HOJA

FECHA

1)

Dado $\bar{v} = 3t\,\hat{i} + xz\,\hat{j} + ty^2\,\hat{k}$

a) $\nabla\cdot\bar{v} = \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = 0 + 0 + 0 = 0$

El campo de $\bar{v}$ corresponde a un flujo incompresible

b) $$\nabla\times\bar{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 3t & xz & ty^2 \end{vmatrix} = (2ty - x)\,\hat{i} + (0-0)\,\hat{j} + (z-0)\,\hat{k}$$

$$\nabla\times\bar{v} = (2ty - x)\,\hat{i} + z\,\hat{k}$$

El flujo no es irrotacional

2) Dado $\bar{v} = 3t\,\hat{i} + xz\,\hat{j} + ty^2\,\hat{k}$

$$\bar{a}_{(x,y,z,t)} = \overbrace{\frac{\partial \bar{v}}{\partial t}}^{\text{Local}} + \overbrace{(\bar{v}\cdot\nabla)\bar{v}}^{\text{convectiva}} = \frac{\partial \bar{v}}{\partial t} + u\cdot\frac{\partial \bar{v}}{\partial x} + v\cdot\frac{\partial \bar{v}}{\partial y} + w\cdot\frac{\partial \bar{v}}{\partial z}$$

$$\bar{a} = \frac{\partial u}{\partial t}\hat{i} + \frac{\partial v}{\partial t}\hat{j} + \frac{\partial w}{\partial t}\hat{k} + \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\left(u\,\hat{i} + v\,\hat{j} + w\,\hat{k}\right)$$

$$\bar{a}_x = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\cdot\overbrace{\frac{dx}{dt}}^{u} + \frac{\partial u}{\partial y}\cdot\overbrace{\frac{dy}{dt}}^{v} + \frac{\partial u}{\partial z}\cdot\overbrace{\frac{dz}{dt}}^{w} = (3+0+0+0)\,\hat{i}$$

$$\bar{a}_y = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial v}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\cdot\frac{dz}{dt} = (0 + z\cdot 3t + 0 + xty^2)\,\hat{j}$$

$$\bar{a}_z = \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt} = (y^2 + 0 + 2tyxz + 0)\,\hat{k}$$

$$\bar{a} = 3\,\hat{i} + (z\cdot 3t + xty^2)\,\hat{j} + (y^2 + 2tyxz)\,\hat{k}$$

NOTA

4) Dado $\bar{v} = (1+2{,}5x+y)\hat{i} + (-0{,}5-1{,}5x-2{,}5y)\hat{j}$

$$\bar{a}_{(x,y,t)} = \frac{\partial \bar{v}}{\partial t} + (\bar{v}\cdot\bar{\nabla})\bar{v}$$

$$\bar{a} = \left(\frac{\partial u}{\partial t}\hat{i} + \frac{\partial v}{\partial t}\hat{j}\right) + \left(u\cdot\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\left(u\hat{i} + v\hat{j} + w\hat{k}\right)$$

$$\bar{a}_x = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\overbrace{\frac{dx}{dt}}^{u} + \frac{\partial u}{\partial y}\cdot\overbrace{\frac{dy}{dt}}^{v} =$$

$$\bar{a}_y = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}\cdot\overbrace{\frac{dx}{dt}}^{u} + \frac{\partial v}{\partial y}\overbrace{\frac{dy}{dt}}^{v} =$$

$$\bar{a}_x = 0 + 2{,}5\,(1+2{,}5x+y) + (-0{,}5-1{,}5x-2{,}5y)\quad\hat{i}$$

$$\bar{a}_y = 0 + -1{,}5\cdot(1+2{,}5x+y) + -2{,}5\,(-0{,}5-1{,}5x-2{,}5y)\;\hat{j}$$

$$\bar{a}_x\Big|_{(2,3)} = 11{,}5\,\frac{m}{s^2}$$

$$\bar{a} = (11{,}5\,\hat{i} + 14\,\hat{j})\,\frac{m}{s^2}$$

$$\bar{a}_y\Big|_{(2,3)} = 14\,\frac{m}{s^2}$$

5) Dado $\bar{v} = (0{,}5 + 1{,}2x)\hat{i} + (-2 - 1{,}2y)\hat{j}$

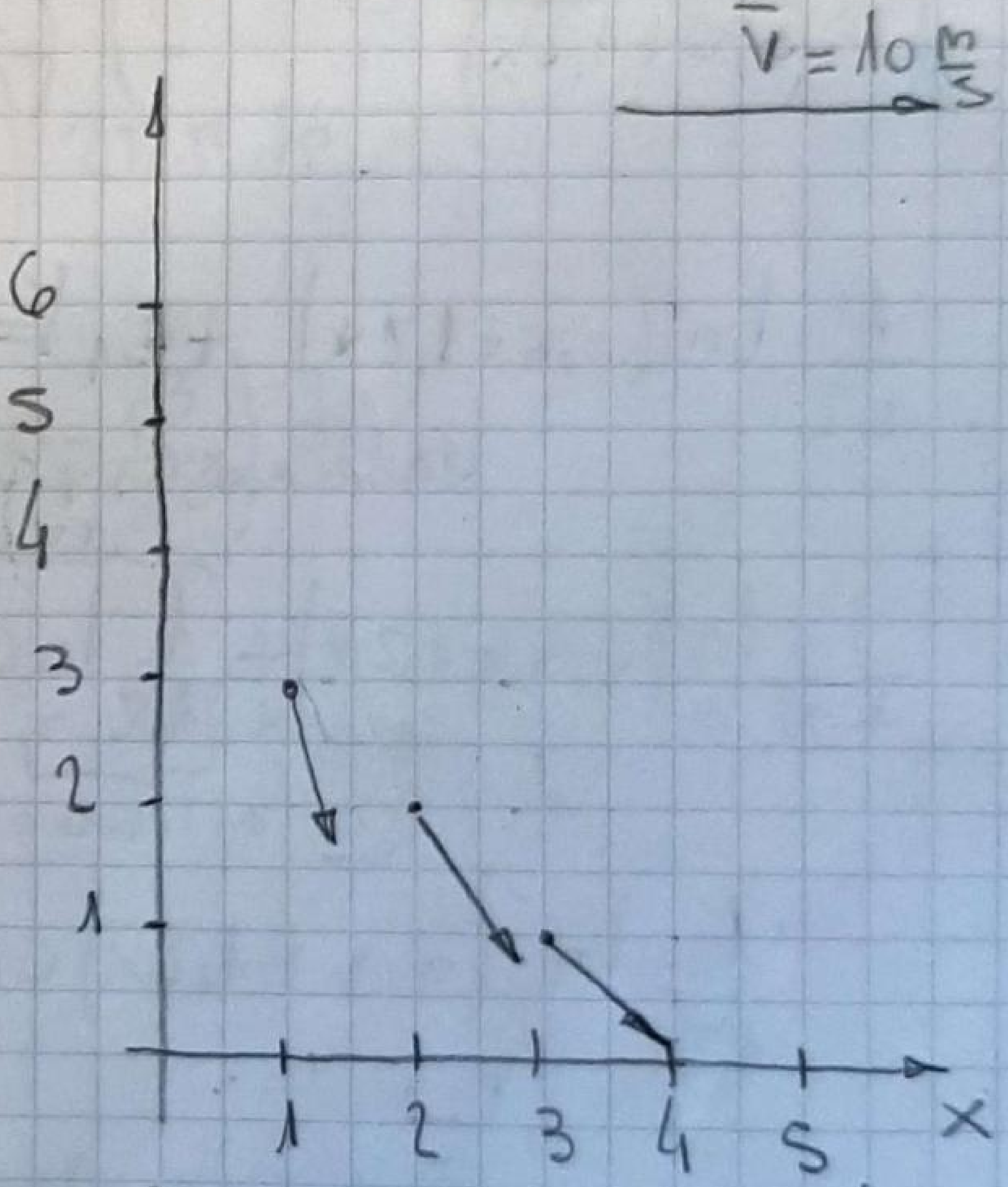
$$\bar{a}_x = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

$$\bar{a}_y = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}\frac{dx}{dt} + \frac{\partial v}{\partial y}\frac{dy}{dt}$$

$$\bar{a}_x = 1{,}2\,(0{,}5 + 1{,}2x)\ \hat{i}$$

$$\bar{a}_y = -1{,}2\,(-2 - 1{,}2y)\ \hat{j}$$

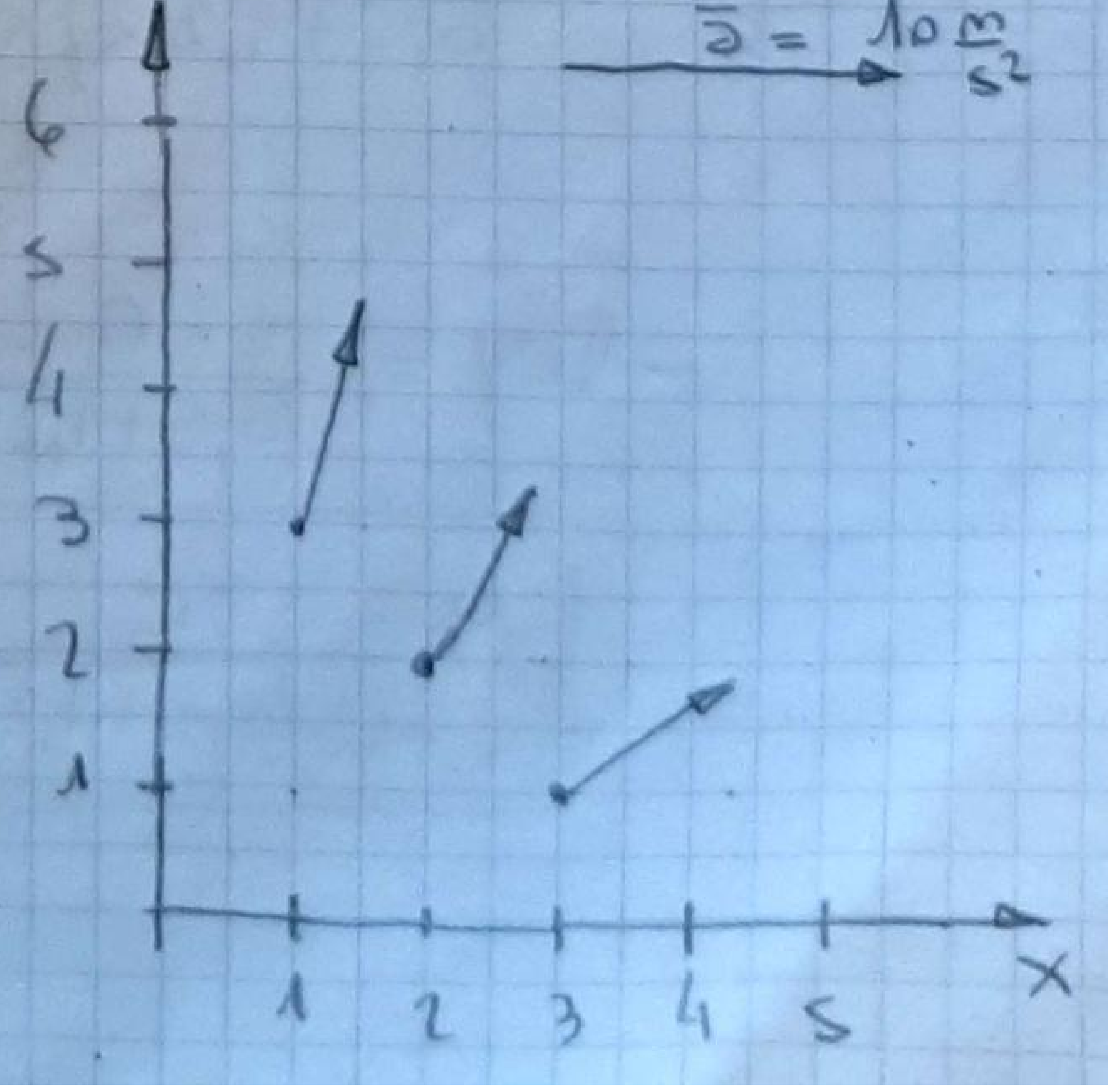
$$\bar{a} = 1{,}2(0{,}5 + 1{,}2x)\,\hat{i} + -1{,}2(-2 - 1{,}2y)\,\hat{j}$$



$$\bar{v}\Big|_{(1,3)} = (1{,}7\,\hat{i} - 5{,}6\,\hat{j})$$

$$\bar{v}\Big|_{(2,2)} = (2{,}9\,\hat{i} - 4{,}4\,\hat{j})$$

$$\bar{v}\Big|_{(3,1)} = (4{,}1\,\hat{i} - 3{,}6\,\hat{j})$$



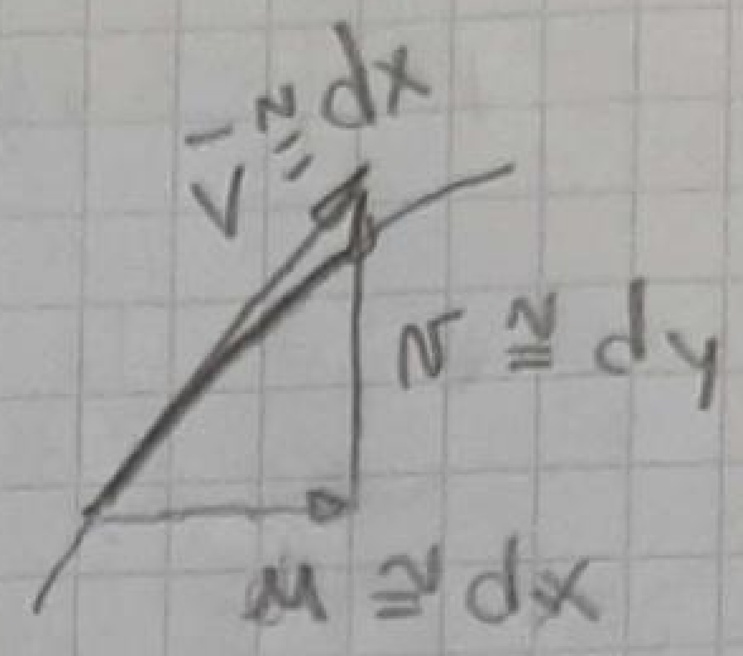
$$\bar{a}\Big|_{(1,3)} = (2{,}04\,\hat{i} + 6{,}72\,\hat{j})$$

$$\bar{a}\Big|_{(2,2)} = (3{,}48\,\hat{i} + 5{,}28\,\hat{j})$$

$$\bar{a}\Big|_{(3,1)} = (4{,}92\,\hat{i} + 3{,}84\,\hat{j})$$

6) Dado $\bar{V} = (0,5 + 1,2x)\hat{i} + (-2 - 1,2y)\hat{j}$

Por definición de líneas de corriente:



$$\frac{u}{v} = \frac{dx}{dy}$$

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\int \frac{1}{(0,5+1,2x)}\,dx = \int \frac{1}{(-2-1,2y)}\,dy$$

$$\frac{1}{1,2}\ln(0,5+1,2x) + C_1 = -\frac{1}{1,2}\cdot\ln(-2-1,2y) + C_2$$

Unificamos C_1 y C_2 en una única constante C.

$$\frac{1}{1,2}\ln(0,5+1,2x) + \overbrace{\underbrace{\frac{1}{1,2}}_{\text{artilugio algebraico}}\ln C} = -\frac{1}{1,2}\cdot\ln(-2-1,2y)$$

$$\cancel{\frac{1}{1,2}}\cdot\ln(0,5+1,2x)\,C = -\cancel{\frac{1}{1,2}}\cdot\ln(-2-1,2y)$$

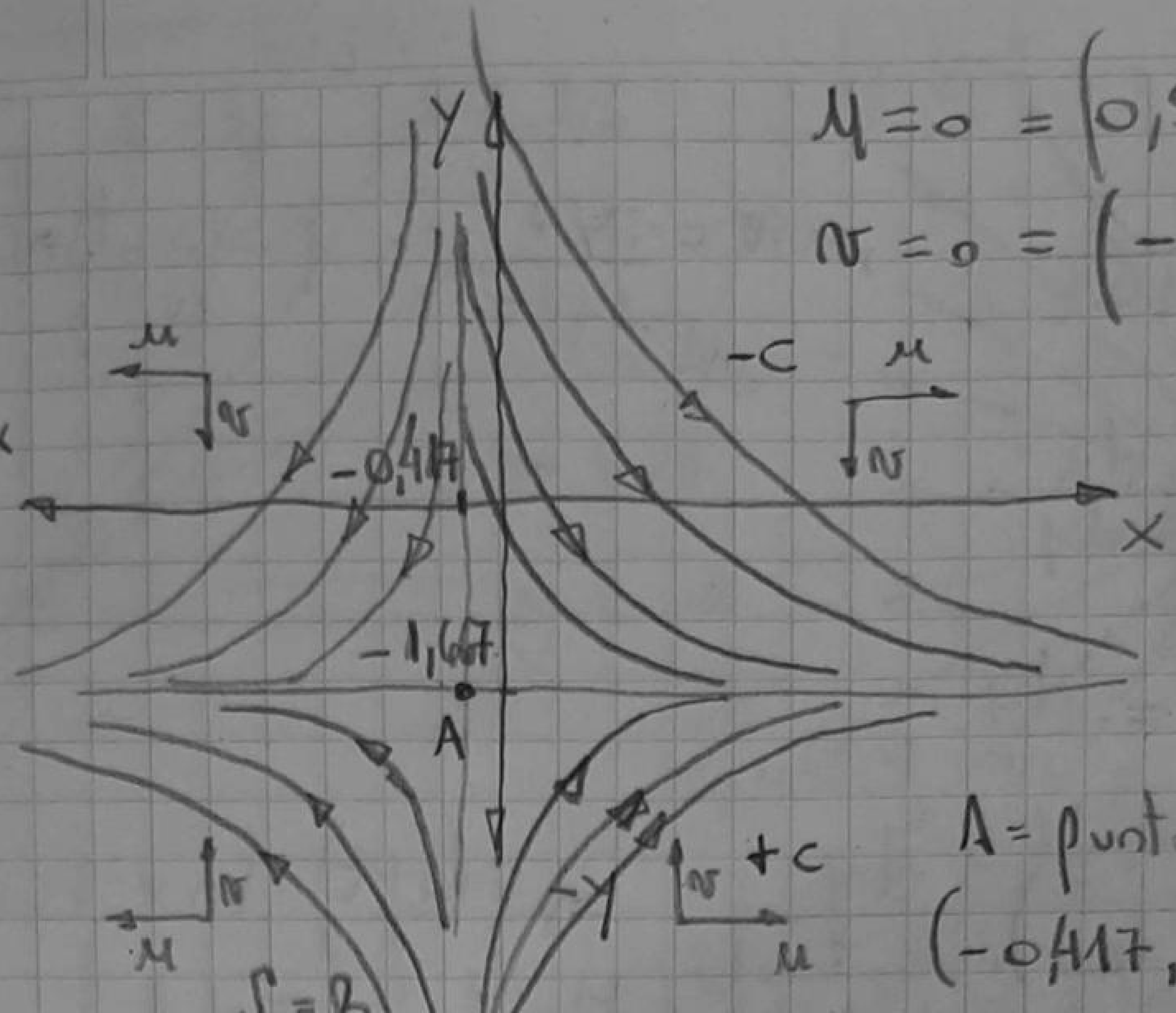
$$(0,5+1,2x)\,C = \frac{1}{(-2-1,2y)}$$

$$-1,2y = \frac{C}{(0,5+1,2x)} + 2$$

$$\boxed{y = \frac{C}{-1,2\cdot(0,5+1,2x)} - 1,667}$$

$u = 0 = (0,5 + 1,2x) \Rightarrow x = -0,417$

$v = 0 = (-2 - 1,2y) \Rightarrow y = -1,667$



A = punto de remanso $\bar{V} = 0$

$(-0{,}417, -1{,}667)$

7) a) $Q_v = \int_A \bar{V} \cdot \hat{n} \, dA = \int_{r=0}^{r=R} u_{max} \left(1 - \frac{r^2}{R^2}\right) \cdot 2\pi \cdot r \, dr$

$Q_v = 2\pi \cdot u_{max} \int_{r=0}^{r=R} \left(r - \frac{r^3}{R^2}\right) dr$

$Q_v = 2\pi u_{max} \cdot \left(\frac{R^2}{2} - \frac{R^{\cancel{4}\,2}}{4\cancel{R^2}}\right) = \cancel{2}\pi u_{max} \frac{R^2}{\cancel{4}\,2} = \pi \cdot u_{max} \frac{R^2}{2}$

$$\boxed{Q_v = \frac{1}{2} \pi u_{max} R^2}$$

b) $Q_v \Big|_{\substack{R = 0,03m \\ u_{max} = 8\frac{m}{s}}} = 0,0113 \ \frac{m^3}{s}$

c) $C\rho = \int_A \rho \cdot \bar{V} \cdot \bar{n} \, dA$

$C\rho = \frac{1}{2} \pi \cdot \rho \cdot u_{max} R^2$

$C\rho \Big|_{\substack{\rho = 1000 \frac{kg}{m^3} \\ u_{max} = 8 \frac{m}{s^2} \\ R = 0,03m}} = 11,3 \ \frac{kg}{s}$

NOTA

8) Dado

a) $u = \frac{x}{1+t}$, $v = \frac{y}{1+2t}$, para t = cte

$$\frac{u}{v} = \frac{dx}{dy}$$

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\int \frac{dx}{\frac{x}{1+t}} = \int \frac{dy}{\frac{y}{1+2t}} \Rightarrow (1+t) \cdot \ln x + c_1 = (1+2t) \cdot \ln y + c_2$$

$$(1+t) \ln x + (1+t) \ln c = (1+2t) \cdot \ln y$$

$$(1+t) \ln x \cdot c = \ln y^{(1+2t)}$$

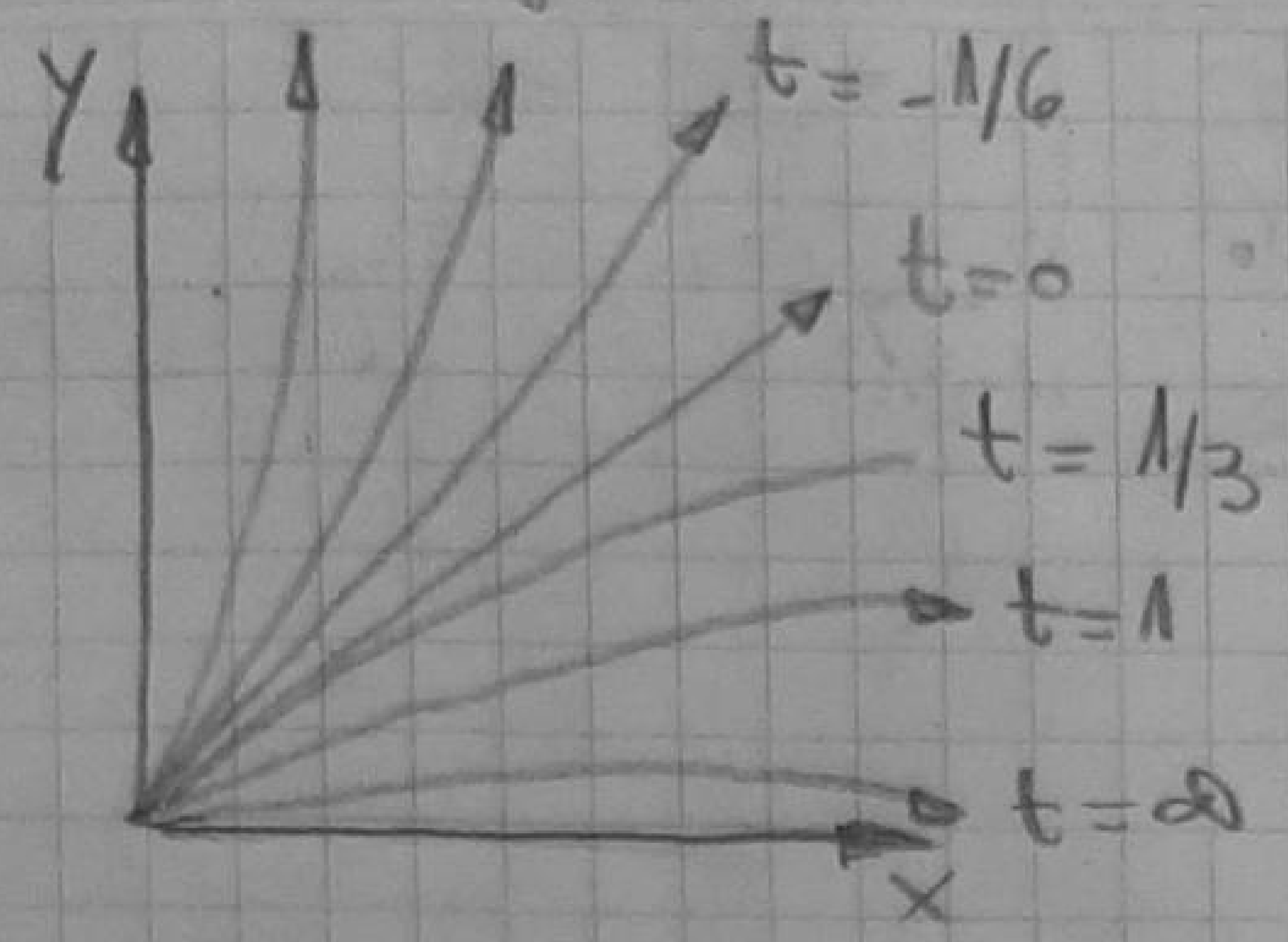
$$\cancel{\ln} x^{(1+t)} \cdot c = \cancel{\ln} y^{(1+2t)}$$

$$x^{(1+t)} \cdot c = y^{(1+2t)}$$

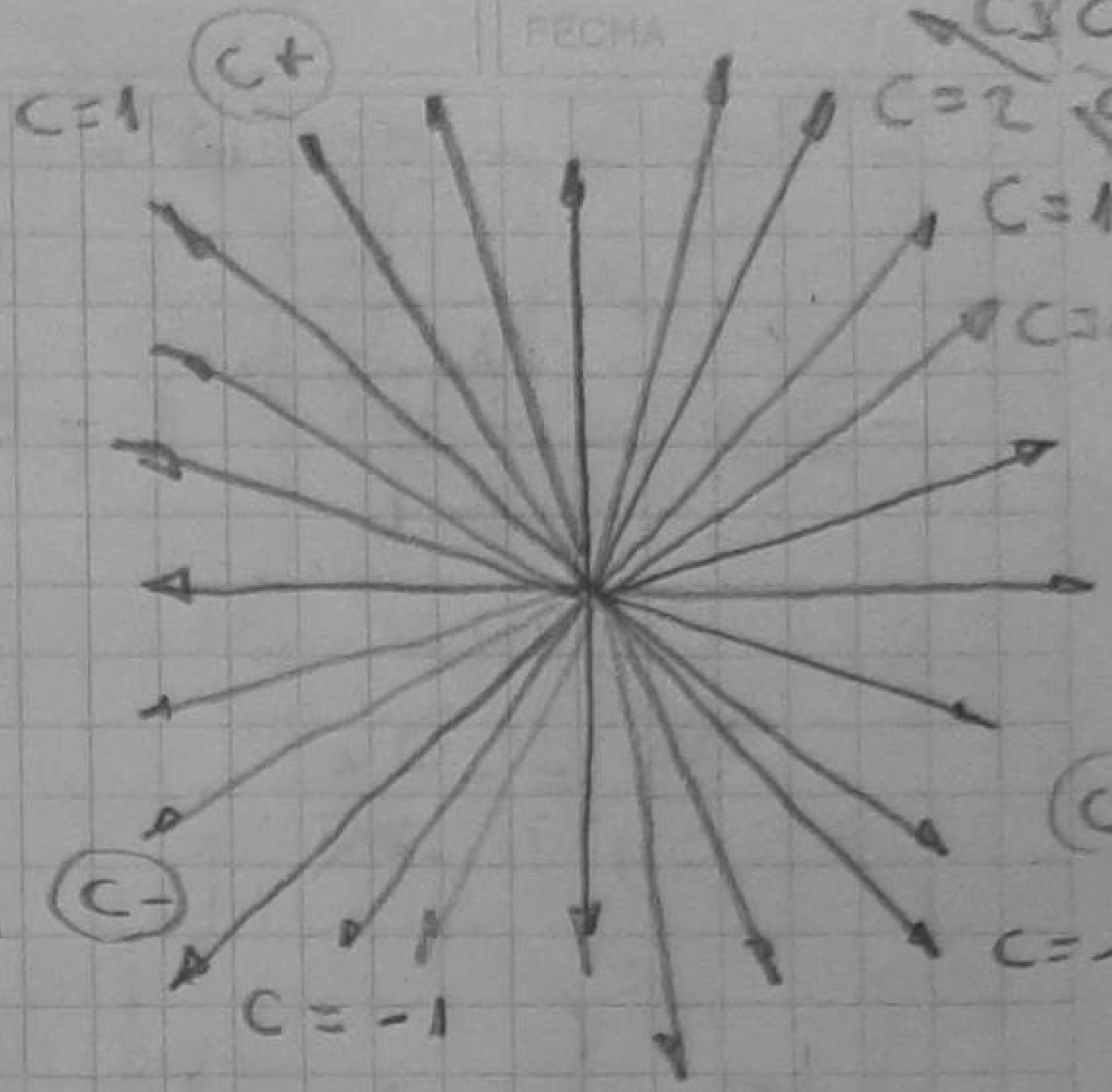
$$\boxed{y = x^{\left(\frac{1+t}{1+2t}\right)} \cdot c}$$

si t = 0 $y = x$, es una linea a 45° (tomando $c = 1$)

Una misma linea de corriente en distintos instantes

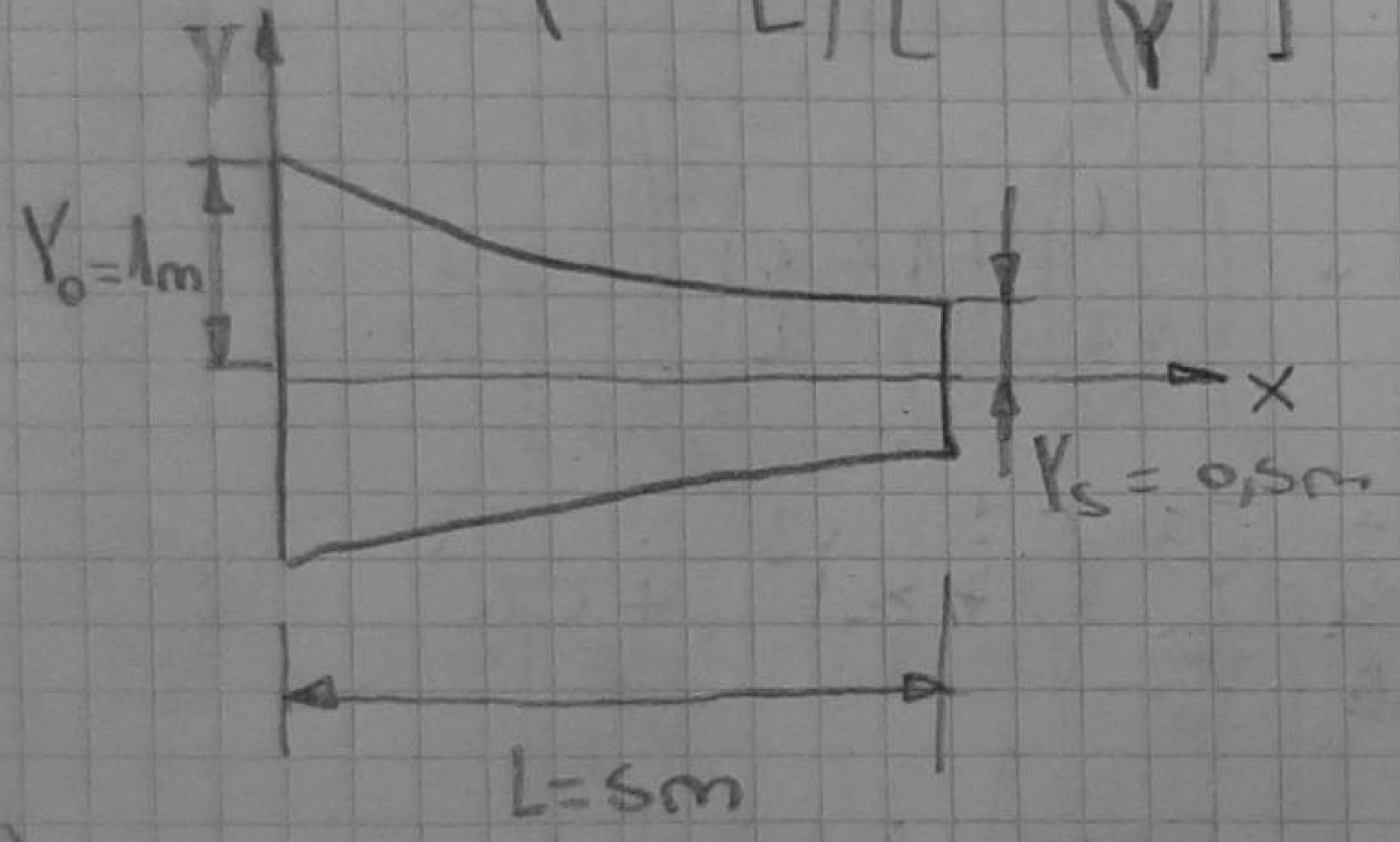


En el instante $t=0$ diferentes lineas de corriente



a)

$$u = u_0\left(1+\frac{x}{L}\right)\left[1-\left(\frac{y}{Y}\right)^2\right] \qquad Y = \frac{Y_0}{\left(1+\left(\frac{x}{L}\right)\right)}$$



a) Por ser un fluido incompresible

$$\nabla\cdot\bar{v} = 0 \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow -\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$u = u_0\left(1+\frac{x}{L}\right)\left[1-\left[\frac{y}{\left(\frac{Y_0}{1+\left(\frac{x}{L}\right)}\right)}\right]^2\right]$$

$$u = u_0\left(1+\frac{x}{L}\right)\left[1-\frac{y^2\cdot\left(1+\frac{x}{L}\right)^2}{Y_0^2}\right]$$

$$u = u_0\left[\left(1+\frac{x}{L}\right)-\frac{y^2}{Y_0^2}\cdot\left(1+\frac{x}{L}\right)^3\right]$$

$$-\frac{\partial u}{\partial x} = -\frac{u_0}{L} + 3 \cdot u_0 \left(\frac{y^2}{Y_0^2}\right)\left(1+\frac{x}{L}\right)^2 \cdot \frac{1}{L}$$

$$-\frac{\partial u}{\partial x} = -\frac{u_0}{L}\left[1 - 3 \left(\frac{y^2}{Y_0^2}\right)\left(1+\frac{x}{L}\right)^2\right]$$

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x}$$

$$\int dv = \int -\frac{\partial u}{\partial x}\, dy$$

$$v = \int -\frac{u_0}{L}\left[1 - 3 \left(\frac{y^2}{Y_0^2}\right)\left(1+\frac{x}{L}\right)^2\right] dy$$

$$v = -\frac{u_0}{L}\, y + \frac{u_0}{L}\,\frac{y^3}{Y_0^2}\left(1+\frac{x}{L}\right)^2 + C(x)$$

$v = 0$ cuando $y = Y$ (en la pared) para todo x

$$0 = -\frac{u_0}{L}\cdot\frac{Y_0}{\left(1+\frac{x}{L}\right)} + \frac{u_0}{L\cdot \cancel{Y_0^2}}\cdot\frac{Y_0^{\cancel{3}}}{\left(1+\frac{x}{L}\right)^{\cancel{3}}}\cancel{\left(1+\frac{x}{L}\right)^2} + C(x)$$

$$\Rightarrow C(x) = 0$$

$$\boxed{v = -\frac{u_0}{L}\, y + \frac{u_0}{L}\,\frac{y^3}{Y_0^2}\left(1+\frac{x}{L}\right)^2}$$

b)

$$\bar{a} = \frac{\partial \bar{V}}{\partial t} + (\bar{V} \cdot \nabla)\, \bar{V}$$

$$u = u_0\left[\left(1+\frac{x}{L}\right) - \frac{y^2}{Y_0^2}\left(1+\frac{x}{L}\right)^3\right]$$

$$v = u_0\left[-\frac{y}{L} + \frac{y^3}{L Y_0^2}\left(1+\frac{x}{L}\right)^2\right]$$

$$\bar{a}_x = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} \cdot u + \frac{\partial u}{\partial y} \cdot v$$

$$\bar{a}_y = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} \cdot u + \frac{\partial v}{\partial y} \cdot v$$

$$\frac{\partial u}{\partial x} = u_0\left[\frac{1}{L} - \frac{3\, y^2}{L\, Y_0^2} \cdot \left(1+\frac{x}{L}\right)^2\right]$$

$$\frac{\partial u}{\partial y} = u_0\left[-\frac{2y}{Y_0^2}\left(1+\frac{x}{L}\right)^3\right]$$

$$\frac{\partial v}{\partial x} = u_0\left[\frac{y^3}{L Y_0^2} \cdot 2\left(1+\frac{x}{L}\right) \cdot \frac{1}{L}\right]$$

$$\frac{\partial v}{\partial y} = u_0\left[\frac{3y^2}{L \cdot Y_0} \cdot \left(1+\frac{x}{L}\right)^2 - \frac{1}{L}\right]$$

$$\bar{a}_x = \frac{u_0^2}{L}\left[\left(1+\frac{x}{L}\right) - 2\left(\frac{y}{Y_0}\right)^2\left(1+\frac{x}{L}\right)^3 + \left(\frac{y}{Y_0}\right)^4\left(1+\frac{x}{L}\right)^5\right]$$

$$\bar{a}_y = \frac{u_0^2}{L}\left[y - 2\frac{y^3}{Y_0^2}\left(1+\frac{x}{L}\right)^2 + \frac{y^5}{Y_0^4}\left(1+\frac{x}{L}\right)^4\right]$$

c)

$$\bar{\omega} = \frac{1}{2}\nabla \times \bar{V}$$

$$\bar{\omega} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$$

$$\bar{\omega} = \frac{y\, u_0}{Y_0^2} \cdot \left(1+\frac{x}{L}\right)\left[\frac{y^2}{L^2} + \left(1+\frac{x}{L}\right)^2\right]$$

d) Vorticidad $\xi = 2\bar{\omega} = \nabla \times \bar{V}$

$$\xi = \frac{2 \cdot y\, u_0}{Y_0^2} \cdot \left(1 + \frac{x}{L}\right)\left[\frac{y^2}{L^2} + \left(1 + \frac{x}{L}\right)^2\right]$$

e)

$$\theta = \nabla \cdot \bar{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$\theta = 0$$

f)

$$\phi_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\phi_{xy} = \frac{y \cdot u_0}{Y_0^2}\left(1 + \frac{x}{L}\right)\left[-\left(1 + \frac{x}{L}\right)^2 + \frac{y^2}{L^2}\right]$$

Nota: Matemáticamente la dilatacion, rotacion y deformacion angular (distorsión) pueden separarse, pero físicamente no pueden separarse, debido a que ocurren al mismo tiempo.

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b) Trayectoria $u = \dfrac{x}{1+t}$ $\quad v = \dfrac{y}{1+2t}$

$$\frac{dx}{dt} = u \Rightarrow \frac{dx}{dt} = \frac{x}{1+t} \Rightarrow \int \frac{dx}{x} = \int \frac{dt}{1+t} \Rightarrow \ln x + C_1 = \ln(1+t) + C_2$$

$$\ln x + \ln C_3 = \ln(1+t) \Rightarrow x C_3 = 1+t \therefore \text{si } t=0 \; x = x_0 \Rightarrow C_3 = \frac{1}{x_0}$$

$$x = (1+t)\, x_0$$

$$\frac{dy}{dt} = v \Rightarrow \int \frac{dy}{y} = \int \frac{dt}{1+2t} \Rightarrow y \cdot C_3 = 1+2t \therefore \text{si } t=0 \; y = y_0 \Rightarrow C_3 = \frac{1}{y_0}$$

$$\boxed{y = (1+2t)\, y_0}$$

c) Flujo

$$x = (1+t)x_0 \Rightarrow t = \frac{x}{x_0} - 1 \Rightarrow y = \left(1 + 2\frac{x}{x_0} - 2\right) y_0 \Rightarrow \boxed{\left(2\frac{x}{x_0} - 1\right) y_0}$$