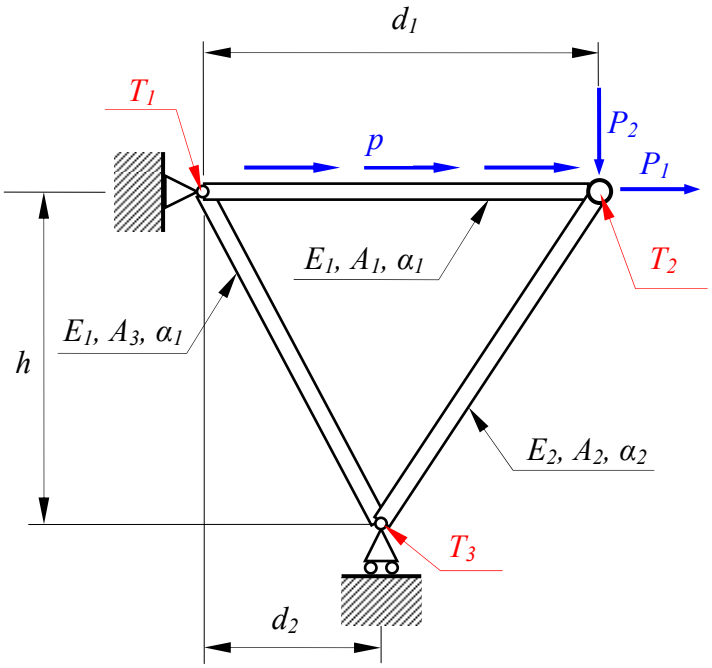
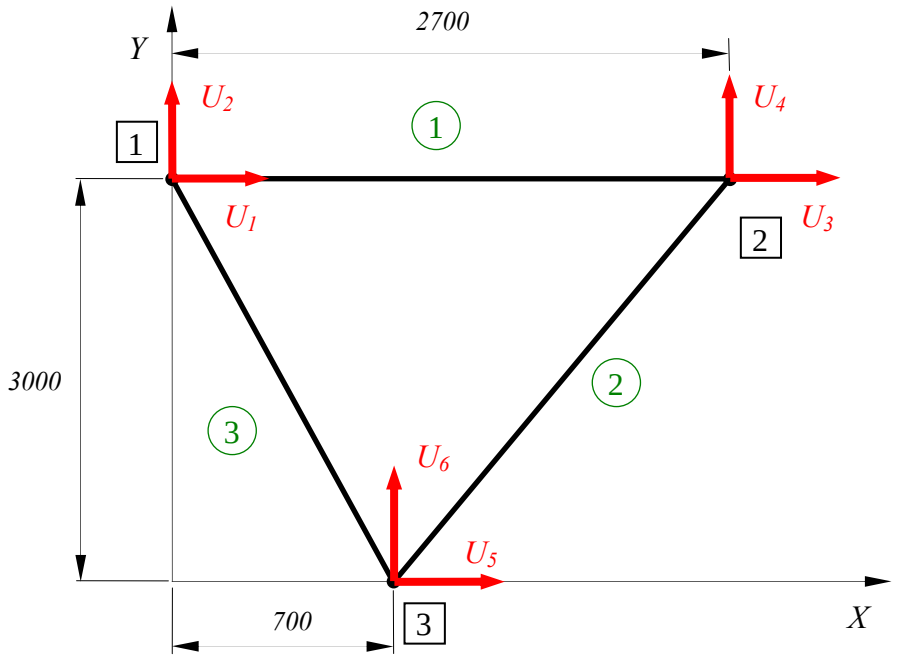
PROBLEMA A RESOLVER



Datos:

$E_1 = 70000\ MPa$	$\alpha_1 = 2{,}3 \times 10^{-5}\ ºC^{-1}$	
$E_2 = 200000\ MPa$	$\alpha_2 = 1{,}2 \times 10^{-5}\ ºC^{-1}$	
$A_1 = 200\ mm^2$	$A_2 = 350\ mm^2$	$A_3 = 150\ mm^2$
$h = 3000\ mm$	$d_1 = 2700\ mm$	$d_2 = 700\ mm$
$P_1 = 7000\ N$	$P_2 = 9000\ N$	$p = 0{,}1\ N/mm$
$T_1 = 200\ ºC$	$T_2 = 300\ ºC$	$T_3 = 100\ ºC$

MODELO DISCRETO



$\boxed{i}$	*Nodo "iésimo"*
$\textcircled{i}$	*Elemento "iésimo"*
U_i	*Grado de Libertad "iésimo"*

DATOS DEL MODELO

1. DATOS ASOCIADOS A LOS NODOS

Nº Nodo	Coord X	Coord Y	Carga Nodal	Restricción	Temperatura
1	0	3000	0	1	200
2	2700	3000	1	0	300
3	700	0	0	2	100

2. DATOS ASOCIADOS A LOS ELEMENTOS

Nº Elemento	Nodo Inicial	Nodo Final	Material	Prop. Geom.	Carga Elem
1	1	2	1	1	1
2	2	3	2	2	0
3	1	3	1	3	0

3. TABLA DE MATERIALES

Número de Material	Módulo Elasticidad	Coeficiente Dilatación
1	70000	2.3 E-5
2	200000	1.2 E-5

4. TABLA DE PROPIEDADES GEOMETRICAS

Número de Propiedad	Area de la barra
1	200
2	350
3	150

5. TABLA DE PATRONES DE CARGAS NODALES

Número de Patrón de Carga Nodal	Componente X	Componente Y
1	7000	-9000

6. TABLA DE PATRONES DE CARGAS ELEMENTALES

Número de Patrón de Carga Elemental	Componente X	Componente Y
1	0.1	0

7. TABLA DE PATRONES DE RESTRICCION

Número de Patrón de Restricción	Restricción X	Desplazamiento X	Restricción Y	Desplazamiento Y
1	1	0	1	0
2	0	0	1	0

DATOS ELEMENTO Nº 1:

Nodo Inicial	Nodo Final	Material	Prop. Geom.	Carga Elem
1	2	1	1	1

Material:

Puntero = 1 $\Rightarrow$ $E = 70000$ $\alpha = 2.3\text{E-}5$

Prop. Geom.:

Puntero = 1 $\Rightarrow$ $A = 200$

Carga Elemental:

Puntero = 1 $\Rightarrow$ $q = 0.1$

DATOS NODOS:

Nodo Inicial = 1 $\Rightarrow$ $X_i = 0$ $Y_i = 3000$ $T_i = 200$

Nodo Final = 2 $\Rightarrow$ $X_f = 2700$ $Y_f = 3000$ $T_f = 300$

CALCULO GEOMETRICO

Longitud Elemento:

$$L = \sqrt{\left(X_f - X_i\right)^2 + \left(Y_f - Y_i\right)^2} = 2700$$

Coseno ángulo:

$$coseno = \frac{X_f - X_i}{L} = 1$$

Seno ángulo:

$$seno = \frac{Y_f - Y_i}{L} = 0$$

CALCULO MATRIZ DE RIGIDEZ

$$[K] = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}$$

$$[K] = \begin{bmatrix} 5185.185 & -5185.185 \\ -5185.185 & 5185.185 \end{bmatrix}$$

CALCULO VECTOR CARGAS MECANICAS

$$\{f_q\} = \begin{Bmatrix} -\frac{qL}{2} \\ -\frac{qL}{2} \end{Bmatrix} = \begin{Bmatrix} -135 \\ -135 \end{Bmatrix}$$

CALCULO VECTOR CARGAS TERMICAS

$$\{f_T\} = \begin{Bmatrix} \dfrac{EA\alpha\left(T_i + T_f\right)}{2} \\ -\dfrac{EA\alpha\left(T_i + T_f\right)}{2} \end{Bmatrix} = \begin{Bmatrix} 80500 \\ -80500 \end{Bmatrix}$$

DATOS ELEMENTO Nº 2:

Nodo Inicial	Nodo Final	Material	Prop. Geom.	Carga Elem
2	3	2	2	0

Material:

Puntero = 2 $\Rightarrow$ $E = 200000$ $\alpha = 1.2\text{E-}5$

Prop. Geom.:

Puntero = 2 $\Rightarrow$ $A = 350$

Carga Elemental:

Puntero = 0 $\Rightarrow$ $q = 0$

DATOS NODOS:

Nodo Inicial = 2 $\Rightarrow$ $X_i = 2700$ $Y_i = 3000$ $T_i = 300$

Nodo Final = 3 $\Rightarrow$ $X_f = 700$ $Y_f = 0$ $T_f = 100$

CALCULO GEOMETRICO

Longitud Elemento:

$$L = \sqrt{\left(X_f - X_i\right)^2 + \left(Y_f - Y_i\right)^2} = 3605.55$$

Coseno ángulo:

$$coseno = \frac{X_f - X_i}{L} = -0.5547$$

Seno ángulo:

$$seno = \frac{Y_f - Y_i}{L} = -0.832$$

CALCULO MATRIZ DE RIGIDEZ

$$[K] = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}$$

$$[K] = \begin{bmatrix} 19414.5 & -19414.5 \\ -19414.5 & 19414.5 \end{bmatrix}$$

CALCULO VECTOR CARGAS TERMICAS

$$\{f_T\} = \begin{Bmatrix} \dfrac{EA\alpha\left(T_i + T_f\right)}{2} \\ -\dfrac{EA\alpha\left(T_i + T_f\right)}{2} \end{Bmatrix} = \begin{Bmatrix} 168000 \\ -168000 \end{Bmatrix}$$

CALCULO MATRIZ DE TRANSFORMACION

$$[T] = \begin{bmatrix} coseno & seno & 0 & 0 \\ 0 & 0 & coseno & seno \end{bmatrix}$$

$$[T] = \begin{bmatrix} -0.5547 & -0.832 & 0 & 0 \\ 0 & 0 & -0.5547 & -0.832 \end{bmatrix}$$

MATRIZ DE RIGIDEZ EN EL SISTEMA GLOBAL

$$\left[\bar{K}\right] = \left[T\right]^{T}\left[K\right]\left[T\right]$$

$$\left[\bar{K}\right] = \begin{bmatrix} 5973.69 & 8960 & -5973.69 & -8960 \\ 8960 & 13439.18 & -8960 & -13439.18 \\ -5973.69 & -8960 & 5973.69 & 8960 \\ -8960 & -13439.18 & 8960 & 13439.18 \end{bmatrix}$$

VECTOR DE CARGAS TERMICAS EN EL SISTEMA GLOBAL

$$\left\{\overline{f}_T\right\} = \left[T\right]^T \left\{f_T\right\}$$

$$\left\{\overline{f}_T\right\} = \begin{Bmatrix} -93189.6 \\ -139776 \\ 93189.6 \\ 139776 \end{Bmatrix}$$

RESUMEN

Elemento *"iésimo"* en Coordenadas Locales

$$\left\{F^{(i)}\right\} = \left[K^{(i)}\right]\{d\} + \left\{f_q^{(i)}\right\} + \left\{f_T^{(i)}\right\}$$

Elemento *"iésimo"* en Coordenadas Globales

$$\left\{\overline{F}^{(i)}\right\} = \left[\overline{K}^{(i)}\right]\left\{U^{(i)}\right\} + \left\{\overline{f}_q^{(i)}\right\} + \left\{\overline{f}_T^{(i)}\right\}$$

Matrices y Vectores Ampliados del Elemento *"iésimo"*

$$\left\{\tilde{F}^{(i)}\right\} = \left[\tilde{K}^{(i)}\right]\{U\} + \left\{\tilde{f}_q^{(i)}\right\} + \left\{\tilde{f}_T^{(i)}\right\}$$

Condición Formal del Ensamblado

$$\sum_{i=1}^{n}\left\{\tilde{F}^{(i)}\right\}=\{P\}=\underbrace{\sum_{i=1}^{n}\left(\left[\tilde{K}^{(i)}\right]\right)}_{[K]}\{U\}+\underbrace{\sum_{i=1}^{n}\left(\left\{\tilde{f}_q^{(i)}\right\}\right)}_{\{F_q\}}+\underbrace{\sum_{i=1}^{n}\left(\left\{\tilde{f}_T^{(i)}\right\}\right)}_{\{F_T\}}$$

$$[K]\{U\}+\{F_q\}+\{F_T\}=\{P\}$$

Introducción de las Condiciones de Contorno

$$\begin{bmatrix} [K_{I,I}] & [K_{I,P}] \\ [K_{P,I}] & [K_{P,P}] \end{bmatrix} \begin{Bmatrix} \{U_I\} \\ \{U_P\} \end{Bmatrix} = \begin{Bmatrix} \{\overline{P}\} \\ \{R\} \end{Bmatrix} - \begin{Bmatrix} \{F_I\} \\ \{F_P\} \end{Bmatrix}$$

Operando resulta:

$$\begin{cases} [K_{I,I}]\{U_I\} + [K_{I,P}]\{U_P\} = \{\overline{P}\} - \{F_I\} \\ [K_{P,I}]\{U_I\} + [K_{P,P}]\{U_P\} = \{R\} - \{F_P\} \end{cases}$$

$$\begin{aligned} \{U_I\} &= [K_{I,I}]^{-1} \left(\{\overline{P}\} - \{F_I\} - [K_{I,P}]\{U_P\} \right) \\ \{R\} &= [K_{P,I}]\{U_I\} + [K_{P,P}]\{U_P\} + \{F_P\} \end{aligned}$$

$$\left\{F^{(1)}\right\} = \begin{bmatrix} 5185.18 & -5185.18 \\ -5185.18 & 5185.18 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} + \begin{Bmatrix} -135 \\ -135 \end{Bmatrix} + \begin{Bmatrix} 80500 \\ -80500 \end{Bmatrix}$$

$$\left\{\overline{F}^{(1)}\right\} = \begin{bmatrix} 5185.18 & 0 & -5185.18 & 0 \\ 0 & 0 & 0 & 0 \\ -5185.18 & 0 & 5185.18 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{Bmatrix} + \begin{Bmatrix} -135 \\ 0 \\ -135 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 80500 \\ 0 \\ -80500 \\ 0 \end{Bmatrix}$$

$$\left\{\tilde{F}^{(1)}\right\} = \begin{bmatrix} 5185.18 & 0 & -5185.18 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -5185.18 & 0 & 5185.18 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix} + \begin{Bmatrix} -135 \\ 0 \\ -135 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 80500 \\ 0 \\ -80500 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\left\{F^{(2)}\right\}=\begin{bmatrix}19414.5 & -19414.5\\ -19414.5 & 19414.5\end{bmatrix}\begin{Bmatrix}d_1\\ d_2\end{Bmatrix}+\begin{Bmatrix}0\\ 0\end{Bmatrix}+\begin{Bmatrix}168000\\ -168000\end{Bmatrix}$$

$$\left\{\bar{F}^{(2)}\right\}=\begin{bmatrix}5973.7 & 8960.5 & -5973.7 & -8960.5\\ 8960.5 & 13440.8 & -8960.5 & -13440.8\\ -5973.7 & -8960.5 & 5973.7 & 8960.5\\ -8960.5 & -13440.8 & 8960.5 & 13440.8\end{bmatrix}\begin{Bmatrix}U_3\\ U_4\\ U_5\\ U_6\end{Bmatrix}+\begin{Bmatrix}0\\ 0\\ 0\\ 0\end{Bmatrix}+\begin{Bmatrix}-93189.6\\ -139784.4\\ 93189.6\\ 139784.4\end{Bmatrix}$$

$$\left\{\tilde{F}^{(2)}\right\}=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 5973.7 & 8960.5 & -5973.7 & -8960.5\\ 0 & 0 & 8960.5 & 13440.8 & -8960.5 & -13440.8\\ 0 & 0 & -5973.7 & -8960.5 & 5973.7 & 8960.5\\ 0 & 0 & -8960.5 & -13440.8 & 8960.5 & 13440.8\end{bmatrix}\begin{Bmatrix}U_1\\ U_2\\ U_3\\ U_4\\ U_5\\ U_6\end{Bmatrix}+\begin{Bmatrix}0\\ 0\\ 0\\ 0\\ 0\\ 0\end{Bmatrix}+\begin{Bmatrix}0\\ 0\\ -93189.6\\ -139784.4\\ 93189.6\\ 139784.4\end{Bmatrix}$$

$$\left\{F^{(3)}\right\} = \begin{bmatrix} 3408.4 & -3408.4 \\ -3408.4 & 3408.4 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 36225 \\ -36225 \end{Bmatrix}$$

$$\left\{\bar{F}^{(3)}\right\} = \begin{bmatrix} 175.99 & -754.24 & -175.99 & 754.24 \\ -754.24 & 3232.45 & 754.24 & -3232.45 \\ -175.99 & 754.24 & 175.99 & -754.24 \\ 754.24 & -3232.45 & -754.24 & 3232.45 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_5 \\ U_6 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 8231.39 \\ -35277.4 \\ -8231.39 \\ 35277.4 \end{Bmatrix}$$

$$\left\{\tilde{F}^{(3)}\right\} = \begin{bmatrix} 175.99 & -754.24 & 0 & 0 & -175.99 & 754.24 \\ -754.24 & 3232.45 & 0 & 0 & 754.24 & -3232.45 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -175.99 & 754.24 & 0 & 0 & 175.99 & -754.24 \\ 754.24 & -3232.45 & 0 & 0 & -754.24 & 3232.45 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 8231.39 \\ -35277.4 \\ 0 \\ 0 \\ -8231.39 \\ 35277.4 \end{Bmatrix}$$

ENSAMBLADO

$$\begin{bmatrix} 5361.17 & -754.24 & -5185.18 & 0 & -175.99 & 754.24 \\ 754.24 & 3232.45 & 0 & 0 & 754.24 & -3232.45 \\ -5185.18 & 0 & 11158.88 & 8960.54 & -5973.7 & -8960.54 \\ 0 & 0 & 8960.54 & 13440.81 & -8960.54 & -13440.81 \\ -175.99 & 754.24 & -5973.7 & -8960.54 & 6149.7 & 8206.3 \\ 754.24 & -3232.45 & -8960.54 & -13440.81 & 8206.3 & 16673.27 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix} + \begin{Bmatrix} -135 \\ 0 \\ -135 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 88731.4 \\ -35277.4 \\ -173689.6 \\ -139784.4 \\ 84958.2 \\ 175061.8 \end{Bmatrix} = \begin{Bmatrix} R_1 \\ R_2 \\ 7000 \\ -9000 \\ 0 \\ R_6 \end{Bmatrix}$$

PARTICIONADO

$$\begin{bmatrix} \begin{bmatrix} 11158.88 & 8960.54 & -5973.7 \\ 8960.54 & 13440.8 & -8960.54 \\ -5973.7 & -8960.54 & 6149.7 \end{bmatrix} & \begin{bmatrix} -5185.18 & 0 & -8960.54 \\ 0 & 0 & -13440.8 \\ -175.99 & 754.24 & 8206.3 \end{bmatrix} \\ \begin{bmatrix} -5185.18 & 0 & -175.99 \\ 0 & 0 & 754.24 \\ -8960.54 & -13440.8 & 8206.3 \end{bmatrix} & \begin{bmatrix} 5361.17 & -754.24 & 754.24 \\ -754.24 & 3232.45 & -3232.45 \\ 754.24 & -3232.45 & 16673.27 \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} U_3 \\ U_4 \\ U_5 \end{Bmatrix} \\ \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \end{Bmatrix} = \begin{Bmatrix} \begin{Bmatrix} 7000 \\ -9000 \\ 0 \end{Bmatrix} \\ \begin{Bmatrix} R_1 \\ R_2 \\ R_6 \end{Bmatrix} \end{Bmatrix} - \begin{Bmatrix} \begin{Bmatrix} -173824.6 \\ -139784.4 \\ 84958.2 \end{Bmatrix} \\ \begin{Bmatrix} 88596.4 \\ -35277.4 \\ 175061.8 \end{Bmatrix} \end{Bmatrix}$$

SOLUCION

$$\begin{Bmatrix} U_3 \\ U_4 \\ U_5 \end{Bmatrix} = \begin{bmatrix} 11158.88 & 8960.54 & -5973.7 \\ 8960.54 & 13440.8 & -8960.54 \\ -5973.7 & -8960.54 & 6149.7 \end{bmatrix}^{-1} \left(\begin{Bmatrix} 7000 \\ -9000 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -173824.6 \\ -139784.4 \\ 84958.2 \end{Bmatrix} \right) = \begin{Bmatrix} 18.058 \\ 6.144 \\ 12.679 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_6 \end{Bmatrix} = \begin{bmatrix} -5185.18 & 0 & -175.99 \\ 0 & 0 & 754.24 \\ -8960.54 & -13440.8 & 8206.3 \end{bmatrix} \begin{Bmatrix} 18.058 \\ 6.144 \\ 12.679 \end{Bmatrix} + \begin{Bmatrix} 88596.4 \\ -35277.4 \\ 175061.8 \end{Bmatrix} = \begin{Bmatrix} -7270 \\ -25714.29 \\ 34714.29 \end{Bmatrix}$$

VERIFICACION DEL EQUILIBRIO

$$\sum F_x = 0 \Rightarrow R_1 + P_1 + p\, d_1 = 0 \Rightarrow -7270 + 7000 + 0.1 \text{ x } 2700 = 0$$

$$\sum F_y = 0 \Rightarrow R_2 + R_6 + P_2 = 0 \Rightarrow -25714.29 + 34714.29 - 9000 = 0$$

$$\sum M_1 = 0 \Rightarrow P_2\, d_1 + R_6\, d_2 = 0 \Rightarrow -9000 \text{ x } 2700 + 34714.29 \text{ x } 700 = 0$$

CALCULO DE CANTIDADES SECUNDARIAS

$$\left\{U^{(1)}\right\} = \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 18.058 \\ 6.144 \end{Bmatrix}$$

$$\left\{d^{(1)}\right\} = \left[T^{(1)}\right]\left\{U^{(1)}\right\} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 18.058 \\ 6.144 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 18.058 \end{Bmatrix}$$

$$\left\{F^{(1)}\right\} = \left[K^{(1)}\right]\left\{d^{(1)}\right\} + \left\{f_q^{(1)}\right\} + \left\{f_T^{(1)}\right\} = \begin{bmatrix} 5185.18 & -5185.18 \\ -5185.18 & 5185.18 \end{bmatrix} \begin{Bmatrix} 0 \\ 18.058 \end{Bmatrix} + \begin{Bmatrix} -135 \\ -135 \end{Bmatrix} + \begin{Bmatrix} 80500 \\ -80500 \end{Bmatrix}$$

$$\left\{F^{(1)}\right\} = \begin{Bmatrix} -13270 \\ 13000 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \sigma_1 \\ \sigma_2 \end{Bmatrix} = \frac{1}{A_1} \begin{Bmatrix} -\left(-13270\right) \\ 13000 \end{Bmatrix} = \begin{Bmatrix} 66.35 \\ 65 \end{Bmatrix}$$

$$\left\{U^{(2)}\right\} = \begin{Bmatrix} U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix} = \begin{Bmatrix} 18.058 \\ 6.144 \\ 12.679 \\ 0 \end{Bmatrix}$$

$$\left\{d^{(2)}\right\} = \left[T^{(2)}\right]\left\{U^{(2)}\right\} = \begin{bmatrix} -0.5547 & -0.832 & 0 & 0 \\ 0 & 0 & -0.5547 & -0.832 \end{bmatrix} \begin{Bmatrix} 18.058 \\ 6.144 \\ 12.679 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -15.129 \\ -7.033 \end{Bmatrix}$$

$$\left\{F^{(2)}\right\} = \left[K^{(2)}\right]\left\{d^{(2)}\right\} + \left\{f_q^{(2)}\right\} + \left\{f_T^{(2)}\right\} = \begin{bmatrix} 19414.5 & -19414.5 \\ -19414.5 & 19414.5 \end{bmatrix} \begin{Bmatrix} -15.129 \\ -7.033 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 168000 \\ -168000 \end{Bmatrix}$$

$$\left\{F^{(2)}\right\} = \begin{Bmatrix} 10816.65 \\ -10816.65 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \sigma_1 \\ \sigma_2 \end{Bmatrix} = \frac{1}{A_2} \begin{Bmatrix} -(10816.65) \\ -10816.65 \end{Bmatrix} = \begin{Bmatrix} -30.9 \\ -30.9 \end{Bmatrix}$$

$$\left\{U^{(3)}\right\}=\begin{Bmatrix}U_1\\U_2\\U_5\\U_6\end{Bmatrix}=\begin{Bmatrix}0\\0\\12.679\\0\end{Bmatrix}$$

$$\left\{d^{(3)}\right\}=\left[T^{(3)}\right]\left\{U^{(3)}\right\}=\begin{bmatrix}0.227 & -0.974 & 0 & 0\\ 0 & 0 & 0.227 & -0.974\end{bmatrix}\begin{Bmatrix}0\\0\\12.679\\0\end{Bmatrix}=\begin{Bmatrix}0\\2.88\end{Bmatrix}$$

$$\left\{F^{(3)}\right\}=\left[K^{(3)}\right]\left\{d^{(3)}\right\}+\left\{f_q^{(3)}\right\}+\left\{f_T^{(3)}\right\}=\begin{bmatrix}3408.4 & -3408.4\\ -3408.4 & 3408.4\end{bmatrix}\begin{Bmatrix}0\\2.88\end{Bmatrix}+\begin{Bmatrix}0\\0\end{Bmatrix}+\begin{Bmatrix}36225\\-36225\end{Bmatrix}$$

$$\left\{F^{(3)}\right\}=\begin{Bmatrix}26405\\-26405\end{Bmatrix}\Rightarrow\begin{Bmatrix}\sigma_1\\\sigma_2\end{Bmatrix}=\frac{1}{A_3}\begin{Bmatrix}-(26405)\\-26405\end{Bmatrix}=\begin{Bmatrix}-176.03\\-176.03\end{Bmatrix}$$