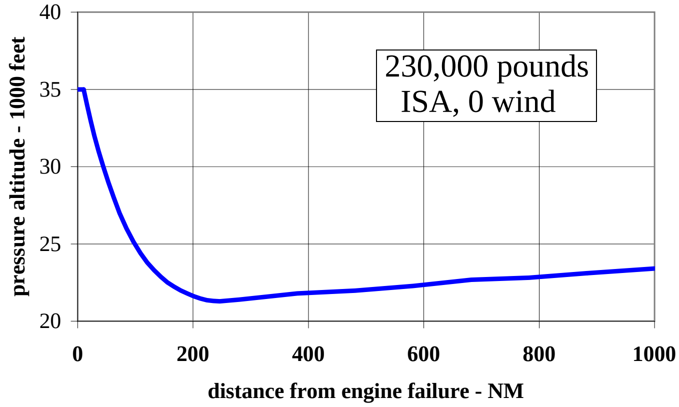


Figure 33-6

Such a profile would have only one clear advantage: keeping the airplane's path as high as possible for as long as possible. In terms of complexity, this profile would impose additional workload on the crew, and might not be acceptable in that air traffic control environment. Further, you can see in figure 33-6 above that the altitude regained during the climbing cruise segment is not much – only about 2,000 feet has been regained after a distance of 1000 NM from the point of engine failure.

If a climbing cruise isn't necessary for reasons of terrain clearance, then a constant-altitude cruise segment following leveloff is more practical.

Descending to an altitude below the altitude for positive net gradient may be desirable when terrain or obstructions aren't a consideration. The flight crew may want to consider establishing engine-out Long Range Cruise at a lower altitude. That will offer faster cruise speeds without a significant fuel mileage penalty.

effect of weight on driftdown

Obviously, weight will affect the driftdown profile. Weight has three effects on the driftdown gradient:

- on the term $\frac{T}{W}$;
- on the lift and drag coefficients and therefore on the term $\frac{(C_D + \Delta C_{D_{WM}} + \Delta C_{D_\psi})}{C_L}$;
- on the descent speed. The descent speed affects the distance flown in a given amount of time, and it also has an effect on the lift coefficient and thus the drag coefficient.

In figure 33-7 to the right, you see the effect of weight on the driftdown profile. First, as you see it produces a lower altitude for positive net gradient; second, it extends the distance to the leveloff point.

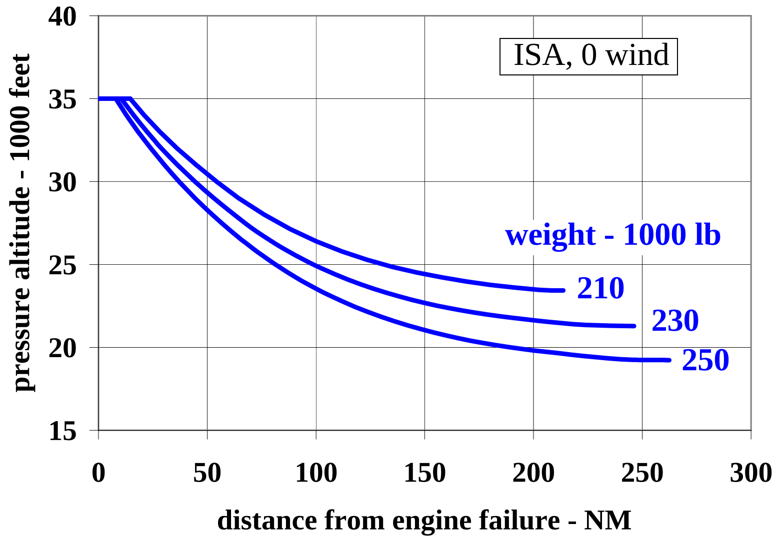


Figure 33-7

effect of temperature on driftdown

Temperature has two effects on driftdown. First, it affects the thrust available, being limited to less thrust on hotter days. Second, it affects the driftdown true airspeed.

You can see to the right the effect on the net driftdown path of a temperature of ISA+20 as compared to a standard day.

Notice that the altitude for positive net gradient is reduced, and the distance to leveloff is increased.

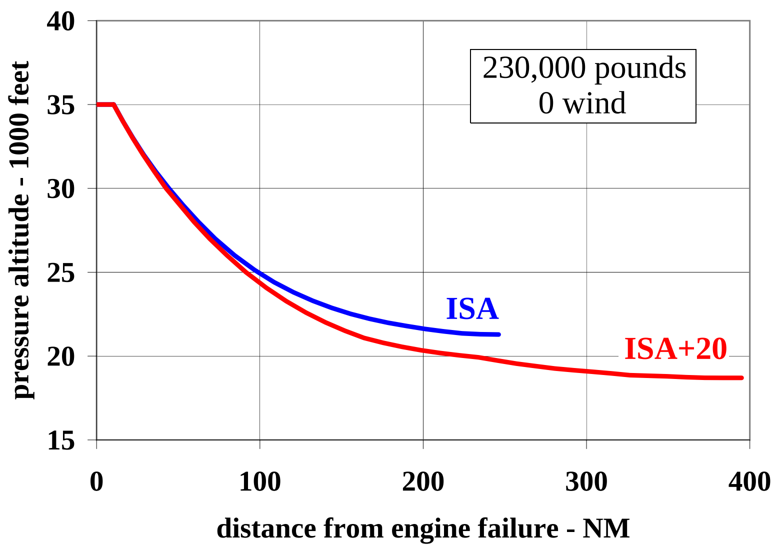


Figure 33-8

effect of wind on driftdown

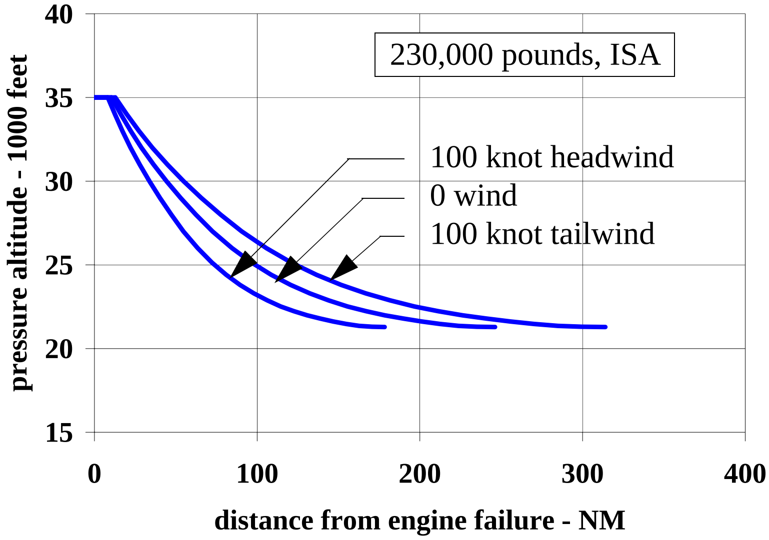


Figure 33-9

In the graph to the right you see the effect of wind on the drift-down net path.

The wind affects only the ground distance flown during the driftdown. The descent time, fuel burnoff, and air distance flown are the same regardless of wind.

A headwind reduces the distance, making the driftdown path appear steeper when it's seen from the ground. A tail-wind, on the other hand, makes the descent path seem less steep, taking a greater ground distance for the same amount of altitude change.

Obviously, when considering the need to clear any enroute terrain or obstructions, it's ground distance traveled from the point of engine failure that's important, not the air distance. For that reason, as you have seen previously, the regulations require that winds, particularly adverse winds, must be accounted for.

analyzing a route considering engine failure

By now you have all of the tools you need to analyze a route for the possible effects of any terrain or obstructions along its length. There are probably as many ways of doing such an analysis as there are performance engineers who have done them – but the following is one way the analysis could be accomplished.

You must begin a route obstacle analysis, of course, by determining the route that will be flown. Then, by reference to the navigation charts, topographical maps, and other available data it will be possible to draw a height-distance profile of the terrain and obstructions along the route.

Figure 33-10 to the right shows a typical height-distance profile.

Remember that all terrain within five statute miles on either side of the intended flight track must be included. (For JAR-OPS operators that becomes five nautical miles. ICAO uses the same width as the FAA. One other regulatory agency – the Civil Aviation Administration of China, CAAC – uses a much more conservative margin of 13.5 NM.)

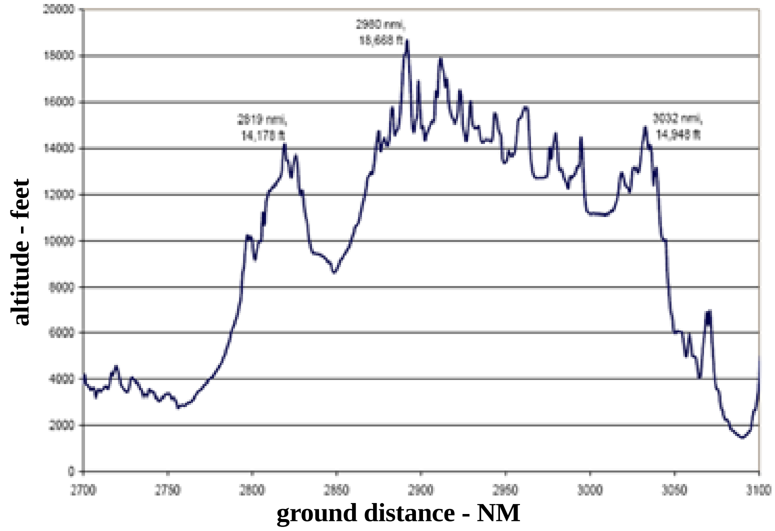


Figure 33-10

determining the allowable takeoff weight

Once the terrain requirements are known, it's possible to determine the maximum takeoff weight that will allow compliance with the terrain clearance requirements.

Relatively few of the world's air routes fly over terrain severe enough to necessitate takeoff weight restrictions based on driftdown performance. It's sensible, therefore, to make a conservative first quick check to see if additional detailed analysis is truly necessary.

Using the Airplane Flight Manual (the "enroute climb weight for positive net gradient" charts) or the Boeing software, it's a simple matter to determine the altitude at which the airplane is capable of zero net gradient as a function of the weight and air temperature.

In the figure to the right, we show a typical chart of weight versus the altitude for a positive net gradient for a 757-200.

You see that even at the higher weights, the altitude capability exceeds 19,000 feet, meaning that any terrain less than 18,000 feet will not be limiting.

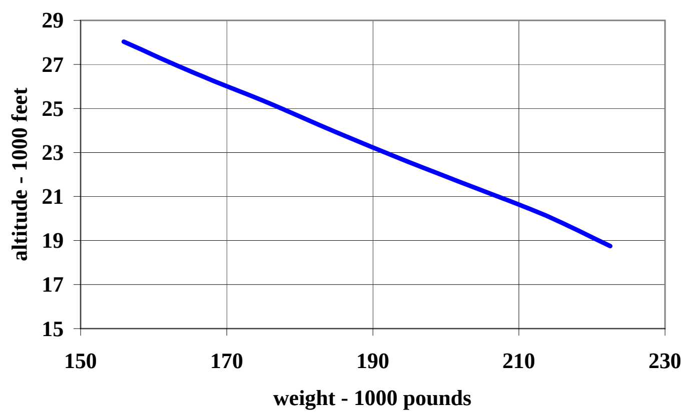


Figure 33-11

A super-quick and very conservative first check would be enter the chart with the expected maximum takeoff weight at the departure airport. If the corresponding altitude from the chart is more than 1000 feet above all of the enroute terrain, then the terrain will never be limiting.

If, on the other hand, the altitude corresponding to the maximum expected takeoff weight is less than 1000 feet above all terrain, then it will be necessary to make a more detailed check.

Knowing the distance to the critical terrain, it's easy enough to calculate an approximate airplane weight when passing over it. If that weight yields an altitude for positive net gradient that's at least 1000 feet higher than the terrain, the terrain will not be limiting.

But if this second simple check still doesn't satisfy the terrain clearance requirements, a full analysis will be needed.

Knowing the route to be flown, the airplane/engine type to be used on the route, the enroute wind and temperature conditions, and the departure airport runway data, it's possible to determine the airplane's weight at every point along the route. For conservatism, we will use the most adverse wind expected along the route – the greatest headwind or least tailwind that can be expected – when calculating the enroute weight. By using those winds we are setting the enroute ground speed at the lowest reasonable value. We will also, again for conservatism, assume that the airplane takes off at the highest weight that can reasonably be expected for this route considering the takeoff runway parameters, the route length, reasonable payload, and so on. The calculations should also account for air temperature effects along the route, since the ISA deviation has a marked effect on the engine-inoperative performance.

Having calculated the airplane's weight along the route, it's possible to draw a series of driftdown profiles along the route in the areas where terrain or obstructions might possibly create a problem. Such a diagram will aid in assessing the effect of the terrain or obstructions on the route.

Let's illustrate what we're discussing here with an example. We will assume that a 757-200 has taken off at a weight of 230,000 pounds and is now cruising at an altitude of 33,000 feet. We have estimated that this is the highest takeoff weight that we will ever reasonably expect for this route. We will assume that an enroute headwind component of 100 knots is the most adverse wind to be expected along the route. We'll also assume that temperatures along this route won't exceed standard day conditions.

We will say that the route of flight must clear some high terrain at distances of approximately 1000 to 1300 nautical miles from the takeoff point. For the example conditions stated above, here's an enroute obstacle analysis chart:

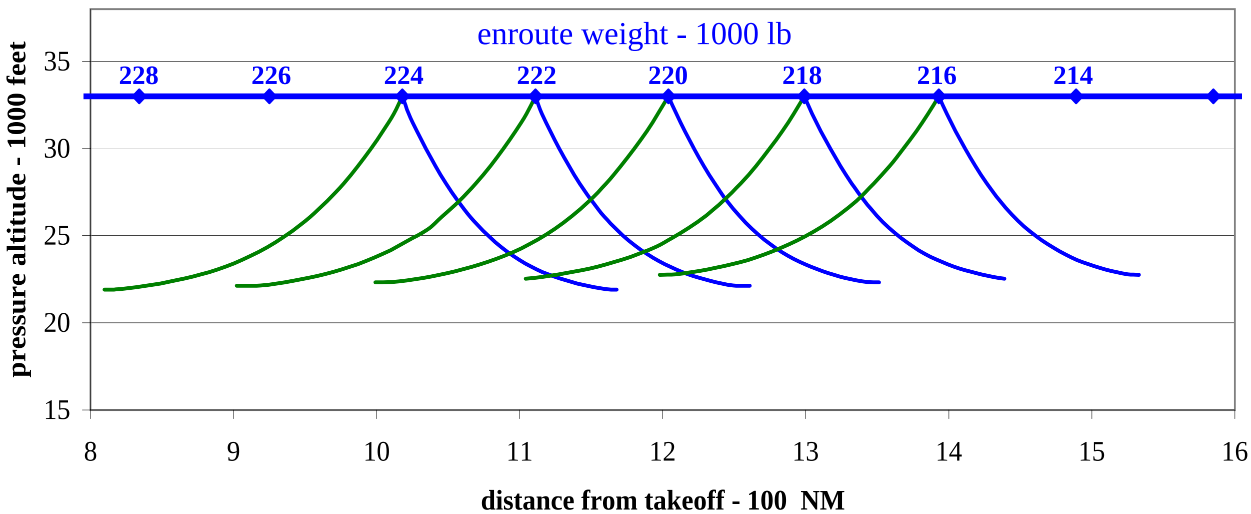


Figure 33-12

As the airplane proceeds along the route, its weight decreases as shown. The lines sloping downward and to the right represent the driftdown profiles assuming that the airplane proceeds forward along the route after an engine fails, assuming a 100-knot headwind.

The lines sloping downward and to the left represent the driftdown profiles assuming that the airplane has reversed course following engine failure and is now heading back toward the takeoff point as it descends. Here we are assuming no wind during the driftdown. Again, that's conservative; in fact, the wind during the driftdown as the airplane proceeds back along the route after reversing course <u>could</u> be 100 knots tailwind since we assumed a 100 knot headwind proceeding forward along the route. However, it would be unconservative to assume a 100-knot tailwind for the driftdown paths backward along the route if the tailwind component is actually less than 100 knots. In this example we have arbitrarily decided to base the backward driftdown paths on zero wind.

You'll notice, if you look carefully, that the driftdown leveloff altitudes are gradually increasing along the route, due to the fuel burnoff.

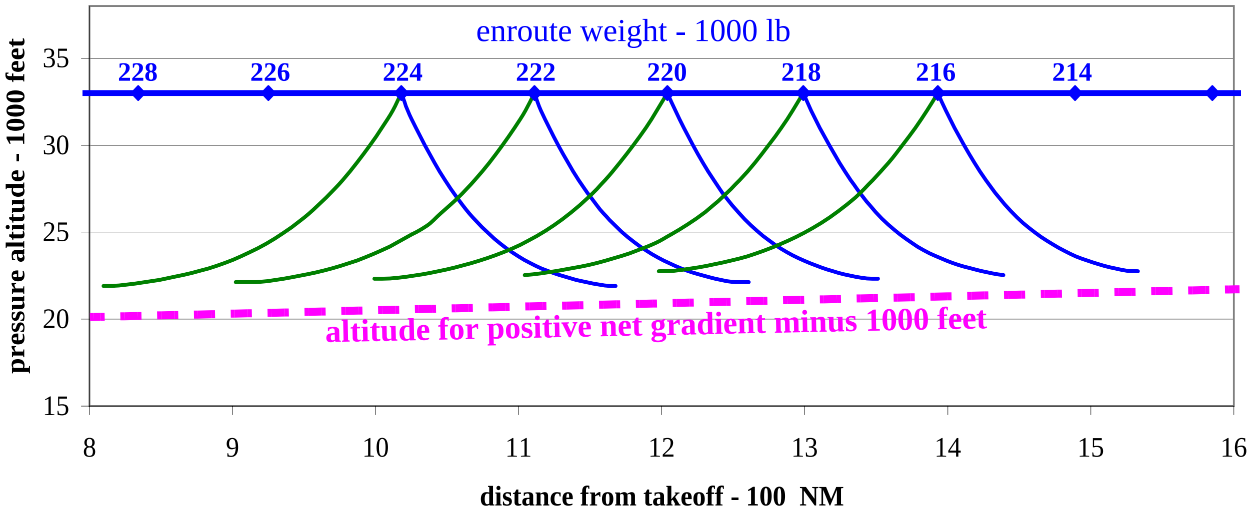


Figure 33-13

If we add a line that is 1000 feet below the leveloff altitudes, as shown in figure 33-13 above, that line would indicate the heights of terrain or obstacles along the route that would allow compliance with FAR 121.191(a)(1). You'll recall that (a)(1) states that if an obstacle can be cleared vertically by at least 1000 feet in level flight (positive net gradient) then the obstacle clearance requirements are met. Thus any obstacles that lie <u>below</u> the dashed line shown in the figure are not limiting to the flight. Any obstacle lying <u>above</u> the dashed line might – or might not – be limiting to the flight under the alternative requirement of FAR 121.191(a)(2).

Let's examine some possible obstacle scenarios to see whether or not they would be able to comply with 121.191(a)(2).

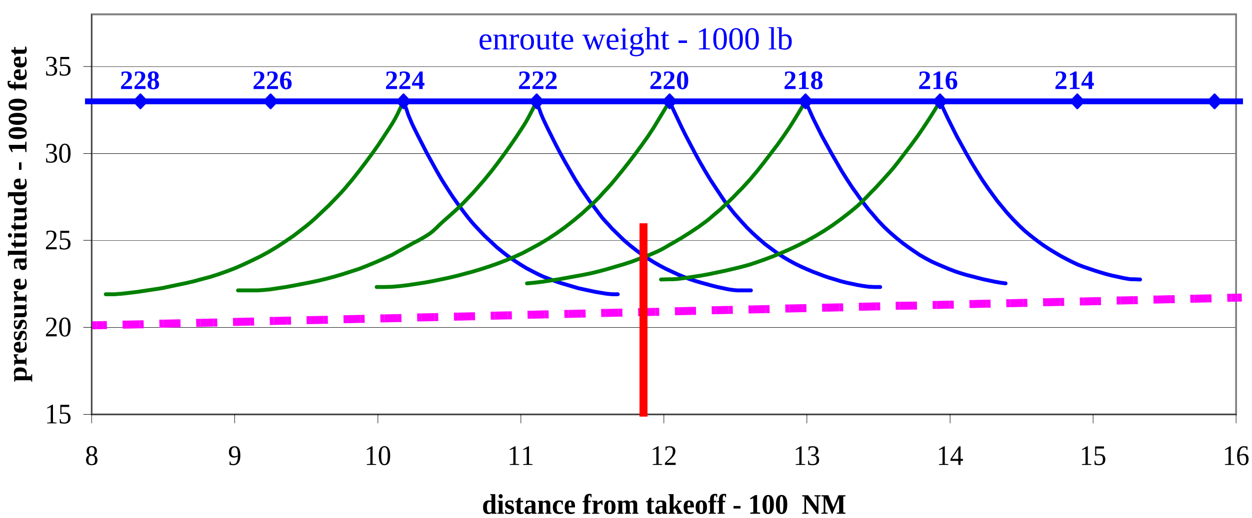


Figure 33-14

In this figure, we'll say that we have a single obstacle that is 24,000 feet high, at a distance of 1190 nautical miles along the route. Since FAR 121.191(a)(2) requires clearing the obstacle during the driftdown by 2000 feet vertically, we are showing the obstacle as having its top at an altitude of 26,000 feet – that is, the altitude that the airplane must be capable of in order to clear the obstacle by 2000 feet.

Would that obstacle be limiting to the flight? In this example, no.

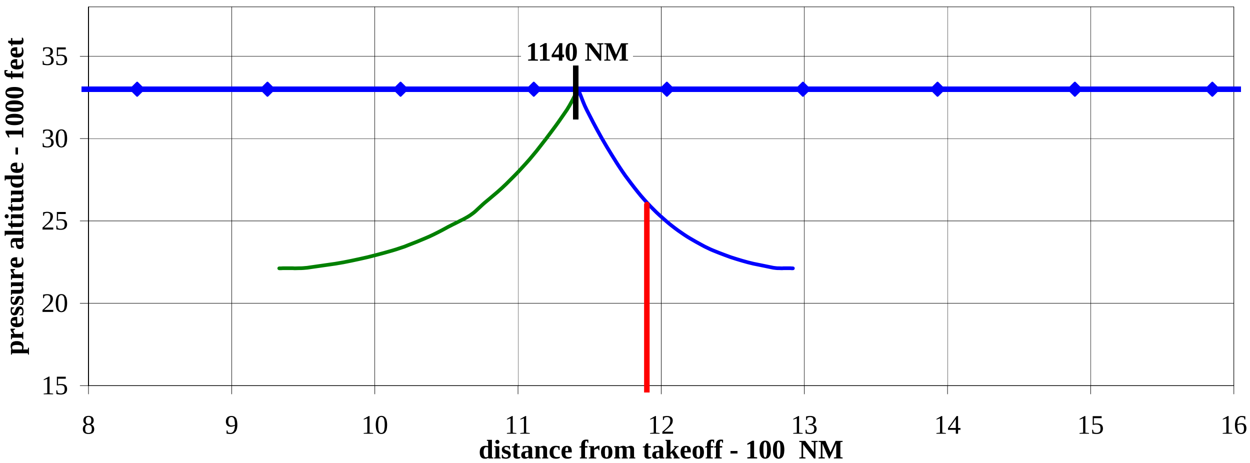


Figure 33-15

You can see in figure 33-15 that if the engine were to fail prior to a point approximately 1140 NM from takeoff, it would be necessary to conduct an air turnback and driftdown in the opposite direction. If, however, the engine were to fail after a point 1140 NM from takeoff, it would be possible to continue along the route as the driftdown path would clear the obstacle by the required 2000 feet vertically. And in our calculation method we have included conservatisms to allow for the highest reasonable driftdown weight and the strongest reasonable wind condition.

You can see that a single obstacle won't limit a route, because it will always be possible to either clear it by the required amount in driftdown, or to conduct an air turnback.

Here's a more critical scenario:

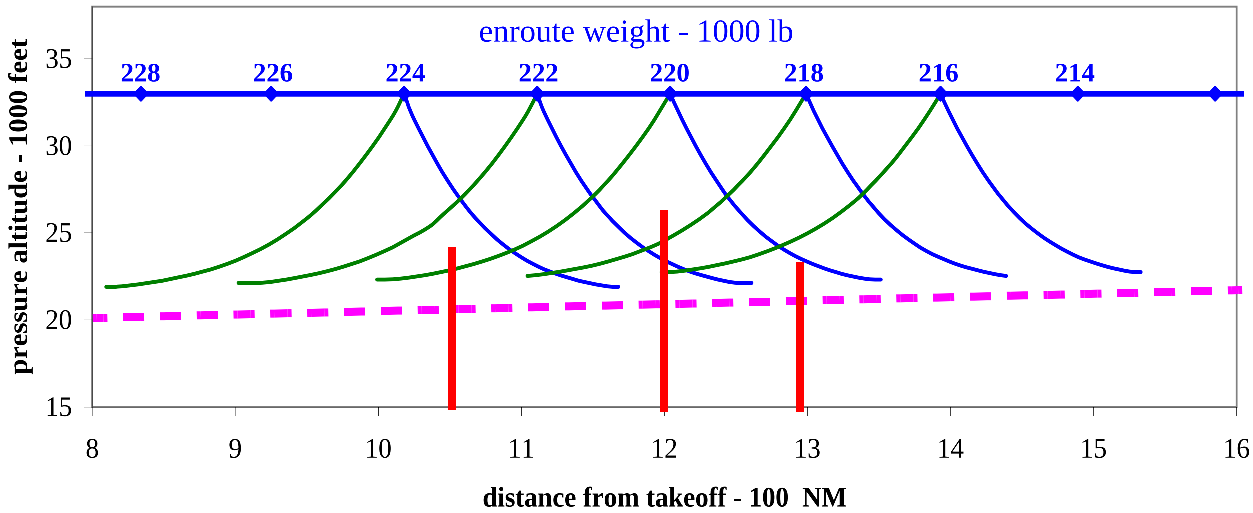


Figure 33-16

We have a mountain range that includes three critical obstacles: a 22,000-foot obstacle at 1050 miles from takeoff, a second obstacle 24,000 feet high at 1200 NM from takeoff, and a third obstacle that is 21,000 feet high at a distance of 1295 miles from takeoff. They're shown in figure 33-16 above as 24,000, 26,000 and 23,000 feet high respectively, since we're including the 2000-foot vertical clearance requirement.

Let's simplify the figure above to make it easier to see:

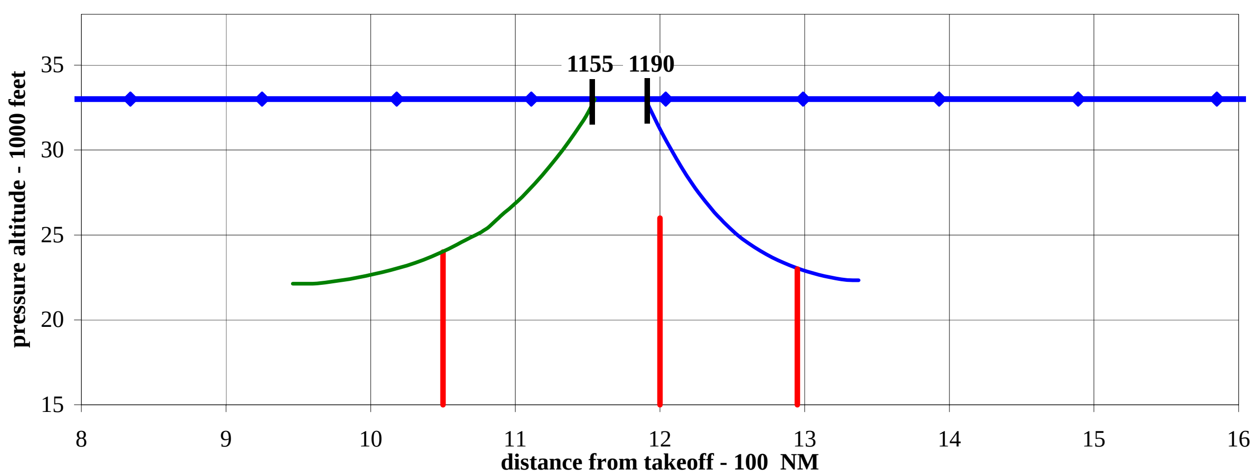


Figure 33-17

We have a problem with this example, don't we?

If we are beyond 1155 NM from takeoff, it's too late to turn back. But at any point before 1190 NM from takeoff, we can't drift down <u>forward</u> along the route, because we couldn't clear the third obstacle.

It appears that we have an impossible situation between 1155 and 1190 miles from takeoff. Within that distance range we can neither make an air turnback and drift down nor proceed forward along the route and drift down. What can we do?

- Try another route between your takeoff and destination airports.
- Reduce the takeoff weight. That has several effects: it raises the leveloff height, so that the airplane might be able to clear the obstructions in level flight; it also makes the driftdown profiles flatter, so that the same amount of descent from the cruise altitude will take more distance. That would bring the two driftdown starting points of 1155 and 1190 NM closer together until they would coincide at some weight.

  The higher of those two weights solutions would be the allowable takeoff weight.
- A third possibility would be to establish an “escape route” to be followed in case an engine failure occurs between 1155 and 1190 NM from takeoff.

Multiple Engine Failure In Flight

For airplanes having three or four engines, there is a second requirement in the regulations that specifies the conditions under which the simultaneous failure of two engines in flight must be considered, and which mandates a minimum level of performance with two engines inoperative.

Let’s look at that.

regulatory requirements

> *Sec. 121.193*
>
> *Airplanes: Turbine engine powered: En route limitations: Two engines inoperative*
>
> *...(c) No person may operate a turbine engine powered transport category airplane along an intended route unless he complies with either of the following:*
> *(1) There is no place along the intended track that is more than 90 minutes (with all engines operating at cruising power) from an airport that meets the requirements of Sec. 121.197.*
> *(2) Its weight, according to the two-engine-inoperative, en route, net flight path data in the Airplane Flight Manual, allows the airplane to fly from the point where the two engines are assumed to fail simultaneously to an airport that meets the requirements of Sec. 121.197, with the net flight path (considering the ambient temperatures anticipated along the track) clearing vertically by at least 2,000 feet all terrain and obstructions within five statute miles (4.34 nautical miles) on each side of the intended track. For the purposes of this subparagraph, it is assumed that--*
> *(i) The two engines fail at the most critical point en route;*
> *(ii) The net flight path has a positive slope at 1500 feet above the airport where the landing is assumed to be made after the engines fail;*

> *(iii) Fuel jettisoning will be approved if the certificate holder shows that the crew is properly instructed, that the training program is adequate, and that all other precautions are taken to ensure a safe procedure;*
> *)iv) The airplane's weight at the point where the two engines are assumed to fail provides enough fuel to continue to the airport, to arrive at an altitude of at least 1,500 feet directly over the airport, and thereafter to fly for 15 minutes at cruise power or thrust, or both; and*
> *(v) The consumption of fuel and oil after the engine failure is the same as the consumption that allowed for in the net flight path data in the Airplane Flight Manual.*

From the above, you see that the requirements are similar to those of Section 121.191: they require that the driftdown net flight path must clear all terrain or obstructions by at least 2000 feet vertically, and the net path must have a positive net gradient 1500 above the airport at which the airplane will land. Notice that it also specifies that there must be a minimum of 15 minutes of fuel remaining when arriving at the airport selected for landing.

The JAR-OPS equivalent to this regulation is 1.505.

Notice also that the requirements of 121.193 apply only when the route of flight contains any point that is more than 90 minutes at normal cruise speed from an acceptable alternate airport where it could land in case of emergency.

calculating the net flight path, two engines inoperative

Manual calculation of a driftdown path with two engines inoperative is the same as for one engine inoperative. In the calculation of the descent gradient and rate of descent, it's necessary to account for the additional drag of two inoperative engines rather one, and the control drag effect will be greater due to the greater asymmetry. For three-engine airplanes such as the 727, DC-10, MD-11, and L-1011 this would not be the case, however, due to the centerline location of the third engine. For conservatism, we should assume that one wing (or pod) engine and the center engine will fail; this will result in a control drag increment ΔC_{D_Ψ}. If we were instead to assume that the symmetric wing (or pod) engines fail, then there would be no control drag penalty. This could be unconservative.

Chapter 34: Cruise - Loss of Pressurization

TBS

Chapter 35: Descent, Approach, and Landing

Introduction

We're grouping three major topics together in this single chapter. You'll see, however, that these three topics are so similar to topics discussed at length in preceding chapters that we really don't need to add much new material here. For example, we have already discussed the climb after takeoff en route to the cruise altitude, in the chapter entitled "Climb To Altitude". Descent from altitude really isn't very different – it's idle thrust instead of climb thrust, and it's downhill instead of uphill – but otherwise the performance techniques are virtually identical. So in this chapter, we'll first discuss the normal descent from the cruise altitude toward the destination airport.

In the chapter entitled "Cruise – Engine Failure and Driftdown" we have already talked about descent with one or two engines inoperative. In the chapter entitled "Cruise – Loss of Pressurization" we have spoken about emergency descents when necessitated by a cabin loss of pressure. Really, all that remains to be discussed is the standard, everyday sort of descent with all engines operating.

We'll then be discussing the approach phase of flight, as the airplane prepares for the landing by changing configuration from "clean" – flaps and gear up – to the over-the-threshold configuration of landing flaps and gear down.

Finally, we'll discuss the subject of landing on dry runways. Here too you'll find that there's not a lot of new material, since the techniques for calculating landing distances are fundamentally the same as those for calculating takeoff distances, which you've already seen in the chapter entitled "Calculating Takeoff Distances".

With this in mind, let's begin the chapter by examining the normal descent from the cruise altitude toward the destination.

Descent From Altitude

In the chapter entitled "Climb To Altitude", we discussed the calculation of the flight profile as an airplane climbs toward its cruise altitude after completing the takeoff. We looked at the climb speeds and also at the calculation of the climb profile using the step-integration method. The performance method for constructing the descent path is essentially the same as the method for constructing a climb path. Also, the descent speeds are very similar to those for climb. Let's look at those first.

descent speeds

These are essentially the same as the speeds for climb, and for the same reasons.

At the higher altitudes, the descent speeds are usually the same as the cruise speed, or close to it. This simplifies the procedures at top of descent, as the pilots need only to retard the throttles to idle thrust and then pitch the airplane down to maintain speed.

As the airplane descends, at some crossover altitude the descent speed will change from being a Mach number to being a calibrated airspeed (CAS). This is just the opposite of the climb. A constant CAS will then be followed. A low-altitude speed restriction may be imposed on the descent, for example a 250-knot restriction when reaching 10,000 feet and then descending below it.

Of course, descent speeds may be dictated by air traffic control constraints as the airplane enters more congested airspace at the lower altitudes, where a large number of airplanes may be maneuvering after takeoff or before landing.

variables affecting the descent parameters

At this point, we should see how the descent parameters – time, fuel and distance – are affected by some of the flight variables. Let's start with the effect of the descent speed.

effect of descent speed

Obviously, the choice of descent speed will have some effect on the descent time, fuel and distance. But you might be surprised to see how little that effect truly is.

In table 35-1 to the right, we show the time, fuel and distance for three different speed schedules. The airplane is a 757, descending from a cruise altitude of 35,000 feet with a weight at top of descent of 190,000 pounds on a standard day with no wind. Idle thrust is assumed throughout the descent profile.

descent speed	descent time (minutes)	descent fuel (pounds)	descent distance (NAM)
0.78/280	23.4	892	116
0.78/350	16.4	680	87
0.78/280/250	25.5	968	122

Table 35-1

The first schedule is Mach 0.78 to 280 knots CAS; the second is a faster descent of Mach 0.78 to 350 knots CAS. In the third descent speed schedule we have added a speed restriction of 250 knots below 10,000 feet.

You can see that the 350-knot descent, compared to the 280-knot descent, produces a shorter descent time and distance, and saves 212 pounds of fuel.

"Ah," you say, "so high-speed descents are better." Well, no. It's true that the descent <u>fuel</u> is less. However: since the descent <u>distance</u> is 29 NM less for a 350-knot descent than it is for the 280-knot descent, the airplane will burn additional fuel <u>in cruise</u> to fly that additional 29 miles before beginning the descent.

At 190,000 pounds gross weight at LRC speed, FL350, the true airspeed is 457 knots and the fuel flow is 3374 pounds per hour per engine. You do the math: you'll see that 29 additional miles of cruise would require 428 additional pounds of fuel. Thus, from a common point, the 350-knot descent is actually worse than the 280-knot descent, by 216 pounds!

The descent speed schedule of .78/280/250, when compared to the same speed schedule but without the speed restriction below 10,000 feet, adds only about two minutes and 80 pounds of fuel, while adding six miles of descent distance. Since the cruise distance is six miles less, the cruise fuel will be 89 pounds less than for the .78/280 descent, so the fuel from a common point is almost exactly the same.

Clearly, the choice of descent speed has only a small effect on the descent parameters, slightly favoring the slower descents.

effect of descent weight

You saw that the choice of descent speed schedule had only a small effect on the descent parameters. But how about the airplane weight's effect on those same parameters?

In table 35-2 to the right, we show you the 757-200 descent parameters again, this time as a function of the airplane's weight at top of descent.

weight at top of descent (pounds)	descent time (minutes)	descent fuel (pounds)	descent distance (NAM)
200,000	23.8	904	119
180,000	22.9	878	114
160,000	21.8	845	107

Table 35-2

You see clearly that the weight effect on time is very small – only two minutes change for a weight change of 40,000 pounds. Similarly, the descent fuel is affected only to a very small degree. The descent distance does see a 12-mile decrease but even that is relatively small.

effect of air temperature

You'll probably expect temperature to have a small effect on descent, since the descent is assumed to be at idle thrust. Let's see the difference in the parameters between ISA and ISA+20, for example.

In table 35-3 to the right you see the effect of a 20-degree change in the air temperature. This is for a weight at top of descent of 190,000 pounds.

air temperature	descent time (minutes)	descent fuel (pounds)	descent distance (NAM)
ISA	23.4	892	116
ISA+20°C	23.9	932	124

Table 35-3

The only appreciable difference is seen in the descent distance, which is eight miles more at ISA+20 than at ISA. Why is the distance greater? Because the descent true airspeed is faster at ISA+20, of course.

effect of wind

This one is easy. You know immediately what the effect of wind will be, right?

Right. Wind will affect only the ground distance covered during the descent. Air distance, time, and fuel are unaffected by wind.

Here's a table showing the effect of wind during the descent. This is again for our 757-200 descending from FL350, with a top-of-descent weight of 190,000 pounds.

average wind during descent (knots)	descent time (minutes)	descent fuel (pounds)	descent air distance (NM)	descent ground distance (NM)
50 headwind	23.4	892	116	101
0	23.4	892	116	116
50 tailwind	23.4	892	116	132

Table 35-4

optimizing the descent

From the preceding, you can see that the descent parameters are quite insensitive to the descent speed, weight or air temperature, and that the ground distance is somewhat sensitive to the wind. So how can one optimize the descent?

The one parameter that does have a marked effect on the overall efficiency of a descent isn't something controlled by the performance engineer. Instead, it's controlled by the pilots flying the airplane, and sometimes by the air traffic environment. It's this: the top of descent point.

To optimize a descent, it's necessary to optimize the point at which the descent is initiated.

The ideal descent is the one that can be made at idle thrust throughout the descent. Starting the descent too soon means that the airplane will be consuming additional fuel at lower altitudes when thrust for additional maneuvering is needed to reach the airport. On the other hand, starting the descent too late means that additional fuel will be consumed at the cruise fuel flow rate, and drag will probably be necessary later in the descent to get the airplane back onto a proper descent profile.

Flight management computers in contemporary airplanes are capable of accurate calculations of the top-of-descent point, provided of course that the FMC is given reasonably accurate information on the winds to be expected during the descent.

When an FMC isn't available, the pilots will need to calculate the approximate effect of the wind on the descent and adjust the top-of-descent point accordingly.

calculating the descent profile

What is there to say? It's the same as for calculating the normal climb profile, except that the engines are at idle thrust or a low thrust setting as needed for descent path control. The equations and techniques are the same.

You have already seen in preceding chapters how to calculate a climb profile, and how to calculate an engine-inoperative descent profile. The only difference between an engine-inoperative descent and an all-engine descent are the amount of idle thrust and the lack of a control drag coefficient increment, since the normal descent is made with symmetric idle thrust.

Ideally, the entire descent will be conducted at idle thrust until the flaps are extended, at which time the added drag of the flaps may necessitate the addition of some thrust in order to maintain airspeed, depending on the descent path angle that's required. For that reason, it's standard practice to base the calculation of the descent data on idle thrust on all engines.

If added drag is needed for a descent profile steeper than that obtainable at idle thrust, spoilers/speedbrakes may be used, and the drag coefficient increment corresponding to the amount of spoiler/speedbrake extension is provided in the Performance Engineer's Manual. Simply add that to the standard drag coefficient.

The Boeing software can be used for descent calculations if you don't want to do the job manually. Either the INFLT program running in a DOS-emulator or the BPS software running in Windows can compute the descent profiles for any selected conditions.

Approach

What does *approach* mean? It's a rather vague term, after all.

In standard airline practice, approach simply refers to that portion of the flight which follows the descent from altitude and ends at the runway threshold. During the approach, then, the airplane is maneuvering to follow some assigned route leading it to the runway, or possibly it's maneuvering following verbal instructions from the air traffic controllers.

By default, the Boeing software assumes that descent ends 1500 feet above the landing airport, and approach begins at 1500 feet and continues to the runway threshold. This definition isn't contained in any sort of regulatory material, it's a purely arbitrary definition that the software uses in the absence of some other definition that may be provided by the user of the software.

During the approach, the airplane will transition from the clean flaps up-gear up configuration to the landing configuration with the flaps at the landing position and the landing gear extended. In doing so, it will follow some flap extension speed schedule that dictates the speeds at which each flap position will be selected. That flap extension speed schedule is designed to maintain adequate margins of speed from stall, even when maneuvering in the traffic control environment.

The performance of an airplane during the approach portion of the flight is subject to regulations that specify minimum allowable levels of climb capability at two particular points during the approach. The intent of these regulations is to ensure that an airplane on approach can, in the event a missed approach maneuver becomes necessary, climb away from the landing surface with an acceptable climb gradient. In some instances, these climb capability requirements may limit the allowable landing weight.

approach regulatory requirements

There are two separate regulatory requirements that are included in the climb-limited landing weight. The first of these two is referred to as the *approach climb* requirement, the second is called the *landing climb* requirement.

The airplane's landing weight must be low enough to meet the legal requirements specified for whichever is the more restrictive case of the two.

The airplane manufacturer, not the operator, will select the flap settings and the associated speeds that are used to establish the approach and landing climb gradients. Those gradients are published in the Airplane Flight Manual or the AFM-DPI, as appropriate.

It's the operator's responsibility to ensure that the airplane's expected weight during the approach will provide climb gradients that comply with the requirements.

Here are the relevant passages from the FARs. For the approach climb:

> *Section 25.121*
>
> *Climb: One-engine-inoperative*
>
> *(d) Approach. In a configuration corresponding to the normal all-engines-operating procedure in which VSR for this configuration does not exceed 110 percent of the VSR for the related all-engines-operating landing configuration:*
> *(1) The steady gradient of climb may not be less than 2.1 percent for two-engine airplanes, 2.4 percent for three-engine airplanes, and 2.7 percent for four-engine airplanes, with--*
> *(i) The critical engine inoperative, the remaining engines at the go-around power or thrust setting;*
> *(ii) The maximum landing weight;*
> *(iii) A climb speed established in connection with normal landing procedures, but not exceeding 1.4 VSR; and*
> *(iv) Landing gear retracted.*
> *(2) The requirements of paragraph (d)(1) of this section must be met:*
> *(i) In non-icing conditions; and*
> *(ii) In icing conditions with the approach ice accretion defined in appendix C. The climb speed selected for non-icing conditions may be used if the climb speed for icing conditions, computed in accordance with paragraph (d)(1)(iii) of this section, does not exceed that for non-icing conditions by more than the greater of 3 knots CAS or 3 percent.]*

The meaning of some of this wording is not immediately obvious. Let's examine one sentence in particular, taken from the beginning of the regulation:

> *In a configuration corresponding to the normal all-engines-operating procedure in which VSR for this configuration does not exceed 110 percent of the VSR for the related all-engines-operating landing configuration:*

"VSR" is the *reference stall speed*. In this context, the reference stall speed is either the $V_{S\ 1g}$ stall speed for all of the more recent airplanes, or $V_{S\ FAR}$ for the older airplanes. Whichever of those two is used as the "reference" speed in determining the minimum V_2 ($1.2V_{S\ FAR}$ or $1.13V_{S\ 1g}$) and the minimum V_{REF} ($1.30V_{S\ FAR}$ or $1.23V_{S\ 1g}$) is considered to be the reference stall speed.

This requirement is saying, in effect, that the flap position selected for calculating the approach climb gradients for compliance with 25.121(d)(1) can not have a stalling speed that is more than 10 percent faster than the stalling speed for the landing flap setting. The intent is to prevent an airplane manufacturer from selecting a very small flap setting (which of course would have superior climb performance) to be the designated approach flap setting published in the Airplane Flight Manual or AFM-DPI, thus offering unrealistically high climb-limited landing weights.

Here is the relevant FAR regulation governing the landing climb requirement:

> *Sec. 25.119*
>
> *Landing climb: All-engines-operating.*
>
> *[In the landing configuration, the steady gradient of climb may not be less than 3.2 percent, with the engines at the power or thrust that is available 8 seconds after initiation of movement of the power or thrust controls from the minimum flight idle to the go-around power or thrust setting--*
> *(a) In non-icing conditions, with a climb speed of VREF determined in accordance with Sec. 25.125(b)(2)(i); and*
> *(b) In icing conditions with the landing ice accretion defined in appendix C, and with a climb speed of VREF determined in accordance with Sec. 25.125(b)(2)(ii).]*

The landing speed V_{REF} specified in subparagraphs (a) and (b) is the *landing reference speed*. Section 25.125 provides criteria for determining the value of V_{REF} that the manufacturer will publish in the AFM and will use in calculating the certified landing distances required.

V_{REF} may not be less than $1.23V_{S\ 1g}$ for the 767-300 and all subsequent models; for earlier models, V_{REF} was not allowed to be less than $1.3V_{S\ FAR}$.

You probably noticed in the regulations quoted above that they make mention of approach and landing considering icing conditions. Let's take a minute to discuss that, because it can cause misunderstanding.

You know that when an airplane is flying in atmospheric conditions that are conducive to accumulations of ice on the wing and airframe, the pilots can activate the airplane's icing protection systems. These systems are typically designed to use either hot engine bleed air or electrical heating to prevent ice accumulation on the flight-critical surfaces: that usually includes the engine cowl and the wing leading edges. Pitot-static probes and other airplane equipment that are sensitive to icing conditions are also protected, most often by electrical heat.

Icing conditions are usually considered to be present when there's visible moisture in the air and the total air temperature is below a specified value. That value is usually 8°C for Boeing airplanes.

When executing an approach for landing in icing conditions, with the anti-ice systems turned on, there is usually a performance penalty. That is due to the effect of extracting hot bleed air from the engines to provide the icing protection, which makes slightly less thrust available for a go-around. A reduction to the climb limit landing weight is thus necessary when operating the anti-ice systems during the approach. That weight penalty is provided by the Airplane Flight Manual.

But there's another possible effect of icing conditions on landing weight, and this one is not quite so obvious. This effect can occur even when the airplane is not making its approach in icing conditions. Here's the explanation:

Let's say that at some point in the flight it has been necessary to use the icing protection systems. That's fine – by doing so we're avoiding the accumulation of ice on the engine inlets (where ice could cause power loss or flameout, or engine damage if ice is shed into the engine intake) and on the wing leading edges (where ice can seriously degrade the wing's aerodynamic qualities including its ability to generate lift).

However, Boeing airplanes do not have any icing protection on the leading edges of the horizontal stabilizer or the vertical fin, and so when flying in icing conditions it's very possible that the horizontal tail and vertical fin may develop some amount of ice accumulation.

Airplane manufacturers are required to prove, if applicable, that ice accumulation on unprotected surfaces can't cause a hazardous situation. But even if the accumulated ice doesn't cause such a situation, at the very least the accumulated ice will present some amount of weight increase and also some additional aerodynamic drag. As a result, there will be a reduction to the airplane's performance.

Here's the important thing to remember:

Even if icing conditions do not exist at the destination airport, if airframe ice has been accumulated during the flight it's possible that some ice may still remain on the airplane during the approach if the air temperatures during the descent and approach aren't warm enough to melt it off.

For that reason, the AFM climb limit landing weight charts contain a note saying:

> *Reduce gross weight by XXXXX pounds when operating in icing conditions during any part of the flight when the forecast landing temperature is below 8 degrees C.*

For some later Boeing models, the temperature criterion is 10°C rather than 8°C. The gross weight reduction shown as *XXXXX* above varies from airplane to airplane, of course.

flap extension speed schedule

In the chapter entitled "Climb To Altitude" you saw the speed schedule for retraction of the flaps as the airplane is configured after takeoff to the clean enroute configuration.

In the table to the right, you see again the flap speed schedule maneuvering speeds for the 757. *Maneuvering speed* in this context simply means the minimum recommended airspeed at which an airplane should carry out any maneuvering after takeoff or before landing.

For each of the airplane's available flap settings, the table shows the appropriate maneuvering speed. In the words of the Flight Crew Training Manual:

> *The schedule provides adequate buffet margin for an inadvertent 15 degree overshoot beyond the normal 25 degree bank angle, when recommended procedures are followed. The flap speed schedule is based on additives to VREF.*
>
> *Using VREF as the basis for the schedule makes it variable as a function of gross weight and will provide adequate maneuver margin above stall at all weights.*
>
> *There are also several other benefits as a result of this schedule. It provides speeds which are close to minimum drag and in climb are close to maximum climb gradient. In level flight it provides relatively constant pitch attitudes and requires little change in thrust required at different flap settings...*
>
> *...During flap retraction and extension, flap movement to the next position should be initiated when within 20 knots of the maneuver speed for the next flap position.*

flap setting	speed
0	$V_{REF30} + 80$
1	$V_{REF30} + 60$
5	$V_{REF30} + 40$
15	$V_{REF30} + 20$
20	$V_{REF30} + 20$
25	V_{REF25}
30	V_{REF30}

Table 35-5

Following this speed schedule, as a 757 decelerates through V_{REF30}+80 knots the pilots will select flaps 1; as it decelerates through V_{REF30} + 60 knots, the pilots will select flaps 5, and so on. The pilots are thus ensured of an adequate margin from buffet even allowing for an inadvertent 15 degree exceedance of the normal 25 degree bank angle limit.

Landing

There are three different restrictions on landing weight. Any one of those three may be the smallest value, depending on the parameters of the landing. Whichever of the three is the smallest becomes the maximum allowable landing weight. Let's take a look at the three requirements.

regulatory requirements

You're already familiar with one of the restrictions on the landing weight, the climb requirements:

the climb limit landing weight

The climb limit landing weight is the weight that satisfies whichever is the more restrictive of the approach climb gradient requirement or the landing climb gradient requirement, as specified in FAR Sections 25.119 and 25.121, quoted above. You have seen that the AFM provides charts giving the approach and climb gradients and also the climb limit weight as a function of the landing altitude and temperature.

Here's another of the three limitations:

the structural limit landing weight

The certified structural limit landing weight is simply that maximum weight, provided by the AFM, which satisfies the design requirements considering the loads that may be imposed on the airplane during the landing. Those requirements specify, among other things, that an airplane landing at its certified landing structural limit weight must be capable of withstanding a rate of sink at touchdown of ten feet per second. [FAR section 25.473(a)(2)]

The third requirement relates to the landing distance:

the field length limit landing weight

This limitation is specified in FAR Section 121:

> *Sec. 121.195*
>
> *Airplanes: Turbine engine powered: Landing limitations: Destination airports.*
>
> *(a) No person operating a turbine engine powered transport category airplane may take off that airplane at such a weight that (allowing for normal consumption of fuel and oil in flight to the destination or alternate airport) the weight of the airplane on arrival would exceed the landing weight set forth in the Airplane Flight Manual for the elevation of the destination or alternate airport and the ambient temperature anticipated at the time of landing.*
> *(b) ...no person operating a turbine engine powered transport category airplane may take off that airplane unless its weight on arrival, allowing for normal consumption of fuel and oil in flight (in accordance with the landing distance set forth in the Airplane Flight Manual for the elevation of the destination airport and the wind conditions anticipated there at the time of landing), would allow a full stop landing at the intended destination airport within 60 percent of the effective length of each runway described below from a point 50 feet above the intersection of the obstruction clearance plane and the runway. For the purpose of determining the allowable landing weight at the destination airport the following is assumed:*
> *(1) The airplane is landed on the most favorable runway and in the most favorable direction, in still air.*

> *(2) The airplane is landed on the most suitable runway considering the probable wind velocity and direction and the ground handling characteristics of the airplane, and considering other conditions such as landing aids and terrain...*

The equivalent for JAR-OPS operators is JAR-OPS 1.515, which imposes the same restriction.

We can simplify the words as written in FAR 121.195 to this: <u>for a dry runway, the actual landing distance required may not exceed 60% of the runway available</u>. That's the basis for determining the runway-limited landing weight.

Thus, before we can find this limit weight, we need to know how to calculate the landing distance.

calculating the certified landing distance

You saw, in the chapter entitled "Calculating Takeoff Distances" that calculating a takeoff distance is a process of step integration: taking incremental changes of airspeed, computing the distance required to accomplish each incremental airspeed change, and then adding together the incremental distances to arrive at the total distance for the takeoff.

Following the step integration process, however, required us to know exactly what kind of physical actions were being carried out in each of these incremental changes: Were the brakes applied? Were we accelerating with all engines operating? Did we have an engine inoperative during this increment? In other words, what was occurring that affected the acceleration, in what sequence, and at what speeds were they occurring during the takeoff.

Only by knowing the sequence of physical actions taking place during the takeoff could we determine the acceleration of the airplane during each speed increment, and therefore the time in each increment and the distance traveled in each increment.

The process for calculating the landing distance is identical to that for calculating the takeoff distance, and it also requires us to know the sequence and timing of the physical actions taking place during the landing. Here they are:

- The airplane crosses the runway threshold at a height of 50 feet.
- Beginning at 50 feet, the airplane conducts a "flare" maneuver, preparing it for touchdown.
- Once on the ground, the airplane passes through a "transition" process during which it's configured to the full braking configuration.
- Once in the full braking configuration, the airplane decelerates to a full stop, <u>without using reverse thrust</u>.

The description above can be seen as defining three distinct segments making up the landing: the "flare" segment, from 50 feet to touchdown; the "transition" segment, from touchdown until the airplane is in the full braking configuration; the "braking" segment, beginning when the airplane is fully configured for stopping and ending when the airplane has come to a complete stop.

The three segments can be shown graphically as you see in figure 35-1 to the right.

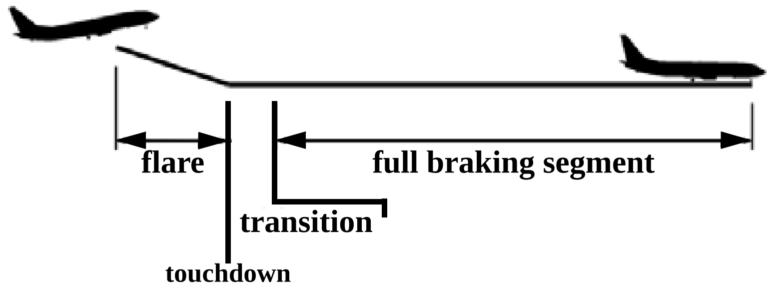


Figure 35-1

To calculate these three distance segments, data collected during the flight testing of the airplane is used to determine the necessary landing distance parameters. A number of landings are conducted during this testing. The landings are made with a variety of approach speeds, approach angles and rates of sink at touchdown to statistically determine the necessary data. The testing covers the expected operational weight range and landing configurations. From all the tests, a number of landing parameters are determined that will be used by the AFM-DPI to calculate the certified landing distances, or will be used in calculating the certified landing distances that appear in the AFM landing distance charts.

First, let's look at the necessary parameters for calculating the first of the three landing distance segments, the flare.

the flare segment

The term *flare* in this context means the distance from the point at which the airplane is at a height of 50 feet above the runway to the point of touchdown.

One of the landing parameters determined during flight testing is the flare time – the time from 50 feet to touchdown. The second of the two parameters needed to calculate the flare distance from 50 feet to touchdown is the flare speed ratio, the ratio of the airspeed at touchdown to the airspeed at 50 feet. Knowing the flare speed ratio allows us to calculate the touchdown speed corresponding to an assumed approach speed. We can then calculate the average airspeed during the flare.

The flare distance that's included in the certified landing distance data is simply the certified representative flare time multiplied by the average speed between 50 feet and touchdown.

Thus, as an example of an air distance calculation, let's say that flight testing has established that the flare time is 4.22 seconds and the flare speed ratio is 0.9818. If we say that the approach speed at 50 feet is 131 knots in zero wind:

$$V_{touchdown} = 131 \times 0.9818 = 128.6 \ knots$$

The average speed between approach and touchdown is therefore 129.8 knots, giving an air distance of

$$air\ distance = 129.8 \times 4.22 \times 1.6878 = 925 \ feet$$

You'll recognize the factor of 1.6878 in the equation above – it's used to convert the approach speed measured in knots into an approach speed in units of feet per second. Doing so makes the units consistent, and feet per second multiplied by seconds yields distance in feet.

the transition segment

"Transition" in this context is the term used to denote the portion of the landing between touchdown and the point at which the airplane is in the full stopping configuration.

The process of configuring the airplane into the full stopping configuration is simple: extension of the spoilers and actuation of the wheel brakes. We are allowed to take credit for automatic actuation of the spoilers if the airplane is equipped with a system providing automatic actuation.

The time intervals used in the transition segment may be demonstrated during flight testing but are not to be less than one second per manual action. No credit may be taken for automatic wheel brake actuation.

Thus if we assume manual braking and automatic spoiler actuation, there will be only one second between touchdown and the full stopping configuration. Manual braking and manual spoiler actuation would require two seconds for configuring to the full stopping condition.

We need another landing parameter in order to calculate the transition distance: the transition speed ratio – the ratio of the brakes-on speed to the touchdown speed. That parameter is also established by flight testing.

For the same airplane as that for which we calculated the air distance a moment ago, the transition speed ratio is 0.9864. Continuing that example, then:

$$V_{brakes\ on} = V_{touchdown} \times 0.9864 = 131 \times 0.9818 \times 0.9864 = 126.9\ knots$$

The average speed between touchdown and brakes on is 127.7 knots. The transition distance, then, for a transition time of one second, would be:

$$transition\ distance = 127.7 \times 1.0 \times 1.6878 = 216\ feet$$

the braking segment

This is the most complex segment because it is the longest and the braking deceleration is not constant. It is similar in that regard to the stopping portion of a rejected takeoff, which you have seen previously in the chapter entitled "Calculating Takeoff Distances". The calculation method is essentially the same: a step integration based on increments of speed, from the brakes-on speed down to zero.

The forces acting on the airplane during the braking segment are simple. They are shown graphically in figure 35-2 to the right.

Contributing forward-acting force are the engines, which are assumed to be at forward idle thrust during the braking segment.

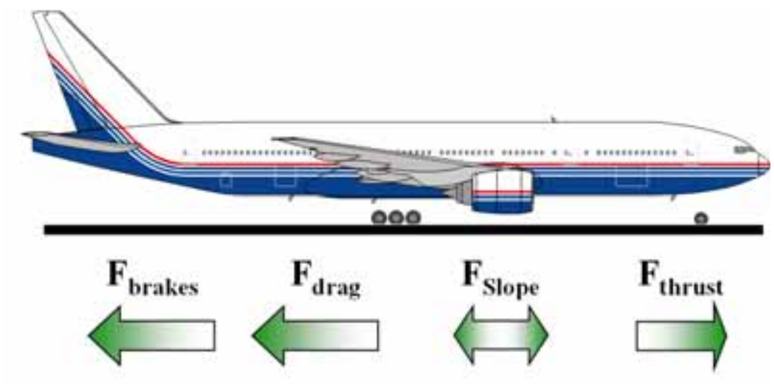


Figure 35-2

Acting to retard the airplane's motion are two forces: the airplane braking force and the aerodynamic drag of the airplane.

The runway slope, if any, will result in a small force which may be acting either forward (in the case of a downhill runway) or rearward (in the case of an uphill runway). Since the runway slope angle is very small, the force resulting from slope can be expressed as:

$$F_{slope} = W \times \phi$$

where $\overline{W}$ is the airplane weight
ϕ is the runway slope <u>expressed in radians</u>

The force supplied by the brakes can be expressed as:

$$F_{brakes} = \mu_B (W - L)$$

where μ_B is the airplane braking coefficient
W is the airplane weight
L is the aerodynamic lift being generated by the airplane in the ground attitude

For a detailed discussion of the airplane braking characteristics and the braking coefficient μ_B, please see Discussion 1 in the Additional Discussion section at the end of this chapter.

The lift force and the drag force are calculated from the airplane's known lift and drag coefficients for the ground attitude, using the usual equations:

$$L = \frac{1}{2} C_L \rho S V_{true}^2 \quad \text{and} \quad D = \frac{1}{2} C_D \rho S V_{true}^2$$

For true airspeeds in knots, those two equations are frequently shown as:

$$L = \frac{C_L \sigma V_{true}^2}{295.369} \quad \text{and} \quad D = \frac{C_D \sigma V_{true}^2}{295.369}$$

The total of the forces acting on the airplane is thus:

$$\Sigma F = T - D - \mu_B W - L - W\phi$$

We can always say that acceleration is:

$$a = \frac{\Sigma F}{m} = \frac{g}{W} \times \Sigma F$$

Thus we can express the equation for the deceleration in the braking segment as:

$$a = \frac{g}{W}[T - D - \mu_B (W - L) - W\phi]$$

where T is the idle thrust force
μ_B is the airplane braking coefficient, <u>which is constant throughout the segment</u>

Since the sum of the retarding forces will be greater than the forward force of the engines at idle thrust, the acceleration term a will be negative, denoting a deceleration condition.

<u>the step integration of distance</u>

At this point, it might be a good idea to do a complete calculation of a landing distance for a set of sample conditions. That will allow you to see the complete process, including the step integration calculation of the braking segment distance.

Here are the sample conditions:

- sea level standard day, no wind (σ = 1.0)
- flaps 30 for landing
- threshold speed is V_{REF30} = 131 knots CAS = 131 knots TAS
- weight = 198,000 pounds
- level runway
- touchdown speed ratio V_{TD}/V_{APP} = 0.982
- touchdown time, 50 feet to touchdown = 4.2 sec
- transition speed ratio V_B/V_{TD} = 0.991
- transition time, touchdown to brakes on = 0.34 sec
- airplane braking coefficient = 0.3701

- lift coefficient in ground attitude = 0.134
- drag coefficient in ground attitude = 0.0.2293
- approach idle thrust is assumed throughout the landing
- reference wing area = 1951 square feet

First, the flare distance: that's the average speed in the flare multiplied by the time. The touchdown speed is:

$$touchdown\ speed\ =\ 131 \times 0.982\ =\ 128.6\ knots$$

The average of the approach speed and the touchdown speed is:

$$average\ speed\ =\ \frac{131 + (131 \times 0.982)}{2}\ =\ 129.8\ knots$$

So the flare distance is:

$$flare\ distance\ =\ 129.8 \times 4.2 \times 1.6878\ =\ 920\ feet$$

Second, the transition distance: that's equal to the average speed in the transition multiplied by the transition time.

The brakes-on speed is:

$$brakes - on\ speed\ =\ 128.6 \times 0.991\ =\ 127.5\ knots$$

The average speed is:

$$average\ speed\ =\ \frac{128.6 + (128.6 \times 0.991)}{2}\ =\ 128.0\ knots$$

Thus the transition distance is:

$$transition\ distance\ =\ 128.0 \times 0.34 \times 1.6878\ =\ 73\ feet$$

Now we can start the step integration, from the brakes-on speed of 127.5 knots down to zero.

At each velocity corresponding to the beginning of the speed increment step, and at the velocity corresponding to the <u>end</u> of that step, we can calculate the deceleration rate, following the equation given above. For example, at the beginning of the first speed increment of the braking segment, the speed is 127.5 knots.

From the thrust tables, we have determined that the approach idle thrust of the two engines combined is 4505 pounds.

For a drag coefficient of 0.2293, the drag force will be:

$$drag = \frac{C_D \times \sigma \times S \times V_{true}^2}{295.369} = \frac{0.2293 \times 1.0 \times 1951 \times 127.5^2}{295.369} = 24{,}662 \text{ pounds}$$

Similarly, the lift force is found to be 14,389 pounds.

For an airplane braking coefficient of 0.3701, the braking force will be:

$$braking\ force = 0.3701 \times (W - L) = 0.3701 \times (198000 - 14389) = 67{,}954 \text{ pounds}$$

Now we can calculate the acceleration:

$$a = \frac{32.174}{198000}(4505 - 24662 - 67954) = -14.3 \text{ feet/sec/sec} = -8.5 \text{ knots/sec}$$

Repeating this process for the speed at the <u>end</u> of the increment, 120 knots, we find an acceleration rate of -8.2 knots per second.

The distance traveled in each step of speed increment can be expressed as:

$$\Delta distance = \frac{V_{average} \times \Delta V}{a_{average}}$$

So for this first speed step of 7.5 knots speed decrease:

$$\Delta distance = \frac{\frac{(127.5 + 120)}{2} \times (120 - 127.5)}{\frac{-(8.5 + 8.3)}{2}} \times 1.6878 = 188 \text{ feet}$$

Now look at the following table:

	true airspeed (knots)	total thrust (pounds)	drag force (pounds)	lift force (pounds)	W-L (pounds)	brake force (pounds)	slope force (pounds)	accel (ft/sec/sec)	accel (knots/sec)	distance (feet)	total distance (feet)
threshold	131										0
touchdown	128.6									920	920
brakes on	127.5	4505	24622	14389	183611	67954	0	-14.3	-8.5	73	993
	120	4666	21810	12746	185254	68563	0	-13.9	-8.2	188	1181
	110	4881	18327	10710	187290	69316	0	-13.4	-7.9	241	1422
	100	5096	15146	8851	189149	70004	0	-13.0	-7.7	227	1649
	90	5371	12268	7169	190831	70627	0	-12.6	-7.5	211	1860
	80	5646	9693	5665	192335	71183	0	-12.2	-7.2	195	2055
	70	5920	7422	4337	193663	71675	0	-11.9	-7.1	177	2232
	60	6195	5453	3186	194814	72101	0	-11.6	-6.9	157	2389
	50	6470	3786	2213	195787	72461	0	-11.3	-6.7	137	2526
	40	6837	2423	1416	196584	72756	0	-11.1	-6.6	114	2640
	30	7204	1363	797	197203	72985	0	-10.9	-6.5	90	2730
	20	7570	606	354	197646	73149	0	-10.8	-6.4	65	2795
	10	7937	151	89	197911	73247	0	-10.6	-6.3	40	2835
	0	8304	0	0	198000	73280	0	-10.6	-6.3	13	2848

Table 35-6

You can see that in the table we have repeated the calculation of distance for each step, and in the column at the extreme right we show the total distance from the threshold to each velocity point. From the threshold to a full stop, then, is 2848 feet.

So is the certified landing distance for these conditions equal to 2848 feet? No, because there's one factor we haven't considered yet.

You may remember that earlier in this chapter we summarized the landing distance requirement of FAR 121.195 as follows: "for a dry runway, the actual landing distance required may not exceed 60% of the runway available".

Let's now define the certified landing distance as 1.67 times the actual landing distance (that is, the actual distance divided by 0.6). Then we can say that "the certified landing distance may not exceed the length of the runway available."

Thus, for our example, the certified landing distance would be 2848 ÷ 0.6 or 4747 feet.

Dry runway landing distances provided by the AFM or the AFM-DPI software follow that definition. All we need to do to ensure compliance with FAR 121.195, is to find the certified landing distance for the expected landing conditions and compare that with the runway length available at the airport where the airplane will land. If the available distance is greater than the certified landing distance, FAR 121.195 is satisfied.

There are several things about the certified landing distances that are worth emphasizing here:

- In the AFM charts or in the AFM-DPI you will not see any entry for runway slope when determining the landing distance. Because of its relatively small contribution, runway slope is ignored in the certified landing distance data.
- The certified dry runway landing fields from either the AFM or the AFM-DPI software are based on standard day temperatures. If you look at an AFM landing distance chart, you'll

observe that there is no temperature effect shown. AFM-DPI output screens display messages stating that the temperature is for standard day and zero slope.

- The winds used in calculating the distances must be equal to 150% of any tailwind component expected, or 50% of any headwind component expected. When using the AFM landing distance charts or AFM-DPI, however, you should not multiply the expected wind by those factors before using the charts or the software, because the 50% / 150% correction is included in the charts or calculation algorithms. Thus, if you calculate a distance for a ten-knot headwind from an AFM chart, or from AFM-DPI, you are actually being given the distance for a five-knot headwind.
- For the purposes of calculating the certified landing distances, the brake forces are based on the assumption that the pilots are using maximum manual braking. No consideration is given at all to any autobrake systems that may be installed on the airplane.

field length limit landing weight

Simply stated, the field length limit landing weight is the landing weight at which the certified landing distance is just equal to the landing distance available.

You have seen how the certified landing distances are calculated. That calculation method can be repeated for a variety of weights and altitudes, allowing one to prepare a chart of certified landing distance that covers all of the expected landing conditions. From such a chart, knowing the available landing distance allows the user to easily determine the field length limit weight.

The AFM presents charts of field length limit weight; the AFM-DPI will calculate a value of field length limit landing weight for any landing conditions entered into the software.

landing brake energy

Way back in a much earlier chapter entitled “Brake Energy Limit Takeoff Weight” we talked at length about airplane brakes and, in particular, about the concept of brakes as devices designed to absorb kinetic energy in order to decelerate a moving vehicle. We showed that the process of absorbing kinetic energy causes the brakes to become hot, and that this heat can create problems if it becomes excessive.

For takeoff, these characteristics of airplane brakes result in the speed V_{MBE}, the speed for maximum brake energy. A rejected takeoff that is initiated (brakes applied) at V_{MBE} will theoretically result in brake kinetic energies that are just equal to the energy absorption limit of the brakes. Application of brakes for an RTO at speeds above VM_{BE} can be expected to result in brake and tire fires. This energy absorption capability is established by flight test.

Of course, these principles apply to a landing also. Although landing brake application speeds and weights are below the conditions that can be expected to cause brake and tire fires, other undesirable consequences can ensue from a high-energy stop.

wheel fuse plugs

A wheel "fusible plug", or "fuse plug" is a safety device installed in the wheels of all Boeing airplanes. A number of holes are drilled in the landing gear wheels; these holes are filled with plugs made of a metal alloy having a low melting point. When the wheels become excessively hot, such as during a high-energy rejected takeoff, the fuse plugs melt and allow the tire pressure to be released. The intent of the fuse plugs is to prevent possible tire explosion that might otherwise occur when wheel temperatures become very high and tire pressures become potentially dangerous.

If you have ever seen the aftermath of a high-energy RTO, you have probably seen that some or all of the tires have deflated. That is due to the action of the wheel fuse plugs, which melted when the wheels were heated excessively by the brakes.

One drawback to the fusible plugs is that there exists an interval of time between use of the brakes and the time at which the fuse plugs melt. If the brake energies are extremely high, the time interval to fuse plug melt is very short, but at brake energies that are only slightly into the range that will cause fuse plug melting – a relatively low-energy RTO, for example – the time interval can become extremely long. This time interval, or "lag", exists because it takes time for the heat to flow from the brakes and time for the metal of the wheels to heat up to the temperature at which the plugs will melt.

By test, it's possible to determine the minimum value of brake energy that will result in fuse plug melting, and also to determine the time lag between brake application and the release of the tire pressure. Just to give you an idea of these values: for a 737-300, the "maximum fuseplug brake energy" (i.e. the upper limit of brake energy that will <u>not</u> result in fuse plug melting) is 23.73 million foot pounds. The time interval to melting, on one particular brake design, was found by test to be 38 minutes.

Here's the problem in a real-world situation: let's say that we're landing at an airport such as Denver, or Mexico City that are at higher altitudes and can have high air temperatures.

Because the pressure altitudes are high and the temperatures are high, the landing <u>true airspeeds</u> are also high. As a result, the kinetic energies absorbed by the wheels during a landing are much greater than at lower altitudes and colder days. Could the landing energies be great enough to melt the fuse plugs?

Yes. It's a very real possibility. And here's the worst aspect of this problem: the time lag between brake application and fuse plug melting. Suppose, for example, that your expected ground time is short – just long enough to disembark some passengers, board some new passengers, and taxi back out again for takeoff. For many airlines, half-hour ground times are quite normal.

We said just above that the time lag might be as long as 38 minutes. What if you were to land at an energy just above the lower limit of energies that will cause melting, and have a short ground time. <u>It's entirely possible that the fuse plugs might melt as the airplane taxies back out for the next takeoff, or even after airborne</u>.

It's difficult to detect a flat tire from the cockpit. If the pilots weren't aware that they had exceeded the safe landing energy during the previous landing, and left the gate for the next takeoff before the fuse plugs melted, they might attempt the next takeoff on one or more flat tires.

This has actually occurred a number of times. The good news is that we are not aware of any airplane incident or accident that was due to operation on deflated tires. The bad news, however, is that attempting a takeoff on flat tires in those instances resulted in airframe damage from pieces of tire carcass thrown outward by the spinning of the wheels as the tires disintegrated during the takeoff. Have you ever driven an automobile on a flat tire? Now imagine making a takeoff in an airplane that has one or more flat tires, at takeoff speeds of more than one hundred knots. It's not a pretty picture.

maximum quick turnaround weight

To avoid this very real possibility, there is a chart in every AFM called the "Maximum Quick Turnaround Weight" chart. Entering the chart with the known landing parameters allows the user to determine the maximum allowable landing weight considering possible fuse plug melting. And a note on the chart reads as follows:

> *After landing at weights exceeding those shown on this chart, wait at least XX minutes, then check wheel thermal plugs before making a subsequent takeoff."*

The time interval of XX minutes in the above quotation varies between airplanes and brake types. On the 737-300 having Bendix wheels and brakes, for example, it's shown as 53 minutes – the demonstrated lag time of 38 minutes plus 15 additional minutes as a conservatism.

Let's make this absolutely clear: the time interval to be observed when exceeding the maximum quick turnaround landing weight is a WAITING time, not a time specified for brake cooling. The intent is to ensure that an airplane in a potential fuse plug melt situation will not leave the gate and taxi out for the next takeoff before waiting enough time for the fuse plug melt to occur. Then, if after the specified fuse plug melt time interval the plugs have not melted, the flight may proceed without fear of plug melting.

possible brake problems on short-haul operations

Operators conducting "short-haul" operations characterized by short flight times and short ground times would do well to keep the following facts in mind:

- Brake energy is cumulative.
- It's easy to make brakes hot.
- It's not easy to make brakes cool.

Imagine an operation having short flight times and short ground times. The first landing won't cause fuse plug melting, but it will pump some kinetic energy into the brakes. Sitting on the ground, brake cooling is very slow, so short ground times will offer little brake cooling. Short flight times won't offer much cooling either, so the next landing after pumping some more kinetic energy into the brakes will probably leave them somewhat hotter than the previous landing.

Sooner or later, if this pattern continues, the fuse plugs will probably reach their melting point unless some actions are taken to avoid that possibility.

What can be done to provide some amount of brake cooling in short-haul operations? Some operators use electric fans blowing cool air onto the wheels and brakes during the time the airplane is parked at the gate. Some airplanes are fitted with electric brake cooling fans, built into the wheel hubs. Some operators may follow a policy of extending the landing gear a few minutes early on each approach, because gear-down brake cooling in flight is roughly ten times more effective than cooling when parked at the gate. Some operators may instruct their crews to leave the gear extended for a few minutes after each takeoff. Each of these will be helpful, and short-haul operators usually employ one or more of them to avoid fuse plug melting.

But the simplest rule is this: when in doubt, wait XX minutes before taxiing back out for the next takeoff. It's that simple.

Additional Discussion

Discussion 1: the airplane braking coefficient

This topic was previously discussed in the chapter entitled "Calculating Takeoff Distances" but we'll repeat it here in a slightly different form for convenience.

The retarding force that can be generated by an airplane's brakes during a stop depends on a number of factors. We speak of the brakes as being either "antiskid-limited" or "torque-limited".

Antiskid-limited means that the brakes are capable of generating more retarding force than the airplane can actually utilize. If there were no protective devices, application of full braking would cause the brakes to "lock up" – that is, to stop all rotation of the wheels. That would leave the airplane skidding along the runway surface on non-rotating wheels, which could result in destruction of the tires and possible loss of directional control.

To prevent the occurrence of such a condition, the airplanes are equipped with antiskid systems, the function of which is to regulate the hydraulic pressure applied to the brakes in such a way as to prevent wheel lockup. In potential wheel lockup conditions the antiskid system becomes the limiting factor on the amount of retarding force that can be developed by the brakes.

An airplane will be antiskid-limited at the lighter landing weights.

Torque-limited means that the brakes can't generate enough torque to cause wheel lockup. This will be the case at the higher landing weights. In this case, the brake force is a function of the friction that is generated between the brake rotors and stators. The antiskid system is not a consideration. An airplane will be torque-limited at the higher landing weights.

At very high values of initial braking energy, the brake force may be affected by *brake fade*. Fade refers to the tendency of some brakes, when operated at extremely high initial kinetic energies, to lose some of their braking torque capability due to actual melting of the surfaces of the rotors and stators at very high friction-generated brake temperatures. This melting causes a film of melted brake material to act as a lubricant between the rotors and stators, reducing the brake force when very hot.

Brake fade is a characteristic of the older brakes on commercial jet transport airplanes, which have rotors and stators made of steel. Newer brakes, using carbon as the braking surfaces, do not exhibit the fade characteristic because even at very high temperatures the carbon material does not melt.

In figure 35-3 to the right we show the amount of brake force that can be generated as a function of two parameters: the average weight on the wheels during the braking segment, and the airplane braking energy at the beginning of the braking segment.

To find the value of μ_B to use in the equation for the stopping deceleration, there are two steps:

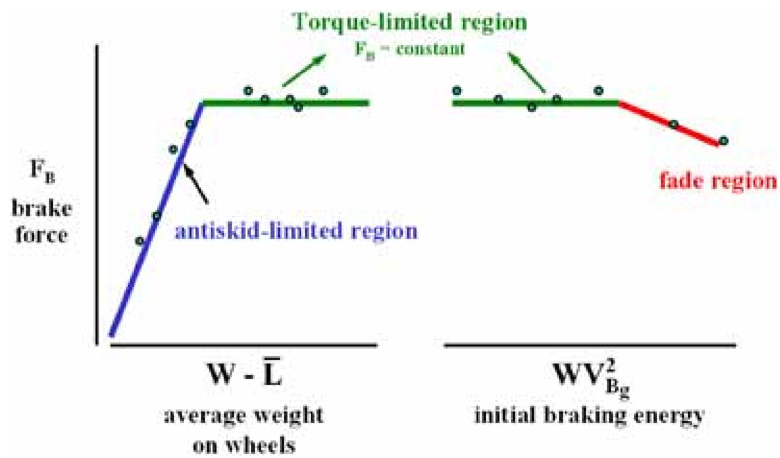


Figure 35-3

First, the user enters the chart at the average weight on the wheels during the segment, $(W - \bar{L})$, and reads the associated value of F_B. This average weight on the wheels is the airplane landing weight minus the average amount of lift generated by the airplane in the ground attitude between the brakes-on speed and the full stop.

Second, the user will calculate the braking energy $WV^2_{B_g}$ at the brakes-on speed and will read from the chart the corresponding value of F_B.

Whichever is the smaller of these two values of F_B will be used to determine the value of μ_B following the equation:

$$\mu_B = \frac{F_B}{(W - \bar{L})}$$

That value of μ_B will be used in the braking deceleration equation. It is a constant throughout the braking segment.

Chapter 36: Landing On Non-Dry Runways

TBS

Chapter 37: Airplane Performance and the FMCS

TBS

37-2 *TBS*

Chapter 38: Miscellaneous Topics

Introduction

As we have written the preceding chapters, we've realized a number of times that certain additional subjects needed to be included – but that these topics somehow didn't belong in the chapter we were then working on. Some subjects could reasonably appear in several different chapters – speed stability, for example, is a consideration in every flight segment.

We decided instead to present those topics in a chapter of their own. These topics are not necessarily related to each other. Each is a subject that stands alone.

We will doubtless be adding more subjects to this chapter as time goes by and more miscellaneous topics come to mind.

Speed Stability

to be supplied

Turbulent Air Penetration

to be supplied

Flight With Unreliable Airspeed

to be supplied

Appendix A: Summary of Useful Information

Table of Contents

Units Conversions

TO USE THESE TABLES: "to obtain X, <u>multiply</u> Y by constant A"
IS THE SAME AS: "to obtain Y, <u>divide</u> X by constant A"

NOTE: All constants are rounded to five significant figures

acceleration

to obtain	multiply	by
feet per second per second	gees	32.174
feet per second per second	knots per second	1.6878
feet per second per second	meters per second per second	3.2808
knots per second	gees	19.063
knots per second	meters per second per second	1.9438
meters per second per second	gees	9.8067

angles

One full circle = 360 degrees of circular arc = 2π radians
1 degree = 60 minutes of arc, 1 minute of arc = 60 seconds of arc

to obtain	multiply	by
degrees	radians	57.296

angular rate

to obtain	multiply	by
degrees per minute	degrees per second	60.000
degrees per minute	radians per second	3437.7
degrees per second	radians per second	57.296

area

to obtain	multiply	by
square feet	square meters	10.764

TO USE THESE TABLES: "to obtain X, multiply Y by constant A"
IS THE SAME AS: "to obtain Y, divide X by constant A"

density (mass)

to obtain	multiply	by
kilograms mass per cubic meter	slugs per cubic foot	515.38

density (weight)

to obtain	multiply	by
kilograms per cubic meter	kilograms per liter	1000.0
kilograms per cubic meter	pounds per cubic foot	16.018
kilograms per cubic meter	pounds per gallon	119.83
pounds per cubic foot	specific gravity (kilograms per liter)	62.428
pounds per cubic foot	pounds per gallon	7.4805
pounds per gallon	specific gravity (kilograms per liter)	8.3454

distance or length

1 meter = 100 centimeters = 1000 millimeters
1 foot = 12 inches, 1 yard = 3 feet

By international agreement, the *nautical mile* is defined as 1852 meters exactly. The distance on the earth's surface subtended by an angle of one minute of latitude at the equator is equal to 1855.34 meters; at the poles it is equal to 1849.12 meters. Thus, for practical purposes, it is acceptable to say that one nautical mile is equal to one minute of latitude (not longitude) on a navigational chart at constant longitude.

to obtain	multiply	by
centimeters	feet	30.480
centimeters	inches	2.5400
feet	kilometers	3280.8
feet	meters	3.2808
feet	nautical miles	6076.1
feet	smoots	5.5833
feet	statute miles	5280.0
kilometers	statute miles	1.6093
kilometers	nautical miles	1.8520
statute miles	nautical miles	1.1508
meters	nautical miles	1852.0
meters	statute miles	1609.3

TO USE THESE TABLES: "to obtain X, multiply Y by constant A"
IS THE SAME AS: "to obtain Y, divide X by constant A"

energy or work

to obtain	multiply	by
calories	BTUs	252.00
foot-pounds	BTUs	778.17
foot-pounds	calories	3.0880
joules (newton-meter)	BTUs	1055.1
joules (newton-meter)	calories	4.1868
joules (newton-meter)	foot-pounds	1.3558

force or weight

1 kilogram = 1000 grams = 1,000,000 milligrams
1 pound = 16 ounces

to obtain	multiply	by
newtons	kilograms	9.8067
newtons	pounds	4.4482
pounds	kilograms	2.2046

length

See "distance or length", above.

mass

to obtain	multiply	by
kilograms mass	slugs	14.594

power (rate of work)

1 watt is defined as 1 newton-meter per second

to obtain	multiply	by
foot-pounds per second	horsepower	550.00
foot-pounds per second	newton-meters per second	1.3558
watts	foot-pounds per second	1.3558
watts	horsepower	745.70

TO USE THESE TABLES: "to obtain X, multiply Y by constant A"
IS THE SAME AS: "to obtain Y, divide X by constant A"

pressure

1 pascal is defined as 1 newton per square meter
1 hectopascal = 100 pascals
1 bar = 100,000 pascals = 1000 hectopascals
1 bar = 1000 millibars
1 millibar = 1 hectopascal

to obtain	multiply	by
hectopascals	atmospheres	1013.2
hectopascals	millimeters of mercury	1.3332
hectopascals	pounds per square inch	68.948
inches of mercury	atmospheres	29.922
millimeters of mercury	atmospheres	760.00
millimeters of mercury	pounds per square inch	51.715
pounds per square inch	atmospheres	14.696

specific gravity

For our purposes, the "specific gravity" of a substance is the same as its weight in kilograms per liter of volume, which is a unit of weight density. For conversions, see "Density (Weight)" above.

speed or velocity

See "velocity or speed", below.

temperature

$$\text{degrees C} = (\text{degrees F} - 32) \times \frac{5}{9}$$

$$\text{degrees F} = 32 + \left(\text{degrees C} \times \frac{9}{5}\right)$$

$$\text{degrees K} = \text{degrees C} + 273.15$$

$$\text{degrees R} = \text{degrees F} + 459.67$$

to obtain	multiply	by
degrees Rankine	degrees Kelvin	1.8000

TO USE THESE TABLES: "to obtain X, multiply Y by constant A"
IS THE SAME AS: "to obtain Y, divide X by constant A"

time

to obtain	multiply	by
minutes	hours	60.000
seconds	minutes	60.000
seconds	microfortnights	1.2096

velocity or speed

1 *knot* is defined as 1 nautical mile per hour

to obtain	multiply	by
feet per second	knots	1.6878
feet per second	statute miles per hour	1.4667
knots	smoots per microfortnight	2.7348
statute miles per hour	knots	1.1508

viscosity

1 pascal-second = 1 newton-second per square meter = 1 kilogram mass per meter-second
1 slug mass per foot-second = 1 pound force-second per square foot
1 kilogram mass per meter-second = 1 newton-second per square meter

to obtain	multiply	by
pascal-second	pound-second per square foot	47.880
pascal-second	kg force-second per sq. meter	9.8067

volume

1 liter is defined as 1000 cubic centimeters

to obtain	multiply	by
cubic feet	cubic meters	35.315
cubic inches	gallons	231.00
cubic inches	liters	61.024
gallons	cubic feet	7.4805
gallons	imperial gallons	1.2010
liters	cubic feet	28.317
liters	gallons	3.7854
liters	imperial gallons	4.546 1

weight

See “force or weight”, above.

work

See “energy or work”, above.

Equations For Atmospheric Parameters

Notes:

1. In the following, the term ISA refers to the International Standard Atmosphere (“standard day”) conditions.
2. The tropopause is defined as occurring at 36089.24 feet, 11000 meters exactly.
3. The term ΔISA refers to the “ISA deviation”, the temperature deviation from the standard day value which may be expressed in °C or °F.

outside air temperature (OAT)

<u>at or below the tropopause:</u>

$$\text{OAT °C} = 15 - (0.0019812 \times h_p) + \Delta\ \text{ISA°C}$$

$$\text{OAT °K} = 288.15 - (0.0019812 \times h_p) + \Delta\ \text{ISA°C}$$

$$\text{OAT °F} = 59 - (0.00356616 \times h_p) + \Delta\ \text{ISA°F}$$

$$\text{OAT °R} = 518.67 - (0.00356616 \times h_p) + \Delta\ \text{ISA°F}$$

where: h_p is the pressure altitude in feet
ΔISA is the temperature deviation from standard day value in degrees

<u>above the tropopause:</u>

$$\text{OAT} = -56.5°\ \text{C} + \Delta\ \text{ISA °C} = 216.65\ \text{°K} + \Delta\ \text{ISA°C}$$

$$\text{OAT} = -69.7\ \text{°F} + \Delta\ \text{ISA°F} = 389.97\ \text{°K} + \Delta\ \text{ISA °F}$$

temperature ratio θ

$$\theta = \frac{T}{T_0}$$

where T is the air temperature in absolute units, °K or °R
T_0 is the sea level standard day temperature in the same absolute units as T
T_0 = 15 °C = 288.15 °K = 59 °F = 518.67 °R

$$\theta = \frac{\text{OAT °K}}{288.15\ \text{°K}} = \frac{\text{OAT °R}}{518.67\ \text{°R}} = \frac{\text{OAT °C} + 273.15}{288.15\ \text{°K}} = \frac{\text{OAT °F} + 459.67}{518.67\ \text{°R}}$$

at or below the tropopause:

$$\theta = \frac{288.15 - (0.0019812 \times h_p) + \Delta\text{ISA°C}}{288.15\ \text{°K}}$$

where h_p is the pressure altitude in feet

or

$$\theta = \frac{518.67 - (0.00356616 \times h_p) + \Delta\text{ISA°F}}{518.67\ \text{°R}}$$

above the tropopause:

$$\theta = \frac{216.65 + \Delta\text{ISA°C}}{288.15\ \text{°K}} = \frac{389.97 + \Delta\text{ISA°F}}{518.67}$$

pressure ratio δ

$$\delta = \frac{p}{p_0}$$

where p is the static air pressure in the same units as p_0
p_0 is the sea level standard day static air pressure, 29.92 lb/in^2 or 1013.2 hPa

at or below the tropopause:

$$\delta = \left(\frac{288.15 - 0.0019812 \times h_p}{288.15}\right)^{5.25588} = (\theta_{ISA})^{5.25588}$$

where h_p is the pressure altitude in feet

above the tropopause:

$$\delta = 0.22336 \times e^{\left(\frac{36089.24 - h_p}{20805.7}\right)}$$

where e is the base of the natural logarithm, e = 2.718281828

density ratio σ

$$\sigma = \frac{\rho}{\rho_0}$$

where ρ is the density of the air in the same units as ρ_0
ρ_0 is the sea level standard day air density, 0.002377 slugs per cubic foot or equivalent

σ is normally found from the ambient air pressure and temperature following the equation

$$\sigma = \frac{\delta}{\theta}$$

For a table of International Standard Atmosphere parameters, see the "Graphs and Tables" section at the end of this document.

Pressure Altitude

when given a value of air pressure or δ

$$\delta = \frac{p}{p_0}$$

where p is the static air pressure in the same units as p_0
p_0 is the sea level standard day static air pressure, 14.696 psi or equivalent

at or below the tropopause (δ equal to or greater than 0.22336):

$$h_p = 145442.15 \times (1 - \delta^{0.190263})$$

above the tropopause (δ less than 0.22336):

$$h_p = 36089.24 - 20805.7 \times \ln\left(\frac{\delta}{0.22336}\right)$$

when given QNH and airport elevation

$$h_p = \text{airport elevation} + 145442.15 \times \left[1 - \left(\frac{QNH}{p_0}\right)^{0.190263}\right]$$

where QNH is the reported altimeter setting for the airport in the same units as p_0
p_0 is the sea level standard day static pressure, 29.92 in. Hg or 1013.2 hPa

---

Speed of Sound and Mach Number

speed of sound

$$a = \sqrt{\gamma RT}$$

where γ is the ratio of specific heats $\frac{C_P}{C_V}$ and for air is equal to 1.4

a is the speed of sound in feet per second or, in metric units, meters per second
R is the specific gas constant, 1716.5619 foot-pounds per slug-degree R
in metric units, R = 287.0529 Newton-meters per kilogram mass-degree K
T is the absolute temperature in degrees R or, in metric units, degrees K

<u>for the speed of sound in feet per second:</u>

$$a = 1116.45\sqrt{\theta}$$

where θ is the temperature ratio

<u>for the speed of sound in knots:</u>

$$a = 661.4786\sqrt{\theta}$$

speed of sound ratio

$$\frac{a}{a_0} = \sqrt{\theta}$$

Mach number

$$M = \frac{TAS}{a} = \frac{\text{TAS (knots)}}{661.4786\sqrt{\theta}} = \frac{\text{EAS (knots)}}{661.4786\sqrt{\delta}}$$

Total Temperature, Pressure and Density

total air temperature (T_{total} or TAT)

$$TAT = OAT \times (1 + 0.2M^2)$$

Note: OAT and TAT <u>must</u> both be in units of absolute temperature, °K or °R

For a table of total temperature at different Mach numbers for ISA conditions, see the "Graphs and Tables" section at the end of this document.

total temperature ratio

$$\theta_{total} = \theta \times (1 + 0.2M^2)$$

where M is the Mach number

total pressure

$$p_{total} = p_{static} \times (1 + 0.2M^2)^{3.5}$$

total pressure ratio

$$\delta_{total} = \delta \times (1 + 0.2M^2)^{3.5}$$

total density

$$\rho_{total} = \rho \times (1 + 0.2M^2)^{2.5}$$

total density ratio

$$\sigma_{total} = \sigma \times (1 + 0.2M^2)^{2.5}$$

Airspeed and Mach Number Conversions

Note: in the following airspeed conversion equations, speeds are in knots

calibrated airspeed to equivalent airspeed

$$V_e = 1479.1\sqrt{\delta\left[\left(\frac{1}{\delta}\left\{\left[1 + 0.2\left(\frac{V_C}{661.4786}\right)^2\right]^{3.5} - 1\right\} + 1\right)^{\frac{1}{3.5}} - 1\right]}$$

calibrated airspeed to Mach number

$$M = \sqrt{5\left[\left(\frac{1}{\delta}\left\{\left[1 + 0.2\left(\frac{V_C}{661.4786}\right)^2\right]^{3.5} - 1\right\} + 1\right)^{\frac{1}{3.5}} - 1\right]}$$

calibrated airspeed to true airspeed

$$V_{true} = 1479.1\sqrt{\theta\left[\left(\frac{1}{\delta}\left\{\left[1 + 0.2\left(\frac{V_C}{661.4786}\right)^2\right]^{3.5} - 1\right\} + 1\right)^{\frac{1}{3.5}} - 1\right]}$$

equivalent airspeed to calibrated airspeed

$$V_C = 1479.1\sqrt{\left[\left(\delta\left\{\left[1 + \frac{1}{\delta}\left(\frac{V_e}{1479.1}\right)^2\right]^{3.5} - 1\right\} + 1\right)^{\frac{1}{3.5}} - 1\right]}$$

equivalent airspeed to Mach number

$$M = \frac{V_e}{661.4786}\sqrt{\frac{1}{\delta}}$$

equivalent airspeed to true airspeed

$$V_{true} = \frac{V_e}{\sqrt{\sigma}} = V_e\sqrt{\frac{\theta}{\delta}}$$

Mach number to calibrated airspeed

$$V_C = 1479.1\sqrt{\left(\{\delta[(0.2M^2 + 1)^{3.5} - 1] + 1\}^{\frac{1}{3.5}} - 1\right)}$$

Mach number to equivalent airspeed

$$V_e = 661.4786 \times M\sqrt{\delta}$$

Mach number to true airspeed

$$V_{true} = 661.4786 \times M\sqrt{\theta}$$

true airspeed to calibrated airspeed

$$V_C = 1479.1\sqrt{\left[\left(\delta\left\{\left[1 + \frac{1}{\theta}\left(\frac{V_{true}}{1479.1}\right)^2\right]^{3.5} - 1\right\} + 1\right)^{\frac{1}{3.5}} - 1\right]}$$

true airspeed to equivalent airspeed

$$V_e = V_{true}\sqrt{\sigma} = V_{true}\sqrt{\frac{\delta}{\theta}}$$

true airspeed to Mach number

$$M = \frac{V_{true}}{661.4786\sqrt{\theta}}$$

compressibility correction ΔV_C

$$V_e = V_C - \Delta V_C$$

For a graph of the compressibility correction ΔV_C, see the “Graphs and Tables” section at the end of this document.

check case for airspeed conversion calculations

Given: FL 350
ISA+10 °C
M = 0.84

$$\theta = \frac{T}{T_0} = \frac{(288.15 - 0.0019812 \times 35000 + 10)}{288.15} = 0.7941$$

$$\delta = \theta_{ISA}^{5.25588} = \left(\frac{T_{ISA}}{T_0}\right)^{5.25588} = \left(\frac{288.15 - 0019812 \times 35000}{288.15}\right)^{5.25588} = 0.2353$$

V_C = 287.1 knots
V_e = 269.6 knots
V_{true} = 495.2 knots

Dynamic Pressure q

Note: the following equations yield the dynamic pressure q in pounds per square foot.

given the true airspeed in feet per second:

$$q = \frac{1}{2}\rho V_{true}^2 = \frac{1}{2}(\rho_0 \times \sigma)V_{true}^2 = \frac{\sigma V_{true}^2}{841.4} \qquad \left[841.4 = \frac{2}{0.002377}\right]$$

where ρ is the air density in slugs per cubic foot
where ρ_0 is the sea level standard day air density in slugs per cubic foot
σ is the density ratio

given the true airspeed in knots:

$$q = \frac{\sigma V_{true}^2}{295.369} \qquad \left[295.369 = \frac{2}{0.002377 \times 1.6878^2}\right]$$

given the equivalent airspeed in knots:

$$q = \frac{V_e^2}{295.369} \qquad [\sigma V_{true}^2 = V_e^2]$$

given the speed in Mach number:

$$q = 1481.4\,M^2\delta \qquad \left[1481.4 = \frac{661.4786^2}{295.369}\right]$$

Lift and Drag Force Coefficients

general expression for lift force

$$L = nW$$

where L is the lift force in the same units as W
n is the normal acceleration in g's; it is equal to 1.0 for level unaccelerated flight
W is the weight

lift and drag coefficients

Note: for the value of S to use in the following coefficients, refer to the "Table of Primary Geometry Parameters" in the "Graphs and Tables" section at the end of this document.

given the true airspeed in feet per second:

$$C_L = \frac{L}{qS} = \frac{L}{\frac{1}{2}\rho V_{true}^2 S} \qquad C_D = \frac{D}{qS} = \frac{D}{\frac{1}{2}\rho V_{true}^2 S}$$

or

$$C_L = \frac{841.4 \times L}{\sigma V_{true}^2 S} \qquad C_D = \frac{841.4 \times D}{\sigma V_{true}^2 S}$$

where L is the lift force in pounds
D is the drag force in pounds
S is the reference wing area in square feet
q is the dynamic pressure in pounds per square foot
ρ is the air density in slugs per cubic foot
σ is the air density ratio

given the true airspeed in knots:

$$C_L = \frac{295.369 \times L}{\sigma V_{true}^2 S} \qquad C_D = \frac{295.369 \times D}{\sigma V_{true}^2 S}$$

given the equivalent airspeed in knots:

$$C_L = \frac{295.369 \times L}{V_e^2 S} \qquad C_D = \frac{295.369 \times D}{V_e^2 S}$$

given the speed in Mach number:

$$C_L = \frac{L}{1481.4\, M^2\, \delta\, S} \qquad C_D = \frac{D}{1481.4\, M^2\, \delta\, S}$$

engine-inoperative corrections to low-speed drag coefficient

$$C_{D_{engine-inop}} = C_{D_{all-eng}} + \Delta C_{D_{windmill}} + \Delta C_{D_{control}}$$

Reynolds number correction to high-speed drag coefficient

High-speed drag coefficient:

$$C_D = C_{D_{nom}} + \Delta C_{D_{RE}}$$

where $C_{D_{nom}}$ is the drag coefficient from the drag polar

$\Delta C_{D_{RE}}$ is the drag coefficient increment due to the Reynolds number effect

Drag increment due to Reynolds number:

$$\Delta C_{D_{RE}} = B \times 10^{-4} \log_{10}\left[\frac{\frac{1}{M}\left(\frac{RE}{ft}\right)}{\frac{1}{M}\left(\frac{RE}{ft}\right)_{nom}}\right]$$

where $\left(\frac{1}{M}\frac{RE}{ft}\right) = 5.13384 \times 10^{6}\left(\frac{\theta + 0.38312}{\theta^2}\right) \times \delta$

For the nominal Reynolds number in the denominator:

θ is the temperature ratio for standard day

δ is the pressure ratio for the reference altitude. Refer to the Performance Engineer's Manual (PEM) for the reference altitude to be used.

For the values of B, see the "Graphs and Tables" section at the end of this document.

Yawing Moment Coefficient

two-engine airplanes

given true airspeed in feet per second:

$$C_N = \left|\frac{(F_{N_1} - F_{N_2}) \times \text{engine moment arm}}{q\,S\,b}\right|$$

where F_{N_1} is the thrust of the left engine in pounds

F_{N_2} is the thrust of the right engine in pounds

b is the wingspan in feet

q is the dynamic pressure in pounds per square foot

S is the reference wing area in square feet

$$C_N = \left|\frac{841.4 \times (F_{N_1} - F_{N_2}) \times \text{engine moment arm}}{\sigma V_{true}^2\,S\,b}\right|$$

given true airspeed in knots:

$$C_N = \left| \frac{295.369 \times (F_{N_1} - F_{N_2}) \times \text{engine moment arm}}{\sigma V_{true}^2 \, S \, b} \right|$$

given equivalent airspeed in knots:

$$C_N = \left| \frac{295.369 \times (F_{N_1} - F_{N_2}) \times \text{engine moment arm}}{V_e^2 \, S \, b} \right|$$

given speed in Mach number:

$$C_N = \left| \frac{(F_{N_1} - F_{N_2}) \times \text{engine moment arm}}{1481.4 \, M^2 \, \delta \, S \, b} \right|$$

four-engine airplanes

$$C_N = \frac{1}{Sqb} \times \left| [(F_{N_1} - F_{N_4}) \times \text{outbd moment arm} + (F_{N_2} - F_{N_3}) \times \text{inbd moment arm}] \right|$$

where F_{N_1} and F_{N_4} are the left and right <u>outboard</u> engine thrusts respectively in pounds
F_{N_2} and F_{N_3} are the left and right <u>inboard</u> engine thrusts respectively in pounds

---

Radius of the Earth

$$r_e = \sqrt{\frac{a^4 + b^4 \tan^2 \varphi}{a^2 + b^2 \tan^2 \varphi}}$$

where r_e is the radius of the earth in feet
a is the radius of the earth at the equator, 20,925,780 feet
b is the radius of the earth at the poles, 20,855,636 feet
φ is the latitude in degrees

Gravitational Acceleration

standard value of gee

g_0 = 32.17405 ft/sec^2 = 9.80665 m/sec^2 at latitude 45.5425 degrees, sea level.

NOTE: in some documents, the standard value of gee is shown as occurring at 45 degrees latitude.

standard gee corrected for latitude: Lambert's equation

The value of $g_{\varphi,\,SL}$ yielded by Lambert's equation includes the effects of the oblateness of the earth and the density distribution of the earth, and the earth's rotation

$$g_{\varphi,\,SL} = 32.17244 \times [1 - 2.6373 \times 10^{-3}\cos(2\varphi) + 5.9 \times 10^{-6}\cos^2(2\varphi)]$$

where $g_{\varphi,\,SL}$ is the acceleration of gravity at any latitude in ft/sec^2
φ is the latitude in degrees

NOTE: the above equation is consistent with Engineering Sciences Data Unit document 77022. In certain other documents, including some Boeing documents, the equation is given as:

$$g_{\varphi,\,SL} = 32.17405 \times [1 - 2.6373 \times 10^{-3}\cos(2\varphi) + 5.9 \times 10^{-6}\cos^2(2\varphi)]$$

standard gee corrected for latitude and altitude

$$g_{\varphi,\,z} = (g_{\varphi,\,SL} + \omega_e^2 r_e \cos^2\varphi) \times \left(\frac{r_e}{r_e + z}\right)^2 - \omega_e^2 (r_e + z)\cos^2\varphi$$

where ω_e is the earth's rotation rate, 7.29212 x 10^{-5} radians per second
r_e is the earth's radius in feet
z is the height above sea level in feet

correction to gee for an airplane in motion

$$g = g_{\varphi,\,z} + \Delta g_{centrifugal}$$

$$\Delta g_{centrifugal} = -\left[\frac{V_G^2}{(r_e + z)} + 2\omega_e V_G \cos\varphi \sin\chi\right]$$

where V_G is the true airspeed *in feet per second*
χ is the true track angle of the flight path in degrees

NOTE: the above equation for $\Delta g_{centrifugal}$ is sometimes shown as two separate corrections. The first term of the above equation is sometimes called the centrifugal correction and the second term may be referred to as the "Coriolis" correction. Whether applied as two separate corrections or one, the result is the same.

Climb Path Angle (Gradient) and Rate of Climb

all-engine climb path angle

$$\gamma = \sin^{-1}\left[\frac{\frac{T-D}{W}}{\left(1+\frac{V}{g}\frac{dV}{dh}\right)}\right] = \sin^{-1}\left[\frac{\frac{T}{W}-\frac{C_D}{C_L}}{\left(1+\frac{V}{g}\frac{dV}{dh}\right)}\right]$$

where γ is the climb path angle relative to horizontal in degrees
T is the total thrust
W is the weight, in the same units as the thrust
the term in the denominator is the "acceleration factor", see below

engine-inoperative climb path angle

$$\gamma = \sin^{-1}\left[\frac{\frac{T-D}{W}}{\left(1+\frac{V}{g}\frac{dV}{dh}\right)}\right] = \sin^{-1}\left[\frac{\frac{(T-D_{WM})}{W}-\frac{(C_D+\Delta C_{D_\Psi})}{C_L}}{\left(1+\frac{V}{g}\frac{dV}{dh}\right)}\right]$$

where D_{WM} is the windmilling drag of the inoperative engine
C_{D_Ψ} is the control drag coefficient

climb gradient

Climb gradient is customarily expressed in percent:

$$\text{climb gradient \%} = 100 \times \tan\gamma$$

acceleration factor

Acceleration factor is defined as $\left(1 + \frac{V}{g}\frac{dV}{dh}\right)$

for constant equivalent airspeed below the tropopause:

$$\frac{V}{g}\frac{dV}{dh} = 0.7M^2\left[1 - 0.190263\left(\frac{T_{ISA}}{T}\right)\right]$$

where M is the Mach number
T_{ISA} is the standard day air temperature at the altitude
T is the ambient air temperature at the altitude

for constant equivalent airspeed above the tropopause:

$$\frac{V}{g}\frac{dV}{dh} = 0.7M^2$$

for constant Mach number below the tropopause:

$$\frac{V}{g}\frac{dV}{dh} = -0.13318\,M^2\left(\frac{T_{ISA}}{T}\right)$$

for constant Mach number above the tropopause:

$$\frac{V}{g}\frac{dV}{dh} = 0$$

for constant calibrated airspeed below the tropopause:

$$\frac{V}{g}\frac{dV}{dh} = 0.7M^2\left[\phi - 0.190263\left(\frac{T_{ISA}}{T}\right)\right]$$

for constant calibrated airspeed above the tropopause:

$$\frac{V}{g}\frac{dV}{dh} = 0.7M^2\phi$$

in the two equations immediately above,

$$\phi = \frac{[(1 + 0.2M^2)^{3.5} - 1]}{0.7M^2(1 + 0.2M^2)^{2.5}}$$

For a graph of acceleration factor for constant calibrated airspeed, see the “Graphs and Tables” section at the end of this document.

rate of climb

In general:

$$\text{rate of climb R/C} = V_{true}\sin\gamma$$

The usual units are speed in knots and R/C in feet per minute, for which:

$$\text{rate of climb R/C} = 101.268 \times V_{true}\sin\gamma \qquad [101.268 = 1.6878 \times 60]$$

Turning Flight

radius of turn

$$R = \frac{V_{true}^2}{g\tan\phi}$$

Where R is the turn radius in feet
V_{true} is the true airspeed in feet per second
g is the local acceleration of gravity
ϕ is the bank angle

<u>for true airspeed in knots and radius in feet:</u>

$$R = 0.08854\left(\frac{V_{true}^2}{\tan\phi}\right) \qquad \left[0.08854 = \frac{1.6878^2}{32.174}\right]$$

rate of turn

$$\frac{d\theta}{dt} = \frac{g\tan\phi}{V_{true}}$$

where $\frac{d\theta}{dt}$ is the rate of turn in radians per second
g is the local acceleration of gravity
ϕ is the bank angle

<u>for true airspeed in knots and rate of turn in degrees per second:</u>

$$\frac{d\theta}{dt} = \frac{1092.1\tan\phi}{V_{true}} \qquad \left[1092.1 = \frac{57.3 \times 32.174}{1.6878}\right]$$

bank angle

for true airspeed in knots and radius in feet:

$$\phi = \tan^{-1}\left(\frac{0.08854V_{true}^2}{R}\right)$$

for true airspeed in knots and rate of turn in degrees per second:

$$\phi = \tan^{-1}\left(\frac{V\dfrac{d\theta}{dt}}{1092.1}\right)$$

normal acceleration in turning flight

$$n = \frac{1}{\cos\phi}$$

where n is the normal acceleration in gees
ϕ is the bank angle

Takeoff Distances and Times

Note: It is standard practice to calculate takeoff distances and times by "step integration", meaning that the acceleration or deceleration segments are divided into small increments of speed change and the corresponding incremental distances and times are added together to obtain the total acceleration or deceleration segment distances and times. Following that method:

acceleration distance

In general terms, when step-integrating to find acceleration distance the equation for the incremental distance ΔS_G will be:

$$\Delta S_G = \frac{\Delta V \times (\overline{V}_{true} - V_{wind})}{\text{acceleration evaluated at } \overline{V}_{true}}$$

where ΔV is the speed increment used for the step integration
$\overline{V}_{true}$ is the average speed in the speed increment step
V_{wind} is the wind velocity along the takeoff direction

further,

$$\text{acceleration} = \frac{g}{W}[T - \mu_R(W - L) - \phi W - D]$$

where W is the weight
L is the lift generated by the wings
T is the thrust
$\mu_R(W - L)$ is the rolling friction retarding the acceleration
ϕ is the runway slope in radians (uphill is +)
D is the drag

More specifically:

$$\Delta S_G = \Delta V \times \frac{(\bar{V}_{true} - V_{wind})}{g\left[\frac{T}{W} - \mu_R - \phi - (C_D - \mu_R C_L)\frac{qS}{W}\right]_{\text{at } \bar{V}_{true}}}$$

where ΔS_G is the incremental acceleration distance in feet
ΔV is the incremental speed change in feet per second
$\bar{V}_{true}$ is the average true airspeed through the incremental step in feet per second
V_{wind} is the wind velocity in feet per second
g is the local acceleration of gravity in feet per second per second
T is the average total thrust through the incremental step
μ_R is the rolling coefficient of friction
q is the dynamic pressure in pounds per square foot
S is the reference wing area in square feet

<u>for true airspeed and wind speed in knots, and acceleration distance in feet:</u>

$$\Delta S_G = \Delta V \times \frac{2.84867(\bar{V}_{true} - V_{wind})}{g\left[\frac{T}{W} - \mu_R - \phi - (C_D - \mu_R C_L)\frac{qS}{W}\right]_{\text{at } \bar{V}_{true}}} \qquad [2.84867 = 1.6878^2]$$

acceleration time

<u>for true airspeed and wind speed in knots, and acceleration time in seconds:</u>

$$\Delta t_G = \frac{1.6878 \times \Delta V}{g\left[\frac{T}{W} - \mu_R - \phi - (C_D - \mu_R C_L)\frac{qS}{W}\right]_{\text{at } \bar{V}_{true}}}$$

flare distance

$$S_{flare} = 1.6878\left(\frac{V_R + V_2}{2} - V_W\right) \times \Delta t$$

where V_R is the rotation speed in knots
V_2 is the speed at 35 feet in knots
V_W is the wind velocity in knots (headwind is +)
Δt is the flight test time from rotation to 35 feet

deceleration distance

In general terms, when step-integrating to find deceleration distance the equation for the incremental distance ΔS_G will be:

$$\Delta S_G = \frac{\Delta V \times (\overline{V}_{true} - V_{wind})}{\text{deceleration evaluated at } \overline{V}_{true}}$$

where ΔV is the speed increment used for the step integration
$\overline{V}_{true}$ is the average speed in the speed increment step
V_{wind} is the wind velocity along the takeoff direction

and

$$\text{deceleration} = \frac{g}{W}[T - \mu_B(W - L) - \phi W - D]$$

where T is the thrust
$\mu_B(W - L)$ is the airplane retarding force due to wheel brakes
ϕ is the runway slope in radians (uphill is +)
D is the drag

More specifically,

$$\Delta S_G = \Delta V \times \frac{2.84867(\overline{V}_{true} - V_{wind})}{\frac{g}{W}[T - \mu_B(W - L) - \phi - D]_{\text{at } \overline{V}_{true}}}$$

where μ_B is the airplane braking coefficient

deceleration time

$$\Delta t_G = \frac{1.6878 \times \Delta V}{\frac{g}{W}[T - \mu_B(W - L) - \phi - D]_{\text{at } \bar{V}_{\text{true}}}}$$

Landing Distances

Note: As is the case with takeoff distances and times, it is standard practice to use step integration methods for calculating the landing deceleration distance.

air distance

$$S_{air} = 1.6878\left(\frac{V_{touchdown} + V_{approach}}{2} - V_W\right) \times \Delta t_{air}$$

where Δt_{air} is the time from the threshold to touchdown

transition distance

$$S_{trans} = 1.6878\left(\frac{V_{touchdown} + V_{\text{brakes on}}}{2} - V_W\right) \times \Delta t_{trans}$$

where Δt_{trans} is the time from touchdown to brakes on

stopping distance

$$\Delta S_G = \Delta V \times \frac{2.84867(\bar{V}_{true} - V_{wind})}{\frac{g}{W}[T - \mu_B(W - L) - \phi - D]_{\text{at } \bar{V}_{\text{true}}}}$$

where $\bar{V}_{true}$ is the average speed through the incremental deceleration step

---

Tire Hydroplaning Speed

<u>for the tire hydroplaning speed in knots:</u>

$$\text{hydroplaning speed} = 8.62\sqrt{P}$$

where P is the tire pressure in pounds per square inch

Wind Velocity Versus Height Above the Ground

The wind velocities V_{W_1} and V_{W_2} at any two heights h_1 and h_2 above the ground are related by:

$$V_{W_1} = V_{W_2}\left(\frac{h_1}{h_2}\right)^{\frac{1}{7}}$$

Great Circle Distance

$$D = 60 \times \cos^{-1}[\sin(lat_1) \times \sin(lat_2) + \cos(lat_1) \times \cos(lat_2) \times \cos(long_2 - long_1)]$$

where lat_1 and $long_1$ are the latitude and longitude respectively of the *starting* point
lat_2 and $long_2$ are the latitude and longitude respectively of the *ending* point
D is the great circle distance in nautical miles

Note: it is necessary to comply with the following sign convention:
North latitudes are positive, south latitudes are negative
West longitudes are positive, east longitudes are negative.

check: San Francisco (N 37°37.0’, W 122°23.0’) to Tokyo (N 35°46.0’, E 140°23.0’)

Distance = 4439.3 nautical miles

Graphs and Tables

table of International Standard Atmosphere parameters

geopotential altitude	temperature		θ	a / a_0	atmospheric pressure			δ	σ
	OAT °F	OAT °C			in. Hg	lb / ft²	mb		
0	59.0	15.0	1.0000	1.0000	29.920	2116.3	1013.2	1.0000	1.0000
1000	55.4	13.0	0.9931	0.9966	28.854	2040.9	977.1	0.9644	0.9711
2000	51.9	11.0	0.9862	0.9931	27.820	1967.7	942.1	0.9298	0.9428
3000	48.3	9.1	0.9794	0.9896	26.816	1896.7	908.1	0.8962	0.9151
4000	44.7	7.1	0.9725	0.9862	25.841	1827.7	875.1	0.8637	0.8881
5000	41.2	5.1	0.9656	0.9827	24.895	1760.8	843.0	0.8320	0.8617
6000	37.6	3.1	0.9587	0.9792	23.977	1695.9	812.0	0.8014	0.8359
7000	34.0	1.1	0.9519	0.9756	23.087	1633.0	781.8	0.7716	0.8106
8000	30.5	-0.8	0.9450	0.9721	22.224	1571.9	752.6	0.7428	0.7860
9000	26.9	-2.8	0.9381	0.9686	21.387	1512.7	724.2	0.7148	0.7620
10000	23.3	-4.8	0.9312	0.9650	20.576	1455.4	696.8	0.6877	0.7385
11000	19.8	-6.8	0.9244	0.9614	19.790	1399.8	670.2	0.6614	0.7156
12000	16.2	-8.8	0.9175	0.9579	19.029	1345.9	644.4	0.6360	0.6932
13000	12.6	-10.8	0.9106	0.9543	18.291	1293.7	619.4	0.6113	0.6713
14000	9.1	-12.7	0.9037	0.9507	17.577	1243.2	595.2	0.5875	0.6500
15000	5.5	-14.7	0.8969	0.9470	16.885	1194.3	571.8	0.5643	0.6292
16000	1.9	-16.7	0.8900	0.9434	16.216	1147.0	549.1	0.5420	0.6090
17000	-1.6	-18.7	0.8831	0.9397	15.568	1101.1	527.2	0.5203	0.5892
18000	-5.2	-20.7	0.8762	0.9361	14.941	1056.8	506.0	0.4994	0.5699
19000	-8.8	-22.6	0.8694	0.9324	14.335	1014.0	485.5	0.4791	0.5511
20000	-12.3	-24.6	0.8625	0.9287	13.750	972.5	465.6	0.4595	0.5328
21000	-15.9	-26.6	0.8556	0.9250	13.183	932.5	446.4	0.4406	0.5150
22000	-19.5	-28.6	0.8487	0.9213	12.636	893.7	427.9	0.4223	0.4976
23000	-23.0	-30.6	0.8419	0.9175	12.107	856.3	410.0	0.4046	0.4807
24000	-26.6	-32.5	0.8350	0.9138	11.596	820.2	392.7	0.3876	0.4642
25000	-30.2	-34.5	0.8281	0.9100	11.103	785.3	376.0	0.3711	0.4481
26000	-33.7	-36.5	0.8212	0.9062	10.627	751.7	359.9	0.3552	0.4325
27000	-37.3	-38.5	0.8144	0.9024	10.168	719.2	344.3	0.3398	0.4173
28000	-40.9	-40.5	0.8075	0.8986	9.725	687.8	329.3	0.3250	0.4025
29000	-44.4	-42.5	0.8006	0.8948	9.297	657.6	314.8	0.3107	0.3881
30000	-48.0	-44.4	0.7937	0.8909	8.885	628.4	300.9	0.2970	0.3741
31000	-51.6	-46.4	0.7869	0.8870	8.488	600.4	287.4	0.2837	0.3605
32000	-55.1	-48.4	0.7800	0.8832	8.105	573.3	274.5	0.2709	0.3473
33000	-58.7	-50.4	0.7731	0.8793	7.737	547.2	262.0	0.2586	0.3345
34000	-62.2	-52.4	0.7662	0.8753	7.382	522.1	250.0	0.2467	0.3220
35000	-65.8	-54.3	0.7594	0.8714	7.040	498.0	238.4	0.2353	0.3099
36000	-69.4	-56.3	0.7525	0.8675	6.712	474.7	227.3	0.2243	0.2981
36089	-69.7	-56.5	0.7519	0.8671	6.683	472.7	226.3	0.2234	0.2971
37000	-69.7	-56.5	0.7519	0.8671	6.397	452.4	216.6	0.2138	0.2844
38000	-69.7	-56.5	0.7519	0.8671	6.097	431.2	206.5	0.2038	0.2710
39000	-69.7	-56.5	0.7519	0.8671	5.810	411.0	196.8	0.1942	0.2583
40000	-69.7	-56.5	0.7519	0.8671	5.538	391.7	187.5	0.1851	0.2462
41000	-69.7	-56.5	0.7519	0.8671	5.278	373.3	178.7	0.1764	0.2346
42000	-69.7	-56.5	0.7519	0.8671	5.030	355.8	170.3	0.1681	0.2236
43000	-69.7	-56.5	0.7519	0.8671	4.794	339.1	162.3	0.1602	0.2131
44000	-69.7	-56.5	0.7519	0.8671	4.569	323.2	154.7	0.1527	0.2031
45000	-69.7	-56.5	0.7519	0.8671	4.355	308.0	147.5	0.1455	0.1936

$T_0 = 15°C = 59°F = 288.15°K = 518.67°$

$\rho_0 = 0.002377$ slugs per subic foot

$a_0 = 1116.45$ feet per second $= 661.4786$ knot

$p_0 = 29.92$ inches Hg $= 1013.2$ millibars $= 760$ mm Hg

table of standard day total temperature

altitude (feet)	OAT (deg C)	Mach number 0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.86	0.88	0.90
0	15	36	37	39	40	42	43	45	47	48	50	52	54	56	58	60	62
1000	13	34	35	36	38	39	41	43	44	46	48	50	52	53	55	57	59
2000	11	31	33	34	36	37	39	41	42	44	46	47	49	51	53	55	57
3000	9	29	31	32	34	35	37	38	40	42	43	45	47	49	51	53	55
4000	7	27	29	30	31	33	35	36	38	39	41	43	45	47	49	50	52
5000	5	25	26	28	29	31	32	34	36	37	39	41	43	44	46	48	50
6000	3	23	24	26	27	29	30	32	33	35	37	38	40	42	44	46	48
7000	1	21	22	24	25	26	28	30	31	33	35	36	38	40	42	44	46
8000	-1	19	20	21	23	24	26	27	29	31	32	34	36	38	39	41	43
9000	-3	17	18	19	21	22	24	25	27	28	30	32	34	35	37	39	41
10000	-5	15	16	17	19	20	21	23	25	26	28	30	31	33	35	37	39
11000	-7	12	14	15	16	18	19	21	22	24	26	27	29	31	33	34	36
12000	-9	10	12	13	14	16	17	19	20	22	23	25	27	29	30	32	34
13000	-11	8	9	11	12	14	15	16	18	20	21	23	25	26	28	30	32
14000	-13	6	7	9	10	11	13	14	16	17	19	21	22	24	26	28	29
15000	-15	4	5	6	8	9	11	12	14	15	17	18	20	22	24	25	27
16000	-17	2	3	4	6	7	8	10	11	13	15	16	18	19	21	23	25
17000	-19	0	1	2	3	5	6	8	9	11	12	14	16	17	19	21	23
18000	-21	-2	-1	0	1	3	4	6	7	9	10	12	13	15	17	18	20
19000	-23	-5	-3	-2	-1	1	2	3	5	6	8	9	11	13	14	16	18
20000	-25	-7	-6	-4	-3	-2	0	1	3	4	6	7	9	10	12	14	16
21000	-27	-9	-8	-6	-5	-4	-2	-1	0	2	3	5	7	8	10	12	13
22000	-29	-11	-10	-9	-7	-6	-5	-3	-2	0	1	3	4	6	8	9	11
23000	-31	-13	-12	-11	-9	-8	-7	-5	-4	-3	-1	0	2	4	5	7	9
24000	-33	-15	-14	-13	-12	-10	-9	-8	-6	-5	-3	-2	0	1	3	5	6
25000	-35	-17	-16	-15	-14	-12	-11	-10	-8	-7	-5	-4	-2	-1	1	2	4
26000	-37	-19	-18	-17	-16	-15	-13	-12	-11	-9	-8	-6	-5	-3	-2	0	2
27000	-38	-22	-20	-19	-18	-17	-15	-14	-13	-11	-10	-8	-7	-5	-4	-2	0
28000	-40	-24	-23	-21	-20	-19	-18	-16	-15	-14	-12	-11	-9	-8	-6	-4	-3
29000	-42	-26	-25	-24	-22	-21	-20	-19	-17	-16	-14	-13	-11	-10	-8	-7	-5
30000	-44	-28	-27	-26	-25	-23	-22	-21	-19	-18	-17	-15	-14	-12	-11	-9	-7
31000	-46	-30	-29	-28	-27	-25	-24	-23	-22	-20	-19	-17	-16	-14	-13	-11	-10
32000	-48	-32	-31	-30	-29	-28	-26	-25	-24	-22	-21	-20	-18	-17	-15	-14	-12
33000	-50	-34	-33	-32	-31	-30	-29	-27	-26	-25	-23	-22	-20	-19	-17	-16	-14
34000	-52	-36	-35	-34	-33	-32	-31	-29	-28	-27	-25	-24	-23	-21	-20	-18	-17
35000	-54	-39	-38	-36	-35	-34	-33	-32	-30	-29	-28	-26	-25	-23	-22	-20	-19
36000	-56	-41	-40	-39	-37	-36	-35	-34	-33	-31	-30	-29	-27	-26	-24	-23	-21
37000	-56	-41	-40	-39	-37	-36	-35	-34	-33	-31	-30	-29	-27	-26	-24	-23	-21
38000	-56	-41	-40	-39	-37	-36	-35	-34	-33	-31	-30	-29	-27	-26	-24	-23	-21
39000	-56	-41	-40	-39	-37	-36	-35	-34	-33	-31	-30	-29	-27	-26	-24	-23	-21
40000	-56	-41	-40	-39	-37	-36	-35	-34	-33	-31	-30	-29	-27	-26	-24	-23	-21
41000	-56	-41	-40	-39	-37	-36	-35	-34	-33	-31	-30	-29	-27	-26	-24	-23	-21
42000	-56	-41	-40	-39	-37	-36	-35	-34	-33	-31	-30	-29	-27	-26	-24	-23	-21
43000	-56	-41	-40	-39	-37	-36	-35	-34	-33	-31	-30	-29	-27	-26	-24	-23	-21
44000	-56	-41	-40	-39	-37	-36	-35	-34	-33	-31	-30	-29	-27	-26	-24	-23	-21
45000	-56	-41	-40	-39	-37	-36	-35	-34	-33	-31	-30	-29	-27	-26	-24	-23	-21

graph of compressibility correction ΔV_C

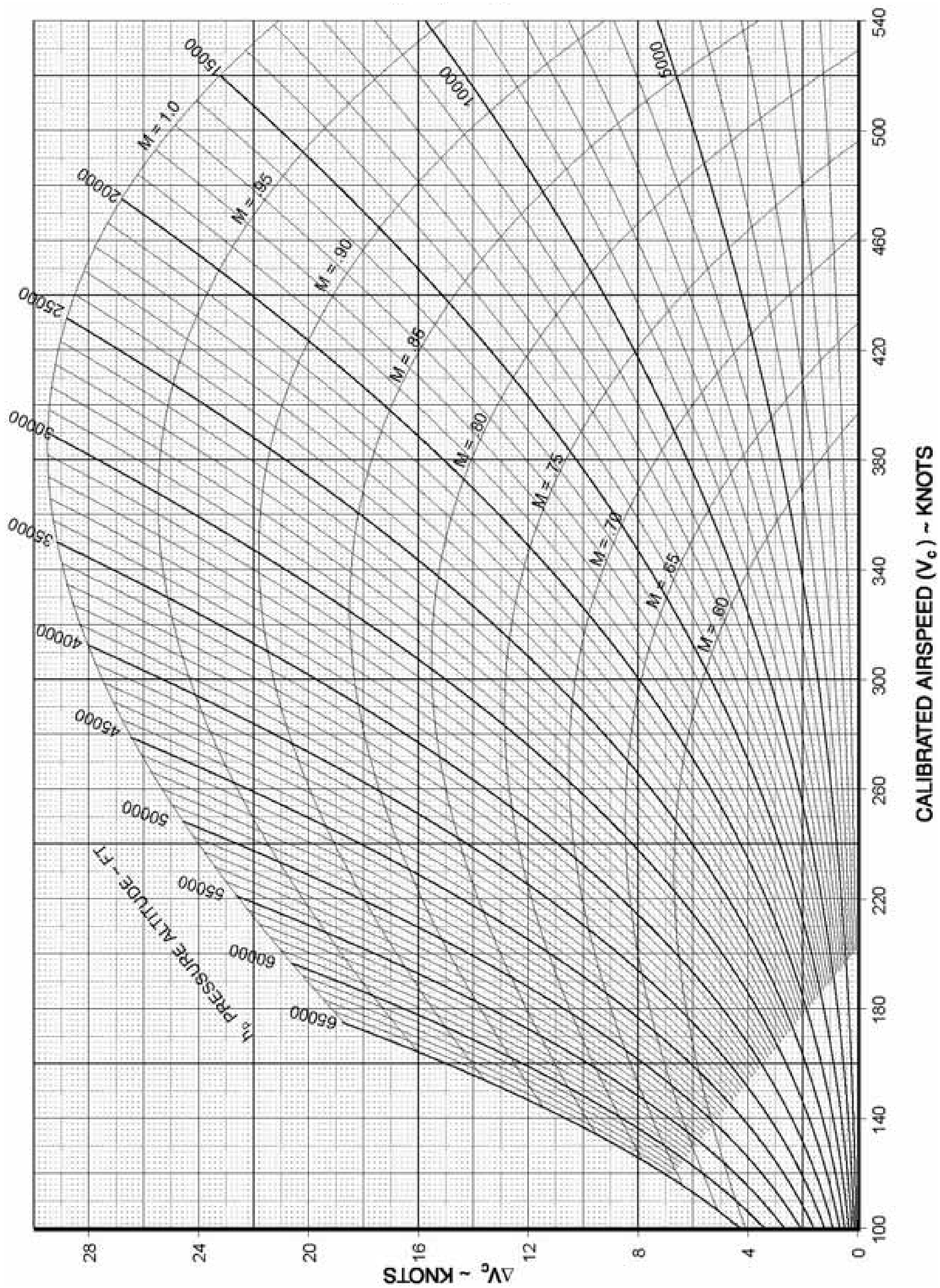


table of B values for Reynolds number correction to high-speed drag

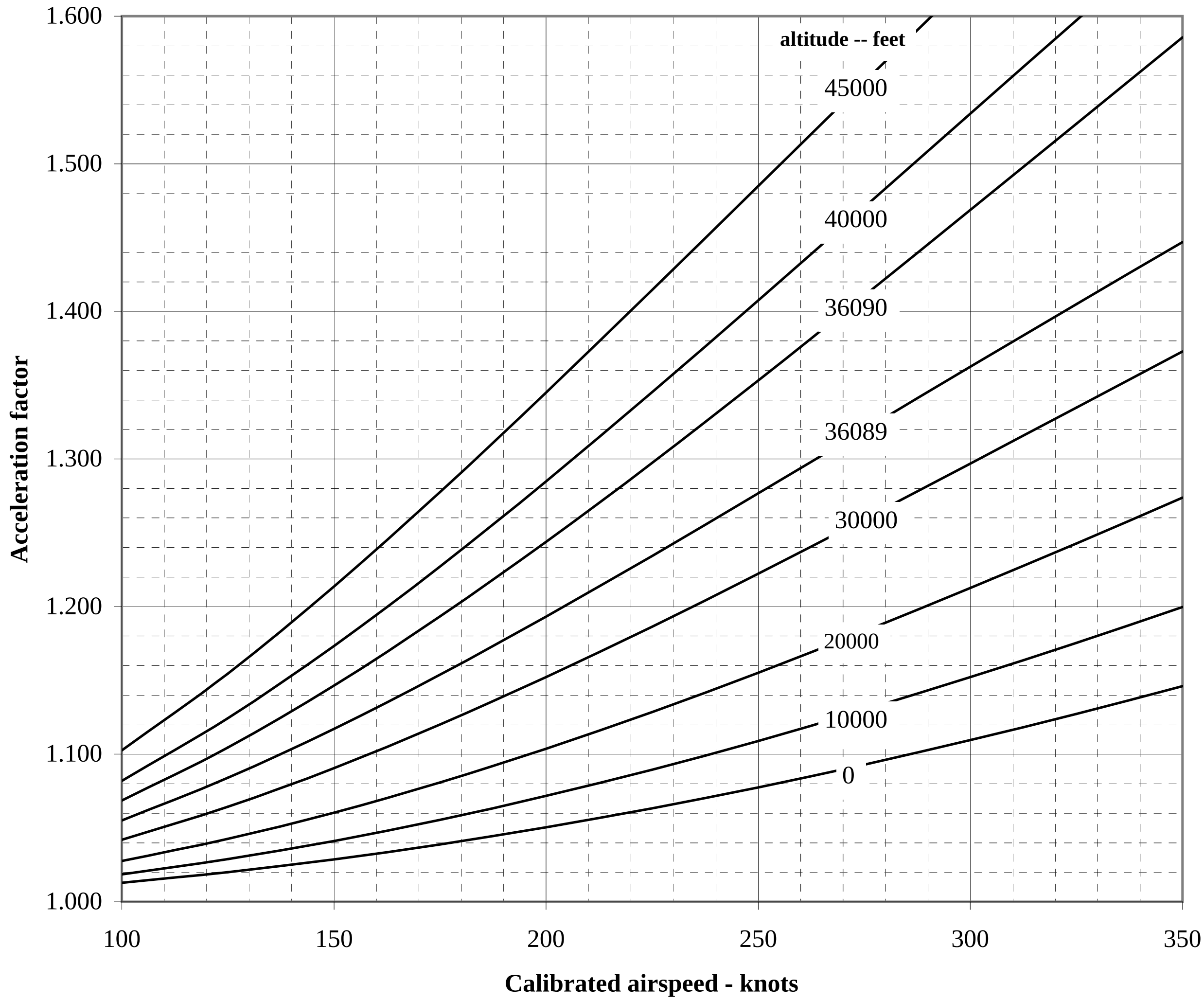
single-aisle models	B	twin-aisle models	B
737-300	-59.57	747-200	-39.826
737-400	-63.99	747-300	-39.826
737-500	-59.58	747-400	-43.60
		747-8	TBD
737-600	-59.98	767-200	-42.222
737-700	-59.623	767-300	-43.65
737-800	-63.99	767-400	-45.23
737-900	-64.80		
757-200	-49.434	777-200	-45.29
757-300	-56.24	777-300	-46.89
		787-3	TBD
		787-8	TBD
		787-9	TBD

table of primary geometry parameters for Boeing airplanes

Many of the following equations require airplane geometry parameters such as the reference wing area $S_{ref,}$ reference wingspan b_{ref} and the engine moment arm. Those are contained in the following table. Additionally, parameters MAC and LEMAC are used in weight and balance calculations and the wing angle of incidence is used in body attitude calculations.

model	S_{ref} (ft^2)	b_{ref} (ft)	MAC (in)	LEMAC (in)	eng mom arm (ft)	wing incid
707-100, -100B	2433	130.83	241.88	786.2	27.17 (inbd) 46.08 (outbd)	2°
707-200	2433	130.83	241.88	786.2	27.17(inbd) 46.08 (outbd)	2°
707-300/B/Badv, C	2892	142.42	272.29	762.97	33.0 (inbd) 52.0 (outbd)	2°
707-400	2892	142.42	272.29	762.97	33.0 (inbd) 52.0 (outbd)	2°
720/B	2433	130.83	241.88	786.2	27.17 (inbd) 46.08 (outbd)	2°
727-100, -200	1560	106	180.0	860.2	10.0 (pod)	2°
737-100 thru -500	980	93.0	134.46	625.6	16.14	1°
737-600 thru -900	1340	112.58	153.8	627.1	16.14	
747-100 thru -300	5500	195.68	327.78	1258.0	39.17 (inbd) 69.5 (outbd)	2°
747-400	5500	211.42	327.78	1258.0	39.17 (inbd) 69.5 (outbd)	2°
747-8	TBS	TBS	TBS	TBS	TBS (inbd) TBS (outbd)	TBS
747SP	5500	195.68	327.78	1258.0	39.17 (inbd) 69.5 (outbd)	2°
757 (all)	1951	124.5	199.7	991.9	21.67	
767 (all)	3050	155.0	237.5	913.2	25.69	
777 (all)	4605	196.9	278.5	1174.5	32.67	
787	TBS	TBS	TBS	TBS	TBS	

graph of acceleration factor at constant calibrated airspeed



index

Symbols

Numerics

A

D

G

H

M

N

T

U

Y

Revision Record

March 2009:
This is a completely new edition of the Jet Transport Performance Methods document. The last revision to this document was in 1989. In view of the many changes that had occurred in the intervening time period, and to take advantage of more up-to-date authoring software (Adobe FrameMaker) and presentation methods such as Adobe Portable Document Files (PDF) it was decided to issue a completely re-written version.