UNIDAD 7

INTEGRACIÓN INDEFINIDA

7-3 Resolver las siguientes integrales usando el método de integración por partes

a) $\int x.senx \ \ dx =$

Recordando $\int u\, dv = u \cdot v - \int v\, du$

Elegimos:

$$u = x \ \Rightarrow du = dx$$

$$dv = sen(x)dx \Rightarrow v = -\cos(x)$$

Reemplazando:

$$\int x\, sen\, x\, dx = -x\, cosx - \int -cosx\, dx$$

Esta última integral es inmediata:

$$\int x\, sen\, x\, dx = -x \cos x + senx + C$$

d) $\int \ln x \ \ dx =$

$$u = lnx \ \Rightarrow du = \frac{1}{x}dx$$

$$dv = dx \Rightarrow v = x$$

$$\int \ln x \ dx = \ln x \,.\, x - \int x \, \frac{1}{x} dx = x \ln x - \int dx = x \ln x - x + C = x.(\ln x - 1) + C$$

g) $\int arc\ \cos(3x)\ dx =$

$$u = arc\ cos(3x) \Rightarrow du = \frac{-1}{\sqrt{1-(3x)^2}}3dx$$

$$dv = dx \Rightarrow v = x$$

$$\int arc\ cos(3x)dx = x.arc\cos(3x) - \int x \cdot \frac{-1}{\sqrt{1-(3x)^2}}3dx = x.arc\cos(3x) + \int x \cdot \frac{1}{\sqrt{1-(3x)^2}}3dx \ \textcircled{1}$$

$\int x \cdot \frac{1}{\sqrt{1-(3x)^2}}3dx$, esta integral debe resolverse por sustitución:

$$t = 1 - 9x^2 \Rightarrow dt = -18x\,dx \Rightarrow \frac{dt}{-18x} = dx$$

$$\int x \cdot \frac{1}{\sqrt{1-(3x)^2}}3dx = \int x \cdot \frac{1}{\sqrt{t}} \cdot 3\frac{dt}{-18x} = -\frac{3}{18}\int \frac{1}{\sqrt{t}}\,dt = -\frac{1}{6} \cdot 2\sqrt{t} + C = -\frac{1}{3}\sqrt{1-9x^2} + C \ \textcircled{2}$$

Reemplazando: ② en ①

$$\int arc\ cos(3x)dx = x.arc\cos(3x) - \frac{1}{3}\sqrt{1-9x^2} + C$$

h) $\int \frac{x}{\cos^2 x}dx =$

$$u = x \Rightarrow du = dx$$

$$dv = \frac{1}{cos^2 x}dx \Rightarrow v = tg\ x$$

$$\int \frac{x}{cos^2 x}\,dx = x \cdot tg\ x - \int tg\ x\,dx = x \cdot tg\ x - \int \frac{sen\ x}{\cos x}\,dx$$

$$= x \cdot tg\ x + \int \frac{1}{t}\,dt = x \cdot tg\ x + \ln|t| + C = x \cdot tg\ x + ln|\cos x| + C$$

Se realizó la sustitución:

$$t = \cos x \Rightarrow dt = -sen\ x\,dx$$

j) $\int 5^x . x^3 dx =$

$$u = x^3 \Rightarrow du = 3x^2 dx$$

$$dv = 5^x dx \Rightarrow v = \frac{5^x}{ln5}$$

$$\int 5^x \cdot x^3 dx = x^3 \cdot \frac{5^x}{ln5} - \int \frac{5^x}{ln5} \cdot 3x^2 dx = x^3 \cdot \frac{5^x}{ln5} - \frac{3}{ln5} \int 5^x \cdot x^2 dx \quad ①$$

$$u = x^2 \Rightarrow du = 2x\, dx$$

$$dv = 5^x dx \Rightarrow v = \frac{5^x}{ln5}$$

$$\int 5^x \cdot x^2 dx = x^2 \cdot \frac{5^x}{ln5} - \int \frac{5^x}{ln5} \cdot 2x dx = x^2 \cdot \frac{5^x}{ln5} - \frac{2}{ln5} \int 5^x \cdot x dx \quad ②$$

Debemos volver a integrar por partes: $\int 5^x \cdot x dx$

$$u = x \Rightarrow du = dx$$

$$dv = 5^x dx \Rightarrow v = \frac{5^x}{ln5}$$

$$\int 5^x \cdot x dx = \frac{5^x}{ln5} \cdot x - \int \frac{5^x}{ln5} dx = \frac{5^x}{ln5} \cdot x - \frac{1}{\ln 5} \int 5^x dx = \frac{5^x}{ln5} \cdot x - \frac{1}{\ln 5} \cdot \frac{5^x}{ln5} + C \quad ③$$

De ①,② y ③

$$\int 5^x \cdot x^3 dx = x^3 \cdot \frac{5^x}{ln5} - \frac{3}{ln5} \cdot \left[x^2 \cdot \frac{5^x}{ln5} - \frac{2}{ln5} \cdot \left(\frac{5^x}{ln5} \cdot x - \frac{1}{\ln 5} \cdot \frac{5^x}{ln5} \right) \right] + C$$

$$\int 5^x \cdot x^3 dx = x^3 \cdot \frac{5^x}{ln5} - \frac{3}{ln5} \cdot \left[x^2 \cdot \frac{5^x}{ln5} - \frac{2 \cdot x \cdot 5^x}{ln^2 5} + \frac{2 \cdot 5^x}{ln^3 5} \right] + C$$

$$\int 5^x \cdot x^3 dx = x^3 \cdot \frac{5^x}{ln5} - \frac{3 \cdot x^2 \cdot 5^x}{ln^2 5} + \frac{6 \cdot x \cdot 5^x}{ln^3 5} - \frac{6 \cdot 5^x}{ln^4 5} + C$$

$$\int 5^x \cdot x^3 dx = \frac{5^x}{ln5} \cdot \left(x^3 - \frac{3x^2}{\ln 5} + \frac{6 \cdot x}{ln^2 5} - \frac{6}{ln^3 5} \right) + C$$

I) $\int e^x . senx \;\; dx =$

$$u = sen\, x \Rightarrow du = \cos x\, dx$$

$$dv = e^x dx \Rightarrow v = e^x$$

$$\int e^x\, sen\, x\, dx = e^x sen\, x - \int e^x \cos x\, dx \;\; ①$$

$\int e^x \cos x\, dx$

$$u = cos\, x \Rightarrow du = -\text{sen}\, x\, dx$$

$$dv = e^x dx \Rightarrow v = e^x$$

$$\int e^x \cos x\, dx \;\; = e^x cos\, x - \int e^x\, (-\text{sen}\, x)\, dx = e^x cos\, x + \int e^x \,\text{sen}\, x\, dx \;\; ②$$

Reemplazando ② en ①

$$\int e^x\, sen\, x\, dx = e^x sen\, x - \left[e^x cos\, x + \int e^x \,\text{sen}\, x\, dx \right]$$

$$\int e^x\, sen\, x\, dx = e^x sen\, x - e^x cos\, x - \int e^x \,\text{sen}\, x\, dx$$

Reuniendo las dos integrales a un lado del igual:

$$\int e^x\, sen\, x\, dx + \int e^x\, sen\, x\, dx = e^x sen\, x - e^x cos\, x$$

$$2 \int e^x\, sen\, x\, dx = e^x sen\, x - e^x cos\, x$$

$$\int e^x\, sen\, x\, dx = \frac{e^x sen\, x - e^x cos\, x}{2}$$

$$\int e^x\, sen\, x\, dx = \frac{e^x}{2} (sen\, x - cos\, x)$$